Tax Compliance, Payment Choice, and Central Bank Digital Currency*

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Abstract

This paper studies the implications of tax evasion for the optimal design of central bank digital currency (CBDC). I build a general equilibrium framework to explicitly allow tax evasion by agents and tax audits by a government. I find that as long as CBDC offers less anonymity than cash, introducing CBDC will decrease tax evasion. However, if CBDC is "*cash-like*" in the sense that it still offers relatively high anonymity but low interest rate, then introducing CBDC will decrease the output from not only agents who evade taxes but also agents who report their income truthfully. If CBDC is instead "*deposit-like*" in the sense that it offers low anonymity but high interest rate, then introducing CBDC will increase output and aggregate welfare. Furthermore, introducing deposit-like CBDC needs not increase the funding costs of private banks or decrease bank lending and investment. However, paying a high interest rate on CBDC will decrease the central bank's net interest revenue, which may jeopardize the central bank's independence.

JEL Codes: H26, E51, E52, E58

Keywords: Tax compliance, Central bank digital currency, Monetary policy, Banking

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1 Introduction

Central bank digital currency (CBDC), which is a digital form of central bank money, has attracted worldwide attention in recent years. A survey by the Bank for International Settlements finds that in 2019, more than 80% of central banks are actively researching CBDC (Boar et al., 2020). Although there is not yet a consensus on how CBDC should be designed, many central bankers argue that the potential use of CBDC in illicit activities should be taken into account.[1] One of the frequently mentioned illicit activities is tax evasion, which is an important problem in many countries. For example, in the US, between 2011 and 2013, the loss in tax revenue due to tax evasion is estimated to be 14.2% of total Federal tax revenue and 2.4% of US GDP.[2] In addition, tax evasion is closely related to the use of cash as a payment instrument. For example, nearly half of the loss in tax revenue between 2011 and 2013 in the US comes from individual businesses that are cash-intensive (Internal Revenue Service, 2019). The goal of this paper is therefore to study an environment where agents have the incentive to evade taxes, and tax evasion motivates the use of cash in transactions. I ask how tax evasion affects the optimal design of CBDC, and how the introduction of CBDC affects the choice of payment methods, output, and aggregate welfare.

To answer these questions, I develop a general equilibrium framework based on the model of Lagos and Wright (2005) to allow tax evasion by agents and tax audits by a government. A key feature of the framework is that the problem of tax evasion and tax audits are studied jointly with the problem of payment choice. Specifically, in the economy, there are buyers and sellers who trade a consumption good, and sellers can choose the payment instrument(s) they accept. After each period, sellers are required to file income reports to the government and pay an income tax. The government may collect the tax based on reported income, or it can conduct costly audits on sellers. After an audit, the probability of the government observing a seller's income depends on the payment instrument(s) in which the income is received. If sellers are found evading taxes, the government can punish them by confiscating their income.

As a benchmark, I study a scenario where only cash and bank deposits are available as payment instruments. In equilibrium, some sellers choose to evade taxes, while other sellers choose to report their income truthfully. The benefit of evading taxes is that sellers can potentially have a higher income, while the cost is the risk of being audited and punished by the government. I find that the risk of punishment acts as a proportional tax on sellers who evade taxes. This means that tax evasion creates distortions in sellers' production decisions on top of any distortions that may have already existed because of the income tax. As a result, sellers who evade taxes produce less and receive smaller payments compared to sellers who report their income truthfully. I also find that as

[1] See Lagarde (2018), Powell (2020), Bank of England (2020), and Bank of Canada (2020). See also Bank of Canada et al. (2020) for an overview of the central banks' motivations for introducing CBDC.

[2] See Internal Revenue Service (2019). Rogoff (2017) argues that tax evasion in Europe is likely to be more severe compared to the US due to Europe's larger informal economy. Canada Revenue Agency (2017, 2019) estimates that in 2014, tax evasion in corporate income tax, Goods and Services Tax, and personal income tax amounts to $26B or 9.6% of the total federal tax revenue. See Rogoff (2017) for discussions about tax evasion in other countries.

long as it is sufficiently easier to hide cash income compared to deposit income, sellers who evade taxes accept cash despite its lower return.

The main results of this paper concern the optimal design of CBDC when tax evasion creates inefficiency and cash facilitates tax evasion. I define central bank digital currency (CBDC) to be another payment instrument (in addition to cash) that is issued by the central bank. It is stored in accounts managed by the central bank, and it can be held by all agents. CBDC can be different from cash and bank deposits depending on two design choices of the central bank: the first is the interest rate on CBDC, and the second is how much anonymity CBDC offers.[3] In the context of tax evasion, the degree of anonymity of CBDC will affect the probability of the income received in CBDC being observed by the government after an audit.

I find is that as long as CBDC offers less anonymity than cash, then introducing CBDC will decrease tax evasion. However, if CBDC is "*cash-like*" in the sense that it still offers relatively high anonymity but low interest rate, introducing CBDC will decrease the output from not only sellers who evade taxes but also sellers who report their income truthfully. Specifically, in equilibrium, although CBDC offers less anonymity, sellers who evade taxes substitute cash with CBDC because the latter pays a positive interest. However, because the visibility of sellers' income increases, the government has more incentive to conduct audits, which decreases tax evasion. For sellers who evade taxes, higher audit probability increases the risk of being punished by the government. Consequently, they produce less for buyers and receive smaller payments.

Perhaps more interestingly, the output from sellers who report their income truthfully will also decrease. In equilibrium, because the interest rate on CBDC is low, sellers who report truthfully prefer to accept bank deposits. The decrease in tax evasion prompts more sellers to switch to bank deposits. As a result, the demand for bank deposits increases, which drives down the deposit rate. This lowers the income of sellers who report truthfully and reduces their output. In addition, a higher interest rate on CBDC will further reduce tax evasion, because it increases the income of sellers who evade taxes, which gives the government more incentive to audit. However, in equilibrium, the higher audit probability offsets the increase in the return on CBDC and causes the output from sellers who evade taxes to decrease. A higher interest rate on CBDC will also worsen the shortage of bank deposits and lower the deposit rate. As a result, the output from sellers who report truthfully will decrease.

It is, however, possible to design CBDC in a way such that tax evasion is lower but a shortage of bank deposits does not happen. Specifically, let the interest rate on CBDC be sufficiently high so that sellers who report their income truthfully are willing to accept CBDC. Meanwhile, let CBDC offer the least amount of anonymity such that sellers who evade taxes are also willing to accept CBDC. In equilibrium, the deposit rate is equal to the interest rate on CBDC. Sellers who report

[3] The anonymity of CBDC can indeed be a choice of the central bank. For example, Darbha and Arora (2020) show that combinations of several cryptographic techniques and operational arrangements can be used to achieve fine-grained privacy designs. ESCB (2019) demonstrates that through "anonymity vouchers", CBDC can offer anonymity while also complying with anti-money laundering and anti-terrorism laws.

their income truthfully are indifferent between accepting deposits and CBDC, while sellers who evade taxes prefer CBDC to cash. I refer to this type of CBDC as "*deposit-like*". Introducing deposit-like CBDC will decrease tax evasion and increase the demand for bank deposits, which is similar to cash-like CBDC. However, because the interest rate on CBDC is equal to the deposit rate, some sellers who report their income truthfully are willing to substitute bank deposits with CBDC. This avoids a shortage of bank deposits and therefore the transactions between buyers and sellers. As a result, introducing CBDC increases the output and aggregate welfare by reducing the distortions created by tax evasion and real resources devoted to tax audits.

It has been argued that CBDC may compete with bank deposits and increase the funding costs of private banks.[4] Indeed, Keister and Sanches (2019) show that while CBDC tends to promote efficiency in exchange, it also crowds out bank deposits and decreases the investment in the economy. However, I find that this is not the case no matter CBDC is cash-like or deposit-like. When CBDC is cash-like, introducing CBDC will lower the deposit rate and decrease the funding costs of private banks. When CBDC is deposit-like, it indeed competes with bank deposits. However, introducing CBDC also increases the demand for payment instruments by reducing the inefficiency resulted from tax evasion and by increasing the output of the economy. I show that it is possible to design CBDC in such a way that private banks' funding costs remain unchanged after the introduction of CBDC.

Although aggregate welfare is higher with deposit-like CBDC, the central bank has to pay higher interest, which may decrease the central bank's net interest revenue. Specifically, the central bank in the model, similar to its counterparts in reality (e.g. the Federal Reserve), earns its revenue through the interest payments on its assets (i.e. government bonds). The central bank's expenses include the interest payments on its liabilities (CBDC) and the costs of operating the central bank (e.g. personnel costs). To ensure a central bank's independence of the fiscal authority, it may be necessary for the central bank to cover the expenses with its revenue rather than rely on transfers from the fiscal authority. For example, when discussing its independence, the Federal Reserve emphasizes that it "does not receive funding through the congressional budgetary process".[5]

I find that if CBDC is cash-like, introducing CBDC decreases the central bank's net interest revenue. This is because, firstly, the demand for central bank money is lower due to fewer sellers evading taxes. Secondly, the decrease in tax evasion increases the demand for bank deposits, which are partially backed by government bonds in equilibrium. Higher demand for government bonds then decreases the bond rate and reduces interest payments the central bank receives. If CBDC is deposit-like, the reduction in the central bank's net interest revenue will be even larger because the interest rate on CBDC is higher. Hence, no matter CBDC is cash-like or deposit-like, introducing CBDC will have a negative impact on the central bank's net interest revenue. While introducing deposit-like CBDC will increase output and aggregate welfare, it may jeopardize the central bank's independence unless the central bank can obtain income from other sources.

[4] See for example Garcia et al. (2020).

[5] See `https://www.federalreserve.gov/faqs/about_12799.htm`.

1.1 Related Literature

This paper is related to the vast theoretical literature on tax compliance.[6] Some of the earliest work includes Allingham and Sandmo (1972) and Reinganum and Wilde (1985, 1986). Allingham and Sandmo (1972) assume the audit probability is exogenous and study how tax evasion responds to tax rates. Reinganum and Wilde (1985, 1986) assume the income distribution is exogenous and study the strategic interactions between taxpayers and the tax authority. Building on Reinganum and Wilde (1986), Erard and Feinstein (1994) assume that a portion of agents are honest and always report truthfully. More recently, Bassetto and Phelan (2008) also assume an exogenous income distribution and study optimal taxation using a mechanism design approach. They find that there exists an equilibrium where households under-report their incomes because other households are expected to do so as well. Compared to the theoretical literature on tax compliance, the main contribution of this paper is to incorporate the choice of payment methods and the audit game into a general equilibrium framework. Such a setup allows me to study how the intention to evade taxation affects the choice of payment methods, and how the characteristics of a payment instrument affect tax evasion.

Papers that also study tax evasion and informal economy in the Lagos and Wright (2005) framework include Gomis-Porqueras et al. (2014), Aruoba (2018), Aït Lahcen (2020), Bajaj and Damodaran (2020), and Kwon et al. (2020).[7] Gomis-Porqueras et al. (2014) assume that cash transactions are not observable to the government but credit transactions are, and that the government cannot audit agents. They find a negative relationship between tax evasion and inflation. Aruoba (2018) studies a Ramsey optimal taxation problem with a focus on tax enforcement capabilities. The government can choose to audit agents but does so randomly because agents do not report their income. Similar to Gomis-Porqueras et al. (2014), Aït Lahcen (2020) does not allow the government to audit agents and obtains a similar negative relationship between tax evasion and inflation. Bajaj and Damodaran (2020) assume that the fiscal authority can observe all transactions in cash, and it only chooses the effort spent in collecting taxes. They find that the effective tax rate is low because cash payments tend to be small, which reduces the fiscal authority's incentive to collect. Lastly, Kwon et al. (2020) also assume the government cannot observe transactions in cash but can observe perfectly transactions in CBDC and deposits. They find that the distortion from tax evasion can be corrected by implementing high inflation on cash and using the seigniorage income to finance a high interest rate on CBDC.

This paper is also related to the emerging literature on CBDC. Using a dynamic general equilibrium model, Barrdear and Kumhof (2016) find that introducing CBDC can stimulate macroeconomic activity as well as bank lending. By focusing on the digital nature of CBDC, Davoodalhosseini (2018) shows that the central bank can in principle cross-subsidize different types of agents and

[6] For reviews of this literature, see Andreoni et al. (1998), Slemrod (2007), and Alm (2019).

[7] For more work on informal economy, see Canzoneri and Rogers (1990), Nicolini (1998), Cavalcanti and Villamil (2003), and Yesin (2004, 2006).

improve welfare, which is not possible with cash. Keister and Sanches (2019) find that CBDC can help alleviate frictions that prevent the efficient level of investment, but it also competes with bank deposits and increases the funding cost of financial institutions. Brunnermeier and Niepelt (2019) derives conditions under which the issuance of CBDC does not alter equilibrium allocations. This suggests that CBDC does not have to reduce credit or crowd out investment. Chiu et al. (2019) and Andolfatto (2020) drop the assumption of competitive banking markets common in the literature. Chiu et al. (2019) find that CBDC can promote the competition in the deposit market and increase output. Andolfatto (2020) shows that although the introduction of CBDC increases the deposit rate and reduces bank revenue, it also increases deposit demand and promotes saving. Williamson (2019a) considers an environment where privacy is demanded in some transactions and banks are subject to limited commitment. CBDC may be designed to offer privacy like cash while being more efficient than bank deposits because the central bank is immune from limited commitment. Fernández-Villaverde et al. (2020), Keister and Monnet (2020), and Williamson (2020) study CBDC and financial stability. Fernández-Villaverde et al. (2020) find that CBDC may attract deposits away from the commercial banking sector because it is more stable during bank runs. Keister and Monnet (2020) show that CBDC is beneficial because real-time information on transactions is available to the central bank and regulators. Such information mitigates the moral hazard problem and improves financial stability. Williamson (2020) studies flight to safety when CBDC is designed to be a safe asset. CBDC is found to reduce the damages resulted from a banking panic as it is less disruptive of retail payments.[8]

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 solves a benchmark model. Section 4 introduces CBDC. Section 5 discuss aggregate welfare and central bank net revenue. Section 6 concludes the paper.

2 Model Environment

The model builds on the Lagos and Wright (2005) framework. Time is discrete and continues forever. Each period is divided into two subperiods: the decentralized market (DM) and the centralized market (CM). There is measure one of infinitely-lived buyers and measure $\alpha > 1$ of infinitely-lived sellers. In the DM, buyers consume a DM good that can only be produced by sellers. In the CM, sellers consume a CM good that can be only produced by buyers. The CM good also serves as the numéraire. A buyer's instantaneous utility is given by

$$u(g_t) - l_t, \tag{2.1}$$

where g_t is the consumption of DM good, and l_t is the labor supplied in the CM. One unit of labor can be turned into one unit of CM good. I assume $u'(g) > 0$, $u''(g) < 0$, $u(0) = 0$, $u'(\infty) = 0$,

[8]For more discussions on the benefits and costs of CBDC, see Bordo and Levin (2017), Berentsen and Schar (2018), Ricks et al. (2018), and Kahn et al. (2020).

$u'(0) = \infty$, and $-gu''(g)/u'(g) < 1$. A seller's instantaneous utility is given by

$$-h_t + x_t, \tag{2.2}$$

where h_t is the labor supplied in the DM, and x_t is the consumption of CM good. I assume one unit of labor in the DM can be turned into one unit of DM good. All agents discount future utility using $\beta \in (0, 1)$. Neither good can be carried across periods.

In the DM, buyers and sellers trade the DM good, and the terms of trade are determined via price posting. Specifically, the market in the DM consists of different submarkets, and each submarket is identified by its terms of trade, (q, p). First, q represents the quantity of DM good a seller in this submarket offers to produce. Second, $p \equiv (m, c, d)$ represents the payment that the seller expects to receive, where m, c, and d denote payments in cash, central bank digital currency (CBDC), and bank deposits, respectively. The terms of trade for the DM of period t are posted in the CM of period $t-1$, and sellers can commit to the terms of trade they post. Buyers observe all terms of trade before they decide which market they will visit and how much cash, CBDC, and bank deposits they will accumulate. Let $n(q, p)$ denote the buyer-to-seller ratio in each submarket. I assume a buyer meets a seller with probability $\min\{1, 1/n(q, p)\}$ and a seller meets a buyer with probability $\min\{1, n(q, p)\}$. Note that because by assumption there are more sellers than buyers, some sellers may not meet a buyer in the DM. This means that although all agents are ex ante homogeneous, sellers may be ex post heterogeneous in their income.

Next, at the beginning of the CM, sellers are required to report their income to a fiscal authority and pay an income tax. After receiving the income reports, the fiscal authority can choose to either collect the tax based on reported income or audit sellers. Each audit costs the fiscal authority C units of CM good. I assume that after an audit, the fiscal authority observes sellers' income received in the form of bank deposits with probability one. However, the income received in cash is only observed with exogenous probability $\rho^m \in (0, 1)$. As for CBDC, I consider various regimes where income in CBDC is more or less likely to be observed compared to other payment methods (see Section 4). If a seller is found evading taxes, the fiscal authority may confiscate his or her income. After the tax is collected and all audits are finished, a perfectly competitive market opens for agents to trade the CM good. Sellers may use their post-tax and post-audit income to purchase the CM good for consumption, while buyers may sell the CM they produce to acquire cash, CBDC, and bank deposits for the next DM.

In the CM, there is also measure one of bankers and entrepreneurs who are active. Bankers create deposits that are used by buyers to purchase the DM good. They derive linear utility from the CM good like sellers, and they can produce the CM good with the same linear technology that buyers use. I assume bankers have limited commitment and can choose to default on their deposit liabilities, and therefore they must be backed by other assets such as cash, CBDC, government bonds, and loans to entrepreneurs. I assume entrepreneurs are born in each CM with a one-period project that takes the CM good as input (denoted by k) and yields the CM good in the CM of the next period as output (denoted by $f(k)$), where $f'(k) > 0$, $f''(k) < 0$, $f'(0) = \infty$, and $f'(\infty) = 0$. Entrepreneurs derive linear utility from consuming the CM good in the second CM of their lives before they die and are replaced with a new set of entrepreneurs. I assume entrepreneurs are born without any funds and they cannot work in the CM. Therefore, they must borrow from bankers and use the output of their projects as collateral. I assume both the loan market and the deposit market are perfectly competitive.

The government in the model consists of the fiscal authority and the central bank. In addition to taxing sellers, the fiscal authority issues one-period nominal government bonds that are traded in a competitive bond market in the CM. Following Andolfatto and Williamson (2015) and Williamson (2016, 2019a, 2019b), I assume the fiscal authority determines the supply of government bonds, while the central bank determines the supply of cash and CBDC through open market purchases and sales of government bonds. Let price of the CM good be p_t. The central bank's objective is to adjust the supply of cash and CBDC to achieve a certain inflation target $\mu = (p_{t+1} - p_t)/p_t$. Now, let M_t and C_t denote the total supply of of cash and CBDC in period t. Let B_t^c denote the government bonds held by the central bank. Let the nominal bond rate and the nominal interest rate on CBDC be R^b and R^c, respectively. The central bank's budget constraint is

$$\frac{M_{t+1} - M_t}{p_t} + \frac{C_{t+1} - C_t}{p_t} + \frac{(1 + R_t^b)B_t^c}{p_t} = \frac{R^c C_t}{p_t} + \frac{B_{t+1}^c}{p_t} + \mathcal{E}_t + \mathcal{T}_t^c. \tag{2.3}$$

The left-hand side of (2.3) represents the per-period income of the central bank, which consists of the revenue from issuing new cash and CBDC, and the revenue from redeeming the government bonds purchased in the last period. The expenses of the central bank consist of the interest payment on CBDC, the purchase of government bonds, and the cost of operating the central bank $\mathcal{E}_t$. After paying the expenses, the central bank transfers the rest of its income to the fiscal authority ($\mathcal{T}_t^c$). If $\mathcal{T}_t^c$ is negative, then it represents a transfer from the fiscal authority to the central bank.

Finally, denote the total supply of government bonds in a period as B_t. The budget constraint of the fiscal authority is

$$\frac{B_{t+1}}{p_t} + \bar{\tau}_t + \mathcal{T}_t^c = T_t + \frac{(1 + R_t^b)B_t}{p_t}. \tag{2.4}$$

The left-hand side of (2.4) denotes the per-period income of the fiscal authority, which consists of the income from issuing new government bonds, net tax revenue, and the transfer from the central bank. The right-hand side denotes the per-period expenses of the fiscal authority, which consists of a non-negative lump-sum transfer to buyers in the CM (T_t) and the redemption value of the government bonds issued in the previous period. Throughout this paper, I assume that the fiscal authority chooses the total amount of government debt ($\mathcal{D} = \frac{(1+R_t^b)B_t}{p_t}$) and the tax income schedule, and lets T_t adjust passively so that (2.4) holds. In Appendix C, I consider an alternative setup where there is no lump-sum transfer (i.e. $T_t = 0$), and the government balances the budget by adjusting $\mathcal{D}$ instead.

3 A Benchmark Model: No CBDC

As a benchmark, I consider a scenario where the only government-issued money is cash. Throughout this section, I restrict my attention to stationary equilibria where all real variables remain constant.

3.1 Tax Compliance and Sellers' Problem

First, let y denote the reported income of a seller, and let $\tau(y) : \mathbb{R}_+ \to \mathbb{R}_+$ denote the tax schedule. In the first part of this section, I solve sellers' and the fiscal authority's problems with a general tax schedule, which is only required to satisfy $\tau(y) \le y$. In the second part, I solve for the optimal tax schedule that maximizes the total surplus in the DM subject to the fiscal authority raising a given amount of net tax revenue.

Let R^d denote the nominal interest rate on bank deposits. Suppose a buyer decides that he or she will visit market (q, p) in the next DM, where $p = (m, d)$ represents the payment in cash and deposits. Accumulating (m, d) requires the following amount of labor in the CM.

$$(1+\mu)m + \frac{(1+\mu)d}{1+R^d}. \tag{3.1}$$

Recall that a buyer meets a seller with probability $\min\{1, 1/n(q, p)\}$. Then, the expected surplus of a buyer choosing to visit (q, p) is given by

$$-(1+\mu)m - \frac{(1+\mu)d}{1+R^d} + \beta \min\{1, 1/n(q,p)\}[u(q) - (m+d)] + \beta(m+d). \tag{3.2}$$

Sellers maximize their utility by choosing q, m, d, n, and y subject to (q, p) providing expected surplus equal to $\mathcal{S}$ to buyers, where $\mathcal{S}$ is expected surplus a buyer can obtain from his or her best alternative. Let $\mathbb{E}_y(\eta)$ denote a seller's expectation about the audit probability η conditional y. A seller's problem is given by

$$\begin{aligned} \max_{q,m,d,y,n} \; & \min\{1,n\}\{[(1-\mathbb{E}_y(\eta))(m+d-\tau(y)) + \mathbb{E}_y(\eta)(1-\rho^m)(m+d-\tau(y))]\mathbb{1}(d \le y < d+m) \\ & + [(1-\mathbb{E}_y(\eta))(m+d-\tau(y)) + \mathbb{E}_y(\eta)(1-\rho^m)m]\mathbb{1}(y < d) \\ & + [m+d-\tau(y)]\mathbb{1}(y = m+d) - q\} \end{aligned} \tag{3.3}$$

$$\text{s.t.} \; -(1+\mu)m - \frac{(1+\mu)d}{1+R^d} + \beta\min\{1, 1/n\}[u(q) - (m+d)] + \beta(m+d) = \mathcal{S}. \tag{3.4}$$

First, note that $\mathbb{1}(.)$ is an indicator function that takes the value of one if the statement in the bracket is true and zero otherwise. Second, recall that if a seller is audited, the fiscal authority can observe directly income in bank deposits, but it can only observe income in cash with probability ρ^m. If $y < d$, the seller always loses his or her income in bank deposits if he or she is audited, but he or she only loses his or her cash income if it is observed. If $d \le y < d + m$, a seller loses his or her entire income when he or she is audited *and* his or her cash income is observed, but does not lose any income if his or her cash income is not observed.

Next, after receiving the income reports, the fiscal authority chooses η to maximize its net tax revenue given $\tau(y)$ and its beliefs about m and d conditional on y.

$$\max_{\eta} \tau(y) + \eta \left[\mathbb{E}_y\{(d + \rho^m m - \tau(y))\mathbb{1}(y < d) + \rho^m(d + m - \tau(y))\mathbb{1}(d \leq y < d + m)\} - C \right], \quad (3.5)$$

where the expectation is taken over m and d. Lastly, recall that there is measure one of buyers and measure α of sellers. Let Φ denote the set of posted terms of trade. Let $F(q,p)$ denote the distribution of (q,p). Then, $n(q,p)$ must satisfy

$$\int_{(q,p)\in\Phi} n(q,p)\, \mathrm{d}F(q,p) = \frac{1}{\alpha}. \quad (3.6)$$

Now, I solve for the equilibrium for a given $\tau(y)$. The equilibrium concept is Perfect Bayesian Equilibrium (PBE), which requires that
(1) the choices of (q, m, d, y, n) and η are sequentially rational given the fiscal authority's beliefs about m and d conditional on y; and
(2) the fiscal authority's beliefs are derived from Bayes' rule whenever possible.
For any y that is not reported in equilibrium, conditions (1) and (2) put no restrictions on the fiscal authority's beliefs. Therefore, there may exist many equilibria supported by various off-equilibrium path beliefs, and some form of equilibrium refinement is necessary. However, standard refinement methods such as the intuitive criterion may not apply in this environment, because sellers' true income is determined by sellers' own choices (i.e. (q, m, d, n)). To refine the equilibrium, I require that for any y, a seller's choices of (q, m, d, n) and the fiscal authority's choice of η constitute a Nash equilibrium. Specifically, given $(\mathcal{S}, y, \eta)$, sellers solve

$$\begin{aligned} \max_{q,m,d,n} \ & \min\{1,n\}\{[(1-\eta)(m+d-\tau(y)) + \eta(1-\rho^m)(m+d-\tau(y))]\mathbb{1}(d \leq y < d+m) \\ & + [(1-\eta)(m+d-\tau(y)) + \eta(1-\rho^m)m]\mathbb{1}(y < d) \\ & + [m+d-\tau(y)]\mathbb{1}(y = m+d) - q\} \quad (3.7) \\ \text{s.t.} \ & -(1+\mu)m - \frac{(1+\mu)d}{1+R^d} + \beta\min\{1, 1/n\}[u(q) - (m+d)] + \beta(m+d) = \mathcal{S}. \quad (3.8) \end{aligned}$$

The fiscal authority's choice of η conditional y and the seller's strategy (q, m, d, n) is optimal.

$$\eta \begin{cases} = 0, \ \text{if } y \geq m + d \text{ or } d + \rho^m m - \tau(y) < C; \\ \in [0,1], \ \text{if } y < d \text{ and } d + \rho^m m - \tau(y) = C, \ \text{or } d \leq y < m + d \text{ and } \rho^m[d + m - \tau(y)] = C; \\ = 1, \ \text{if } y < d \text{ and } d + \rho^m m - \tau(y) > C, \ \text{or } d \leq y < m + d \text{ and } \rho^m[d + m - \tau(y)] > C. \end{cases} \quad (3.9)$$

Note that for any y that is on the equilibrium path, (q, m, d, n) must solve problem (3.7) and η must satisfy expression (3.9). The proposed refinement simply requires the same to be true for any off-equilibrium choice of y. This refinement is in the same vein of the Reordering Invariance (RI) equilibrium proposed by In and Wright (2018). See Appendix A for more discussion on the equilibrium refinement.

In the next proposition, I describe all possible equilibria of the game between sellers and the fiscal authority given $\tau(y)$, R^d, μ, and ρ^m.

Proposition 3.1 *Assume $R^d > 0$ and $C/\rho^m < \tilde{q}$ where $\tilde{q}$ solves $u'(q) = \frac{1+\mu}{\beta}$. Then,*
(1) *Sellers who fail to meet buyers in the DM report $y = 0$ and produce $q = 0$.*
(2) *Sellers who meet buyers in the DM randomize over strategies that include:*
(a) *$q > 0$, $m > 0$, $d > 0$, and $y = d$;* (b) *$q > 0$, $m = 0$, $d > 0$, and $0 < y < d$;*
(c) *$q > 0$, $m = 0$, $d > 0$, and $y = d$;* (d) *$q > 0$, $m = 0$, $d > 0$, and $y = 0$;*
(e) *$q > 0$, $m > 0$, $d = 0$, and $y = 0$.*
In particular, in any equilibrium, either (d) *or* (e) *is played with a probability strictly between zero and one.*
Proof: see Appendix B.

Note that if $R^d = 0$, there is no benefit from using bank deposits. To make the problem interesting, I assume $R^d > 0$. If a seller fails to meet a buyer in the DM, it is his or her dominant strategy to report $y = 0$ and produce $q = 0$ since sellers do not derive utility from the DM good.

Now, suppose a seller meets a buyer in the DM. Consider strategies (a)-(e). Strategy (a) says that the seller under-reports his or her income, and he or she accepts both cash and bank deposits as payment. The amount of income the seller reports, y, is equal to the amount he or she accepts in bank deposits. Accepting both cash and deposits allows the seller to benefit from both the higher return of deposits and the feature of cash that allows it to be hidden with probability $1 - \rho^m$. To see why $y = d$, note that for all $d \leq y$, the larger d is, the more the seller benefits from the higher return of deposits. If the seller is audited, then as long as the fiscal authority fails to observe his cash income, he or she does not lose any income (bar the tax due). However, if $d > y$ and the seller is audited, he or she will lose all income in bank deposits with certainty. If the following relationship holds in equilibrium

$$R^d < \frac{(1 - \rho^m)\eta}{1 - \eta}, \tag{3.10}$$

then the tax-evasion benefit of cash outweighs the higher return of deposits. Hence, the seller chooses to receive the rest of the payment in cash. Note that (3.10) implies that for cash to be used in equilibrium, the return on deposits must not be too high.

If R^d is instead high, i.e. $R^d > \frac{(1-\rho^m)\eta}{1-\eta}$, then it is beneficial to accept only bank deposits (strategy (b)). In this case, even though the seller loses all of his or her income if he or she is audited, the higher return on deposits outweighs the tax-evasion benefit of cash. Depending on $\tau(y)$, it may be that reporting truthfully (i.e. strategy (c)) offers higher surplus compared to the under-reporting strategies of (a) and (b). In this case, cash loses its tax-evasion benefit. Then, as long as $R^d > 0$, the seller accepts only bank deposits.

Lastly, consider strategies (d) and (e). The proposition says that either one of (d) and (e) must be played with probability strictly between 0 and 1. To see why, first note that because there are more sellers than buyers, a positive measure of sellers do not meet buyers in the DM and will report $y = 0$. If the fiscal authority does not audit sellers who report $y = 0$, then all sellers including those who have met buyers will report $y = 0$. However, if the fiscal authority audits sellers who report $y = 0$ with probability one, all sellers will report truthfully, and then the fiscal authority will not have the incentive to audit sellers. Hence, in equilibrium, it must be that the fiscal authority audits sellers who report $y = 0$ with probability strictly between 0 and 1, and some (but not all) sellers who have met buyers report $y = 0$.

Now, I am ready to solve for the optimal tax schedule that maximizes the total surplus in the DM subject to raising a given amount of net tax revenue.

Proposition 3.2 *The optimal tax schedule is* $\tau^*(y) = \min\{y, \tilde{\tau}\}$.
Proof: see Appendix B.

The tax schedule $\tau^*(y)$ is optimal for two reasons. First, it does not distort sellers' decisions in the DM as long as sellers report their income truthfully. Second, as shown in the proof of Proposition 3.2, it can eliminate strategies (a) and (b) from the equilibrium. That is, under $\tau^*(y)$, sellers who meet buyers in the DM either report $y = 0$ or report truthfully. This way, $\tau^*(y)$ lowers the audit costs and increases the fiscal authority's net tax revenue.

The equilibrium of the audit game under $\tau^*(y)$ is given by the following.

(1) With probability $1 - \gamma$, a matched seller chooses $q = q^h$, $m = 0$, and $d = y = d^h$, where q^h solves

$$u'(q) = \frac{1+\mu}{\beta(1+R^d)}, \tag{3.11}$$

and d^h is given by

$$d^h = \frac{1+R^d}{1+\mu}[\beta u(q^h) - \mathcal{S}]. \tag{3.12}$$

(2) With probability γ, a matched seller chooses $y = 0$. Given R^b and $\tilde{\tau}$, there exists $\rho^{m\prime}$ such that if $\rho^m < \rho^{m\prime}$, the seller accepts only cash and is audited with probability η^0. The seller chooses $q = q^0$ and $m = m^0$, where q^0 solves

$$u'(q) = \frac{1+\mu}{\beta(1-\rho^m\eta^0)}, \tag{3.13}$$

and m^0 is given by

$$m^0 = \frac{1}{1+\mu}[\beta u(q^0) - \mathcal{S}]. \tag{3.14}$$

(3) $\mathcal{S}$ is such that $d^h - q^h - \tilde{\tau} = 0$. η^0 is such that $(1 - \rho^m \eta^0)m^0 - q^0 = 0$. The fiscal authority does not audit sellers who report $y = d^h$.
(4) γ solves

$$\frac{\gamma \rho^m m^0}{\alpha - 1 + \gamma} = C. \tag{3.15}$$

It should be noted that if $\rho^m \geq \rho^{m\prime}$, then sellers who report $y = 0$ may also use bank deposits, and hence the demand for cash is zero. In what follows, I assume that ρ^m is sufficiently small so that sellers who evade taxes use cash. Note also that the term $\frac{\gamma}{\alpha-1+\gamma}$ in (3.15) represents the probability that a seller who reports $y = 0$ has met a buyer in the DM. Hence, (3.15) guarantees that the fiscal authority is indifferent between auditing and not auditing a seller who reports $y = 0$.

It is worth noting that because (by assumption) there are more sellers than buyers in the economy and matching is efficient, sellers' surplus is driven to zero in equilibrium. To see why, first note that in equilibrium there must not exist a submarket where sellers are strictly better off while buyers are at least as well off. Second, because there are more sellers than buyers, it must be that $n(q, p) < 1$ for some (q, p). If sellers' surplus is positive, then consider a submarket with (q, p, n') where $n' > n(q, p)$. In this submarket, buyers are as well off but sellers are strictly better off, which is a contradiction. If matching is not efficient, then both $\mathcal{S}$ and seller's surplus may be strictly positive. In such case, the determination of $\mathcal{S}$ is more complex, but the main findings in this paper remain unchanged.

The next proposition shows the effects of $\tilde{\tau}$, R^d, and ρ^m on equilibrium outcomes.

Proposition 3.3 (1) *An increase in $\tilde{\tau}$ leads to a decrease in q^0, no change in q^h, and an increase in γ.* (2) *An increase in R^d leads to an increase in q^h, an increase in q^0, and a decrease in γ.* (3) *An increase in ρ^m leads to a decrease in γ but no changes in q^0 and q^h.*
Proof: see Appendix B.

An increase in $\tilde{\tau}$ makes tax evasion more attractive. Therefore, the audit probability on sellers who report $y = 0$ must increase. This leads to a decrease in production by these sellers (i.e. q^0) and a decrease in payment to these sellers (i.e. m^0). This means that there must be more sellers who meet buyers in the DM reporting $y = 0$ so that (3.15) holds. An increase in R^d has the opposite effect: it makes reporting truthfully more attractive. Therefore, the audit probability on sellers who report $y = 0$ must decrease. This then leads to an increase in q^0, and a decrease in γ.

Notice that even though the tax schedule $\tau^*(y)$ is optimal, it mitigates but does not eliminates distortions in the economy. Specifically, for sellers who report $y = 0$ but have met buyers in the DM, the risky of being audited by the fiscal authority acts effectively as a proportional tax. As a result, these sellers produce less in equilibrium. This means that when tax evasion is possible, distortions caused by taxes cannot be eliminated solely through the optimal design of the tax schedule.

Lastly, result (3) shows that a change in ρ^m does not have any effect on q^0 and q^h. This is because any change in ρ^m is offset by a change in η^0 in the opposite direction so that $1-\rho^m\eta^0$, the probability of successful tax evasion, is kept constant. In fact, q^0 only depends on $\mathcal{S}$ in equilibrium. To see this, note that the surplus of sellers who report $y = 0$ satisfies

$$(1-\rho^m\eta^0)m^0 - q^0 = 0, \tag{3.16}$$

where

$$m^0 = \frac{1}{1+\mu}[\beta u(q^0) - \mathcal{S}], \tag{3.17}$$

$$u'(q^0) = \frac{1+\mu}{\beta(1-\rho^m\eta^0)}, \tag{3.18}$$

$$\mathcal{S} = \beta u(q^h) - \frac{(1+\mu)(q^h+\tilde{\tau})}{1+R^d}. \tag{3.19}$$

Hence, we have

$$\mathcal{S} = \beta u(q^0) - \frac{1+\mu}{1-\rho^m\eta^0}q^0 = \beta u(q^0) - \beta q^0 u'(q^0). \tag{3.20}$$

That is, as long as $\mathcal{S}$ does not change, q^0 will not change. And $\mathcal{S}$ will not change as long as μ, R^d, and $\tilde{\tau}$ do not change. This means that even if the increase in ρ^m is sufficiently large so that sellers who report $y = 0$ switch to accepting bank deposits, it will have no effect on q^0 and q^h. However, it is easy to show that γ will decrease and net tax revenue will increase.

3.2 Bankers' Problem in the CM

Recall that the markets for bank deposits bank loans are perfectly competitive, so bankers take the nominal deposit rate R^d and the nominal loan rate R^k as given.

I assume that if a banker chooses to default, he or she can abscond with a fraction θ of the collateral. Bankers' problem is given by

$$\max_{d^B,m^B,b^B,k^B} \{\beta(k^B+m^B+b^B-d^B) - e\} \tag{3.21}$$

$$\text{s.t. } \frac{(1+\mu)k^B}{1+R^k} + (1+\mu)m^B + \frac{(1+\mu)b^B}{1+R^b} = \frac{(1+\mu)d^B}{1+R^d} + e, \tag{3.22}$$

$$(k^B+m^B+b^B)(1-\theta) \geq d^B, \tag{3.23}$$

$$k^B, m^B, b^B, d^B \geq 0. \tag{3.24}$$

The amount of bank deposits (in real term) created by a banker is d^B, and e is the amount of CM good produced by the banker using the same technology as buyers. This means that, to satisfy the incentive constraint (3.23), bankers supply its own capital (i.e. "sweat equity"). Finally, a banker's holdings of cash, government bonds, and loans are m^B, b^B, and k^B, respectively.

Next, given R^d, the demand for deposits by a buyer who decides to visit (q^h, d^h) is

$$d^h = q^h + \tilde{\tau}, \tag{3.25}$$

where

$$u'(q^h) = \frac{1+\mu}{\beta(1+R^d)}. \tag{3.26}$$

Since a fraction $1-\gamma$ of sellers accept deposits in the DM, to clear the deposit market, R^d must be such that $d^B = (1-\gamma)d^h$.

Given R^k, the demand for loans by entrepreneurs is given by

$$f'(k) = \frac{1+R^k}{1+\mu}. \tag{3.27}$$

Recall that there is measure one of entrepreneurs and bankers. Hence, R^k must be such that $\frac{(1+\mu)k^B}{1+R^k} = k$.

Finally, depending on the return on loans, government bonds, and cash, the bankers may hold one or more types of assets as collateral. In what follows, I focus on equilibria where (3.23) binds. If bankers hold both government bonds and loans, then R^k and R^b must satisfy

$$\frac{1+\mu}{1+R^b} = \frac{1+\mu}{1+R^k} = \beta\theta + \frac{(1-\theta)(1+\mu)}{1+R^d}. \tag{3.28}$$

3.3 Equilibrium

An equilibrium is a vector $\{q^h, q^0, k, \gamma, \eta^0, R^d, R^b, R^k\}$ that solves the audit game between sellers and the fiscal authority and bankers' problems in the CM, and satisfies the budget constraints (2.3) and (2.4) given fiscal and monetary policies $(\tilde{\tau}, \mathcal{D}, \mu)$.

Proposition 3.4 *There exists μ' such that if (1) $\beta - 1 < \mu < \mu'$ and (2) $\frac{C}{\rho^m} \leq \tilde{\tau}$, then there exists a unique equilibrium.*
Proof: see Appendix B.

To solve the equilibrium, one may first derive the total demand for cash from (3.15).

$$\bar{m} = \frac{(\alpha - 1 + \gamma)C}{\rho^m}. \tag{3.29}$$

This shows that the demand for cash only depends on other parameter values through its dependence on γ, the proportion of sellers who evade taxes. Next, the total demand for government bonds by bankers is given by

$$\bar{b} = \frac{1-\gamma}{1-\theta}(q^h + \tilde{\tau}) - k^B. \tag{3.30}$$

In the proof, I show that if $\frac{C}{\rho^m} \leq \tilde{\tau}$, then $\bar{m} + \bar{b}$ is strictly increasing in R^b. Hence, there exists a unique R^b such that $\bar{m} + \bar{b} = \mathcal{D}$. For an equilibrium to exist, it is also necessary that μ is not too large. This is because if μ is large, the cost of holding cash may be so high that even if the fiscal authority conducts no audits, sellers accepting cash will not be able to compete with sellers who accept bank deposits. In such case, the demand for cash is zero.

Next, I consider the effects of increasing the inflation target μ on the equilibrium. In the model, this is achieved through open market purchases of government bonds.

Proposition 3.5 *Suppose the assumptions in Proposition* 3.4 *hold. Then, an increase in μ leads to increases in both q^h and q^0. In addition, γ and $\frac{1+R^b}{1+\mu}$ increase, while η^0 and k decreases.*
Proof: see Appendix B.

Proposition 3.5 says that when μ is small, increasing μ will increase q^h, q^0, and the proportion of sellers who evade taxes. This because higher inflation reduces the income of sellers who evade taxes. Tax evaders' lower cash income means that even if they are found evade taxes, the punishment the fiscal authority can impose is small, This decreases the fiscal authority's incentive to conduct audits. The drop in the level of tax enforcement (i.e. η^0) leads to a larger share of sellers evading taxes. It can be shown that as long as $\frac{C}{\rho^m} \leq \tilde{\tau}$, the drop in enforcement always offsets the increase in inflation. As a result, sellers who accept cash and evade taxes can produce more for buyers. More sellers accepting cash also means a lower demand for bank deposits. Consequently, the demand for government bonds decreases, and the real bond rate increases. This allows sellers who accept bank deposits and report truthfully to produce more for buyers. However, a higher government bond rate crowds out bank lending to entrepreneurs, so k decreases.

The general equilibrium effects of inflation on real allocations in Proposition 3.5 emerge in this environment for two reasons. First, the value of cash as a payment instrument is determined not only by inflation but also by the fiscal authority's audit strategy. A change in inflation leads not only to a change in the cost of carrying cash but also to a change in the audit probability. Second, the total supply of government liabilities, $\mathcal{D} = \bar{m} + \bar{b}$, is too low to support efficient consumption in the DM. This means that a decrease in demand for bank deposits will lead to an increase in government bond rate and the deposit rate, and hence a higher q^h. In comparison, if $\mathcal{D}$ is sufficiently large so that $R^d = R^b = (1+\mu)/\beta - 1$, constraint (3.23) will not bind and bankers will be indifferent between holding or not holding one extra unit of government bonds. In this case, an increase in inflation will increase γ and q^0, but will have no effect on q^h.

4 Central Bank Digital Currency: Equilibrium

I define central bank digital currency (CBDC) to be another type of payment instrument (in addition to cash) that is issued by the central bank. It is stored in accounts managed by the central bank, and it can be held by all agents. Throughout this section, I assume that the central bank does not withdraw cash from circulation. However, I will show that depending on the characteristics of CBDC, cash may not be used in equilibrium because CBDC is a superior payment instrument.

It is also important to be clear about how CBDC is introduced into the economy. Recall that in the benchmark model, the central bank can increase or decrease the supply of cash through open market purchases or sales of government bonds. The central bank's balance sheet is

Assets of the central bank	**Liabilities of the central bank**
Government bonds	Cash

Similar to cash, the central bank introduces CBDC by using it to purchase government bonds from the bond market. The central bank's balance sheet after the introduction of CBDC is

Assets of the central bank	**Liabilities of the central bank**
Government bonds	Cash
	Central bank digital currency

Now, I discuss the characteristics of CBDC. I assume that CBDC can be (potentially) different from cash and bank deposits in two dimensions: the first is the interest rate on CBDC (R^c), and the second is how much anonymity CBDC offers. In the context of tax evasion, the degree of anonymity of CBDC will affect the probability of the income received in CBDC being observed by the fiscal authority after an audit (ρ^c). I assume that both R^c and ρ^c are the choices of the central bank.[9] Specifically, I consider the scenarios where $(R^c, \rho^c) \in \mathbb{R}_+ \times [\rho^m, 1]$. Recall that ρ^m is the probability of income received in cash being observed by the fiscal authority after an audit. Since CBDC competes with cash, if $\rho^c \geq \rho^m$, then it must be that $R^c \geq 0$, because otherwise sellers will strictly prefer cash over CBDC.[10]

In what follows, I first consider a scenario where both the interest rate on CBDC and the degrees of anonymity of CBDC are too low so that CBDC is not used in equilibrium. I then show what other types of equilibria may exist depending on R^c and ρ^c.

[9]Advances in computer science have made it possible for CBDC to offer many different levels of anonymity. Darbha and Arora (2020) show that combinations of several cryptographic techniques and operational arrangements can be used to achieve fine-grained privacy designs. ESCB (2019) demonstrates that through "anonymity vouchers", CBDC can offer anonymity while also complying with anti-money laundering and anti-terrorism laws.

[10]If $\rho^c < \rho^m$, then even if the interest rate on CBDC is negative, it may still be accepted by sellers. I discuss this case in Appendix D. I also consider a scenario where CBDC offers *less* anonymity than bank deposits. Specifically, I assume that the fiscal authority can costlessly observe any income sellers receive in CBDC, but audits are necessary for the fiscal authority to observe cash and deposit income.

4.1 Type-1 Equilibrium: CBDC is not Used

Denote deposit rate in the benchmark as R^{bench}. Denote the audit probability in the benchmark as η^{bench}. Recall that in the benchmark model, sellers who choose to evade taxes accept only cash, and they solve the following problem

$$\max_{q,m}\{(1-\rho^m\eta^{bench})m-q\} \text{ s.t. } -(1+\mu)m+\beta u(q)=\mathcal{S}, \tag{4.1}$$

where $\mathcal{S}$ is highest surplus buyers can obtain elsewhere in the market, and sellers take it as given. If a seller chooses to accept CBDC, then he or she solves

$$\max_{q,c}\{(1-\rho^c\eta^{bench})c-q\} \text{ s.t. } -\frac{1+\mu}{1+R^c}c+\beta u(q)=\mathcal{S}, \tag{4.2}$$

where c represents the payment in CBDC. Then, it is easy to see that as long as

$$\frac{1+\mu}{(1-\rho^c\eta^{bench})(1+R^c)} > \frac{1+\mu}{1-\rho^m\eta^{bench}}, \tag{4.3}$$

sellers who evade taxes will strictly prefer cash over CBDC. The left-hand side of (4.3) represents the marginal cost of accepting CBDC, while the right-hand side represents the marginal cost of accepting cash. While CBDC may offer a positive interest rate (i.e. $R^c > 0$), it may also offer less anonymity compared to cash (i.e. $\rho^c > \rho^m$). If R^c is too low and ρ^c is too high, then after factoring the risk of being audited and punished by the fiscal authority, the cost of accepting cash is strictly lower compared to the cost of accepting CBDC.

Next, consider sellers who report their income truthfully. In the benchmark model, these sellers accept bank deposits, and they solve

$$\max_{q,d}\{d-\tilde{\tau}-q\} \text{ s.t. } -\frac{1+\mu}{1+R^{bench}}d+\beta u(q)=\mathcal{S}. \tag{4.4}$$

If a seller chooses to accept CBDC, then he or she solves

$$\max_{q,c}\{c-\tilde{\tau}-q\} \text{ s.t. } -\frac{1+\mu}{1+R^c}d+\beta u(q)=\mathcal{S}. \tag{4.5}$$

Then as long as $R^c < R^{bench}$, sellers who report their income truthfully will have no incentive to accept CBDC. If neither sellers who evade taxes nor sellers who report truthfully accept CBDC, then the equilibrium will be identical to the benchmark equilibrium in Section 3.

Now, suppose that R^c and ρ^c are such that

$$\frac{1+\mu}{(1-\rho^c\eta^{bench})(1+R^c)} < \frac{1+\mu}{1-\rho^m\eta^{bench}}. \tag{4.6}$$

This means that for sellers who evade taxes, the marginal cost of accepting CBDC is lower compared to the marginal cost of accept cash. In this case, depending on R^c and ρ^c, sellers who report their income truthfully may accept only bank deposits (a type-2 equilibrium; see Section 4.2), or some of them may switch to accepting CBDC (a type-3 equilibrium; see Section 4.3).

Finally, suppose that

$$\frac{1+\mu}{(1-\rho^c\eta^{bench})(1+R^c)} > \frac{1+\mu}{1-\rho^m\eta^{bench}} \tag{4.7}$$

but $R^c > R^{bench}$. In this case, sellers who report their income have the incentive to accept CBDC since it offers a higher interest rate. However, for sellers who evade taxes, the marginal cost of accepting cash is still lower compared to the marginal cost of accepting CBDC. I discuss this type equilibria (type-4 equilibria) in Section 4.4.

4.2 Type-2 Equilibrium: CBDC Replaces Cash

Assume that

$$\frac{1+\mu}{(1-\rho^c\eta^{bench})(1+R^c)} < \frac{1+\mu}{1-\rho^m\eta^{bench}}. \tag{4.8}$$

Then, sellers who evade taxes have the incentive to switch to accepting CBDC. Now, suppose that R^c is low so that sellers who report truthfully prefer to accept bank deposits. The equilibrium is given by the following.

(1) With probability $1-\gamma$, a matched seller produces $q = q^h$, demands deposit payment $d = d^h$, and reports $y = d$, where q^h solves

$$u'(q) = \frac{1+\mu}{\beta(1+R^d)}, \tag{4.9}$$

and d^h is given by

$$d^h = \frac{1+R^d}{1+\mu}[\beta u(q^h) - \mathcal{S}]. \tag{4.10}$$

(2) With probability γ, a matched seller produces $q = q^0$, demands CBDC payment $c = c^0$, and reports $y = 0$, where q^0 solves

$$u'(q) = \frac{1+\mu}{\beta(1-\rho^c\eta)(1+R^c)}, \tag{4.11}$$

and c^0 is given by

$$c^0 = \frac{1+R^c}{1+\mu}[\beta u(q^0) - \mathcal{S}]. \tag{4.12}$$

(3) $\mathcal{S}$ is such that $d^h - q^h - \tilde{\tau} = 0$. η^0 is such that $(1-\rho^c\eta)c^0 - q^0 = 0$.

(4) γ solves

$$\frac{\gamma\rho^c c^0}{\alpha - 1 + \gamma} = C. \tag{4.13}$$

A banker's problem is similar to the benchmark model, but now the bankers can hold CBDC. However, note that similar to the benchmark, in equilibrium,

$$\frac{1+\mu}{1+R^b} = \frac{1+\mu}{1+R^k} = \beta\theta + \frac{(1-\theta)(1+\mu)}{1+R^d}, \tag{4.14}$$

where R^b and R^k are government bond rate and nominal loan rate, respectively. Since $R^b > R^d$ and $R^d > R^c$, the interest rate on CBDC is lower than the interest rate on government bonds. This means that bankers will not hold CBDC. Finally, let $\bar{c}$ denote the the demand for CBDC. Note that it is equal to the demand for government bonds from the central bank. Let $\bar{b}$ denote the demand for government bonds from bankers. Then

$$\bar{c} = \frac{(\alpha - 1 + \gamma)C}{\rho^c}, \tag{4.15}$$

$$\bar{b} = \frac{(1-\gamma)(q^h + \tilde{\tau})}{1-\theta} - f'(k)k, \tag{4.16}$$

where $\frac{(1-\gamma)(q^h+\tilde{\tau})}{(1-\theta)}$ is the total demand for bank deposits, k is the investment in each entrepreneur's project, and $f'(k)k$ is the total value of loans. In equilibrium, R^d is such that $\mathcal{D} = \bar{c} + \bar{b}$, where $\mathcal{D}$ is the total supply of government bonds.

Now I derive the effects of R^c and ρ^c on the equilibrium.

Proposition 4.1 *Increasing R^c or ρ^c while holding μ constant lowers q^h and q^0. In addition, γ, R^d, and R^b will decrease, while k will increase.*
Proof: see Appendix B.

First, a higher return on CBDC leads to larger payments to sellers who evade taxes, which gives the fiscal authority more incentive to audit these sellers. Such an increase in tax enforcement leads to fewer sellers evading the tax and lower output from these sellers. Because more sellers are accepting deposits, the demand for deposits increases. This drives down the deposit rate and the government bond rate. Consequently, the output produced by sellers who report truthfully decreases as well. A lower government bond rate also means that the loan rate will decrease and bank lending will increase. Second, a higher ρ^c increases the visibility of sellers' income, which also makes the fiscal authority's more willing to audit sellers. This decreases tax evasion and increases the demand for deposits. Similar to increasing R^c, the deposit rate will be lower, and the income and output of sellers who report truthfully will decrease.

Figure 1 illustrates the mechanism. Note that an increase in R^c or ρ^c does not directly affect q^h, since sellers who report truthfully do not use CBDC, and they are not audited. In a partial equilibrium where the deposit rate is taken as given, increasing R^c or ρ^c will only affect the level of tax evasion (γ) and the output from sellers who evade taxes (q^0). However, in a general equilibrium, the decrease in tax evasion raises the demand for bank deposits, which then affects the output from sellers who report truthfully through its effect on the deposit rate.

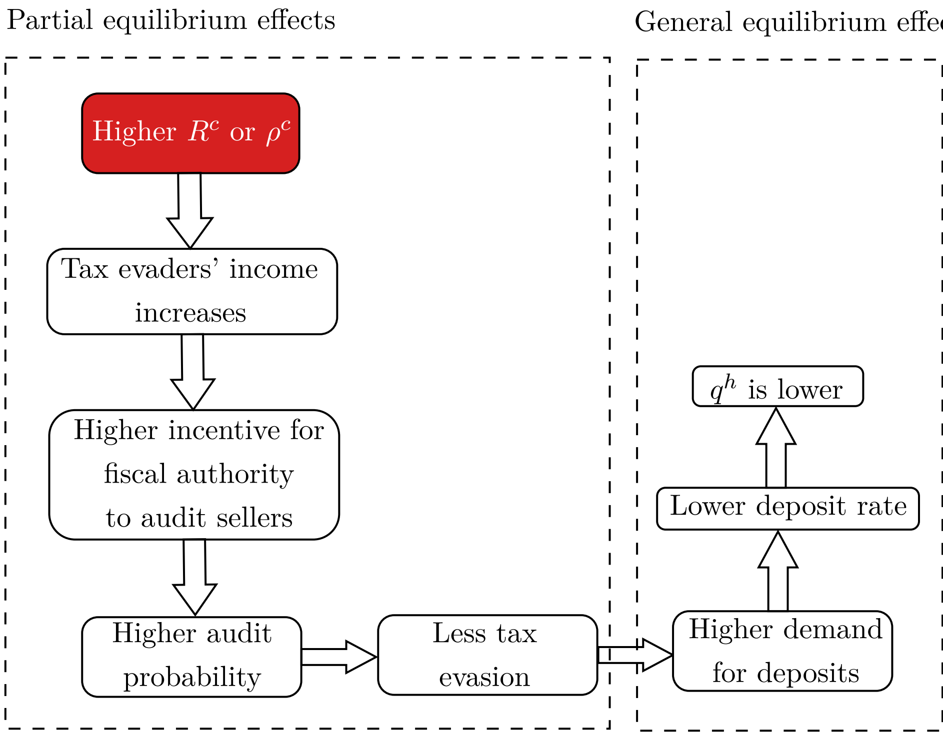


Figure 1: Effects of Increasing R^c in a Type-2 Equilibrium

Because R^d is decreasing in R^c and ρ^c, for sufficiently large R^c and ρ^c, we will have $R^c = R^d$. In such case, sellers who report truthfully will be willing to switch to accepting CBDC as well. I discuss this case in the next sub-section.

4.3 Type-3 Equilibrium: CBDC Replaces Cash and (some) Bank Deposits

Before I discuss type-3 equilibria, it should be made clear that while CBDC may completely replace cash (a type-2 equilibrium), it will not completely replace bank deposits. The reason is that if bank deposits are not used in equilibrium, bankers will have to fund the loans to entrepreneurs through working in the CM. In such case, the real loan rate $\frac{1+R^k}{1+\mu}$ will be equal to $\frac{1}{\beta}$, and the real deposit rate will be equal to $\frac{1}{\beta}$ as well. This means that unless the real interest rate on CBDC is greater or equal to $\frac{1}{\beta}$, sellers will prefer to use bank deposits. Note that the demand for CBDC will be infinite if the real interest rate on CBDC is greater than $\frac{1}{\beta}$. Hence, in any equilibrium, sellers who report their income truthfully will either strictly prefer bank deposits, or be indifferent between accepting CBDC and accepting bank deposits.

Now, I discuss a type-3 equilibrium where some sellers who report truthfully accept CBDC, while other sellers who report truthfully accept bank deposits. In such case, the interest rate on CBDC must be equal to the deposit rate. Let $R \equiv R^c = R^d$. The equilibrium is given by the following.

(1) With probability $1-\gamma$, a matched seller reports truthfully and produces $q = q^h$, where q^h solves

$$u'(q) = \frac{1+\mu}{\beta(1+R)}. \tag{4.17}$$

Among sellers who report truthfully, a fraction ϵ accept bank deposits, while the rest accept CBDC. Let a^h denote the payment received in bank deposits or CBDC by these sellers. Then a^h is given by

$$a^h = \frac{1+R}{1+\mu}[\beta u(q^h) - \mathcal{S}]. \tag{4.18}$$

(2) With probability γ, a matched seller produces $y = 0$, reports an income equal to zero, and demands CBDC payment $c = c^0$, where q^0 solves

$$u'(q) = \frac{1+\mu}{\beta(1-\rho^c\eta^0)(1+R)}, \tag{4.19}$$

and c^0 is given by

$$c^0 = \frac{1+R}{1+\mu}[\beta u(q^0) - \mathcal{S}]. \tag{4.20}$$

(3) $\mathcal{S}$ is such that $a^h - q^h - \tilde{\tau} = 0$, and η^0 is such that $(1-\rho^c\eta^0)c^0 - q^0 = 0$.
(4) γ solves

$$\frac{\gamma\rho^c c^0}{\alpha - 1 + \gamma} = C. \tag{4.21}$$

(5) The demand for government bonds from bankers is given by

$$\bar{b} = \frac{\epsilon(1-\gamma)(q^h+\tilde{\tau})}{1-\theta} - f'(k)k, \tag{4.22}$$

where $\frac{\epsilon(1-\gamma)(q^h+\tilde{\tau})}{1-\theta}$ is the total demand for bank deposits, k is the investment in each entrepreneur's project, and $f'(k)k$ is the total value of loans.
(6) The total demand for CBDC is given by

$$\bar{c} = (1-\gamma)(1-\epsilon)(q^h+\tilde{\tau}) + \frac{(\alpha-1+\gamma)C}{\rho^c}, \tag{4.23}$$

where the first term represents the demand from sellers who report truthfully, and the second term represents the demand from sellers who evade taxes.

(7) The fraction of sellers who report truthfully and accept deposits, ϵ, is such that $\bar{b} + \bar{c} = \mathcal{D}$.

Finally, a banker's problem is similar to the benchmark model. In equilibrium,

$$\frac{1+\mu}{1+R^b} = \beta\theta + \frac{(1-\theta)(1+\mu)}{1+R^d}. \tag{4.24}$$

Hence, $R^b > R^d = R^c$. This means that the interest rate on CBDC is lower than the interest rate on government bonds, so bankers will not hold CBDC.

Now, I discuss the effects of R^c and ρ^c on the equilibrium.

Proposition 4.2 (1) *Increasing R^c while holding μ and ρ^c constant increases q^h and q^0. In addition, γ, ϵ, and k will decrease.* (2) *Increasing ρ^c while holding μ and ρ^c constant has no effect on q^h, q^0, and k. However, γ, η^0, and ϵ will decrease.*
Proof: see Appendix B.

First, a higher R^c increases the income of sellers who report truthfully and makes them willing to produce more for buyers. However, if sellers who evade taxes choose to produce more (and receive more payment in exchange), the fiscal authority will have a higher incentive to conduct audits. That is, tax evasion prevents these sellers from taking advantage of the higher R^c. In equilibrium, this means that more sellers will choose to report truthfully. The decrease in tax evasion is sufficiently large to lower the fiscal authority's incentive to audit, which allows sellers who evade taxes to eventually produce more as well. Second, a higher ρ^c has no effect on sellers who report their income truthfully because the interest rate on CBDC and bank deposits are unchanged. However, because ρ^c is larger, the visibility of sellers' income increases. This decreases sellers' incentive to evade taxes and reduces tax evasions. Since fewer sellers are evading taxes, the audit probability decreases, which compensates for the increase in ρ^c. As a result, q^0 is also unchanged.

To understand why ϵ decreases with R^c and ρ^c, first recall that CBDC is created by the central bank through purchasing government bonds with CBDC, so one unit of government bonds can be used to create one unit of CBDC. However, because bankers have limited commitment and can abscond with a fraction θ of assets, they can only create $1-\theta$ units of bank deposits with one unit of government bonds. This means that the central bank is more efficient at creating payment assets compared to bankers.[11] In equilibrium, an increase in R^c or ρ^c will lead to an increase in the demand for bank deposits and CBDC. The central bank then increases the supply of CBDC by purchasing more government bonds from the bond market. Since the total supply of government bonds is fixed, bankers decrease their holdings of government bonds and reduce the supply of deposits. As a result, in equilibrium, a larger share of sellers who report truthfully accept CBDC.

The mechanisms for the results in Proposition 4.2 are summarized in Figure 2.

[11] I discuss this property in more details in Section 5.1.

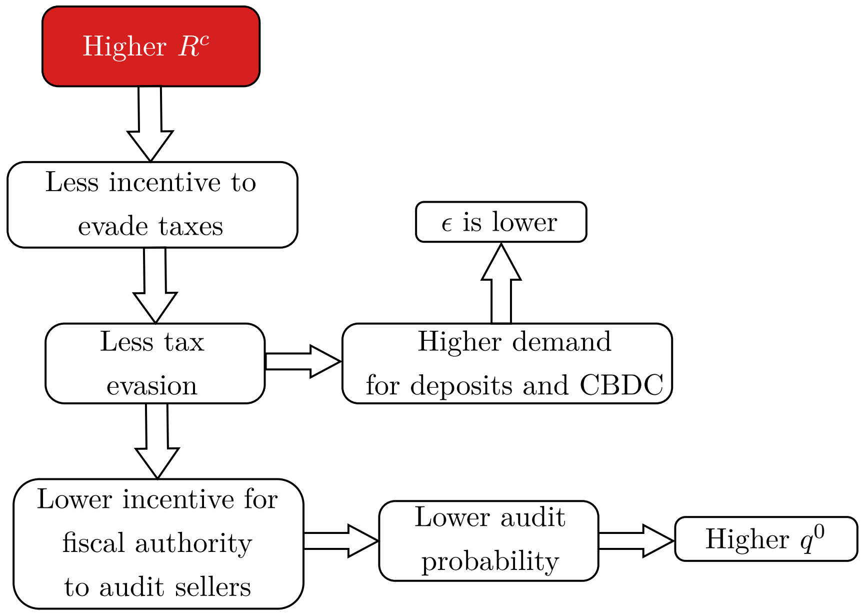


Figure 2: Effects of Increasing R^c in a Type-3 Equilibrium

It should be noted that in both type-2 and type-3 equilibria, a higher R^c decreases tax evasion. However, in a type-3 equilibrium, a higher R^c increases rather than decreases R^d, q^h, and q^0. To see why, first note that in a type-2 equilibrium, CBDC is not accepted by sellers who report their income truthfully. A higher R^c leads to more sellers accepting deposits, which decreases the deposit rate and lowers the income and output of sellers who report truthfully. In a type-3 equilibrium, an increase in R^c also increases the demand for bank deposits, but because the interest rate on CBDC is high, some sellers who report truthfully substitute CBDC for bank deposits. Since the central bank is more efficient at creating payment assets, the total supply of CBDC *and* bank deposits increases. As a result, the deposit rate and the government bond rate increase rather than decrease, and all sellers produce more in equilibrium.

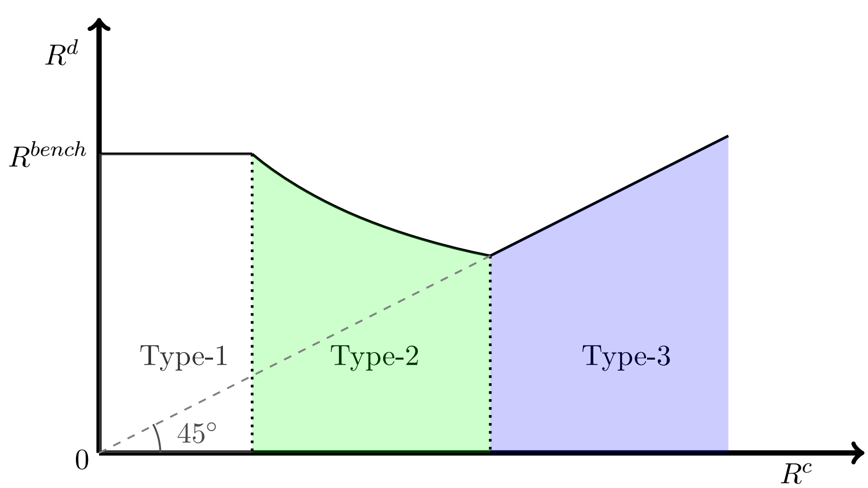


Figure 3: Relationship Between R^d and R^c

In Figure 3, I fix ρ^c and show how R^d varies with R^c for a given ρ^c. When R^c is low, CBDC is not used in equilibrium, and R^d is equal to the deposit rate in the benchmark. When R^d is large enough for sellers who evade taxes to be willing to accept CBDC, the economy is in a type-2 equilibrium. In such case, R^d is decreasing in R^c. When $R^d = R^c$, the economy transitions into a type-3 equilibrium, and R^d is decreasing in R^c. Notice that $R^c < R^{bench}$ when the equilibrium switches from type-2 to type-3. This means that even if the interest rate on CBDC is lower than the deposit rate in the benchmark, sellers who report their income truthfully may still accept CBDC after its introduction.

Finally, suppose $R^c < R^{bench}$ and consider an increase in ρ^c. If R^c and ρ^c satisfy

$$\frac{1+\mu}{(1-\rho^c\eta^{bench})(1+R^c)} > \frac{1+\mu}{1-\rho^m\eta^{bench}}, \tag{4.25}$$

then sellers who evade taxes will switch back to accepting cash. Since $R^c < R^{bench}$, sellers who report their income truthfully will not accept CBDC either. That is, the equilibrium transitions from type-3 to type-1. Now, suppose $R^c > R^{bench}$ and consider again an increase in ρ^c. If (4.25) holds, then sellers who report truthfully will accept CBDC but sellers who evade taxes will not. I discuss this type of equilibria in the next sub-section.

4.4 Type-4 Equilibrium: CBDC Replaces (some) Bank Deposits

Assume that $R^c > R^{bench}$, and that R^c and ρ^c satisfy

$$\frac{1+\mu}{(1-\rho^c\eta^{bench})(1+R^c)} > \frac{1+\mu}{1-\rho^m\eta^{bench}}. \tag{4.26}$$

Then, sellers who evade taxes prefer to accept cash, while sellers who report their income truthfully have the incentive to accept CBDC. Recall that in any equilibrium, sellers who report their income truthfully must either strictly prefer bank deposits or be indifferent between accepting CBDC and accepting bank deposits (see Section 4.3). This means that R^d must increase in equilibrium so that $R^d = R^c$. Now, define $R \equiv R^d = R^c$. The equilibrium is given by the following.

(1) With probability $1-\gamma$, a matched seller reports truthfully and produces $q = q^h$, where q^h solves

$$u'(q) = \frac{1+\mu}{\beta(1+R)}. \tag{4.27}$$

Among sellers who report truthfully, a fraction ϵ accept bank deposits, while the rest accept CBDC. Let a^h denote the payment received in bank deposits or CBDC. Then a^h is given by

$$a^h = \frac{1+R}{1+\mu}[\beta u(q^h) - \mathcal{S}]. \tag{4.28}$$

(2) With probability γ, a matched seller produces $y = 0$, reports an income equal to zero, and

demands CBDC payment $m = m^0$, where q^0 solves

$$u'(q) = \frac{1+\mu}{\beta(1-\rho^m\eta^0)}, \tag{4.29}$$

and m^0 is given by

$$m^0 = \frac{1}{1+\mu}[\beta u(q^0) - \mathcal{S}]. \tag{4.30}$$

(3) $\mathcal{S}$ is such that $a^h - q^h - \tilde{\tau} = 0$, and η^0 is such that $(1-\rho^m\eta^0)m^0 - q^0 = 0$.
(4) γ solves

$$\frac{\gamma\rho^m m^0}{\alpha - 1 + \gamma} = C. \tag{4.31}$$

(5) The demand for government bonds from bankers is given by

$$\bar{b} = \frac{\epsilon(1-\gamma)(q^h+\tilde{\tau})}{1-\theta} - f'(k)k, \tag{4.32}$$

where $\frac{\epsilon(1-\gamma)(q^h+\tilde{\tau})}{1-\theta}$ is the total demand for bank deposits, k is the investment in each entrepreneur's project, and $f'(k)k$ is the total value of loans.
(6) The total demand for CBDC is given by

$$\bar{c} = (1-\gamma)(1-\epsilon)(q^h+\tilde{\tau}). \tag{4.33}$$

(7) The total demand for cash is given by

$$\bar{m} = \frac{(\alpha-1+\gamma)C}{\rho^m}. \tag{4.34}$$

(8) The fraction of sellers who report truthfully and accept deposits, ϵ, is such that $\bar{m}+\bar{b}+\bar{c} = \mathcal{D}$.
In equilibrium, R^b and R^d satisfy

$$\frac{1+\mu}{1+R^b} = \beta\theta + \frac{(1-\theta)(1+\mu)}{1+R^d}. \tag{4.35}$$

Hence, $R^b > R^d = R^c$. This means that the interest rate on CBDC is lower than the interest rate on government bonds, so bankers will not hold CBDC.

In a type-4 equilibrium, CBDC is not accepted by sellers who evade taxes. This means that how much anonymity CBDC offers will not have any impact on the equilibrium. Now, consider the effects of R^c on equilibrium outcomes.

Proposition 4.3 *Increasing R^c while holding μ constant increases q^h and q^0. In addition, γ, ϵ, and k will decrease.*
Proof: see Appendix B.

Increasing R^c has similar effects in type-3 and type-4 equilibria. Specifically, a higher R^c increases the income and output of sellers who report truthfully and lowers the incentive for sellers to evade taxes. The decrease in tax evasion lowers the fiscal authority's incentive to audit, which allows sellers who evade taxes to also produce more for buyers. The decrease in tax evasion also increases the demand for bank deposits and CBDC. To increase the supply of CBDC, the central bank purchases more government bonds from the bond market. Recall that because the central bank is immune from the limited commitment issue, for the same quantity of government bonds the central bank can create more payment assets compared to bankers. Since the total supply of government bonds is fixed, the competition between the central bank and bankers drives out deposits. As a result, ϵ decreases in equilibrium.

I summarize the four equilibrium types discussed in Section 4.1-4.4 in the following table.

Equilibrium Types	**Payment instrument(s) accepted by sellers who report truthfully**	**Payment instrument(s) accepted by sellers who evade taxes**
Type-1 (benchmark)	Bank deposits	Cash
Type-2	Bank deposits	CBDC
Type-3	CBDC and bank deposits	CBDC
Type-4	CBDC and bank deposits	Cash

Table 1: Equilibrium Types

In Figure 4, I show how equilibrium types depend on R^c (the horizontal axis) and ρ^c (the vertical axis). The origin represents $(R^c, \rho^c) = (0, \rho^m)$. In general, CBDC replaces only cash if it offers high anonymity but low interest rate (a type-2 equilibrium). CBDC replaces (some) bank deposits if it offers low anonymity but high interest rate (a type-4 equilibrium). If CBDC offers a combination of high interest rate and *higher* anonymity than deposits, then it replaces cash and (some) bank deposits (a type-3 equilibrium).

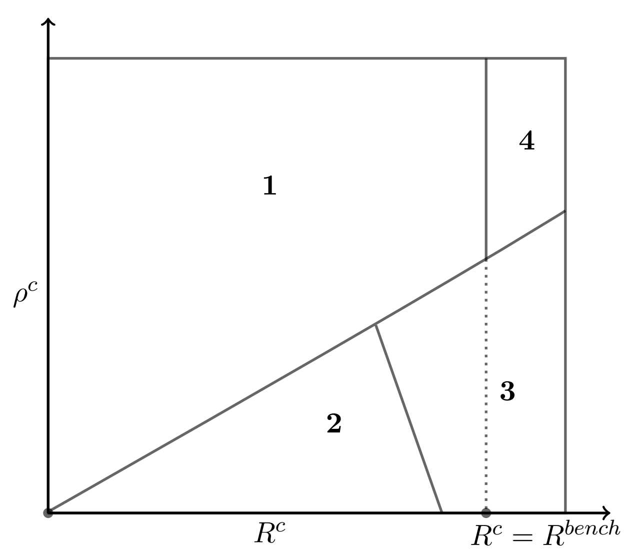


Figure 4: Equilibrium Types

5 Aggregate Welfare and Central Bank Net Revenue

In the first part of this section, I discuss the welfare effects of introducing a central bank digital currency (CBDC). Aggregate welfare is measured using an equal-weighted sum of all agents' utility. Specifically, let x^b denote the net consumption of bankers in the DM (recall that bankers may both produce and consume in the CM). Let x^e denote the consumption of entrepreneurs born in the previous period. Recall that l is the labor supplied by buyers in the CM, and x is the consumption of sellers in the CM. Furthermore, q^h and q^0 are the output from sellers who report their income truthfully and the output from sellers who evade taxes, respectively. Finally, γ is the proportion of sellers who evade taxes. Then, aggregate welfare is given by

$$\mathcal{W} = (1-\gamma)[u(q^h) - q^h] + \gamma[u(q^0) - q^0] + x^b + x^e + x - l. \tag{5.1}$$

In the second part of this section, I discuss the effect of introducing CBDC on the central bank's net revenue. The central bank in the model, similar to its counterparts in reality (e.g. the Federal Reserve), earns revenue through the interest payments on its assets (government bonds). The expense of the central bank is the interest payments on its liabilities (CBDC and/or cash). Since the central bank does not pay interest on cash, the expense of the central bank is zero in the benchmark. In practice, the expenses of central banks may also include costs of producing and replacing physical cash, and the costs of operating the central bank (e.g. personnel costs).[12] To ensure a central bank's independence of the fiscal authority, it may be necessary for the central bank to cover its own expenses rather than rely on transfers from the fiscal authority. For example, when discussing its independence, the Federal Reserve emphasizes that it "does not receive funding through the congressional budgetary process".[13] It is therefore of interest to understand how introducing CBDC, especially interest-bearing CBDC, affects the central bank's net revenue.

5.1 Aggregate Welfare

Recall that η^0 is the audit probability and C is the cost per audit. In equilibrium, the fiscal authority only audits sellers who report zero income, which include measure $\alpha - 1$ of sellers who did not meet buyers in the DM, and measure γ of sellers who choose to evade taxes. Hence, the total audit costs are equal to $(\alpha - 1 + \gamma)\eta^0 C$. Recall also that k is the amount of CM good invested in each entrepreneur's project. Then, $f(k) - k$ represents the net output from entrepreneurs' projects. Since the CM good cannot be carried into the next period, the resource constraint in the CM

[12]The interest expense of the Federal Reserve in 2019 is \$41B, and the operating expense is \$7B. Together, they account for 85% of the Federal Reserve's total expenses. In comparison, the interest expense and the operating expense of the Bank of Canada in 2019 are \$406M and \$519M, respectively, and they account for 94% of the Bank of Canada's total expenses.

[13]See `https://www.federalreserve.gov/faqs/about_12799.htm`. Currently, both the Federal Reserve and the Bank of Canada earn more than enough to cover their expenses, and they transfer the remaining revenue to the fiscal authorities at the end of each year. In 2019, the Federal Reserve and the Bank of Canada transferred \$54.8B and \$1.2B to the federal governments of the US and Canada, respectively.

implies that net CM consumption $x^b + x^e + x - l$ must satisfy

$$x^b + x^e + x - l = f(k) - k - (\alpha - 1 + \gamma)\eta^0 C. \tag{5.2}$$

Aggregate welfare can then be divided into three components.

$$\mathcal{W} = \underbrace{(1-\gamma)[u(q^h) - q^h] + \gamma[u(q^0) - q^0]}_{\text{Total surplus in the DM}} + \underbrace{f(k) - k}_{\text{Net output from entrepreneurs}} - \underbrace{(\alpha - 1 + \gamma)\eta^0 C}_{\text{Total audit costs}}. \tag{5.3}$$

In what follows, to study the effect of introducing CBDC on aggregate welfare, I compare the three components with their counterparts in the benchmark equilibrium.

I. Total surplus in the DM

Recall that R^c denotes the interest rate on CBDC, and ρ^c denotes the probability of the income received in CBDC being observed by the fiscal authority after an audit. Depending on the central bank's choices of R^c and ρ^c, there are four types of equilibria (see Table 1 and Figure 4). Let R^{bench} denote the deposit rate in the benchmark equilibrium.

First, consider the effect of introducing CBDC on γ.

Proposition 5.1 *Compared to the benchmark, γ is smaller in type-2, type-3, and type-4 equilibria. In addition, γ is smaller in type-3 equilibria than in type-2 equilibria. Given R^c, γ is also smaller in type-3 equilibria than in type-4 equilibria.*
Proof: see Appendix B.

In a type-2 equilibrium, CBDC is only accepted by sellers who evade taxes. Recall that ρ^m is the probability of cash being observed by the fiscal authority after an audit. Because $\rho^c > \rho^m$, introducing CBDC increases the visibility of sellers' income. This gives the fiscal authority more incentive to audit sellers, which decreases tax evasion. In a type-4 equilibrium, CBDC is only accepted by sellers who report their income truthfully, so the anonymity of CBDC is irrelevant to tax evasion. However, in this case, CBDC competes with bank deposits and increases the deposit rate, which attracts more sellers to report their income truthfully. Finally, in a type-3 equilibrium, CBDC is accepted by both sellers who report their income truthfully and sellers who evade taxes. Therefore, CBDC not only competes with bank deposits but also makes tax audits more effective. The result is that the reduction in tax evasion is larger compared to any type-4 equilibria given R^c and any type-2 equilibria.

Next, consider the effects of introducing CBDC on q^0 and q^h.

Proposition 5.2 *Compared to the benchmark, (1) q^0 and q^h are smaller in type-2 equilibria, and larger in type-4 equilibria; and (2) q^0 and q^h are smaller in type-3 equilibria if $R^c < R^{bench}$ but larger if $R^c > R^{bench}$.*
Proof: see Appendix B.

In equilibrium, q^0 and q^h only depend on the deposit rate in the economy. To see this, note that q^h solves

$$u'(q^h) = \frac{1+\mu}{\beta(1+R^d)}. \tag{5.4}$$

Recall that sellers who evade taxes and sellers who report truthfully offer the same surplus $\mathcal{S}$ to buyers. By following (3.16)-(3.20), it is easy to derive that

$$\mathcal{S} = \beta u(q^h) - \beta u'(q^h)(q^h + \tilde{\tau}) = \beta u(q^0) - \beta u'(q^0)q^0. \tag{5.5}$$

In type-3 and type-4 equilibria, because sellers who report truthfully are indifferent between CBDC and bank deposits, $R^c = R^d$. Hence, q^0 and q^h are larger than their counterparts in the benchmark equilibrium if and only if $R^c \geq R^{bench}$. In type-2 equilibria, the decrease in tax evasion causes a shortage of bank deposits, which lowers the deposit rate. Hence, in any type-2 equilibrium, $R^d < R^{bench}$, and q^0 and q^h are smaller compared to their counterparts in the benchmark equilibrium.

The intuition for the relationship between q^0 and q^h (i.e equation (5.5)) is as follows. First, q^h does not depend on the fiscal authority's audit strategy because it is the output from sellers who report truthfully. Second, if q^h changes, the fiscal authority must change the audit probability so that sellers who evade taxes produce exactly the q^0 that satisfies (5.5). This ensures that sellers who evade taxes have no disadvantage nor advantage when competing with sellers who report truthfully, because neither sellers strictly preferring to evade taxes nor sellers strictly preferring *not* to evade taxes can be part of an equilibrium.

Finally, consider how introducing CBDC affects the total surplus in the DM.

Proposition 5.3 *There exists $R^{c\prime} < R^{bench}$ such that for any $R^c \geq R^{c\prime}$, the total surplus in DM is higher in type-3 equilibria than in the benchmark equilibrium, any type-4 equilibria given R^c, and also any type-2 equilibria.*
Proof: see Appendix B.

Proposition 5.3 follows from Proposition 5.1 and 5.3. In particular, it says that even if $R^{c\prime} \leq R^c < R^{bench}$, as long as R^c and ρ^c are such that the equilibrium is type-3, introducing CBDC will increase DM surplus compared to the benchmark. In such case, although q^0 and q^h are lower than their counterparts in the benchmark equilibrium, γ is also smaller. Recall that $q^0 < q^h$ because the risk of being audited and punished by the fiscal authority acts as a proportional tax on sellers who evade taxes. Hence, the total surplus in the DM increases even though the deposit rate is lower than the benchmark.

If $R^c \geq R^{c\prime}$, then increasing R^c in a type-3 equilibrium will increase q^0 and q^h and decrease γ (see Proposition 4.2). Hence, the total surplus in the DM will be higher. However, it will also increase the funding costs of bankers, which will in turn raise the loan rate and lower the investment in entrepreneurs' projects. I discuss this effect in Part II.

II. Net output from entrepreneurs' projects

The investment in an entrepreneur's project, k, is given by

$$f'(k) = \frac{1+R^k}{1+\mu}, \tag{5.6}$$

where R^k is the nominal loan rate. From bankers' problem in Section 3.2, we know that the relationship between R^k and the deposit rate R^d is given by

$$\frac{1+\mu}{1+R^k} = \beta\theta + \frac{(1-\theta)(1+\mu)}{1+R^d}. \tag{5.7}$$

In a type-2 equilibrium, the deposit rate is lower than the benchmark because the decrease in tax evasion causes a shortage in bank deposits. Hence, k is larger than the benchmark. In a type-3 equilibrium, the deposit rate is equal to the interest rate on CBDC, so the deposit rate is larger than the benchmark if $R^c < R^{bench}$ and smaller if $R^c > R^{bench}$. In a type-4 equilibrium, CBDC competes with bank deposits and increases the deposit rate. Hence, k is smaller than the benchmark.

However, a higher k does not necessarily mean higher net output from entrepreneurs' projects ($f(k) - k$). When the supply of assets (government bonds plus loans to entrepreneurs) is low, the deposit rate carries a large liquidity premium (i.e. R^d is low). This means the loan rate is low and investment k is inefficiently high. Specifically, if $R^{bench} < \mu$, then an increase in k compared to the benchmark will decrease the net output from entrepreneurs' projects. The next proposition follows directly from the above arguments.

Proposition 5.4 *Assume $R^{bench} < \mu$. Compared to the benchmark, $f(k) - k$ is lower in type-2 equilibria. In addition, there exists $R^{c\prime\prime}$ such that if $R^{bench} < R^c \leq R^{c\prime\prime}$, $f(k) - k$ is higher in both type-3 and type-4 equilibria. If $R^{bench} < R^c$, $f(k) - k$ is lower in type-3 equilibria.*

If $R^{bench} > \mu$ and $R^c > R^{bench}$, then compared to benchmark, $f(k) - k$ is lower in both type-3 and type-4 equilibria. Recall that under the same conditions, the total surplus in the DM surplus is higher compared to the benchmark. Hence, a trade-off exists in such cases: introducing CBDC promotes the trade in the DM, but it also increases the funding costs of bankers and decreases the output from entrepreneurs.

III. Total audit costs

The number of audits that the fiscal authority conducts depends on the measure of sellers who report zero income ($\alpha - 1 + \gamma$) and the audit probability (η^0). Proposition 5.1 shows that introducing decreases tax evasion and reduces the measure of sellers who report zero income. However, the audit probability may be higher or lower compared to the benchmark depending on R^c and ρ^c.

Proposition 5.5 *Compared to the benchmark, η^0 is lower in type-4 equilibria but higher in type-2 and type-3 equilibria.*

Proof: see Appendix B.

Hence, compared to the benchmark, the total audit costs are lower in type-4 equilibria. For type-2 and type-3 equilibria may increase or decrease depending on R^c and ρ^c.[14]

Because all three components of aggregate welfare can in principle move in different directions, in order to derive clear welfare implications, I restrict my attention to subsets of CBDC configurations. First, assume $R^c > R^{bench}$. Recall that in this case, only type-3 and type-4 equilibria exist (see Figure 4). Now, define $\rho^c(R^c)$ to be the highest ρ^c such that given (R^c, ρ^c), the equilibrium is type-3. Specifically, $\rho^c(R^c)$ solves

$$\frac{1+\mu}{(1-\rho^c\eta^0)(1+R^c)} = \frac{1+\mu}{1-\rho^m\eta^0}, \tag{5.8}$$

where η^0 is given by the solution to a type-3 equilibrium in Section 4.3.

Proposition 5.6 *For any $R^c > R^{bench}$, $(R^c, \rho^c(R^c))$ offers the highest aggregate welfare.*
Proof: see Appendix B.

First, in type-3 and type-4 equilibria, $R^c = R^d$. From Part I and II, we know that q^0, q^h, and k only depend on R^d. This means that for any given $R^c > R^{bench}$, aggregate welfare varies only because of γ and η^0. By following Proposition 4.2, it is easy to plot the relationship between γ and ρ^c and the relationship between η^0 and ρ^c.

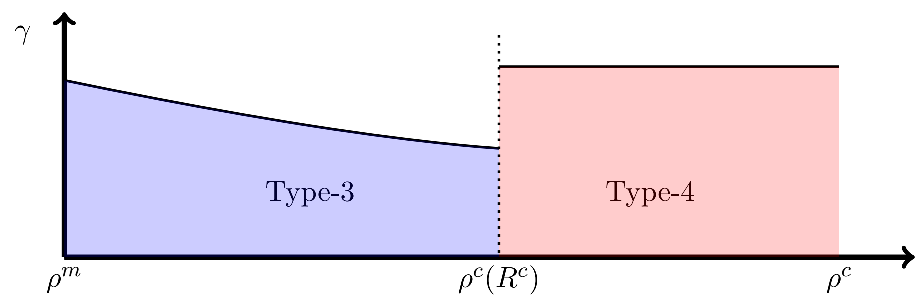


Figure 5: Relationship Between γ and ρ^c when $R^c > R^{bench}$

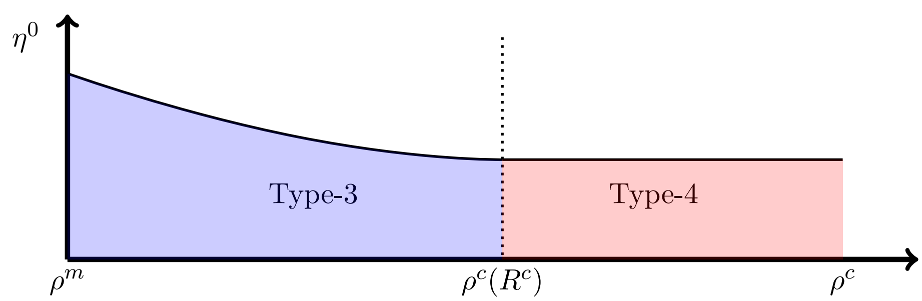


Figure 6: Relationship Between η^0 and ρ^c when $R^c > R^{bench}$

[14]See Figure 22 in Appendix E for a numerical example that illustrates this result.

That is, for any given $R^c > R^{bench}$, γ and η^0 reach minima when $\rho^c = \rho^c(R^c)$. Hence, for any given $R^c > R^{bench}$, aggregate welfare reaches a unique maximum when $\rho^c = \rho^c(R^c)$. Note that in type-4 equilibria, equilibrium outcomes do not depend on ρ^c since CBDC is only accepted by sellers who report truthfully.

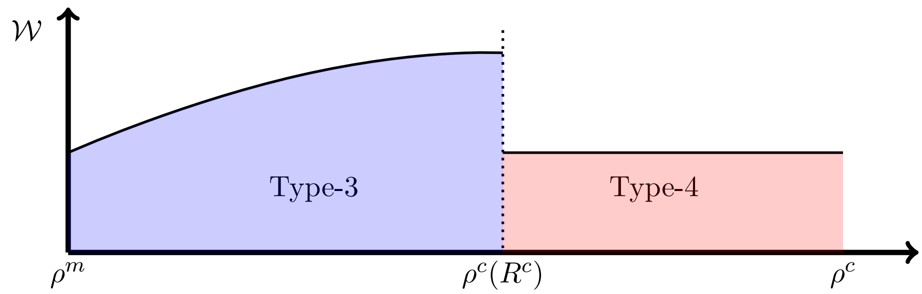


Figure 7: Relationship Between $\mathcal{W}$ and ρ^c when $R^c > R^{bench}$

How introducing CBDC affects aggregate welfare is in general ambiguous. The reason is that the changes in net output from entrepreneurs are often in the opposite direction of the changes in DM surplus. Nevertheless, if the question we wish to answer is whether introducing CBDC *can* increase aggregate welfare, then we may restrict our attention to R^c's that are close to R^{bench}. In such cases, k will be close to its counterpart in the benchmark equilibrium. I refer to this type of CBDC as "**deposit-like**" CBDC. Formally, a CBDC is deposit-like if
(1) (R^c, ρ^c) is in the neighborhood of $B \equiv (R^{bench}, \rho^c(R^{bench}))$, and
(2) the equilibrium given (R^c, ρ^c) is type-3.

I also compare deposit-like CBDC with "**cash-like**" CBDC. Formally, a CBDC is **cash-like** if
(1) (R^c, ρ^c) is in the neighborhood of $A \equiv (0, \rho^m)$, and
(2) the equilibrium given (R^c, ρ^c) is type-2.
The following figure shows the locations of A and B in the (R^c, ρ^c) space.

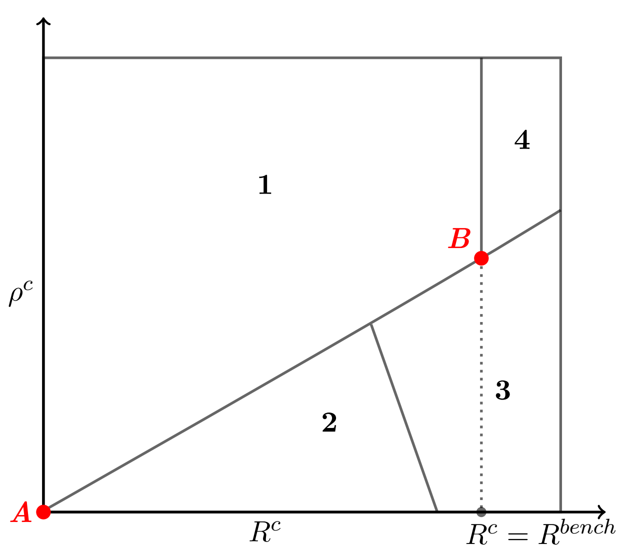


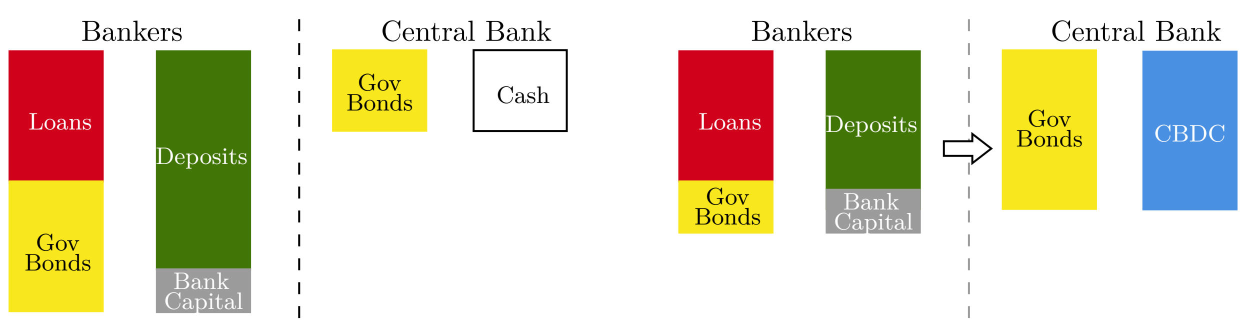
Figure 8: Locations of A and B

Proposition 5.7 *Introducing deposit-like CBDC increases aggregate welfare. Furthermore, aggregate welfare is higher with deposit-like CBDC than with cash-like CBDC.*
Proof: see Appendix B.

By following Proposition 5.3, it is easy to show that compared to the benchmark equilibrium and any type-2 equilibria with cash-like CBDC, the total surplus in the DM is higher. Furthermore, the total audit costs is also lower with deposit-like CBDC because the reduction in tax evasion is larger in type-3 equilibria than in type-2 equilibria.

I conclude this subsection by providing some intuition for why deposit-like CBDC provides higher aggregate welfare compared to cash-like CBDC. First, because tax evasion is distortionary, ceteris paribus, a decrease in tax evasion always increases aggregate welfare. However, if CBDC is cash-like, sellers who report their income truthfully will accept only bank deposits, so a decrease in tax evasion will increase the demand for deposits. The resulted shortage of bank deposits will lower the deposit rate and hinder the transactions between buyers and sellers. To solve this problem, the design of CBDC must achieve two objectives. First, introducing CBDC must increase the supply of payment assets in the economy. Second, the interest rate on CBDC must be sufficiently high for sellers who report truthfully to accept CBDC. Note that achieving only one of the two objectives is not enough: the shortage of bank deposits will remain if either CBDC is not accepted by sellers who report truthfully, or introducing CBDC does not increase the total supply of payment assets.

Achieving the second objective requires the interest rate on CBDC to be sufficiently high, but how does introducing CBDC increase the supply of payment assets? Recall that bankers have limited commitment so they can abscond with a portion θ of their assets. By assumption, the central bank is immune from the limited commitment problem. This means that bankers must hold more assets than their liabilities so that they do not have the incentive to default. The difference between bankers' assets and liabilities is bank capital, which bankers accumulate through working in the CM. In comparison, the central bank holds the same quantity of assets (government bonds) and liabilities (cash and/or CBDC). See Figure 9(a) for an illustration.



(a) Before Introducing Deposit-like CBDC (b) After Introducing Deposit-like CBDC

Figure 9: The Balance Sheet Effects of Introducing Deposit-like CBDC

Recall that the central bank introduces CBDC by purchasing government bonds from the bond market with CBDC (and it introduces cash in the same way). Since there are fewer government bonds available, bankers issue fewer deposits. See Figure 9(b) for an illustration.

Next, note that with each unit of government bonds, the central bank can create one unit of CBDC, while bankers can only create $1-\theta$ units of deposits. As a result, after the introduction of CBDC, the total supply of bank deposits and deposit-like CBDC is larger compared to the supply of deposits in the benchmark. In other words, the introduction of CBDC increases the supply of payment assets.

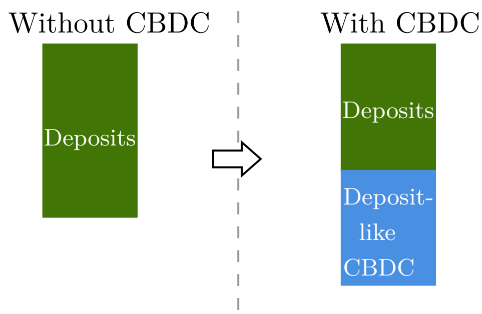


Figure 10: Supply of Bank Deposits and Deposit-like CBDC

Regardless of whether CBDC is cash-like or deposit-like, introducing CBDC will decrease tax evasion and increase the demand for payment assets. However, with deposit-like CBDC, the increase in the demand for payment assets is satisfied by the increase in the supply of payment assets (see Figure 12). This effect is what allows the total output and surplus in the DM to increase after the introduction of CBDC. Without this effect, the increase in demand for deposits will drive down the deposit rate, which will then decrease the income and output of sellers who accept bank deposits (see Figure 11).

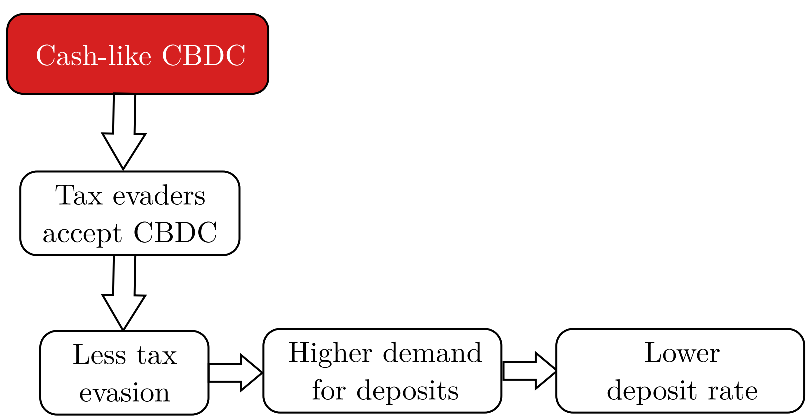


Figure 11: Effects of Introducing Cash-like CBDC

It should be noted that the above results do not imply that the central bank should replace private banking because bankers also make loans to entrepreneurs. In practice, central banks may lack the expertise to make such loans. It should also be noted that if $R^c = R^{bench}$, the supply of loans is not affected by the introduction of CBDC. Although bank deposits are crowded out by CBDC, they are only (a portion of) the deposits that are backed by government bonds (see Figure 9(b)).

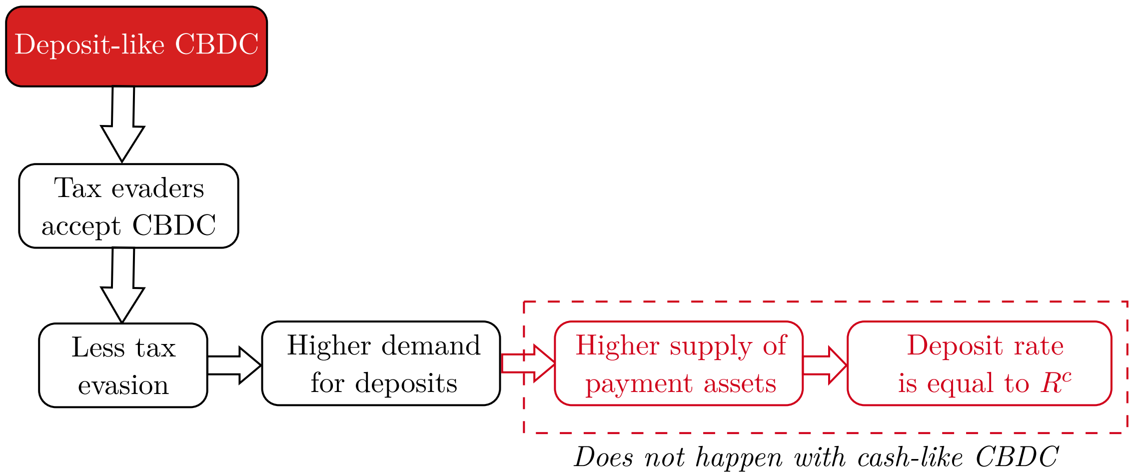


Figure 12: Effects of Introducing Deposit-like CBDC

5.2 Central Bank Net Revenue

Although aggregate welfare is higher with deposit-like CBDC, the central bank also has to pay higher interest. The goal of this section is to understand the impact of an interest-bearing CBDC on the central bank's net revenue. The central bank's budget constraint is given by the following.

$$\frac{M_{t+1} - M_t}{p_t} + \frac{C_{t+1} - C_t}{p_t} + \frac{(1 + R^b)B_t^c}{p_t} = \frac{R^c C_t}{p_t} + \frac{B_{t+1}^c}{p_t} + \mathcal{E}_t + \mathcal{T}_t^c. \tag{5.9}$$

Recall that $\mathcal{E}$ is the central bank's operating expenses. Since the central bank uses the cash and CBDC it creates to purchase government bonds, we have $M_t + C_t = B_t^c$ for all t. Now, define $\tilde{c} = \frac{C_t}{p_t}$ and $\tilde{m} = \frac{M_t}{p_t}$. The central bank's net revenue can be written as

$$\Pi = \frac{R^b - R^c}{1 + \mu}\tilde{c} + \frac{R^b}{1 + \mu}\tilde{m} - \mathcal{E}. \tag{5.10}$$

Note that $\tilde{m} = 0$ in type-2 and type-3 equilibria, but $\tilde{m} > 0$ in type-4 equilibria. Note also that Π is equal to the transfer to the fiscal authority, $\mathcal{T}^c$. In principle, Π can be negative, in which case the central bank receive transfers from the fiscal authority. In practise, the operating expenses of a central bank can be large. For example, the operating expenses of the Bank of Canada is equal to 28% of its net interest income.

The effect of introducing CBDC on central bank net revenue depends on the interest rate spread $R^b - R^c$ and the demand for central bank liabilities, $\tilde{c}$ and $\tilde{m}$. In a type-2 equilibrium, R^b is smaller compared to the benchmark (see Proposition 5.2). The demand for CBDC is also lower compared to the demand for cash in the benchmark, because CBDC is accepted by sellers who evade taxes, and there is less tax evasion. Hence, the central bank's net revenue is lower. In a type-3 equilibrium, the interest rate spread $R^b - R^c$ is smaller than in a type-2 equilibrium because R^c is higher. However, the demand for CBDC is also higher because CBDC is accepted by both sellers who evade taxes and (some) sellers who report truthfully.

Now, denote the deposit rate and government bond rate in the benchmark equilibrium as R^{bench} and $R^{b\prime}$, respectively. They satisfy the following relationship

$$\frac{1+\mu}{1+R^b} = \beta\theta + \frac{(1-\theta)(1+\mu)}{1+R^{bench}}. \tag{5.11}$$

Let the level of tax evasion in the benchmark equilibrium be γ^{bench}. Recall that $\mathcal{D} = \frac{(1+R^b)B_t}{p_t}$ is the total supply of government debt. I show that assuming the following assumption about the benchmark equilibrium holds, the central bank's net revenue is lower with deposit-like CBDC than with cash-like CBDC.

Assumption 5.1 $\frac{(R^b - R^{bench})\mathcal{D}}{(1+R^d)(1+\mu)} < \frac{R^b(\alpha - 1 + \gamma^{bench})C}{\rho^m}$.

The assumption is satisfied as long as θ is sufficiently small. Recall that the central bank creates CBDC by purchasing government bonds with CBDC. In the extreme case where the central purchases all government bonds from the market, the supply of CBDC is equal to the total supply of government bonds, $\mathcal{D}/(1+R^b)$. Then, the left-hand side of the inequality represents the highest possible central bank net revenue when $R^c = R^{bench}$. The right-hand side represents the central bank's net revenue in the benchmark. Intuitively, Assumption 5.1 ensures that in a type-3 equilibrium, the increase in demand for CBDC does not offset the decrease in the interest rate spread $R^b - R^c$.

Proposition 5.8 *Assume Assumption* 5.1 *holds. Then, introducing either deposit-like CBDC or cash-like CBDC decreases central bank net revenue. Furthermore, central bank net revenue is lower with deposit-like CBDC than with cash-like CBDC.*
Proof: see Appendix B.

If the decrease in net revenue is significant, the central bank may not be able to cover its own expense and may require transfers from the fiscal authority. Such transfers may be feasible because the reduction in tax evasion increases the fiscal authority's tax revenue. However, relying on the fiscal authority's transfers may undermine the independence of the central bank.

6 Conclusion

The goal of this paper is to study the implications of tax evasion for the optimal design of central bank digital currencies (CBDC). To accomplish this goal, I incorporate an audit game between taxpayers and the fiscal authority into a general equilibrium model. As a benchmark, I consider a scenario where only cash and bank deposits are available as payment. Bank deposits have a higher return than cash, but it is easier to conceal cash from the fiscal authority. I find that under the optimal tax schedule, an increase in inflation prompts agents to substitute away from cash, but this also lowers the fiscal authority's incentive to audit agents. In equilibrium, the decrease in tax enforcement leads to more agents evading taxes and hence a higher demand for cash.

When CBDC is introduced as a new payment instrument, the effect on tax evasion depends crucially on the degree of anonymity of CBDC, which determines the probability of the income received in CBDC being observed by the government. I find that as long as CBDC offers less anonymity than cash, introducing CBDC will decrease tax evasion. However, if CBDC is cash-like in the sense that it still offers relatively high anonymity but low interest rate, then introducing CBDC will decrease the output from not only agents who evade taxes but also agents who report their income truthfully. If CBDC is instead deposit-like in the sense that it offers low anonymity but high interest rate, then introducing CBDC will increase output and aggregate welfare. Furthermore, introducing deposit-like CBDC needs not increase the funding costs of private banks or decrease bank lending and investment. However, paying the high interest rate on CBDC will decrease the central bank's net interest revenue, which may jeopardize the central bank's independence.

To emphasize the mechanisms through which tax evasion affects payment choice and CBDC design, I abstract from several important issues that could be addressed in future research. First, in this paper, all agents are assumed to be ex ante homogeneous. The only uncertainty in the economy comes from search friction, and it only affects agents who are risk-neutral. These simplifying assumptions remove the income redistribution concerns when designing the tax schedule. Income redistribution is certainly an important issue, and CBDC may be a useful tool in this regard as well (see Davoodalhosseini (2018)). Second, in this paper, there are no concerns of privacy other than tax evaders trying to conceal their income from the fiscal authority. In practice, there are many reasons why individuals may want to hide their transactions from other individuals and the government (see Kahn et al. (2020)). The origins of such privacy concerns will likely determine the type(s) of transactions where CBDC will be accepted. This in turn can have important implications for tax compliance and the central bank's revenue. Finally, it has been argued that CBDC may help promote financial inclusion (see Lagarde (2018)). Specifically, individuals may prefer cash not because they want to evade taxes, but because the transaction costs are lower compared to bank deposits and bank credit (see Aït Lahcen and Gomis-Porqueras (2019)). If CBDC can be easy and cheap to use, it may promote both transaction efficiency and tax compliance at the same time.

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Appendix A Discussion: Endogenous Signaling Games

Consider an "endogenous signaling" game where player 1 makes an unobservable move (l or r) and an observable move (u or d) before player 2 moves (y or n).

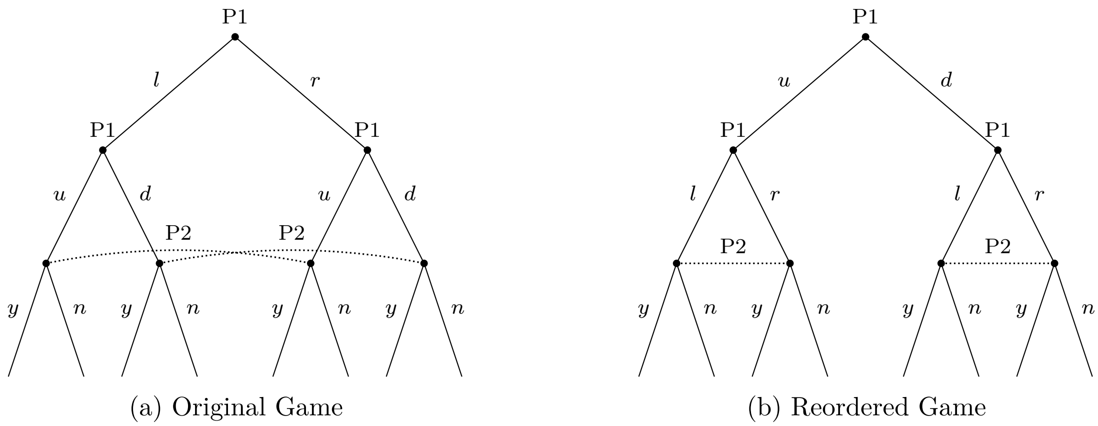


Figure 13: Reordering Invariance Equilibrium

In this game, PBE also does not put any restrictions on player 2's beliefs about player 1's unobservable move on off-equilibrium paths. In and Wright (2018) note that the order of player 1's two moves does not matter for equilibrium outcomes, because the player does not gain any payoff-relevant information between moves. However, if player 1 makes the observable move first, his subsequent unobservable move and player 2's move constitutes a proper subgame of the reordered game. By solving the Nash equilibrium of the subgame, one can confine player 2's beliefs about player 1's unobservable move in a logically consistent way. The Reordering Invariance (RI) equilibrium then selects the equilibrium outcomes that are also equilibrium outcomes of the reordered game. Reordering Invariance equilibrium has been used in Li et al. (2012), Gomis-Porqueras et al. (2014), Kang (2017), and Berentsen et al. (2017) to study the problem of counterfeit assets.

In this paper, the game between sellers and the fiscal authority is similar to the endogenous signaling game. A seller makes an unobservable action (posting (q, p)) and an observable action (reporting y) before the fiscal authority decides η. For any give y, the game between sellers and the fiscal authority, which is described by (3.7) and (3.9), is similar to the subgame in the reordered endogenous signaling game. The main difference is that between between a seller's unobservable action and observable action, nature moves to decide whether a seller meets a buyer or not. However, the same logic still applies: for any equilibrium (y, q, p) and η, it must be that given y, (q, p) and η constitute a Nash equilibrium. Similar to the Reordering Invariance (RI) equilibrium, the proposed refinement requires the same to be true for any off-equilibrium choice of y.

Appendix B Proofs

Proof of Proposition 3.1: This proof has two parts. In part I, I solves for the Nash equilibrium for any given $\tau(y)$ and y. In part II, I solve for the optimal choice of y. In part III, I solve for the optimal tax schedule. Throughout this proof, I assume $R^d > 0$. The case with $R^d => 0$ is straightforward to solve using this proof.

Part I. In this part, I first solve the Nash equilibrium for any given y. It is easy to see that (3.8) must bind because if it does not bind, a seller can be better off by choosing a smaller q. Also, it must be that $n \leq 1$ because otherwise a seller can lower n and make (3.8) slack while not affecting his surplus.

Next, there are two possible cases. First, suppose that $R^d < (1 - \rho^m)\eta/(1 - \eta)$. Then the marginal cost of using cash is lower than bank deposits. However, since $R^d > 0$, the seller benefits from using some bank deposits. Specifically, the seller will choose $d = y$. The seller solves the following problem

$$\max_{q,m}\{(1-\rho^m\eta)(y+m-\tau(y))-q\} \text{ s.t. } -(1+\mu)m - \frac{1+\mu}{1+R^d}y + \beta u(q) = \mathcal{S}. \tag{B.1}$$

The first order condition is

$$u'(q) = \frac{1+\mu}{\beta(1-\rho^m\eta)}. \tag{B.2}$$

In a Nash equilibrium, it must be that $\rho^m(d+m-\tau(y)) = C$ and $\eta \in (0,1)$. Then, η must be such that

$$\frac{1}{1+\mu}\left[\beta u(q) - \mathcal{S}\right] = \tau(y) - \frac{R^d y}{1+R^d} + \frac{C}{\rho^m}. \tag{B.3}$$

Hence, q is increasing y and therefore η is decreasing in y. Then, there exists $y^\dagger$ such that for all $y \geq y^\dagger$, if the seller continue using both bank deposits and cash, then $R^d \geq (1-\rho^m)\eta/(1-\eta)$. In such case, the marginal cost of using cash is higher than bank deposits. This means the seller has the incentive to deviate and use bank deposits. Suppose the seller chooses to use bank deposits. The seller solves

$$\max_{q,d}\{(1-\eta)(d-\tau(y))-q\} \text{ s.t. } -\frac{(1+\mu)d}{1+R^d} + \beta u(q) = \mathcal{S}. \tag{B.4}$$

The first order condition is

$$u'(q) = \frac{1+\mu}{\beta(1-\eta)(1+R^d)}. \tag{B.5}$$

In a Nash equilibrium, it must be that $d - \tau(y) = C$ and $\eta \in (0,1)$. That is, η solves

$$d = \frac{1+R^d}{1+\mu}[\beta u(q) - \mathcal{S}] = \tau(y) + C. \tag{B.6}$$

It is easy to see that q is increasing y and therefore η is decreasing in y. This means that $y^\ddagger$ such that for all $y < y^\ddagger$, if the seller continue using only bank deposits, then $R^d < (1-\rho^m)\eta/(1-\eta)$. This means the seller has the incentive to deviate and use both bank deposits and cash. Note that $y^\dagger < y^\ddagger$. To see this, suppose the seller uses bank deposits and $y = y^\ddagger$. If the seller switches to a mixture of bank deposits and cash, then q will not change because $R^d = (1-\rho^m)\eta/(1-\eta)$. However, since $\rho^m(y+m-\tau(y)) < d-\tau(y)$ because $R^d > 0$ and $\rho^m < 1$, the fiscal authority has to lower η so that $\rho^m(y+m-\tau(y)) = C$. Then it must be that $R^d > (1-\rho^m)\eta/(1-\eta)$, i.e. $y > y^\dagger$. This means that for any $y^\dagger < y < y^\ddagger$, there does not exist an equilibrium where the seller plays a pure strategy. Instead, for all $y^\dagger < y < y^\ddagger$, $\eta = \eta' = R^d/(1-\rho^m+R^d)$. For any $y^\dagger < y < y^\ddagger$, let $(q^\dagger, m^\dagger)$ solve

$$u'(q^\dagger) = \frac{1+\mu}{\beta(1-\rho^m\eta')}, \tag{B.7}$$

$$m^\dagger = \frac{1}{1+\mu}\left[\beta u(q^\dagger) - \mathcal{S} - \frac{1+\mu}{1+R^d}y\right]. \tag{B.8}$$

Let $(q^\ddagger, d^\ddagger)$ solve

$$u'(q^\ddagger) = \frac{1+\mu}{\beta(1-\eta')(1+R^d)}, \tag{B.9}$$

$$d^\ddagger = \frac{1+R^d}{1+\mu}[\beta u(q^\ddagger) - \mathcal{S}]. \tag{B.10}$$

It is clear that $\rho^m(y+m^\dagger-\tau(y)) < C$ and $d^\ddagger - \tau(y) > C$ for any $y^\dagger < y < y^\ddagger$. In addition, both $\rho^m(y+m^\dagger-\tau(y)) < C$ and $d^\ddagger - \tau(y) > C$ are decreasing in y. The former is because $y + m^\dagger - \tau(y) = \frac{1}{1+\mu}\left[\beta u(q^\dagger) - \mathcal{S}\right] + R^d/(1+R^d)y - \tau(y)$ and by assumption, $R^d/(1+R^d)y - \tau(y)$ is decreasing in y. Hence, for any $y^\dagger < y < y^\ddagger$, there exists $\epsilon(y) \in (0,1)$ such that

$$\epsilon(y)\rho^m(y+m^\dagger-\tau(y)) + (1-\epsilon(y))(d^\ddagger-\tau(y)) = C. \tag{B.11}$$

And $\epsilon(y)$ is decreasing in y. Then, for all $y^\dagger < y < y^\ddagger$, the seller chooses $(q^\dagger, m^\dagger)$ with probability $\epsilon(y)$ and $(q^\ddagger, d^\ddagger)$ with probability $1-\epsilon(y)$.

To summarize, for any $\tau(y)$ and y, the Nash equilibrium is given by the following. Define $y^\dagger$ to be such that $R^d = (1-\rho^m)\eta/(1-\eta)$ where η is given by

$$u'(q) = \frac{1+\mu}{\beta(1-\rho^m\eta)}, \tag{B.12}$$

$$\frac{1}{1+\mu}\left[\beta u(q) - \mathcal{S}\right] = \tau(y) - \frac{R^d y}{1+R^d} + \frac{C}{\rho^m}. \tag{B.13}$$

Define $y^\ddagger$ to be such that $R^d = (1-\rho^m)\eta/(1-\eta)$ where η is given by

$$u'(q) = \frac{1+\mu}{\beta(1-\eta)(1+R^d)}, \tag{B.14}$$

$$\frac{1+R^d}{1+\mu}[\beta u(q) - \mathcal{S}] = \tau(y) + C. \tag{B.15}$$

Then for all $y \leq y^\dagger$, $d = y$. m and q solve

$$u'(q) = \frac{1+\mu}{\beta(1-\rho^m\eta)}, \tag{B.16}$$

$$m = \frac{1}{1+\mu}\left[\beta u(q) - \mathcal{S} - \frac{1+\mu}{1+R^d}y\right] = \tau(y) - y + \frac{C}{\rho^m}. \tag{B.17}$$

For all $y \geq y^\ddagger$, $m = 0$. d and q solve

$$u'(q) = \frac{1+\mu}{\beta(1-\eta)(1+R^d)}, \tag{B.18}$$

$$d = \frac{1+R^d}{1+\mu}[\beta u(q) - \mathcal{S}] = \tau(y) + C. \tag{B.19}$$

For all $y^\dagger < y < y^\ddagger$, the seller chooses $(q^\dagger, m^\dagger)$ with probability $\epsilon(y)$ and $(q^\ddagger, d^\ddagger)$ with probability $1-\epsilon(y)$, where $(q^\dagger, m^\dagger)$ solve (B.7) and (B.8) and $(q^\ddagger, d^\ddagger)$ solve (B.9) and (B.10). $\epsilon(y)$ is given by (B.11).

Part II. In this part, I solve for the optimal y. First, suppose that the seller plays a pure strategy and accepts both cash and bank deposits. Note that in such case, we have $\rho^m(d + m - \tau(y)) = C$. Hence, the seller's surplus is then

$$\frac{(1-\rho^m\eta)C}{\rho^m} - q = \frac{(1+\mu)C}{\beta\rho^m u'(q)} - q, \tag{B.20}$$

where the equality uses the first order condition. Take the derivative of the RHS with respect to q and substitute in the assumption $-qu''(q)/u'(q) = \xi$. We have

$$\frac{(1+\mu)\xi C}{\beta\rho^m q u'(q)} - 1. \tag{B.21}$$

If $C/\rho^m < \tilde{q}$ where $\tilde{q}$ is given by $u'(q) = \frac{1+\mu}{\beta}$, there exists a unique $q^\diamond$ such that

$$\frac{(1+\mu)\xi C}{\beta\rho^m q u'(q)} = 1. \tag{B.22}$$

Now, let $y^\diamond$ be such that

$$\frac{1}{1+\mu}\left[\beta u(q^\diamond) - \mathcal{S}\right] = \tau(y^\diamond) - \frac{R^d y^\diamond}{1+R^d} + \frac{C}{\rho^m}. \tag{B.23}$$

If $y^\diamond < y^\dagger$, then in such case the seller will choose $y = y^\diamond$. Otherwise the seller will choose $y = y^\dagger$.

Second, suppose that the seller plays a pure strategy and accepts only bank deposits. Note that in such case, we have $d - \tau(y) = C$. The seller's surplus is then

$$(1-\eta)C - q = \frac{(1+\mu)C}{\beta(1+R^d)u'(q)} - q.$$

Take the derivative with respect to q to get

$$\frac{(1+\mu)\xi C}{\beta(1+R^d)qu'(q)} - 1. \tag{B.24}$$

Suppose that $C < \hat{q}$ where $\hat{q}$ solves $u'(q) = \frac{1+\mu}{\beta(1+R^d)}$. Then, there exists a unique $q^{\odot}$ such that

$$\frac{(1+\mu)\xi C}{\beta(1+R^d)qu'(q)} = 1. \tag{B.25}$$

Now, let $y^{\odot}$ be such that

$$\frac{1+R^d}{1+\mu}[\beta u(q^{\odot}) - \mathcal{S}] = \tau(y^{\odot}) + C. \tag{B.26}$$

If $y^{\odot} > y^{\ddagger}$, then the seller will choose $y = y^{\odot}$. Otherwise the seller will choose $y = y^{\ddagger}$.

Third, suppose the seller plays a mixed strategy. In such case, his surplus is equal to when $y = y^{\dagger}$ and the seller accepts both cash and deposits, or $y = y^{\ddagger}$ and the seller accepts only deposits. In conclusion, if a seller chooses to misreport income, the optimal y can be $y^{\diamond}$, $y^{\odot}$, or any $y \in [y^{\dagger}, y^{\ddagger}]$.

Fourth, suppose the seller report truthfully. Since $R^d > 0$, the seller will only accept deposits. The seller's problem becomes

$$\max_{q,d}\{d - \tau(d) - q\} \;\text{ s.t. }\; -\frac{1+\mu}{1+R^d}d + \beta u(q) = \mathcal{S}. \tag{B.27}$$

Then, to determine the equilibrium, we can compare the seller's surplus when he or she misreports his income and when he or she reports truthfully.

Lastly, in equilibrium it must be that $n(q,p) < 1$ for some (q,p) since there are more sellers than buyers. This means that (1) $\mathcal{S}$ must be such that sellers' surplus in equilibrium is zero; and (2) some sellers do not meet buyers in the DM. For these sellers, the dominant strategy is to produce $q = 0$ and report $y = 0$. If the fiscal authority does not audit sellers who report zero income, then all sellers will report zero income. Hence, these sellers must be audited with a positive probability so that sellers who meet buyers are indifferent between reporting an income of zero or a positive income. Consider one of such sellers. If the seller chooses to accept bank deposits, he or she solves

$$\max_{q,d}\{(1-\eta)d - q\} \;\text{ s.t. }\; -\frac{(1+\mu)d}{1+R^d} + \beta u(q) = \mathcal{S}. \tag{B.28}$$

For this to be part of an equilibrium, it must be that $R^d > (1-\rho^m)\eta/(1-\eta)$. If the seller chooses to accept cash, he or she solves

$$\max_{q,m}\{(1-\rho^m\eta)m - q\} \;\text{ s.t. }\; -(1+\mu)m + \beta u(q) = \mathcal{S}. \tag{B.29}$$

For this to be part of an equilibrium, it must be that $R^d \leq (1-\rho^m)\eta/(1-\eta)$. Now let γ denote the share of sellers who post either (q,d) or (q,m) that solves the above problems. In equilibrium, if the seller chooses to accept bank deposits, it must be that

$$\frac{\gamma d}{\alpha - 1 + \gamma} = C. \tag{B.30}$$

If the seller chooses to accept cash, it must be that

$$\frac{\gamma \rho^m m}{\alpha - 1 + \gamma} = C. \tag{B.31}$$

□

Proof of Proposition 3.2: I prove the optimality of $\tau^*(y)$ by showing that for any $\tau(y)$, there exists a $\tilde{\tau}$ such that all agents are (weakly) better off while the fiscal authority receives (weakly) higher net tax revenue.

First, for any $\tau(y)$, let Y denote the set of income reported in equilibrium. Let y° be given by

$$y^{\circ} \in \arg\max_{y \in Y} \tau(y). \tag{B.32}$$

There are two possible cases: (1) strategies (a) and (b) (see Proposition 3.1) are not played in the equilibrium; and (2) either strategy (a) and/or (b) are played in the equilibrium.

First, in case (1), let $\tilde{\tau} = \tau(y^{\circ})$. Then, as long as no sellers who meet buyers report $y \in (0, \tilde{\tau})$, agents under $\tau^*(y)$ are (weakly) better-off compared to $\tau(y)$ and the fiscal authority receives (weakly) higher net tax revenue. Now, suppose sellers under $\tau^*(y)$ have the incentive to report some $y^{\ominus} \in (0, \tilde{\tau})$. Assume sellers under $\tau(y)$ also report $y^{\ominus}$. If $\tau(y^{\ominus}) = y^{\ominus}$, then the seller obtain the same surplus by reporting $y^{\ominus}$ under $\tau(y)$, which means the seller must have the incentive to report $y^{\ominus}$ under $\tau(y)$ as well, a contradiction. If $\tau(y^{\ominus}) < y^{\ominus}$, consider strategy (a). It is easy to show that η must be larger so that $\rho^m(d + m - \tau(y)) = C$ holds. This means q must be smaller. In other words, the seller obtains a higher surplus reporting $y^{\ominus}$ under $\tau(y)$ compared to reporting $y^{\ominus}$ under $\tau^*(y)$. If the seller plays strategy (b), then again the seller obtain the same surplus when reporting $y^{\ominus}$ under $\tau(y)$ or $\tau^*(y)$. In both cases, the seller has the incentive to report $y^{\ominus}$ under $\tau(y)$ as well, a contradiction.

Second, in case (2), let $\tilde{\tau} = \tau(y^{\circ}) - \upsilon$ where $\upsilon > 0$ is a small constant. Then, as long as no sellers who meet buyers report $y \in (0, \tilde{\tau})$, agents under $\tau^*(y)$ are strictly better-off compared to $\tau(y)$ and the fiscal authority receives a strictly higher net tax revenue. To see the latter point, note that if all sellers who meet buyers report truthfully, the fiscal authority saves the audit costs. Then as long as υ is small, the fiscal authority receives strictly higher net tax revenue. Now, suppose sellers under $\tau^*(y)$ have the incentive to report some $y^{\ominus} \in (0, \tilde{\tau})$. Similar to case (1), sellers under $\tau(y)$ can make the same report and receive at least the same surplus. Since sellers who report truthfully are better-off under $\tau^*(y)$, this means sellers who reports $y^{\ominus}$ under $\tau(y)$ are better-off than they are under $\tau(y)$, a contradiction. □

Proof of Proposition 3.3: First, it is easy to see that an increase in $\tilde{\tau}$ leads to a decrease in $\mathcal{S}$. Now consider sellers who meet buyers in the DM but report $y = 0$. Since it must be that $(1 - \rho^m \eta^0)m^0 - q^0 = 0$ and m^0 is given by

$$m^0 = \frac{1}{1+\mu}[\beta u(q^0) - \mathcal{S}], \tag{B.33}$$

then η^0 must decrease. We also have

$$\mathcal{S} = \beta u(q^0) - \frac{1+\mu}{1 - \rho^m \eta^0} q^0 = \beta u(q^0) - \beta q^0 u'(q^0) \tag{B.34}$$

and

$$\frac{\mathrm{d}q^0}{\mathrm{d}\mathcal{S}} = -\frac{1}{\beta q^0 u''(q^0)}. \tag{B.35}$$

Hence,

$$\frac{\mathrm{d}m^0}{\mathrm{d}\mathcal{S}} = \frac{1}{1+\mu}\left[-\frac{u'(q^0)}{q^0 u''(q^0)} - 1\right] > 0. \tag{B.36}$$

Since

$$\frac{\gamma \rho^m m^0}{\alpha - 1 + \gamma} = C, \tag{B.37}$$

then γ must increase.

Second, suppose R^b increases. It is easy to see that it leads to increases in q^h and $\mathcal{S}$. Then from the last case, we know that η^0 must decrease and q^0 must increase. In addition, m^0 will increase, and hence γ must decrease. Lastly, suppose ρ^m increases. Then η^0 must decrease to keep $(1 - \rho^m \eta^0)$ constant. There are otherwise no changes in the equilibrium. □

Proof of Proposition 3.4: First, given R^d and μ, q^h solves

$$u'(q) = \frac{1+\mu}{\beta(1+R^d)}. \tag{B.38}$$

Next, given $\tilde{\tau}$, η^0 is such that

$$\frac{1-\rho^m\eta^0}{1+\mu}\left[\beta u(q^0)-\beta u(q^h)+\frac{(1+\mu)(q^h+\tilde{\tau})}{1+R^d}\right]-q^0=0, \tag{B.39}$$

$$u'(q^0)=\frac{1+\mu}{\beta(1-\rho^m\eta^0)}. \tag{B.40}$$

γ is such that

$$\frac{\gamma\rho^m m^0}{\alpha-1+\gamma}=C. \tag{B.41}$$

The demand for cash is given by

$$\bar{m}=\gamma m^0=\frac{(\alpha-1+\gamma)C}{\rho^m}. \tag{B.42}$$

Note that $\frac{1+\mu}{1+R^b}=\beta\theta+\frac{(1-\theta)(1+\mu)}{1+R^d}$. Then, R^b is such that

$$f'(k)=\frac{1+R^b}{1+\mu}, \tag{B.43}$$

$$(1-\theta)(k^B+\bar{b})=(1-\gamma)(q^h+\tilde{\tau}). \tag{B.44}$$

Note that (B.44) defines $\bar{b}$ as a function of R^b. Then,

$$\bar{m}+\bar{b}=\frac{q^h+\tilde{\tau}}{1-\theta}-\frac{\gamma}{1-\theta}\left(q^h+\tilde{\tau}-\frac{C(1-\theta)}{\rho^m}\right)-f'(k)k+\frac{(\alpha-1)C}{\rho^m}. \tag{B.45}$$

Assume $\tilde{\tau}-\frac{C(1-\theta)}{\rho^m}\geq 0$. Then $\bar{m}+\bar{b}$ is decreasing in γ. Note that γ is decreasing in m^0, which is increasing in $\mathcal{S}$. q^h and $\mathcal{S}$ are increasing in R^b. Hence, γ is decreasing in R^b. If $kf'(k)$ is increasing in k, k is decreasing in R^b. In conclusion, $\bar{m}+\bar{b}$ is increasing in R^b. Hence, there exists a unique equilibrium where $\bar{m}+\bar{b}=\mathcal{D}$. □

Proof of Proposition 3.5: Consider an increase in μ and assume that $\frac{1+R^d}{1+\mu}$ is unchanged. Then $\mathcal{S}$ is unchanged. This means that q^0 is unchanged as well, Since

$$u'(q^0)=\frac{1+\mu}{\beta(1-\rho^m\eta^0)} \tag{B.46}$$

and μ is higher, $1-\rho^m\eta^0$ must increase. Because $m^0=q^0/(1-\rho^m\eta^0)$, it is lower. Since

$$\frac{\gamma\rho^m m^0}{\alpha-1+\gamma}=C, \tag{B.47}$$

it means γ is larger. Now, recall that

$$\bar{m}+\bar{b}=\frac{q^h+\tilde{\tau}}{1-\theta}-\frac{\gamma}{1-\theta}\left(q^h+\tilde{\tau}-\frac{C(1-\theta)}{\rho^m}\right)-k+\frac{(\alpha-1)C}{\rho^m} \tag{B.48}$$

is decreasing in γ so long as $\tilde{\tau}-\frac{C(1-\theta)}{\rho^m}\geq 0$. This means that if μ is higher and $\frac{1+R^d}{1+\mu}$ is unchanged, $\bar{m}+\bar{b}<\mathcal{D}$. Because $\bar{m}+\bar{b}$ is increasing in R^b, in equilibrium it must be that $\frac{1+R^d}{1+\mu}$ is larger. Then, $\mathcal{S}$ and q^h must be larger in equilibrium, and so is q^0, which means that η^0 must be smaller. Now, if γ is unchanged, then $\bar{m}$ is unchanged. However, because q^h is larger and $\frac{1+R^d}{1+\mu}k$ is smaller, (B.44) suggests that $\bar{b}$ is larger. This means $\bar{m}+\bar{b}>\mathcal{D}$, or that R^b needs to be smaller. Since γ is decreasing in R^b, this means in equilibrium, γ must be larger. This means that $\bar{m}$ is larger and $\bar{b}$ is smaller, which means an open market purchase of government bonds is necessary for μ to increase.

Lastly, the above results hold as long as there exist $\eta^0\in(0,1)$ and $\gamma\in(0,1)$ that are part of an equilibrium. To see what this means, note that as μ increases, $\frac{1+R^d}{1+\mu}$ increases and η^0 decreases. Now let $\frac{1+R^d}{1+\mu}=\frac{1}{\beta}$. Then even if $\eta^0=0$, it is not possible for (B.39) to hold. Hence, there exists μ' such that if

$\mu \geq \mu'$, the equilibrium described above does not exist. $\square$

Proof of Proposition 4.1: First, given R^d and μ, q^h solves

$$u'(q) = \frac{1+\mu}{\beta(1+R^d)}. \tag{B.49}$$

Next, given $\tilde{\tau}$, η^0 is such that

$$\beta u(q^h) - \beta u'(q^h)(q^h + \tilde{\tau}) = \beta u(q^0) - \beta u'(q^0)q^0, \tag{B.50}$$

$$u'(q^0) = \frac{1+\mu}{\beta(1-\rho^c\eta^0)(1+R^c)}. \tag{B.51}$$

Let $c^0 = q^0/(1-\rho^c\eta^0)$. Then, γ is such that

$$\frac{\gamma\rho^c c^0}{\alpha - 1 + \gamma} = C. \tag{B.52}$$

The demand for CBDC is given by

$$\bar{c} = \gamma c^0 = \frac{(\alpha - 1 + \gamma)C}{\rho^c}. \tag{B.53}$$

Note that $\frac{1+\mu}{1+R^b} = \beta\theta + \frac{(1-\theta)(1+\mu)}{1+R^d}$. Then, R^b is such that

$$f'(k) = \frac{1+R^b}{1+\mu}, \tag{B.54}$$

$$(1-\theta)(k^B + \bar{b}) = (1-\gamma)(q^h + \tilde{\tau}). \tag{B.55}$$

Then,

$$\bar{c} + \bar{b} = \frac{q^h + \tilde{\tau}}{1-\theta} - \frac{\gamma}{1-\theta}\left(q^h + \tilde{\tau} - \frac{C(1-\theta)}{\rho^c}\right) - f'(k)k + \frac{(\alpha-1)C}{\rho^c}. \tag{B.56}$$

Now, consider an increase in R^c. If R^d is unchanged, then q^h and q^0 will be unchanged. But then $1 - \rho^c\eta^0$ will decrease, which means c^0 will increase and γ will decrease. Hence, R^d must decrease, which means q^h and q^0 will decrease. Finally, consider an increase in ρ^c. If R^d is unchanged, then q^h, q^0, and c^0 will be unchanged, but γ will decrease. Hence, R^d must decrease, which means q^h and q^0 will decrease. $\square$

Proof of Proposition 4.2: First, given R and μ, q^h solves

$$u'(q) = \frac{1+\mu}{\beta(1+R)}. \tag{B.57}$$

Then $d^h = q^h + \tilde{\tau}$. Next, given $\tilde{\tau}$, η^0 is such that

$$\beta u(q^h) - \beta u'(q^h)(q^h + \tilde{\tau}) = \beta u(q^0) - \beta u'(q^0)q^0, \tag{B.58}$$

$$u'(q^0) = \frac{1+\mu}{\beta(1-\rho^c\eta^0)(1+R)}. \tag{B.59}$$

Let $c^0 = q^0/(1-\rho^c\eta^0)$. Then, γ is such that

$$\frac{\gamma\rho^c c^0}{\alpha - 1 + \gamma} = C, \tag{B.60}$$

The demand for government bonds is given by

$$f'(k) = \frac{1+R^b}{1+\mu}, \tag{B.61}$$

$$(1-\theta)(k^B + \bar{b}) = \epsilon(1-\gamma)(q^h + \tilde{\tau}). \tag{B.62}$$

where R^b is given by $\frac{1+\mu}{1+R^b} = \beta\theta + \frac{(1-\theta)(1+\mu)}{1+R}$. Then, we have

$$\bar{c} + \bar{b} = \frac{(\alpha - 1 + \gamma)C}{\rho^c} + (1-\epsilon)(1-\gamma)(q^h + \tilde{\tau}) + \frac{\epsilon(1-\gamma)(q^h + \tilde{\tau})}{1-\theta} - f'(k)k. \tag{B.63}$$

Now, consider an increase in R. Then q^h and q^0 will increase. In addition, c^0 will increase because $c^0 = q^0/(1-\rho^c\eta^0) = \beta(1+R)q^0u'(q^0)/(1+\mu)$. This means γ and ϵ will decrease. Finally, consider an increase in ρ^c. If R^c is unchanged, then q^h, q^0, and c^0 will be unchanged. However, this means that γ will decrease. $\square$

Proof of Proposition 4.3: First, given R^c and μ, q^h solves

$$u'(q) = \frac{1+\mu}{\beta(1+R^c)}. \tag{B.64}$$

Next, given $\tilde{\tau}$, η^0 is such that

$$\beta u(q^h) - \beta u'(q^h)(q^h+\tilde{\tau}) = \beta u(q^0) - \beta u'(q^0)q^0, \tag{B.65}$$

$$u'(q^0) = \frac{1+\mu}{\beta(1-\eta^m\eta^0)}. \tag{B.66}$$

Let $m^0 = q^0/(1-\rho^m\eta^0)$. And γ is such that

$$\frac{\gamma\rho^m m^0}{\alpha-1+\gamma} = C. \tag{B.67}$$

The demand for government bonds is given by

$$f'(k) = \frac{1+R^b}{1+\mu}, \tag{B.68}$$

$$(1-\theta)(k^B+\bar{b}) = \epsilon(1-\gamma)(q^h+\tilde{\tau}). \tag{B.69}$$

Then,

$$\bar{m}+\bar{c}+\bar{b} = \frac{(\alpha-1+\gamma)C}{\rho^m} + (1-\epsilon)(1-\gamma)(q^h+\tilde{\tau}) + \frac{\epsilon(1-\gamma)(q^h+\tilde{\tau})}{1-\theta} - f'(k)k. \tag{B.70}$$

Now, consider an increase in R^c while μ is held constant. Then q^h and q^0 will increase. In addition, m^0 will increase because $m^0 = q^0/(1-\rho^m\eta^0) = \beta q^0u'(q^0)/(1+\mu)$. Hence, γ will decrease. Since the demand for government liabilities increases, ϵ must decrease. $\square$

Proof of Proposition 5.1: Let γ^{bench} and η^{bench} denote the share of sellers who evade taxes and the audit probability in the benchmark equilibrium, respectively. First, in a type-2 equilibria, assume

$$\frac{1+\mu}{(1-\rho^c\eta^{bench})(1+R^c)} = \frac{1+\mu}{1-\rho^m\eta^{bench}}. \tag{B.71}$$

Then q^0 and q^h will be unchanged. However, $c^0 = q^0/(1-\rho^c\eta^{bench}) > m^0 = q^0/(1-\rho^m\eta^{bench})$ because either $R^c > 0$ or $\rho^c > \rho^m$. This means that

$$\frac{\gamma^{bench}\rho^c c^0}{\alpha-1+\gamma} > C. \tag{B.72}$$

Hence, R^d must decrease and $\gamma < \gamma^{bench}$. Now, from Proposition 4.1 we know that the γ is decreasing in R^c and ρ^c. This means that γ in a type-3 equilibrium must be smaller than in a type-2 equilibrium.

Finally, given R^c, q^0 are the same in both type-3 equilibria and type-4 equilibria. However, $m^0 = q^0/(1-\rho^m\eta^0)$ in type-4 equilibria but $c^0 = q^0/(1-\rho^c\eta^0)$. Since $\rho^c \geq \rho^m$, $m^0 \geq c^0$. In a type-4 equilibrium, γ is given by

$$\frac{\gamma\rho^m m^0}{\alpha-1+\gamma} = C. \tag{B.73}$$

In a type-3 equilibrium, γ is given by

$$\frac{\gamma\rho^c c^0}{\alpha-1+\gamma} = C. \tag{B.74}$$

Hence, γ is smaller in a type-3 equilibrium than in a type-3 equilibrium. $\square$

Proof of Proposition 5.2: Note that q^h is given by

$$u'(q^h) = \frac{1+\mu}{\beta(1+R^d)}. \tag{B.75}$$

And q^0 is given by

$$\mathcal{S} = \beta u(q^h) - \beta u'(q^h)(q^h + \tilde{\tau}) = \beta u(q^0) - \beta u'(q^0)q^0. \tag{B.76}$$

Hence, q^h and q^0 only depend on R^d. From the proof of Proposition 5.1, we know that R^d is lower in a type-2 equilibrium than in the benchmark equilibrium. In a type-3 equilibrium or a type-4 equilibrium, $R^d = R^c$. Hence, compared to the benchmark, q^0 and q^h are smaller in type-2 equilibria, and larger in type-4 equilibria. Also, q^0 and q^h are smaller in type-3 equilibria if $R^c < R^{bench}$ but larger if $R^c > R^{bench}$. $\square$

Proof of Proposition 5.3: Since given R^c, γ is smaller in type-3 equilibria than in type-4 equilibria while q^0 and q^h are equal in the two types of equilibria, it is clear that the total surplus in DM is the higher in type-3 equilibria than type-4 equilibria given R^c. Next, because when $R^c = R^{bench}$, q^0 and q^h are equal in type-3 equilibria and the benchmark equilibrium but γ is smaller in type-3 equilibria, the total surplus in DM is the higher in type-3 equilibria. This means that there exists $R^{c\dagger} < R^{bench}$ such that for any $R^c \geq R^{c\dagger}$, the total surplus in DM is higher in type-3 equilibria than in the benchmark equilibrium. Finally, recall that q^0 and q^h are lower in type-2 equilibria than in the benchmark equilibrium, while γ is lower in type-3 equilibria than in type-2 equilibria. This means that there exists $R^{c\ddagger} < R^{bench}$ such that for any $R^c \geq R^{c\ddagger}$, the total surplus in DM is higher in type-3 equilibria than in type-2 equilibria. Define $R^{c\prime} = \max\{R^{c\dagger}, R^{c\ddagger}\}$, then we have the desired result. $\square$

Proof of Proposition 5.5: First, in type-2 equilibria, suppose that for some $\rho^c \geq \rho^m$, R^c is such that

$$\frac{1+\mu}{(1-\rho^c\eta^{bench})(1+R^c)} = \frac{1+\mu}{1-\rho^m\eta^{bench}}. \tag{B.77}$$

Because $c^0 = q^0/(1-\rho^c\eta^{bench}) > m^0 = q^0/(1-\rho^m\eta^{bench})$, it must be that

$$\frac{\gamma^{bench}\rho^c c^0}{\alpha - 1 + \gamma} > C. \tag{B.78}$$

Hence, R^d must decrease and $\eta^0 > \gamma^{bench}$. Next, from the proof of Proposition 4.1, it is easy to see that η^0 will further increase as R^c increases. Hence, η^0 is larger in type-2 equilibria than in the benchmark equilibrium.

Second, on the boundary of type-1 and type-3 equilibria, it must be that

$$\frac{1+\mu}{(1-\rho^c\eta^{bench})(1+R^c)} = \frac{1+\mu}{1-\rho^m\eta^{bench}}. \tag{B.79}$$

From Proposition 4.2, we know that η^0 will further increase as ρ^c decreases. Hence, η^0 is larger in type-3 equilibria than in the benchmark equilibrium.

Finally, in a type-4 equilibrium, q^0 is larger than its counterpart in the benchmark. Since

$$u'(q^0) = \frac{1+\mu}{\beta(1-\rho^m\eta^0)}, \tag{B.80}$$

it must be that η^0 is larger in type-4 equilibria than in the benchmark equilibrium. $\square$

Proof of Proposition 5.6: I only need to show that for any given R^c, when $\rho^c = \rho^c(R^c)$, γ is larger in a type-4 equilibrium than in a type-3 equilibrium, while η^0 is equal in type-3 and type-4 equilibria. Note that on the boundary of type-3 and type-4 equilibria, it must be that

$$\frac{1+\mu}{(1-\rho^c\eta^0)(1+R^c)} = \frac{1+\mu}{1-\rho^m\eta^0}. \tag{B.81}$$

Hence, η^0 is equal in type-3 and type-4 equilibria. Next, note that $c^0 = q^0/(1-\rho^c\eta^{bench}) > m^0 = q^0/(1-$

$\rho^m \eta^{bench}$). In addition, in a type-4 equilibriumγ is given by

$$\frac{\gamma \rho^m m^0}{\alpha - 1 + \gamma} = C. \tag{B.82}$$

In a type-3 equilibrium, γ is given by

$$\frac{\gamma \rho^c c^0}{\alpha - 1 + \gamma} = C. \tag{B.83}$$

Hence, γ is larger in a type-4 equilibrium than in a type-3 equilibrium. □

Proof of Proposition 5.7: Following Proposition 5.1-5.5, we know that if $(R^c, \rho^c) = (R^{bench}, \rho^c(R^{bench}))$, then η^0, q^0, q^h, and k in a type-3 equilibrium are all identical to their counterparts in the benchmark equilibrium. However, γ is strictly lower. This means that the DM surplus is strictly higher, while the audit costs are strictly lower. Hence, the aggregate welfare is strictly higher in a type-3 equilibrium than in the benchmark equilibrium. In other words, if CBDC is deposit-like, then introducing CBDC increases aggregate welfare.

Next, consider cash-like CBDC. We know that as long as (R^c, ρ^c) are close to point A, then the change in γ and k will be small compared to the benchmark. However, η^0 is strictly higher, while q^0 and q^h are strictly lower. Hence, aggregate welfare is higher with deposit-like CBDC than with cash-like CBDC. □

Proof of Proposition 5.8: If Assumption 5.1 holds, because the demand for CBDC is strictly lower than $\mathcal{D}/(1+R^d)$, compared to the benchmark, the central bank's net revenue is lower after introducing deposit-like CBDC. Next, consider cash-like CBDC. As long as (R^c, ρ^c) are close to point A, the central bank's revenue will be close to (but strictly lower than) its revenue in the benchmark. □

Appendix C Extension: Zero Lump-sum Transfer

In this appendix, I consider an alternative setup where the fiscal authority sets $T = 0$ and fixes the tax on sellers. It balances the budget through issuing government bonds. I restrict my attention to the benchmark case.

The equilibrium characterization is similar to the original model. The main difference is how the nominal government bond rate, R^b, is determined in equilibrium. In the original model, the demand for cash, $\bar{m}$, and the demand for government bonds by bankers, $\bar{b}$, must satisfy

$$\bar{m} + \bar{b} = \mathcal{D}, \tag{C.1}$$

where $\mathcal{D}$ is determined by the fiscal authority. It can be shown that $\bar{m}$ is given by

$$\bar{m} = \frac{(\alpha - 1 + \gamma)C}{\rho}, \tag{C.2}$$

where γ is the proportion of sellers who evade taxes. $\bar{b}$ is given by

$$\bar{b} = \frac{1-\gamma}{1-\theta}(q^h + \tilde{\tau}) - \frac{1+R^d}{1+\mu}k. \tag{C.3}$$

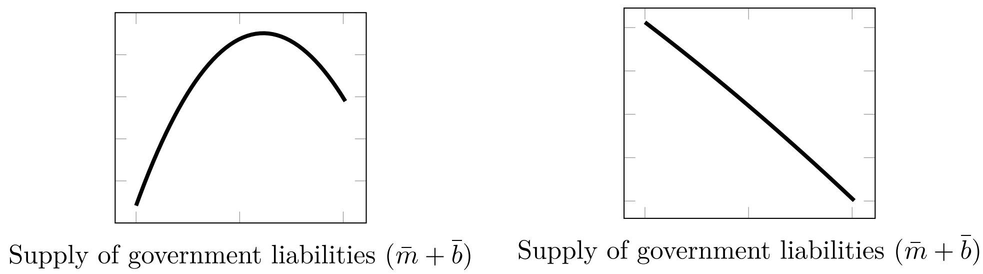
where q^h is the amount of DM good produced by sellers who report their income truthfully, and k is loans to entrepreneurs. It can be shown that γ and k is decreasing in R^b, while q^h is increasing in R^b. If $\frac{C}{\rho} \leq \tilde{\tau}$, then $\bar{m} + \bar{b}$ is strictly increasing in R^b. Hence, there exists a unique R^b such that $\bar{m} + \bar{b} = \mathcal{D}$.

Now, suppose that $\tilde{\tau}$ is fixed, while $\bar{b}$ is adjusted to balance the budget. Consider the function $\mathcal{G}(\bar{b})$ given by

$$\mathcal{G}(\bar{b}) = \mu \bar{m} - \left(1 - \frac{1+\mu}{1+R^b}\right)\bar{b} + (1-\gamma)\tilde{\tau}. \tag{C.4}$$

Then $\mathcal{G}(\bar{b})$ represents the fiscal authority's net revenue. One can conclude from the benchmark model that $\bar{b}$ is increasing in R^b. Hence, $\bar{m}$ is decreasing in $\bar{b}$. Because γ is decreasing in R^b, the net tax revenue is increasing in $\bar{b}$. Depending on parameter values, $\mathcal{G}(\bar{b})$ may not be monotonic in $\bar{b}$, as demonstrated by the numerical examples in Figure 14.[15]

First, in panel (a) of Figure 14, $\tilde{\tau}$ is relatively large. Net government revenue first increases with $\bar{b}$, because the increase in net tax revenue offsets the increase in the cost of servicing government bonds (the second term in (C.4)). However, as $\bar{b}$ continues to increase, the cost of servicing government bonds becomes the dominant force and net government revenue decreases. In panel (b) of Figure 14, $\tilde{\tau}$ is low. In this case, the increase in the cost of servicing government bonds is always larger than the increase in net tax revenue. Hence, the net government revenue is always decreasing in $\bar{b}$. In both cases, seigniorage income (the first term in (C.4)) also decreases with $\bar{b}$. However, as long as the seigniorage income is small, which happens when γ is small, its impact on net government revenue is insignificant.



(a) High $\tilde{\tau}$ (b) Low $\tilde{\tau}$

Figure 14: Net Government Revenue and Nominal Bond Rate

If net government revenue is a non-monotonic function of government liabilities, then there may exist multiple equilibria as shown in Figure 15. Specifically, there may exist an equilibrium where $\bar{b}$ and $\bar{b} + \bar{m}$ are small and R^b is low, and another equilibrium where $\bar{b}$ and $\bar{b} + \bar{m}$ are large and R^b is high. Multiple equilibria exist because a higher supply of government bonds drives up R^b. For bankers, a higher bond return decreases the cost of holding government bonds as collateral, which translates into a higher deposit rate. Then, fewer sellers choose to under-report their income, and the net tax revenue increases. The fiscal authority can then use the additional tax revenue to pay for the increase in the cost of servicing government bonds, which is why the fiscal authority can maintain the net revenue to be zero. Note that buyers are better off in the high R^b equilibrium.

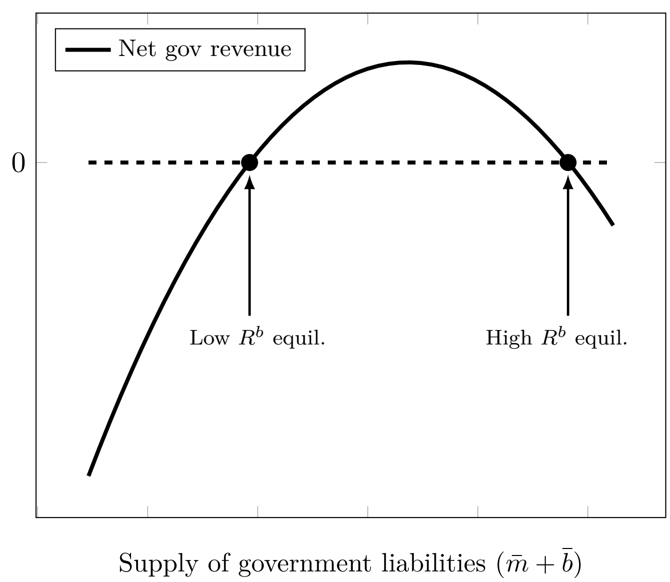


Figure 15: Multiple Equilibria

[15]In both panels, $u(c) = 2x^{0.5}$, $f(k) = k^{0.3}$, $\mu = 1\%$, $\beta = 0.99$, $\rho = 0.5$, $C = 0.1$, and $\alpha = 1.5$. In panel (a), $\tilde{\tau} = 0.45$. In panel (b), $\tilde{\tau} = 0.2$.

Appendix D Extension: Other Equilibria with CBDC

D.1 CBDC offers less Anonymity than Bank Deposits

I assume that the fiscal authority can costlessly observe any income sellers receive in CBDC. However, to observe income in bank deposits, the fiscal authority has to audit sellers. In equilibrium, CBDC is accepted by sellers who report truthfully, while bank deposits compete with cash in transactions involving tax evasion.

The strategies of sellers who evade taxes can be divided into the following three cases:
(1) Only cash is accepted by sellers who evade taxes: for this to be part of an equilibrium, it must be that $1 + R^d < (1 - \rho\eta^0)/(1 - \eta^0)$.
(2) Only bank deposits are accepted by sellers who evade taxes: for this to be part of an equilibrium, it must be that $1 + R^d > (1 - \rho\eta^0)/(1 - \eta^0)$.
(3) Sellers who evade taxes are indifferent between cash and bank deposits: for this to be part of an equilibrium, it must be that $1 + R^d = (1 - \rho\eta^0)/(1 - \eta^0)$.

In what follows, I focus my attention on case (3). Let d^0 and m^0 represent the deposit and cash payments to sellers. Let the proportion of sellers who use bank deposits be γ^d and let the proportion of sellers who use cash be γ^m. In equilibrium, the fiscal authority must be indifferent between auditing sellers or not. Hence, γ^d and γ^m satisfy

$$\frac{\gamma^d d^0}{\alpha - 1 + \gamma^d + \gamma^m} + \frac{\gamma^m \rho m^0}{\alpha - 1 + \gamma^d + \gamma^m} = C. \tag{D.1}$$

The next proposition shows the existence and uniqueness of the equilibrium.

Proposition D.1 *There exist $\mathcal{D}^\diamond$ and $\mathcal{D}^{\diamond\diamond}$ such that if $\mathcal{D} \in [\mathcal{D}^\diamond, \mathcal{D}^{\diamond\diamond}]$ and C is small, there exists a unique equilibrium where both cash and bank deposits are used by sellers who evade taxes. In addition, increasing R^c while holding μ constant leads to increases in q^h, q^0, $\gamma^c + \gamma^m$, and decreases in R^b and η^0.*

Proof: First, given R^c and μ, q^h solves

$$u'(q) = \frac{1 + \mu}{\beta(1 + R^c)}. \tag{D.2}$$

Next, given $\tilde{\tau}$, η^0 is such that

$$\frac{1 - \rho\eta^0}{1 + \mu}\left[\beta u(q^0) - \beta u(q^h) + \frac{(1 + \mu)(q^h + \tilde{\tau})}{1 + R^d}\right] - q^0 = 0, \tag{D.3}$$

$$u'(q^0) = \frac{1 + \mu}{\beta(1 - \rho\eta^0)}. \tag{D.4}$$

And R^d is given by $1 + R^d = (1 - \rho\eta^0)/(1 - \eta^0)$. It is easy to see that the following conditions hold.

$$\frac{(1 - \eta^0)(1 + R^d)}{1 + \mu}\left[\beta u(q^0) - \beta u(q^h) + \frac{(1 + \mu)(q^h + \tilde{\tau})}{1 + R^c}\right] - q^0 = 0, \tag{D.5}$$

$$u'(q^0) = \frac{1 + \mu}{\beta(1 - \eta^0)(1 + R^d)}. \tag{D.6}$$

γ^d and γ^m are such that

$$\frac{\gamma^d d^0}{\alpha - 1 + \gamma^d + \gamma^m} + \frac{\gamma^m \rho m^0}{\alpha - 1 + \gamma^d + \gamma^m} = C, \tag{D.7}$$

where $d^0 = q^0/(1 - \eta^0)$ and $m^0 = q^0/(1 - \rho\eta^0)$. The demand for government liabilities is given by

$$\tilde{\mathcal{D}} = (1 - \gamma^d - \gamma^m)(q^h + \tilde{\tau}) + \frac{\gamma^d d^0}{1 - \theta} + \gamma^m m^0. \tag{D.8}$$

where $\tilde{\mathcal{D}} = \mathcal{D} + k$. Then γ^d and γ^m solve (D.7) and (D.8). We have

$$\gamma^d = \frac{(\rho m^0 - C)(\tilde{\mathcal{D}} - q^h - \tilde{\tau}) - (m^0 - q^h - \tilde{\tau})(\alpha - 1)C}{(d^0/(1-\theta) - q^h - \tilde{\tau})(\rho m^0 - C) - (d^0 - C)(m^0 - q^h - \tilde{\tau})}, \tag{D.9}$$

$$\gamma^m = \frac{(d^0/(1-\theta) - q^h - \tilde{\tau})(\tilde{\mathcal{D}} - q^h - \tilde{\tau}) - (d^0 - C)(\alpha - 1)C}{(d^0/(1-\theta) - q^h - \tilde{\tau})(\rho m^0 - C) - (d^0 - C)(m^0 - q^h - \tilde{\tau})} \tag{D.10}$$

Then it is easy to see that there exist $\mathcal{D}^\diamond$ and $\mathcal{D}^{\diamond\diamond}$ such that if $\mathcal{D} \in [\mathcal{D}^\diamond, \mathcal{D}^{\diamond\diamond}]$, γ^d and γ^m will take values between 0 and 1. □

An equilibrium where sellers who evade taxes play a mixed strategy exists provided the total supply of government debt, $\mathcal{D}$, is neither too large nor too small. This is because if $\mathcal{D}$ is too large, R^b may be so high that sellers who evade taxes only accept bank deposits. Then, the equilibrium is the same as when CBDC replaces cash. If $\mathcal{D}$ is too small, government liabilities may not be able to support DM consumption, which is determined solely by R^c, ρ, and μ but not $\mathcal{D}$.

Now, suppose the central bank increases R^c while holding μ constant. An increase in R^c attracts sellers to accept CBDC and report their income truthfully. This lowers the fiscal authority's incentive to audit sellers, which leads to an increase in $\gamma^c + \gamma^m$. A lower η^0 first increases q^0 and d^0, but the subsequent increase in the demand of government bonds drives down R^b. Nevertheless, the decrease in audit probability outweighs the decrease in R^b. In equilibrium, q^0 increases.

D.2 CBDC offers More Anonymity than Cash

I assume that if an agent is audited, the fiscal authority observes his or her income in CBDC with probability $\rho^c < \rho^m$. Now, suppose a seller choose to evade taxes. If the seller chooses to accept cash, he or she solves

$$\max_{q,m} \{(1 - \rho\eta^0)m - q\} \text{ s.t. } -(1+\mu)m + \beta u(q) = \mathcal{S}. \tag{D.11}$$

For this to be part of an equilibrium, it must be that $1 + R^d \leq (1 - \rho\eta^0)/(1 - \eta^0)$ and $1 + R^c \leq (1 - \rho\eta^0)/(1 - \rho^c\eta^0)$. The first condition guarantees that the seller does not have the incentive to use bank deposits, and the second condition guarantees that the seller does not have the incentive to use CBDC. If the seller chooses to accept CBDC, he or she solves

$$\max_{q,d} \{(1 - \rho^c\eta^0)c - q\} \text{ s.t. } -\frac{(1+\mu)c}{1+R^c} + \beta u(q) = \mathcal{S}. \tag{D.12}$$

For this to be part of an equilibrium, it must be that $(1 + R^d)/(1 + R^c) \leq (1 - \rho\eta^0)/(1 - \eta^0)$ and $1 + R^c \geq (1 - \rho\eta^0)/(1 - \rho^c\eta^0)$. The first condition guarantees that the seller does not have the incentive to use bank deposits, and the second condition guarantees that the seller does not have the incentive to use cash.

There are three possible equilibria:
(1) Only cash is accepted by sellers who evade taxes: for this to be part of an equilibrium, it must be that $1 + R^c < (1 - \rho\eta^0)/(1 - \rho^c\eta^0)$.
(2) Only CBDC is accepted by sellers who evade taxes: for this to be part of an equilibrium, it must be that $1 + R^c > (1 - \rho\eta^0)/(1 - \rho^c\eta^0)$.
(3) Sellers who evade taxes are indifferent between cash and CBDC: for this to be part of an equilibrium, it must be that $1 + R^c = (1 - \rho\eta^0)/(1 - \rho^c\eta^0)$.

In what follows, I focus on case (3). Let c^0 and m^0 represent the CBDC and cash payments to sellers. Let the proportion of sellers who use CBDC be γ^c and let the proportion of sellers who use cash be γ^m. In equilibrium, it must be that the fiscal authority is indifferent between auditing sellers or not. Hence, γ^c and γ^m must satisfy

$$\frac{\gamma^c\rho^c c^0}{\alpha - 1 + \gamma^c + \gamma^m} + \frac{\gamma^m\rho m^0}{\alpha - 1 + \gamma^c + \gamma^m} = C, \tag{D.13}$$

where $\frac{\gamma^c}{\alpha-1+\gamma^c+\gamma^m}$ ($\frac{\gamma^m}{\alpha-1+\gamma^c+\gamma^m}$) is the probability that a seller who reports an income of zero received CBDC (cash). To solve for the equilibrium, note that given R^c, ρ, and ρ^c, the condition $1+R^c = (1-\rho\eta^0)/(1-\rho^c\eta^0)$

determines η^0. This means that q^0 and $\mathcal{S}$ are determined by μ:

$$u'(q^0) = \frac{1+\mu}{\beta(1-\rho\eta^0)}, \tag{D.14}$$

$$\mathcal{S} = \beta u(q^0) - \frac{(1+\mu)q^0}{1-\rho\eta^0}. \tag{D.15}$$

Then, because sellers who report truthfully use bank deposits, $\mathcal{S}$ and μ determine R^d:

$$u'(q^h) = \frac{1+\mu}{\beta(1+R^d)}, \tag{D.16}$$

$$\mathcal{S} = \beta u(q^h) - \frac{(1+\mu)q^h}{1+R^d}. \tag{D.17}$$

This means that the terms of trade in the DM are determined by only four parameters: R^c, ρ, ρ^c, and μ. Because $1+R^c = (1-\rho\eta^0)/(1-\rho^c\eta^0)$, sellers who evade taxes produce the same amount of DM good regardless of the payment method they use. Lastly, γ^c and γ^m are such that (D.13) holds, and the total demand for government liabilities (cash, CBDC, and government bonds) is equal to the supply of government liabilities prescribed by the fiscal authority, $\mathcal{D}$. The next proposition shows the effects of R^c on the equilibrium.

Proposition D.2 *Suppose the central bank increases R^c while holding μ constant. If $\rho^c < \rho^m$, then η^0 and k will decrease, while q^h, q^0, $\gamma^c + \gamma^m$, and R^b will increase.*

Proof: First, if $\rho^c < \rho$, then η^0 is decreasing in R^c. This means q^0 and $\mathcal{S}$ must increase because

$$u'(q^0) = \frac{1+\mu}{\beta(1-\rho\eta^0)}, \tag{D.18}$$

$$\mathcal{S} = \beta u(q^0) - \frac{(1+\mu)q^0}{1-\rho\eta^0}. \tag{D.19}$$

Since

$$u'(q^h) = \frac{1+\mu}{\beta(1+R^d)}, \tag{D.20}$$

$$\mathcal{S} = \beta u(q^h) - \frac{(1+\mu)q^0}{1+R^b}, \tag{D.21}$$

q^h and R^b will increase as well. Next, note that because

$$c^0 = \frac{q^0}{1-\rho^c\eta^0} = \frac{\beta(1+R^c)q^0 u'(q^0)}{1+\mu}, \tag{D.22}$$

$$m^0 = \frac{q^0}{1-\rho\eta^0} = \frac{\beta q^0 u'(q^0)}{1+\mu}, \tag{D.23}$$

and q^0 increases, then m^0 will increase but c^0 will decrease. $\square$

If $\rho^c < \rho$, the effect of an increase in R^c on the equilibrium is the opposite because $(1-\rho\eta^0)/(1-\rho^c\eta^0)$ is decreasing in η^0. Intuitively, because $\rho^c < \rho$, the payment in CBDC is smaller than the payment in cash (in real terms). An increase in R^c attracts sellers who evade taxes to accept CBDC and decreases the left-hand side of (D.13). Hence, η^0 must decrease. As a result, more sellers evade taxes, and $\gamma^c + \gamma^m$ increases. Note that in this case, R^c must be strictly negative for sellers who evade taxes to accept cash.

Appendix E Extension: Numerical Examples

In this section, I provide some numerical examples of how equilibrium outcomes change following the introduction of CBDC. I assume the following functional forms utility function and entrepreneurs' production function.

$$u(g) = \frac{g^{1-\sigma}}{1-\sigma}; \quad f(k) = k^{\upsilon}$$

The choices of parameter values are given by the following table.

Parameters	Notation	Value
Discount factor	β	0.96
Probability of cash income being observed	ρ^m	0.5
Buyers' preference	σ	0.5
Curvature of production	υ	0.3
Measure of sellers	α	1.5
Cost per audit	C	0.1
Tax	$\tilde{\tau}$	0.2
Bankers' limited commitment	θ	0.1

In this example, $R^{bench} = 0.85\%$. In what follows, I show how γ, q^0, q^h, k, total DM surplus, net entrepreneur output, total audit costs, aggregate welfare, and central bank net revenue change after the introduction of CBDC.

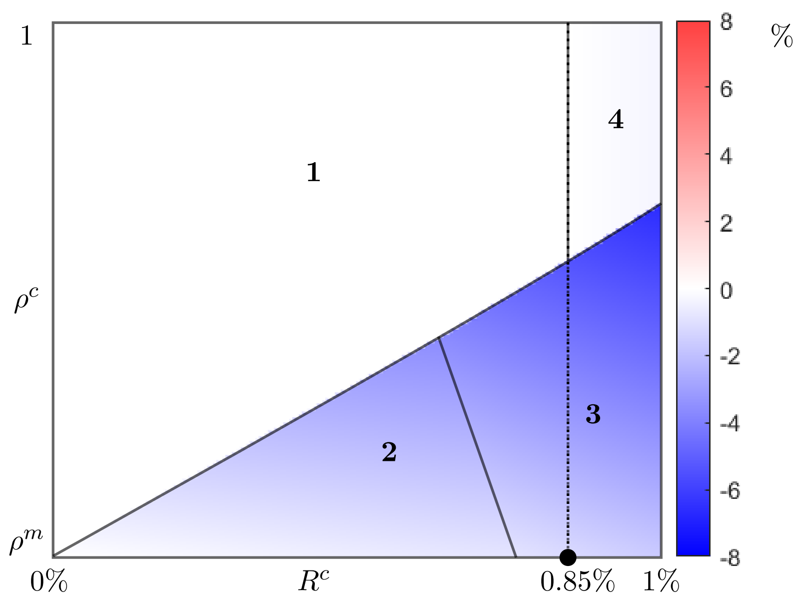


Figure 16: Percentage Change in Tax Evasion (γ) (relative to the benchmark)

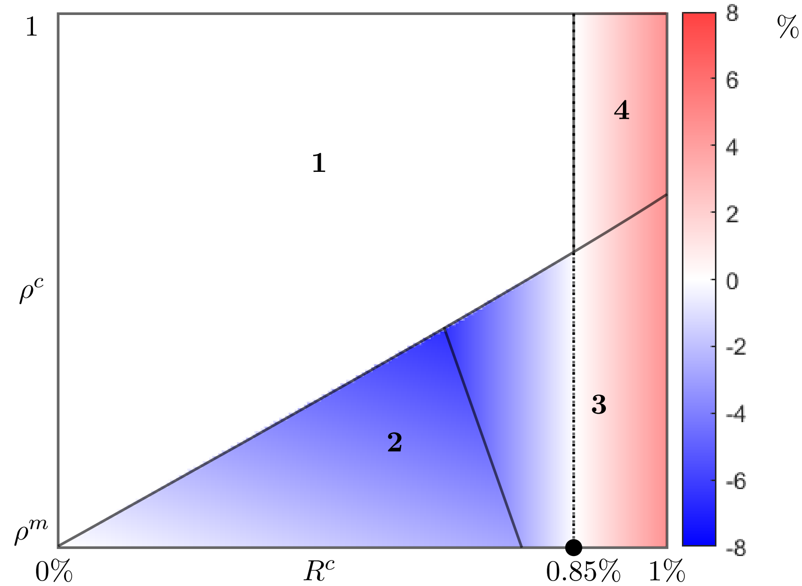


Figure 17: Percentage Change in q^0 (relative to the benchmark)

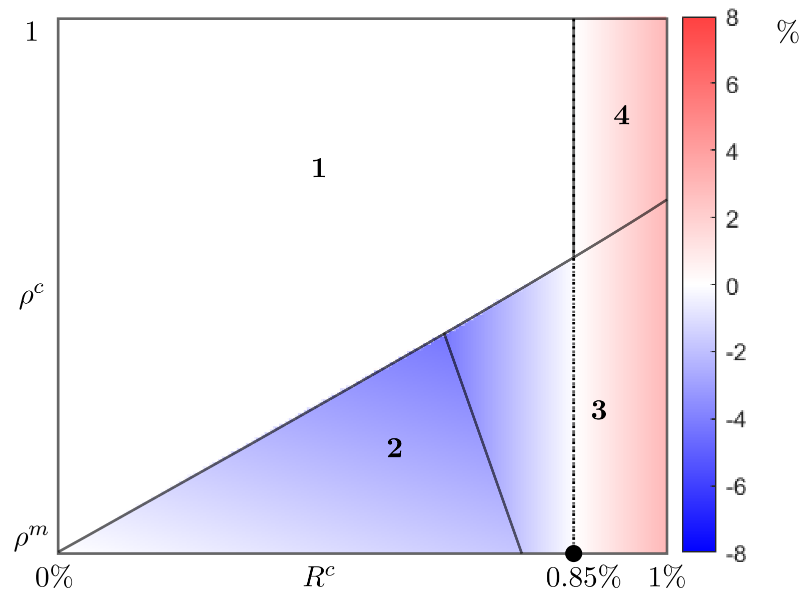


Figure 18: Percentage Change in q^h (relative to the benchmark)

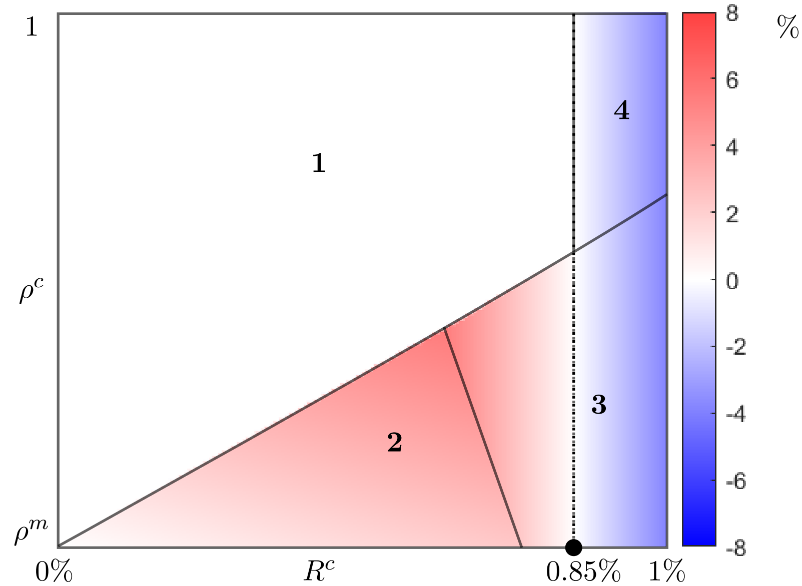


Figure 19: Percentage Change in Investment (k) (relative to the benchmark)

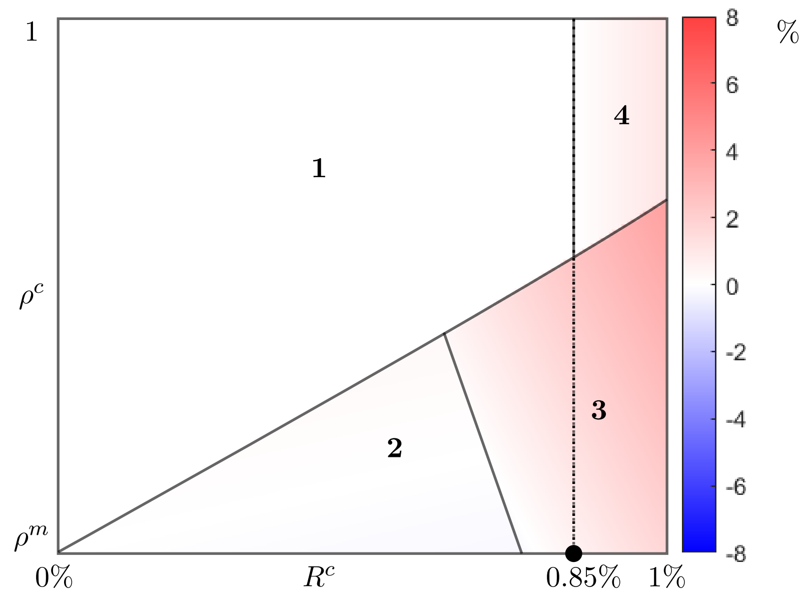


Figure 20: Percentage Change in Total DM Surplus (relative to the benchmark)

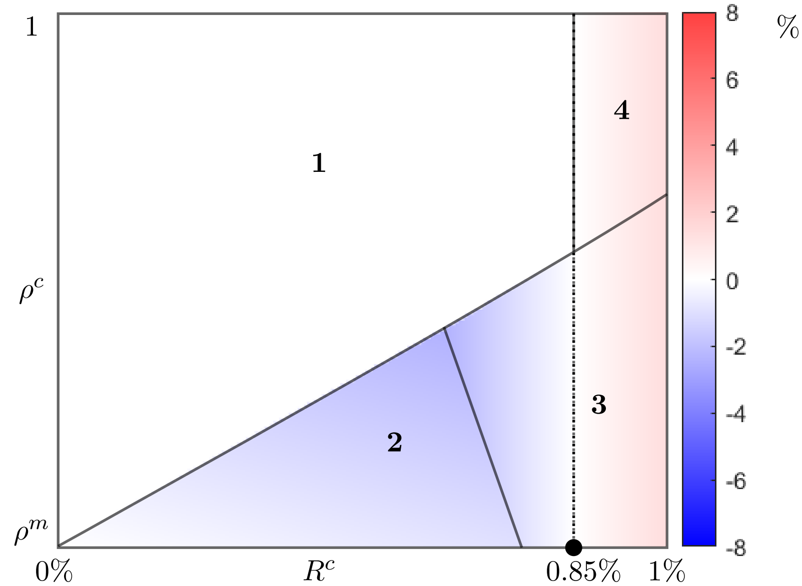


Figure 21: Percentage Change in Net Entrepreneur Output (relative to the benchmark)

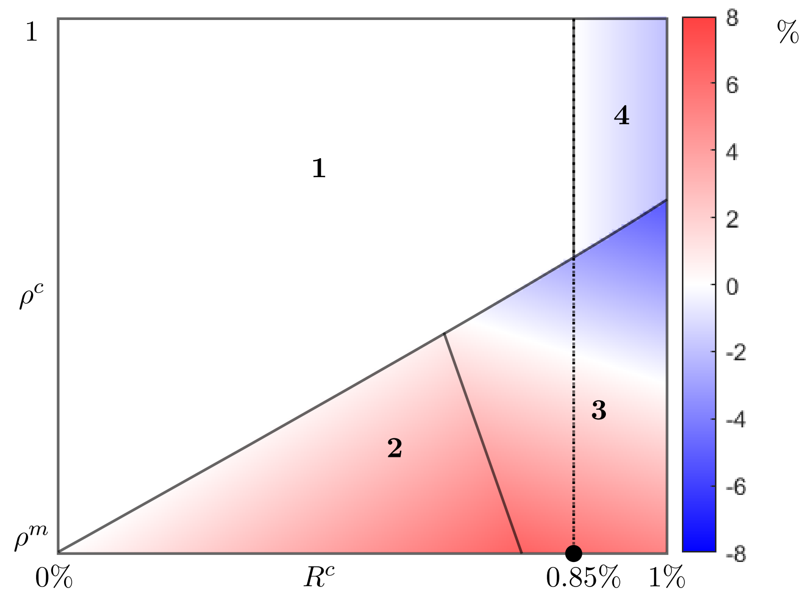


Figure 22: Percentage Change in Total Audit Costs (relative to the benchmark)

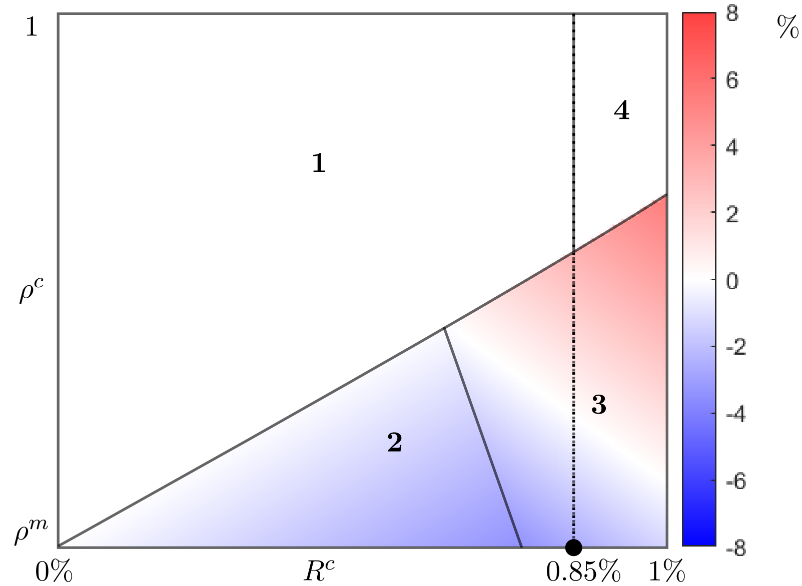


Figure 23: Percentage Change in Aggregate Welfare ($\mathcal{W}$) (relative to the benchmark)

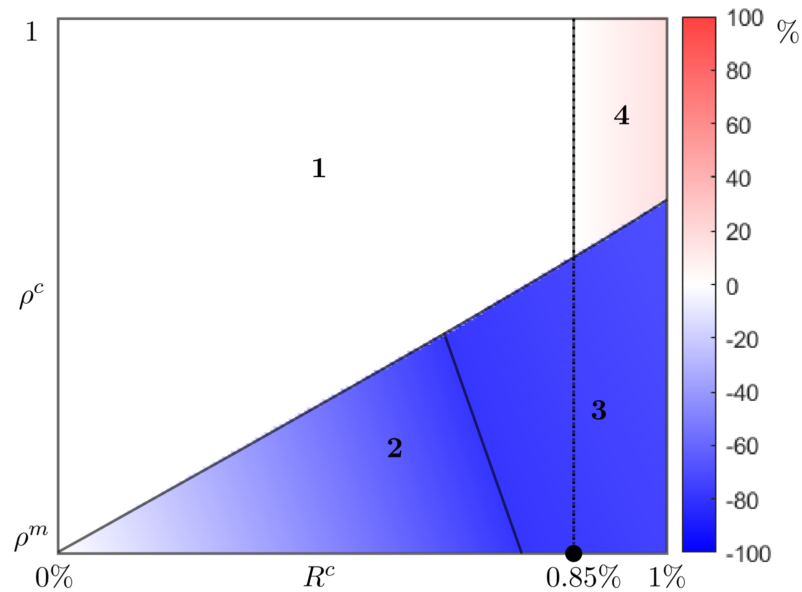


Figure 24: Percentage Change in Central Bank Net Revenue (Π) (relative to the benchmark)