Data-enabled learning, network effects and competitive advantage*

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Abstract

We model competition between firms which improve their products through learning from customer data. We explore the implications for competitive dynamics of three new features of such learning compared to traditional learning-by-doing: (i) learning increases willingness-to-pay rather than reducing marginal cost, (ii) firms can improve their products for each customer based on their individual usage experience, (iii) products can improve while customers are still consuming them. The model allows us to analyze data-sharing, the shape of the learning curves, and other factors affecting an incumbent's competitive advantage. Finally, we show when and how network effects arise endogenously from data-enabled learning.

Keywords: machine learning, dynamic competition, data sharing, switching costs.

1 Introduction

In recent years, much attention has been focused on the role data can play in providing incumbent firms with a competitive advantage. Digitization, connectivity to cloud-based infrastructures, together with cheaper storage and more effective use of data (i.e. improvements in machine learning algorithms), have made it possible for firms in many industries to learn from their customer data and rapidly improve their products. And with better products, these firms can attract more customers (or more usage from existing customers), and therefore obtain more data, potentially creating a self-reinforcing cycle that can make

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it difficult for any new entrant to compete. We call this self-reinforcing cycle "data-enabled learning".[1]

As an example, consider Google Maps, which relies on learning across users. As its traffic predictions improve, more drivers adopt it, resulting in more data to improve traffic predictions, more drivers adopting, and so on. Or consider Google's Nest smart thermostat, which relies on personalized learning. As customers use their thermostat more, it learns their preferences better, resulting in improved temperature setting, meaning users tend to keep using it, and so on. Other notable examples that exhibit data-enabled learning include search engines (Google, Bing, Baidu), speech recognition applications (e.g. dictation software, virtual assistants, chatbots), smart connected devices (e.g. smart beds by Eight Sleep and the Tonal home gym), online services that use recommender systems (e.g. Netflix, Stitchfix, Spotify, Tinder, TrueFit), advanced driver assistance and autonomous vehicle systems (Cruise, Mobileye, Waymo), as well as a myriad of startups developing AI-based real-time monitoring and predictions across sectors as diverse as agriculture (Blue River Technology and Prospera), healthcare (Notable Labs and SkinVision), law (Genie AI and Luminance) and security (SeeTrue and Vaak).

Data-enabled learning would seem to give incumbent firms a competitive advantage. But how strong is this advantage and how does it differ from that obtained from more traditional mechanisms, such as (i) learning-by-doing, in which an incumbent lowers its costs as it produces (and sells) more, thereby being able to offer a lower price, and attract more sales, and (ii) standard network effects, in which an incumbent firm becomes more valuable to consumers as it attracts more consumers, thereby attracting even more consumers and offering even higher value?

To address these questions, and determine the drivers of competitive advantage with data-enabled learning, we develop a theory of dynamic competition in which firms can improve their products via data gleaned from their customers. The theory is based on Bertrand competition between two infinitely lived firms, which can differ in the amount of their prior learning as well as in the shape of their respective learning functions. We allow firms to set prices to compete for consumers every period, taking into account how attracting consumers (and therefore more data) increases the value they can offer to consumers in subsequent periods. This differs from traditional learning-by-doing because learning works through willingness-to-pay rather than costs. Aside from differences in standalone utility and learning functions, the two firms in our model are assumed to be identical. An attractive feature of

[1]Some practitioners have called this phenomenon "data network effects". Our use of "data-enabled learning" is intended to be more general since, as discussed below, the phenomenon need not involve any network effect (for example, if the learning is specific to each customer). Indeed, one of the contributions of this paper is to determine when data-enabled learning creates genuine network effects.

our setup is that, despite allowing for general learning functions, we are able to completely characterize the equilibrium outcomes, determining which firm will win and each firm's value for any arbitrary starting position of the two firms.

As a baseline, we consider the case with across-user learning only (Section 3), in which the two firms only learn based on the total number of consumers they served in the past. Since consumers' willingness-to-pay can increase over time with more users even if prices remain unchanged, there is a type of dynamic network effect at play. As a result, if a firm is able to win in one period, it will enjoy an increasing advantage, and be able to win in all subsequent periods.

We then consider the case of within-user learning only (Section 4), in which any given consumer's willingness-to-pay for a given firm's product only depends on how many times the firm has served that particular consumer in the past. This highlights a second key difference between data-enabled learning and traditional mechanisms, which is the role of customization: firms can often improve their products for each individual customer based on that customer's particular usage experience. Within-user learning implies the more a customer uses a firm's product, the greater is the effective cost of switching to use the rival's product. Such data-enabled learning therefore creates an endogenous switching cost that grows over time, but not any type of network effect.

In these baseline models, we show that despite the fact that sometimes both the winning firm and the losing firm price below cost to consumers, the competitive outcome always coincides with the socially optimal outcome. This is not at all an obvious result despite our Bertrand competition setting: the determination of which firm wins in a particular period depends on how much each firm is willing to subsidize consumers in the current period, which in turn depends on how much each firm's value increases by winning this period as opposed to losing this period, both of which are only defined recursively. A second fundamental result comes from comparing across-user learning and within-user learning. We find the conditions for either firm to win are identical in the two cases. However, all else equal, the winning firm makes higher profits when learning is across users rather than within users. Fundamentally, this is because within-user learning creates endogenous switching costs, so each individual consumer takes into account that they would benefit if the losing firm had a stronger competitive position in the future, which in turn makes the winning firm have to price more competitively. We also explain how various factors, such as how far each firm is along its learning curve and the nature of the learning curve, affect the condition for either firm to win.

In these two baseline models, consumers do not face any coordination problem arising from network effects. Our third main contribution consists in showing how network effects that lead to a coordination problem among consumers and thus create a scope for beliefs to affect the competitive outcome can arise endogenously (Section 5) when we introduce either one of two realistic features. The first is that for cloud-based products and services that benefit from across-user learning, firms can often improve their products while their customers are still consuming them. In these cases, consumers need to form expectations over how many other consumers will purchase the same product in order to determine the value they will get from the product.

The second feature that creates a coordination problem from network effects is the combination of across-user and within-user learning. Remarkably, this feature gives rise to a consumer coordination problem even in the absence of any within-period learning, exogenous switching costs or prices. Specifically, even if the product is available for free and consumers are free to change firms each period, within-user learning creates an endogenous switching cost. This means consumers also want to choose the product that other consumers are expected to choose in order to also benefit from across-user learning in subsequent periods. For example, this means Google likely benefits from real network effects in organic search, even though it is "costless" for users to switch to Bing.

A key implication of our analysis is that even though data can create a type of dynamic network effect, there is no systematic inefficiency in the market outcome unless the dynamic network effect leads to a consumer coordination problem and consumer beliefs favor one of the firms. On the other hand, even absent any such coordination problem, our analysis implies that increasing the learning of the losing firm generally increases consumer surplus. This might suggest that a policy that forces incumbents to share their superior data with entrants would be good for consumers. We use our dynamic framework to explore how data sharing works, and find that if firms anticipate such a policy, they may compete less aggressively in the first place, which in our model means subsidizing consumers less, and potentially lowering consumer surplus. Finally, we also show that whether data protects an incumbent from an entrant or not depends on a comparison of the two firms' learning curves. Even if the entrant starts with less data, if it can ultimately create more value from data, through dynamic pricing it may actually be able to attract consumers and become the market leader.

2 Related literature

Our paper combines aspects from three large and established literatures. First, it relates to works that provide theoretical models of competition with learning-by-doing (Fudenberg and Tirole, 1983, Dasgupta and Stiglitz, 1988, Salant, 1990, Cabral and Riordan, 1994, Besanko et al., 2010). Aside from the fact learning increases consumer's willingness to pay rather than decreases firms' costs, our particular model of learning is quite distinct from any in the existing literature. The baseline model with pure across-user learning is closest to that in the classic paper of Cabral and Riordan (1994). A key difference is that they allow for horizontal differentiation between the two firms, whereas we allow for vertical differentiation (both in standalone values and via asymmetric learning functions). Because of the tractability of our particular framework, we are able to obtain several new results, including a general efficiency result, the implications of learning for consumer surplus, and comparative statics on the shape of the learning curves. And we apply the framework to study questions that do not arise in the learning-by-doing setting: the implications of data sharing policies, the difference in competitive outcomes between across-user and within-user learning, and conditions under which data-enabled learning leads to meaningful network effects.

Second, our paper relates to the substantial literature on network effects. From this literature we borrow the modelling of consumer "beliefs" to select equilibria when consumers play coordination games (Katz and Shapiro, 1986, and Caillaud and Jullien, 2001). At a high level, our model with across-user and within-period learning is related to dynamic models with network effects (e.g. Mitchell and Skrzypacz, 2006, Cabral, 2011, Biglaiser and Cremer 2020, Halaburda et al., 2020), since the combination of these two features works in the same way as exogenous network effects within a period. However, a key difference is that in our setting with learning, willingness-to-pay depends not only on the current stock of consumers, as it would in network effects models, but also on past consumers even if they are no longer consuming from the firm.

Third, our paper relates to IO models of switching costs, such as the classic papers of Klemperer (1987) and Beggs and Klemperer (1992). As noted in the introduction, within-user learning creates an endogenous switching cost that grows the longer a consumer is with a particular firm. To the best of our knowledge, such a feature has not previously been considered in the switching cost literature even though it seems relevant more generally.

Finally, there are some recent works that also study aspects of data-enabled learning, albeit using approaches that are very different from ours. Prufer and Schottmüller (2017) model a dynamic feedback loop in which a firm's current cost of investing in quality is decreasing in the firm's previous period sales, and greater investment in quality leads to higher demand in the current period. This differs from our approach in a few ways. They do not explicitly model consumers' choices or firms' pricing decisions, but focus instead on firms' investment decisions. While they allow for learning across users, they do not consider either within-period learning or within-user learning, and so user coordination issues do not arise.

Finally, they explore the interesting issue of how learning from data allows the dominant firm to enter connected markets when the existing data the firm has collected can be reused, an issue we do not consider. Farboodi et al. (2019) provide a model of data-enabled learning in which data helps firms choose the best production technique and so higher quality products. They abstract from any strategic firm behavior or consumer decisions, and instead focus on the industry dynamics generated from a continuum of price-taking firms. De Cornière and Taylor (2020) also study the effect of data on competition but focus on showing how different uses of data (including improving a firm's product, but also ad targeting and price discrimination) determine whether data is pro- or anti-competitive. While their framework is much more general in other ways, they don't analyze a fully dynamic setting in which data learning can accumulate. Schaefer et al. (2018) investigate how data drives the quality of internet search results, providing evidence that the quality of search results improve with more data on previous searches, and that personalized information is particularly valuable. Finally, Biglaiser et al. (2019) provides a high-level discussion of the different ways firms can enjoy incumbency advantages, including access to more data: they note that across-user learning and within-user learning are two distinct ways in which data generates a competitive advantage for incumbents.

3 Across-user learning

In this section we focus on across-user learning—each firm improves the product for each consumer based on what it learns from the usage of all its consumers. For example, traffic information in Google Maps updates based on information collected from all drivers, and the traffic information shown at any point in time is likely to be the same (or very similar) for everyone using Google Maps. The learning technology we consider in this section has the general property that the value to consumers of a given firm's product in a period is an increasing function of the measure of consumers who have purchased the product previously (past consumers) from the same firm. We will refer to the model described below as our baseline setting because it turns out that much of the model setup and analysis, as well as many of the results in this section carry over (with suitable adjustments) to when we allow for within-user learning (Section 4 and Section 5.3) and when we allow for learning to happen within the period(s) that users consume products in (Sections 5.1 and 5.2).

Two firms, which we refer to as the incumbent I and the entrant E, compete over infinitely many periods. Both firms face a marginal cost of production equal to c and compete in prices. We assume consumers have unit demand in each period and c is high enough that prices are never negative, thus ensuring consumers would only ever want to purchase one product in a given period. The (common) one period ahead discount factor is denoted δ, and satisfies $0 < \delta < 1$. We normalize the measure of consumers in each period to one, and so the number of periods that a firm has previously won also represents the total measure of consumers it can learn from. In this formulation of across-user learning, there is no distinction between consumers who are purchasing again (repeat consumers) vs. entirely new consumers. Thus, our setting in this section is compatible with two interpretations:

- Consumers live for one period only. New consumers in a period make a single buying decision and then exit.

- The same consumers are active and make purchasing decisions in every period, i.e. they are infinitely lived, but there is no cost for each of these individual consumers to switch from one firm to the other across periods. In this case, the learning from consumers when they purchase again is assumed to be the same as that from new consumers.

We assume all consumers are atomistic, which is reasonable given the large number of consumers firms are learning from in most of our examples. This means even if consumers are of the second type above, they do not need to consider the impact of their purchase decision on the pricing game or the options they will face in subsequent periods. Later, when we look at within-user learning (or more generally, endogenous switching costs), this will no longer be true.

The standalone value of firm i's product (or service) is denoted s_i, where $i \in \{I, E\}$. This value is augmented by learning. In any period, firms are asymmetric Bertrand competitors, so each firm will either sell to all (measure one) of the consumers or none. As a result, the measure of consumers a firm has learnt from is equivalent to the number of periods in which it has made positive sales. If firm i has sold in N_i periods in the past, then the value current consumers obtain from firm i's product is $s_i + f_i\left(\min\left\{N_i, \overline{N}_i\right\}\right)$, where f_i is firm i's learning function and $\overline{N}_i$ is the number of periods firm i must sell in to reach its maximum threshold of learning.[2]

This specification captures that firms learn more as they obtain more consumer data, and this allows them to increase the value of the product they offer, but at some point they cannot use consumer data to increase the value of the product offered any further (i.e. learning is bounded). Later in this section we will show how our results extend to handle unbounded learning functions. The only requirements on f_i are that it is everywhere weakly increasing,

[2] In what follows, since it will be equivalent, we will refer to the number of periods of positive sales as the measure of consumers the firm has sold to.

and without loss of generality, strictly increasing in the last step to reach its maximum threshold (i.e. $f_i(\overline{N}_i) > f_i(\overline{N}_i - 1)$). In particular, f_i need not be a continuous function. Thus, at any point in time there are potentially three sources of asymmetry between firms: (i) differences in standalone value ($s_I \neq s_E$), (ii) differences in the learning curve ($f_I \neq f_E$ and/or $\overline{N}_I \neq \overline{N}_E$), and (iii) differences in how far along each firm is on its learning curve in the current period ($N_I \neq N_E$).

Consumers and firms have perfect and complete information. Our focus is on Markovian strategies and a Markov-perfect equilibrium (MPE). Among MPEs, we also rule out equilibria which are supported by the losing firm pricing in such a way that its value would be negative if it instead won consumers at the prices charged in a particular period and then firms followed their equilibrium pricing strategies in all subsequent periods. This is equivalent to the standard approach of focusing on a Nash equilibrium in which the losing firm does not price below its marginal cost in the one-shot asymmetric Bertrand game. Thus, when we refer to a unique MPE we mean the MPE is unique after eliminating MPEs that involve such weakly dominated strategies. Finally, we use the convention that E wins in case consumers are indifferent when both firms price at the lowest level they would still be willing to sell at.

The relevant state at the start of any period is defined by (N_I, N_E), the measures of past consumers for each firm. The corresponding value functions for I and E (i.e. the present discounted values of future profit flows starting with the current period when firms follow their equilibrium pricing strategies in every period) are denoted by $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$. An implicit assumption in our setup is that a firm will continue to compete even if in equilibrium its value is zero. This is consistent with our assumption that firms do not face any fixed cost to remain in operation and ensures firms always face some form of competition. If instead firms incur fixed costs of operating, the competitive dynamics we model are best thought of as playing out in one segment of one market, but each of the two firms may also serve other market segments, so each firm covers its fixed costs via these other market segments. In practice the firms may also be subject to shocks which allow the losing firm to sometimes become the winner (e.g. the discovery of a new technology that allows them to use their data to reach the learning threshold) and this possibility provides another reason for a losing firm to remain in the market. We will analyze a version of our model with one such shock in Section 3.2, when we consider a data sharing policy.

Consider firms starting from the state $(\overline{N}_I, \overline{N}_E)$, so both firms have reached their respective learning thresholds. Then I offers $s_I + f_I(\overline{N}_I)$ to consumers in every period and E offers $s_E + f_E(\overline{N}_E)$ to consumers in every period. Given the resulting asymmetric Bertrand competition, and taking into account that the competitive situation will be the same in every subsequent period, E wins every period starting with the current one if and only if

$s_E + f_E(\overline{N}_E) \geq s_I + f_I(\overline{N}_I)$. Furthermore, for each firm $i \in \{I, E\}$, we must have

$$V^i(\overline{N}_I, \overline{N}_E) = \frac{1}{1-\delta} \max\left\{s_i - s_j + f_i(\overline{N}_i) - f_j(\overline{N}_j), 0\right\},$$

where $i \neq j$.

When a firm has not yet reached its learning threshold, in order to win in the current period it will be willing to offer a subsidy equal to the difference in the discounted value of its future profits if it were to win the current period and its future profits if it were to lose the current period. Taking into account the maximum subsidy each firm is willing to offer in order to win, we can work out which firm will win in each period and its corresponding value function, working backwards from $(N_I, N_E) = (\overline{N}_I, \overline{N}_E)$. Using this approach, we show that for any state (N_I, N_E), there is a cutoff level of $s_E - s_I$ above which E wins and below which I wins. We use two-dimensional backwards induction (in N_I and N_E) to establish the following Proposition (unless stated otherwise, all proofs are in the Appendix).

Proposition 1. *Suppose I has previously sold to* $0 \leq N_I \leq \overline{N}_I$ *consumers and E has previously sold to* $0 \leq N_E \leq \overline{N}_E$ *consumers. There exists a unique MPE in which E wins in all periods if and only if* $s_E - s_I \geq \Delta(N_I, N_E)$*, and I wins in all periods otherwise, where*

$$\begin{aligned} \Delta(N_I, N_E) &= (1-\delta)\left(\sum_{j=0}^{\overline{N}_I - N_I} \delta^j f_I(N_I + j) - \sum_{j=0}^{\overline{N}_E - N_E} \delta^j f_E(N_E + j)\right) \\ &\quad + \delta^{\overline{N}_I - N_I + 1} f_I(\overline{N}_I) - \delta^{\overline{N}_E - N_E + 1} f_E(\overline{N}_E). \end{aligned} \tag{1}$$

Proposition 1 characterizes I's competitive advantage, which is captured by $\Delta(N_I, N_E)$. The higher is $\Delta(N_I, N_E)$, the higher is the level of $s_E - s_I$ required for E to win. This function is proportional to the difference in the present discounted value (PDV) of gross surplus generated from learning across the two firms, comparing the paths where each firm wins in every period from the current period onwards. It is easily verified that $\Delta(N_I, N_E)$ is increasing in N_I and decreasing in N_E. This captures that along the equilibrium path, if a firm wins in the current period, because it benefits from an additional learning period, $\Delta(N_I, N_E)$ shifts such that the same firm will continue to win in each subsequent period, and indeed does so even more easily. This last feature is reminiscent of the "increasing increasing dominance" property explored by Cabral and Riordan (1994). Yet in our setting, E may win and so move ahead of I even though it is currently behind (e.g. $N_E < N_I$). Trivially, this could be because it learns more despite having less data (so $f_E(N_E) > f_I(N_I)$). More interestingly, even if $f_E(N_E) < f_I(N_I)$, this can be true if its learning curve is steeper and/or is increasing for longer, and so ultimately generates more value from getting ahead. For example, inspecting (1), it is easily seen that if $f_E(\overline{N}_E) > f_I(\overline{N}_I)$, then for all δ sufficiently close to 1 (i.e. whenever agents are sufficiently patient), we have $\Delta(N_I, N_E) < 0$, so E has an overall competitive advantage. Thus, with asymmetric learning curves, increasing dominance does not necessarily hold in our setting.

Proposition 1 was worked out assuming maximum learning thresholds $\overline{N}_I$ and $\overline{N}_E$ exist. The next Proposition extends these results to the case of any increasing learning curves by taking the limit of our results as $\overline{N}_I \to \infty$ and $\overline{N}_E \to \infty$, showing that our framework can also allow for unbounded learning functions.

Proposition 2. *Suppose I has previously sold to $0 \leq N_I < \overline{N}_I$ consumers, E has previously sold to $0 \leq N_E < \overline{N}_E$ consumers. The equilibrium defined in Proposition 1 remains an equilibrium in the limit as $\overline{N}_I \to \infty$ and $\overline{N}_E \to \infty$ (i.e. such that the learning functions continue to increase forever) provided there exists some power function which the learning curves lie below in the limit.*

The condition in Proposition 2 involves a very weak requirement that allows us to capture any reasonable learning function. Moreover, by the same logic underlying this result, all our subsequent characterizations of $\Delta(N_I, N_E)$, $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$ also extend to this unbounded case, by taking the limit as $\overline{N}_I \to \infty$ and $\overline{N}_E \to \infty$.

As part of the proof of Proposition 1 we characterize the value function for each firm, which we summarize here.

Corollary 1. *Suppose I has previously sold to $0 \leq N_I \leq \overline{N}_I$ consumers and E has previously sold to $0 \leq N_E \leq \overline{N}_E$ consumers. In the unique MPE, the firms' value functions are determined as follows:*

- *if $s_E - s_I < \Delta(N_I, N_E + 1)$ then $V^E(N_I, N_E) = 0$ and $V^I(N_I, N_E) = \frac{s_I - s_E}{1-\delta} + \frac{\Delta(N_I,N_E) - \delta\Delta(N_I,N_E+1)}{(1-\delta)^2}$ (no subsidization by E)*
- *if $\Delta(N_I, N_E + 1) \leq s_E - s_I < \Delta(N_I, N_E)$, then $V^E(N_I, N_E) = 0$ and $V^I(N_I, N_E) = \frac{s_I - s_E + \Delta(N_I,N_E)}{(1-\delta)^2}$ (subsidization by E)*
- *if $\Delta(N_I, N_E) \leq s_E - s_I < \Delta(N_I + 1, N_E)$, then $V^E(N_I, N_E) = \frac{s_E - s_I - \Delta(N_I,N_E)}{(1-\delta)^2}$ and $V^I(N_I, N_E) = 0$ (subsidization by I)*
- *if $s_E - s_I \geq \Delta(N_I + 1, N_E)$, then $V^E(N_I, N_E) = \frac{s_E - s_I}{1-\delta} - \frac{\Delta(N_I,N_E) - \delta\Delta(N_I+1,N_E)}{(1-\delta)^2}$ and $V^I(N_I, N_E) = 0$ (no subsidization by I),*

where we define $\Delta(\overline{N}_I+1, N_E) \equiv \Delta(\overline{N}_I, N_E)$ *for any* $0 \leq N_E \leq \overline{N}_E$ *and* $\Delta(N_I, \overline{N}_E+1) \equiv \Delta(N_I, \overline{N}_E)$ *for any* $0 \leq N_I \leq \overline{N}_I$.

Corollary 1 characterizes the PDV of the firms' equilibrium profits for different states (N_I, N_E) and different ranges of $s_E - s_I$. An interesting implication of the Corollary is that the losing firm is still willing to offer a subsidy to consumers for some range of parameters when the winning firm's competitive advantage is not too large. Thus, if $\Delta(N_I, N_E+1) \leq s_E - s_I < \min\left\{\Delta\left(\overline{N}_I, N_E+1\right), \Delta\left(N_I, N_E\right)\right\}$, then I wins and E subsidizes for a finite number of periods,[3] whereas if $\Delta(\overline{N}_I, N_E+1) \leq s_E - s_I < \Delta(N_I, N_E)$, then I wins and E subsidizes forever, i.e. even after I reaches its learning threshold. And symmetrically when E wins and I subsidizes. This reflects that in the off-equilibrium situation that the losing firm actually wins the current period, it expects to have a positive value (via some future profits). This implies the winning firm's competitive advantage will be limited by the losing firm's willingness to subsidize consumers in such cases. The willingness of a firm to offer a subsidy reflects the "prize" that the firm can obtain from winning, which is not only that it obtains an improvement in its product (or moves closer to obtaining this improvement) but that the other firm does not. This underlying logic is similar to that in standard learning-by-doing settings (e.g. Cabral and Riordan, 1994) but the price dynamics are different: the winning firm can charge more with data-enabled learning, whereas the winning firm would charge less in a setting equivalent to our model except that learning would decrease marginal costs instead. Finally, note that Corollary 1 does not rule out the winning firm also subsidizing consumers. Specifically, if $\Delta(N_I, N_E) \leq s_E - s_I < f_I(N_I) - f_E(N_E)$, then E wins and subsidizes in the current period (and symmetrically for I).

Finally, we analyze how the prospect of future learning impacts the firms' profits. Specifically, starting from (N_I, N_E), we ask whether firm profits are higher when $\overline{N}_I > N_I$ and $\overline{N}_E > N_E$, so there is scope for future learning, or when $\overline{N}_I = N_I$ and $\overline{N}_E = N_E$, so there is no scope for future learning. We answer this question for two special cases in the following proposition.

Proposition 3. *If* $f_E = f_I = f$, $\overline{N}_I = \overline{N}_E = \overline{N}$ *and* $N_E = N_I < \overline{N}$, *then for any* $s_E \neq s_I$, *the PDV of profits of the winning firm (and of joint profits) is higher than if there were no scope for future learning. If, instead, there is some asymmetry in the learning functions and* $s_E - s_I \neq f_I(N_I) - f_E(N_E)$, *then there exists* $\varepsilon > 0$, *such that for any* (s_E, s_I) *such that* $|s_E - s_I - \Delta(N_I, N_E)| < \varepsilon$, *the PDV of joint profits is lower than if there were no scope for future learning.*

[3] Specifically, E subsidizes for the first k periods, where $k \leq \overline{N}_I - N_I$ is uniquely defined by the inequality $\Delta(N_I+k, N_E+1) < s_E - s_I < \Delta(N_I+k+1, N_E+1)$.

The first part of the proposition says that when firms have identical learning functions and start on the same point on the learning function, removing the prospect of future learning decreases the profits of the winner. This is in contrast with the result in Cabral and Riordan (1994), according to which symmetric firm are always better off when the prospect of future learning-by-doing is removed. The difference is explained by the presence of vertical differentiation independent of learning in our model ($s_E \neq s_I$), which is absent from Cabral and Riordan. Thus, when firms are symmetric in their learning functions in our model, the availability of future learning necessarily increases the magnitude of vertical differentiation that is eventually achieved by the winner (I if $s_I > s_E$ and E if $s_E > s_I$). This effect co-exists with the effect identified in Cabral and Riordan, namely that future learning increases the firms' willingness to subsidize, i.e. price more aggressively, which tends to decrease the winner's profits. When firms are symmetric in their learning functions in our setting, the former effect dominates the latter (unless $s_I = s_E$), so we obtain the opposite result from Cabral and Riordan. However, when firms are asymmetric, the availability of future learning no longer necessarily increases vertical differentiation between the two firms. In fact, it may even decrease it, as illustrated by the second part of the proposition.

3.1 Welfare analysis

Since the cutoff (1) in Proposition 1 is directly proportional to the difference in the PDV of gross surplus generated from learning if I wins in every period vs. if E wins in every period, we have the following result.

Proposition 4. *The unique MPE outcome characterized in Proposition 1 is socially optimal.*

Starting from any point, whichever firm creates greater PDV of gross surplus from standalone value and learning assuming it wins every period should be able to offer a higher level of current utility plus subsidy to consumers, and so given the Bertrand setting should be able to win in the current period. And if a particular firm wins in the current period, this just further increases the value it can offer in future periods, thereby ensuring the same firm wins in all future periods. However, the logic behind the efficiency result in Proposition 4 is more subtle than this. Indeed, the winning firm does not always capture the full PDV of gross surplus that it creates over and above that created by the losing firm because it has to compete with a firm that will sometimes be willing to subsidize in an attempt to win. Nevertheless, the PDV of the subsidy offered by the losing firm is just the PDV of its own future profit assuming it is able to win in the current period. Thus, despite the possibility of subsidies, it turns out that the only way a firm wins (and has a positive value) is if the

PDV of gross surplus that it creates by winning every period is greater than that created by the rival firm. This is equivalent to the condition for efficiency to be maximized.

Further emphasizing that the result does not just follow automatically from Bertrand competition, the cutoff for E to win in a finite period version of the model is not generally socially optimal. Indeed, in an *Online Appendix* we show that the equilibrium cutoff when the firms start with (N_I, N_E) previous sales and the game ends after $T \geq 1$ periods is

$$\Delta(N_I, N_E, T) = (1-\delta)\left(\sum_{j=0}^{T-1} \frac{\delta^j(1-\delta^{T-j})}{1-(T+1)\delta^T + T\delta^{T+1}}(f_I(N_I+j) - f_E(N_E+j))\right),$$

while the socially efficient cutoff in the T-period model is

$$\Delta^S(N_I, N_E, T) = (1-\delta)\left(\sum_{j=0}^{T-1} \frac{\delta^j}{1-\delta^T}(f_I(N_I+j) - f_E(N_E+j))\right).$$

When $T > 1$, we generally have $\Delta(N_I, N_E, T) \neq \Delta^S(N_I, N_E, T)$, however it is easily verified that when $f_i(N_i) = f_i\left(\min\left\{N_i, \overline{N}_i\right\}\right)$ for $i = I, E$ and T tends to infinity, both $\Delta(N_I, N_E, T)$ and $\Delta^S(N_I, N_E, T)$ converge to $\Delta(N_I, N_E)$ defined in (1).

In the finite case with $T > 1$, the distortion $\Delta(N_I, N_E, T) - \Delta^S(N_I, N_E, T)$ can go in either direction depending on the shapes of the learning curves. For example, it is straightforward to verify that with $f_I = f_E \equiv f$ and $N_I > N_E$, this distortion is positive if f is concave, negative if f is convex, and zero if f is linear. On the other hand, if $f_i(x) = \theta_i x$ and $N_I \geq N_E$, the distortion is positive if $\theta_I < \theta_E$, negative if $\theta_I > \theta_E$, and zero if $\theta_I = \theta_E$.

Clearly prices play a key role in the efficiency result of Proposition 4. If prices were constrained to be zero in every period, there would be no subsidies, meaning consumers would compare $s_I + f_I(N_I)$ and $s_E + f_E(N_E)$ in the current period, and choose E in every period if and only if $s_E - s_I \geq f_I(N_I) - f_E(N_E)$. Future learning benefits would not be taken into account and the equilibrium choice of firm would diverge from the socially optimal one. This emphasizes that competitive subsidies play a key role in ensuring atomistic consumers who make their purchase decisions in every period solely on the basis of current period utility (because even in the case that they are long-lived, they face no cost of switching firms in any future period and can ignore the implications of their individual choice on the future options they will face), end up taking into account the future benefits of their choices in deciding which firm to buy from.

With respect to consumer surplus, note that in equilibrium, the firms' prices in any given period leave consumers indifferent between the two firms. Since along the equilibrium path, once a firm wins, it will win in all future periods, this implies that the PDV of consumer surplus is determined by the surplus offered by the losing firm in each period along the equilibrium path. As a result, learning by the winning firm as it sells to additional consumers each period makes consumers weakly worse off. To see this note first that if the losing firm has already stopped offering a subsidy (e.g. the no-subsidization case in Corollary 1), then learning by the winner does not change the PDV of consumer surplus offered by the loser, which in case firm i is the losing firm is fixed at

$$CS\left(N_I, N_E\right) = \frac{s_i - c + f_i(N_i)}{1-\delta}.$$

In case the losing firm currently offers a subsidy (either one of the two subsidization cases in Corollary 1), the PDV of consumer surplus can be determined by the PDV of total surplus less the PDV of the winning firm's profit stream, i.e.

$$CS\left(N_I, N_E\right) = \frac{s_i - c}{1-\delta} + \sum_{k=0}^{\overline{N}_i - N_i} \delta^k f_i(N_i + k) + \frac{\delta^{\overline{N}_i - N_i + 1}}{1-\delta} f_i(\overline{N}_i) - \left(\frac{s_i - s_j + \Delta(N_I, N_E)}{(1-\delta)^2}\right)$$

in case firm i is the winning firm and firm j is the losing firm. It is straightforward to check that in this case $CS\left(N_I, N_E\right)$ is decreasing in N_i, so consumer surplus is strictly decreasing as the winning firm learns more along the equilibrium path. This reflects that the losing firm's subsidy gets smaller over time as the winning firm's learning advantage continues to increase along the equilibrium path.

The fact that additional learning by the winning firm as it sells to more consumers does not help consumers reflects the fact that in this model, the winning firm extracts all the additional learning benefit above the surplus that the losing firm can offer. While with a more general model of competition or with elastic aggregate demand we may expect some of the winning firm's learning benefit to be shared with consumers, the result that consumers can be made worse off by the winning firm's learning because the losing firm is no longer willing to subsidize as much in an attempt to compete is likely to be much more general. It suggests consumers would be better off if they could somehow deviate by coordinating on joining the losing firm instead of the winning firm for a number of periods. However, given each individual consumer is atomless and can freely switch in each period, she has no reason to internalize this effect since her own decision about which firm to join will not affect the consumer surplus she can obtain next period (in the case she is long lived). This suggests a possible role for a data sharing policy, which we consider in the next section.

3.2 Data sharing

Requiring the incumbent to share its data with the entrant would be one way to help E catch up to the learning of I, and based on the analysis in the previous section, would seem to be good for consumers. However, there is potentially a countervailing effect highlighted by our model. If E anticipates being able to catch up to I's learning through such a policy, it may compete less aggressively in the first place (a form of free-riding), which in our model can mean subsidizing consumers less.

To formalize this tradeoff in the simplest possible way and determine the effect of data sharing on consumer surplus, we assume both firms face the same learning function f and learning threshold $\overline{N}$, and that I has achieved maximal learning while E has not. For the anticipatory effect of data sharing on E's pricing to be captured, there should be at least one period in which firms compete before I's data is shared with E. To reflect this and the likely uncertainty over the time until E actually obtains I's data under a data sharing policy, we assume that each period, with probability $0 < \lambda \leq 1$, I shares its superior data with E whenever E remains behind at the end of that period.

In the following proposition, we compare the implied PDV of consumer surplus with data sharing to that arising without data sharing. To proceed, we define for every $0 \leq N_E < \overline{N}$ the cutoffs[4]

$$\Delta(\overline{N}, N_E, \lambda) = (1 - \delta(1-\lambda)) \sum_{j=0}^{\overline{N}-N_E-1} (\delta(1-\lambda))^j \left(f(\overline{N}) - f(N_E + j)\right). \tag{2}$$

and

$$\Theta^a\left(\overline{N}, N_E, \lambda\right) = \frac{\frac{\Delta(\overline{N}, N_E, 0)}{(1-\delta)^2} - \frac{\Delta(\overline{N}, N_E, \lambda)}{(1-\delta(1-\lambda))^2}}{\frac{1}{(1-\delta)^2} - \frac{1}{(1-\delta(1-\lambda))^2}}. \tag{3}$$

It can be verified that

$$0 \leq \Delta(\overline{N}, N_E + 1, \lambda) \leq \Theta^a\left(\overline{N}, N_E, \lambda\right) < \Delta(\overline{N}, N_E, \lambda).$$

In the appendix, we show that when I is at the learning threshold and E can get the full benefit of data sharing with probability λ, E wins the current period if and only if $s_E - s_I \geq \Delta(\overline{N}, N_E, \lambda)$. Otherwise, I wins the current period. However, unlike the baseline model, the same firm need not win all periods. Indeed, if $0 \leq s_E - s_I < \Delta(\overline{N}, N_E, \lambda)$, then I wins every period as long as E does not get access to I's data, but once E shares that data, the outcome switches to E winning forever. We then obtain the following result.

[4] We use the convention that $\Delta(\overline{N}, \overline{N}, \lambda) = 0$.

Proposition 5. *Suppose the learning functions are identical, i.e. $f_I = f_E = f$, and assume $N_I = \overline{N} > N_E$. Then:*

- *if $s_E - s_I \leq \Theta^a\left(\overline{N}, N_E, \lambda\right)$, data sharing with probability λ increases the PDV of consumer surplus*
- *if $\Theta^a\left(\overline{N}, N_E, \lambda\right) < s_E - s_I \leq \Delta(\overline{N}, N_E, \lambda)$, data sharing with probability λ decreases the PDV of consumer surplus*
- *if $s_E - s_I \geq \Delta(\overline{N}, N_E, \lambda)$, data sharing with probability λ does not change the PDV of consumer surplus.*

Proposition 5 compares the PDV of consumer surplus when $\lambda > 0$ (i.e. under a data sharing policy) with $\lambda = 0$ (so there is no chance of I being forced to share its data with E). It confirms that data sharing is not always good for consumers, and indeed shows that it can be bad. For data sharing to be bad for consumers, E must have a sufficiently strong product advantage (i.e. $s_E - s_I > \Theta^a\left(\overline{N}, N_E, \lambda\right)$). This ensures the subsidy it offers absent data sharing is large, which means the extent to which data sharing can reduce E's subsidy is also large. But the advantage cannot be so strong that E would win prior to any data being shared (i.e. we require $s_E - s_I \leq \Delta(\overline{N}, N_E, \lambda)$). In this range, the anticipation of possible data sharing leads E to reduce the extent of its subsidy to consumers. For example, in the extreme case in which data sharing happens for sure at the end of the first period ($\lambda = 1$), E completely removes its subsidy to consumers under a data sharing policy. Given we are considering the range of $s_E - s_I$ for which E loses prior to any data being shared, the higher pricing of E reduces the competitive pressure on I, and so increases the price consumers pay for I's offering. On the other hand, consumers will be better off *after* I's data is shared, reflecting that I will share more of the surplus it creates with consumers—this is either because I still wins but E now provides a stronger competitive constraint on I's pricing, or because I now loses and so is willing to offer all of the surplus it creates in an attempt to attract consumers. In the range of $\Theta^a\left(\overline{N}, N_E, \lambda\right) < s_E - s_I < \Delta(\overline{N}, N_E, \lambda)$, the former negative ex-ante effect of data sharing on consumers outweighs the latter ex-post positive effect, reflecting that data sharing leads to a large reduction in E's subsidy in this range. In the range $\Delta(\overline{N}, N_E+1, \lambda) \leq s_E - s_I < \Theta^a\left(\overline{N}, N_E, \lambda\right)$ this result reverses because data sharing only leads to a relatively small reduction in E's subsidy. In case $s_E - s_I < \Delta(\overline{N}, N_E+1, \lambda)$, E never subsidizes consumers, so we only get the positive ex-post effect of data sharing. Finally, when $s_E - s_I \geq \Delta(\overline{N}, N_E, \lambda)$, E wins prior to any data being shared, and it continues to win after I's data is shared, so consumers obtain the surplus created by I which is unchanged by data sharing.

While consumers may be better or worse off under data sharing, it is straightforward to confirm that the PDV of expected welfare is strictly higher with this type of data sharing whenever it implies a positive probability of E winning (since E's surplus is shifted up in the case any data is shared), and is otherwise unchanged.

3.3 Determinants of competitive advantage

Recall I's competitive advantage is measured by $\Delta(N_I, N_E)$ since a higher value of $\Delta(N_I, N_E)$ means E requires a higher standalone benefit to win. Some general comparative static results are summarized in the following Proposition.

Proposition 6. *Consider the general learning functions f_I and f_E. Suppose $0 \leq N_I \leq \overline{N}_I$ and $0 \leq N_E \leq \overline{N}_E$. Then I's competitive advantage is (i) increasing in the number of previous consumers N_I that I has sold to and decreasing in the number of previous consumers N_E that E has sold to; (ii) increasing in the maximum threshold for I's learning $\overline{N}_I$ and decreasing in the maximum threshold for E's learning $\overline{N}_E$; (iii) increasing in anything that increases I's learning function f_I and decreasing in anything that increases E's learning function f_E; and (iv) independent of an equal (additive) increase in both firms learning functions.*

The result in (i) was already noted earlier and is entirely intuitive—being higher up its learning curve gives I more of an advantage. The results in (ii) and (iii) follow directly from an inspection of (1). Simply put, the higher a firm's learning threshold or the value the firm gains from learning, the bigger that firm's competitive advantage. Adding the same constant to both f_I and f_E does not change $\Delta(N_I, N_E)$, so any additive change to the value of learning that affects both firms equally cancels out, which explains the result in (iv).

To explore the implications of changes that affect both firms at the same time, we now resort to a specific functional form by assuming both firms face S-shaped learning functions. In general, with an S-shaped learning curve, the value initially increases little with additional data since it takes some initial minimum corpus of data to get things started, then increases rapidly once the minimum training data has been gathered, and finally no longer increases much after most learning has been exhausted. We approximate such a learning curve by assuming there is no learning until a minimum amount of past sales is obtained, after which there is linear learning until the maximum threshold level of learning is achieved. Formally, we assume when firm i has sold to N_i past consumers, the value it offers from learning is $f_i\left(\min\left\{N_i, \overline{N}_i\right\}\right) = \theta_i \max\left\{\min\left\{N_i, \overline{N}_i\right\} - \underline{N}_i, 0\right\}$, where $\underline{N}_i$ and $\overline{N}_i$ are the minimum and maximum thresholds for firm i and $0 \leq \underline{N}_i \leq \overline{N}_i$. Note that this formulation of S-shaped learning functions includes as special cases that one or both of the learning functions are linear ($\underline{N}_i = 0$) and one or both are step functions ($\overline{N}_i - \underline{N}_i = 1$ and $\underline{N}_i > 0$).

Applying these functions to (1) reveals that the cutoff for E to win in this case is

$$\begin{aligned} \Delta(N_I, N_E) &= \theta_I \left(\max\{N_I - \underline{N}_I, 0\} + \frac{\delta^{\max\{\underline{N}_I - N_I, 0\}+1} - \delta^{\overline{N}_I - N_I + 1}}{1-\delta} \right) \\ &\quad -\theta_E \left(\max\{N_E - \underline{N}_E, 0\} + \frac{\delta^{\max\{\underline{N}_E - N_E, 0\}+1} - \delta^{\overline{N}_E - N_E + 1}}{1-\delta} \right), \end{aligned} \tag{4}$$

for all $0 \leq N_I \leq \overline{N}_I$ and $0 \leq N_E \leq \overline{N}_E$. Note (4) implies that when neither firm has reached the increasing part of its learning curve yet, what really matters is how far each firm is from being able to move up its learning curve $\underline{N}_i$, rather than which firm has sold to more customers in the past. Thus, even though I may begin with more data ($N_I > N_E$), if E's learning begins sooner ($\underline{N}_E - N_E < \underline{N}_I - N_I$), it may enjoy a competitive advantage. Likewise, if additional data translates into a better product for E at a faster rate than for I ($\theta_E > \theta_I$).

Using this cutoff we are able to show the following comparative statics.

Proposition 7. *Suppose both firms have S-shaped learning functions. (i) I's competitive advantage is increasing in an equal additive increase in θ_I and θ_E if I is (weakly) closer to reaching its maximum threshold and the increasing part of I's learning curve lasts (weakly) longer (i.e. $\overline{N}_I - N_I \leq \overline{N}_E - N_E$ and $\overline{N}_I - \underline{N}_I \geq \overline{N}_E - \underline{N}_E$, with one inequality strict). (ii) If the learning functions are identical (i.e. $\theta_I = \theta_E$, $\underline{N}_I = \underline{N}_E = \underline{N}$, $\overline{N}_I = \overline{N}_E = \overline{N}$) and $N_I > N_E$, then I's competitive advantage is increasing in how close both firms are to obtaining some positive value from learning when neither firm has learnt anything yet (i.e. is larger when both N_I and N_E increase by k, provided $N_I + k \leq \underline{N}$ and $N_E + k \leq \underline{N}$) and decreasing in how close both firms are to obtaining maximum learning when both have already obtained some positive value from learning (i.e. is smaller when both firms are k periods closer to the maximum learning threshold provided $N_I > \underline{N}$ and $N_E > \underline{N}$).*

The first result covers several different cases including the case when the firms have the same thresholds but I is just further along its learning curve, and the case when both firms are the same number of periods away from reaching their respective minimum thresholds but the increasing part of I's learning curve lasts longer. The logic for the result is that if I has had more periods of learning and/or will have more periods of future learning, then an equal increase in the value obtained from each learning period for the two firms provides an advantage for I. The result in (ii) is less obvious. The first part can be interpreted as considering what happens when both firms experience some improvement in their learning (e.g. access to some new data or some new technology that allows them to speed up their initial learning) so that both firms find themselves further along the initial flat part of their learning curve (i.e. closer to where they can offer positive value from learning). Alternatively, it can be interpreted as considering what happens when the learning technology improves so firms require fewer periods of initial consumer data until the value from learning starts to increase. Either way, it means that this common improvement in learning increases I's competitive advantage. To understand this result note that if firms are yet to move onto the increasing part of the learning curve, requiring fewer periods of customer data to reach the increasing part brings forward (i.e. discounts less) the advantage that I obtains from being closer to the threshold, thereby increasing its competitive advantage. This result reverses when firms are already on the increasing part of the learning curve. Here, a reduction in the number of learning periods both firms need to reach the maximum threshold decreases the number of periods over which I can enjoy its learning advantage, so helps E.

4 Within-user learning

In this section we focus on within-user learning. To be comparable to the case of across-user learning, we assume the same measure one of consumers are present in every period. Thus, consumers are infinitely lived but can switch firms in any period.

Like the previous section, the value to a given consumer of firm i's product in the current period is $s_i + f_i\left(\min\left\{N_i, \overline{N}_i\right\}\right)$ for $i \in \{I, E\}$. The difference is that here N_i refers to the number of times the consumer has purchased from the same firm before, rather than the total number of consumers that the firm has sold to. Another key difference that naturally arises in the case of within-period learning is the possibility for firms to price discriminate based on an individual consumer's history. Unlike the case with across-user learning, here the individual consumer's history affects the value offered by each firm's product. Consistent with our perfect information setup, we assume that both firms can observe how many periods a customer has consumed from each firm previously, and can price discriminate accordingly.[5] The fact that there is no across-user learning and that firms can price discriminate with respect to individual consumers ensures that the analysis can be done independently for each individual consumer.

[5]Obviously, each firm will know how many times a consumer has purchased from it before, so can condition its prices on this information. Given the model setup in which the same consumers are present in every period, each firm would therefore also be able to work out how many times a consumer has purchased from the rival firm in the past. In practice, firms may elicit this information from consumers by requiring a consumer supply their past purchase records so as to provide them with a better deal if they switch.

We can then characterize when E wins, when I wins, the social efficiency of the outcome, and the value obtained by firms.

Proposition 8. *Suppose I has sold N_I times in the past to a given consumer and E has sold N_E times to that consumer, where $0 \leq N_I \leq \overline{N}_I$ and $0 \leq N_E \leq \overline{N}_E$. Then a unique MPE exists in which E wins in all periods if and only if $s_E - s_I \geq \Delta(N_I, N_E)$, and I wins in all periods otherwise, where $\Delta(N_I, N_E)$ is defined by (1). The outcome is socially optimal. Moreover, the firms' value functions are determined as follows:*

- *if $s_E - s_I < \Delta(N_I, N_E)$, then $V^E(N_I, N_E) = 0$ and $V^I(N_I, N_E) = \frac{s_I - s_E + \Delta(N_I, N_E)}{1-\delta}$ (with subsidization by E if and only if $\Delta(N_I, N_E + 1) \leq s_E - s_I < \Delta(N_I, N_E)$)*
- *if $s_E - s_I \geq \Delta(N_I, N_E)$, then $V^E(N_I, N_E) = \frac{s_E - s_I - \Delta(N_I, N_E)}{1-\delta}$ and $V^I(N_I, N_E) = 0$ (with subsidization by I if and only if $\Delta(N_I, N_E) \leq s_E - s_I < \Delta(N_I + 1, N_E)$).*

Proposition 8 shows that the cutoff in $s_E - s_I$ that determines which firm wins in each period coincides with that obtained in the across-user case, and therefore remains socially optimal. As in the case with across-user learning, all the results in the Proposition remain valid when $\overline{N}_I \to \infty$ and $\overline{N}_E \to \infty$, provided learning functions are bounded by some power function. This means the comparative static results in Section 3.3 carry over to the case of within-user learning. However, despite this equivalence, the logic behind the result and its proof are quite different. With across-user learning, we never had to solve the consumers' forward-looking problem given their choice in any given period didn't affect their options in subsequent periods. In contrast, here consumers have to take into account the effect of their current choice of firm on their future options (within-user learning acts as an endogenous switching cost), and so we need to determine the value functions for consumers as well as for firms. In the proof of Proposition 8, we construct the consumers' value functions, and show that consumers never expect to switch firms in equilibrium.

In contrast to the case of across-user learning, the logic for why the equilibrium outcome under within-user learning coincides with the socially optimal outcome is quite straightforward. Here consumers are forward looking, so they take into account all future subsidies and utility they will receive from their chosen firm. Starting from any point, whichever firm creates greater PDV of gross surplus from learning assuming it wins every period will be able to offer a higher level of utility plus subsidy to consumers, and so given the Bertrand setting will win in the current period. And if a particular firm wins in the current period, this just further increases the value it can offer in future periods, thereby ensuring the same firm wins in all future periods. Consistent with this, in the Online Appendix we confirm that the outcome with within-user learning and a finite time horizon is also always socially optimal, in contrast to what we found for across-user learning.

To further highlight the fact that although the cutoff is the same, the rationale for the result is different, note that prices no longer drive the equilibrium result. Indeed, if prices were set to zero in all periods (which obviously rules out any subsidies), consumers would simply compare their respective present discounted willingness-to-pay between the two firms, so the condition for E to win would remain the same.

Another particularly interesting difference relative to the results with across-user learning is that the winning firm always does better with across-user learning than with the equivalent setting under within-user learning.

Proposition 9. *Comparing the case of within-user learning with across-user learning for the same learning curves, the same learning thresholds, and the same current state* (N_I, N_E)*, then provided either* $N_I < \overline{N}_I$ *or* $N_E < \overline{N}_E$*, the PDV of the winning firm's profit is strictly higher under across-user learning than in the case of within-user learning.*

The main reason behind the result in Proposition 9 is that the winning firm needs to price more aggressively under within-user learning because consumers need to take into account future utility when making their choice of firm. The logic is that within-user learning acts as an endogenous switching cost: consumers take into account that the surplus they will be left with in any period is the surplus offered by the losing firm (due to Bertrand competition), and that this surplus will be lower if they buy from the winning firm this period versus if they buy from the losing firm this period. Thus, the winning firm must compensate consumers by pricing lower in the current period compared to the case with across-user learning, in which consumers did not need to take into account future utility when making their choice of firm. Indeed, with across-user learning, each individual consumer's choice in the current period had no impact on the relative competitiveness of the two firms next period (since consumers are atomistic). Obviously, Proposition 9 implies that consumers are better off under within-user learning than under the comparable situation with across-user learning. This result can also be interpreted as meaning that under within-user learning, forward-looking consumers are better off than myopic consumers. Indeed, the setting with within-user learning but myopic consumers would result in exactly the same outcome as the case with across-user learning, where consumers had no reason to consider more than their current surplus when deciding which firm to buy from.

Finally, we can compare the effect of anticipated data sharing under within-user learning with that under across-user learning. To do so, we use exactly the same set-up for data sharing as in Section 3.2. Following very similar steps to Proposition 5, we show in the Appendix that data sharing decreases consumer surplus if and only if

$$\Theta^w\left(\overline{N}, N_E, \lambda\right) < s_E - s_I < \Delta(\overline{N}, N_E, \lambda), \tag{5}$$

where $\Delta(\overline{N}, N_E, \lambda)$ is still given by (2) and

$$\Theta^w\left(\overline{N}, N_E, \lambda\right) = \frac{\frac{\Delta(\overline{N}, N_E, 0)}{1-\delta} - \frac{\Delta(\overline{N}, N_E, \lambda)}{1-\delta(1-\lambda)}}{\frac{1}{1-\delta} - \frac{1}{1-\delta(1-\lambda)}}. \tag{6}$$

If $s_E - s_I$ is less than the lower bound in (5), then data sharing increases consumer surplus, while if $s_E - s_I$ is greater than the upper bound in (5), then data sharing has no impact on consumer surplus. It is easily confirmed that $\Theta^w\left(\overline{N}, N_E, \lambda\right)$ is lower than the corresponding cutoff $\Theta^a\left(\overline{N}, N_E, \lambda\right)$ for the case with across-user learning given by (3), which means that the region under which data sharing decreases consumer surplus is larger under within-user learning. This is consistent with the result above that within-user learning makes the two firms compete more aggressively in the absence of data sharing, which diminishes the potential benefits of data sharing.

5 Endogenous network effects and consumer beliefs

So far network effects have not played any role in the equilibrium analysis. In particular, consumers have not had to form beliefs about which firm other consumers will coordinate on in any given period. In Section 4 this was because learning was specific to each consumer and firms could price discriminate with respect to individual consumers. With across-user learning in Section 3, although there was a self-reinforcing dynamic across periods in which a firm that attracts consumers this period improves its product and finds it easier to attract consumers in subsequent periods, current consumers still did not face the usual coordination problem that arises under classic network effects. Indeed, even with long-lived consumers, because each consumer was atomistic and could costlessly switch each period, a consumer's decision in one period had no effect on the opportunities they could enjoy in subsequent periods, so there was no reason to take into account the decisions of contemporary consumers.

The purpose of this section is to show when network effects which create a coordination problem for consumers can arise endogenously in our framework. We will consider three ways in which this can happen. In the first two subsections, we assume that the product continues to improve during the period (Section 5.1) or periods (5.2) over which consumers derive utility from the product. This means each consumer needs to anticipate how many other contemporary or future consumers will choose the same product. Meanwhile (and most interestingly), in Section 5.3 we show network effects arise endogenously once we combine across-user and within-user learning, even in the absence of any exogenous switching cost or price (the products are assumed to be available for free).

5.1 Across-user learning and within-period learning

Consider our benchmark model of Section 3 in which there is across-user learning, but now allow firms to learn immediately from consumers purchasing in the current period, i.e. we allow for "within-period" learning. As argued in the introduction, within-period learning is a key distinguishing feature of cloud-based learning from consumers, and contrasts with earlier types of learning from consumers, where the new product features were only embedded in the next releases of the product which would be sold to *future* consumers. Within-period learning is assumed to work in exactly the same way as learning from past sales to consumers: it is just like another period of learning, except it provides value to current consumers. Specifically, if firm i has previously sold to a total measure of N_i consumers and sells to $n_i \leq 1$ consumers in the current period, then the value current consumers obtain from firm i's product is $s_i + f_i\left(\min\left\{N_i + n_i, \overline{N}_i\right\}\right)$, where $\overline{N}_i$ is the number of periods of sales (including the current period) it takes for firm i to reach the maximum threshold of learning.

A consequence of combining within-period learning and across-user learning is that there can be multiple equilibria in consumers' decision of which firm to buy from, given prices in any period, just like with regular network effects. Specifically, consumers may prefer buying from I if they expect all other consumers to buy from I in the current period, and they may prefer to buy from E if they expect all other consumers to buy from E in the current period. We will compare the outcomes under two types of beliefs. First, we assume that consumers believe all other consumers will coordinate on the equilibrium outcome that is best for them in the given period. This selects the equilibrium which maximizes the consumers' surplus (i.e. net utility) in that period. In the case consumers live for one period only, which recall is one interpretation of our across-user learning setup, this will indeed correspond to the Pareto optimal equilibrium for consumers, and with this in mind, we refer to these beliefs as "Pareto" beliefs. We will then contrast Pareto beliefs with beliefs that always favor the incumbent. The latter means that in each period consumers believe all other consumers will buy from I whenever for given prices, buying from I is an equilibrium outcome.

Relative to Proposition 1, the case of within-period learning and Pareto beliefs is as if both firms were one period closer to their respective maximum learning thresholds, whereas the case of within-period learning and beliefs favoring I is as if only I were one period closer to its maximum learning threshold. Then following an almost identical proof to that of Proposition 1 we obtain the following result.

Proposition 10. *Suppose I has previously sold to $0 \leq N_I \leq \overline{N}_I$ consumers and E has previously sold to $0 \leq N_E \leq \overline{N}_E$ consumers. With Pareto beliefs, E wins in all periods if and only if $s_E - s_I \geq \Delta(N_I + 1, N_E + 1)$, and I wins in all periods otherwise, where $\Delta(N_I, N_E)$ is defined by (1). With favorable beliefs for I, E wins in all periods if and only if $s_E - s_I \geq \Delta(N_I + 1, N_E)$, and I wins in all periods otherwise. The equilibrium outcome is socially efficient with Pareto beliefs and involves I winning too often with beliefs favoring I.*

The intuition for these results is the same as before. Note the socially optimal outcome takes into account the higher willingness-to-pay created from learning within the period, just as the equilibrium conditions do. Moreover, Corollary 1 and Corollary **??** continue to apply, given that the cutoffs that apply here are defined in the same way as (1), but with the terms in N_I and N_E increased by zero or one depending on the type of beliefs. By the same logic, other results similarly carry over.

Taking the difference in cutoffs when beliefs favor I vs. when beliefs are Pareto, we obtain

$$\Delta(N_I+1, N_E) - \Delta(N_I+1, N_E+1) = (1-\delta)\left(\sum_{j=0}^{\overline{N}_E - N_E - 1} \delta^j \left(f_E(N_E + j + 1) - f_E(N_E + j)\right)\right), \tag{7}$$

which is strictly positive whenever $N_E < \overline{N}_E$.[6] The expression in (7) is a measure of the distortion that arises in the cutoff for E to win due to I's favorable beliefs, as compared to the socially optimal outcome. It shows that whether I is close or far away from its learning threshold is irrelevant to the distortion implied by I enjoying favorable beliefs. This is because under both types of beliefs, consumers evaluate the PDV of future product improvements that can be offered by I assuming I will win in every period. The expression (7) also illustrates that the impact of beliefs on I's competitive advantage reflects that E is effectively viewed by consumers as being one period of learning behind in every period under beliefs that favor I compared to under Pareto beliefs.

Taking the difference between (7) evaluated at $N_E - k$ and (7) evaluated at N_E, the following result follows almost immediately.

Proposition 11. *Suppose there is across-user learning and within-period learning. If E's learning function f_E is weakly concave, the distortion in the cutoff resulting from beliefs favoring I is higher the further away E is from its maximum learning threshold (i.e. the lower N_E is).*

[6] When $N_E = \overline{N}_E$, beliefs are irrelevant, and (7) is zero.

This result reflects that shifting E back by one period of learning has a bigger effect when E is on the steeper (i.e. earlier) part of its learning function, and that the corresponding disadvantage for E arises over additional periods when E is further away from its maximum learning threshold. Put more directly, under the conditions of Proposition 11, the concern that beliefs favoring I give I an inefficient advantage is greater the further away E is from its maximum learning threshold.

5.2 Across-user learning with one-time purchase

Consider the model of pure across-user learning from Section 3, but now assume consumers only need to purchase once to derive consumption utility over all subsequent periods based on the value of the product offered in each period. Thus, we have in mind products that can be used over multiple periods of time, e.g. operating systems, software, apps and connected devices that are purchased upfront and can be used repeatedly at no additional cost thereafter. We assume there is a measure one of new consumers joining in each period and all consumers are infinitely lived. These consumers purchase one of the two firms' products and get to enjoy the stream of future utility associated with their choice of product. Consumers could, if they wanted, switch to use the other product in any period, but this would require purchasing that product as well. Finally, consistent with our baseline model of across-user learning with new consumers in each period, a firm only learns once from any consumer it sells to. Formally, if firm i has sold to a total measure of N_i unique consumers in the past, then the value current consumers obtain from firm i's product is $s_i + f_i\left(\min\left\{N_i, \overline{N}_i\right\}\right)$, where $\overline{N}_i$ is the total measure of unique past sales it takes for firm i to reach the maximum threshold of learning.

Once again, there can be multiple equilibria in consumers' decision of which firm to buy from, given prices in any period. Here, network effects arise despite the absence of any within-period learning, and are due to the repeated usage consumers enjoy from a purchase, which creates a type of switching cost. If a consumer buys a product and then decides to switch to the other product in a later period, she needs to purchase the other product, whereas she could have continued to consume the existing product without any further payment. This means that, all other things equal, consumers prefer to buy the product which they expect will also be bought by current and future consumer cohorts, because that product will benefit from the most learning in the future. Because of this switching cost, each consumer now has an incentive to coordinate her purchase decision with the purchase decisions of the other consumers arriving in the same period.

Beliefs once again play an important role and, like in the previous subsection, we highlight their effect by comparing the outcomes with Pareto beliefs vs. beliefs that always favor I. The difference relative to the previous section is that here the beliefs of the current cohort of consumers must also include the behavior of future cohorts of consumers. Specifically, the current cohort of consumers believe future cohorts will choose the same firm as the current cohort. These beliefs always turn out to be fulfilled in equilibrium because the firm that wins the current period has a higher competitive advantage and therefore always wins in all future periods.

The proof, which is lengthy, follows similar steps as the proof of Proposition 1. We relegate it to the Online Appendix, where we establish the following proposition.

Proposition 12. *Suppose I has previously sold to* $0 \leq N_I \leq \overline{N}_I$ *unique consumers and E has previously sold to* $0 \leq N_E \leq \overline{N}_E$ *unique consumers. Under Pareto beliefs, E wins if and only if* $s_E - s_I \geq \Delta(N_I, N_E)$*, where*

$$\Delta(N_I, N_E) = (1-\delta)^2 \left(\sum_{j=0}^{\overline{N}_I - N_I - 1} (j+1)\delta^j f_I(N_I + j) - \sum_{j=0}^{\overline{N}_E - N_E - 1} (j+1)\delta^j f_E(N_E + j) \right)$$
$$+\delta^{\overline{N}_I - N_I}(1 + (1-\delta)(\overline{N}_I - N_I))f_I(\overline{N}_I) - \delta^{\overline{N}_E - N_E}(1 + (1-\delta)(\overline{N}_E - N_E))f_E(\overline{N}_E).$$

In contrast, under favorable beliefs for I, E wins if and only if $s_E - s_I \geq \Delta^I(N_I, N_E)$*, where*

$$\Delta^I(N_I, N_E) = (1-\delta)^2 \left(\sum_{j=0}^{\overline{N}_I - N_I - 1} (j+1)\delta^j f_I(N_I + j) - \sum_{j=0}^{\overline{N}_E - N_E - 1} \delta^j f_E(N_E + j) \right)$$
$$+\delta^{\overline{N}_I - N_I}(1 + (1-\delta)(\overline{N}_I - N_I))f_I(\overline{N}_I) - \delta^{\overline{N}_E - N_E} f_E(\overline{N}_E).$$

The cutoff in $s_E - s_I$ *above which E wins with Pareto beliefs is socially efficient and lower than the cutoff in* $s_E - s_I$ *above which E wins with beliefs favorable to I, i.e.* $\Delta(N_I, N_E) < \Delta^I(N_I, N_E)$ *for all* (N_I, N_E) *such that* $0 \leq N_I \leq \overline{N}_I$ *and* $0 \leq N_E \leq \overline{N}_E - 1$*.*

To understand this result, note that in each state, consumers evaluate the firms based on the utility they offer in the current period (which reflects the firms' additional learning), plus the subsidy each firm offers in the current period (reflecting how much each firm stands to benefit if it is able to capture consumers in the current period relative to the case when it loses the current period), plus the PDV of the future utility stream that the consumer expects will be offered in all subsequent periods. The firms' value functions are defined in the proof of Proposition 12. They turn out to be identical to the expressions in Corollary 1 except that the $\Delta(N_I, N_E)$ is now defined as above (and depends on whether beliefs are

Pareto or favor I), and that we need to divide everywhere by an additional $1-\delta$ in order to reflect that consumers get an infinite stream of future benefits from the product when they purchase. The $(j+1)$ factors in the expression of $\Delta(N_I, N_E)$ in the proposition reflect that now the learning levels $f_I(N_I+j)$ and $f_E(N_I+j)$ are enjoyed not just by the current cohort of consumers, but also by the next j cohorts.

Proposition 12 shows that beliefs clearly affect E's ability to win: E wins more often under Pareto beliefs relative to favorable beliefs for I. Moreover, like in the case of within-period learning analyzed in the previous subsection, favorable beliefs for I lead to a socially inefficient cutoff rule, in which I sometimes wins even though it would be efficient for consumers to buy from E.

5.3 Across-user and within-user learning

In this final section we demonstrate that by combining across-user and within-user learning we can generate network effects that create a coordination problem for consumers. In particular, we shut down the channels that led to such network effects in the previous two sections—within-period learning (Section 5.1), and switching costs that arise via prices (Section 5.2). We do so by combining the models of Sections 3 and 4 in the following way. First, we assume that the value a given consumer obtains from firm i's product in the current period depends on the two types of learning—the total measure of consumers who have previously purchased from the same firm, and the number of times the same consumer has previously purchased from the same firm. Second, we set the prices of both firms to zero in all periods to show that prices are not necessary to obtain our result (meaning our theory can also apply to free products, such as Google search). Thus, while consumers can only consume from one firm in a given period, they are free to switch between the two firms across periods. And finally, we assume there is just one fixed set of consumers of measure one who are present in all periods, consistent with the model of within-user learning used in Section 4.

A consumer who has previously consumed n_i times from firm $i \in \{I, E\}$ when firm i has previously served $N_i \leq 1$ of all consumers in the past derives utility $s_i + f^A(N_i) + f^W(\min\{n_i, \overline{n}\})$ in the current period if she chooses firm i's product, where $\overline{n} \geq 1$ is the threshold level of within-user learning for either firm. We assume the learning functions f^A and f^W are everywhere weakly increasing and identical across firms. Thus, the maximum utility that can ever be offered by firm i to an individual consumer in a period is $s_i + f^A(1) + f^W(\overline{n})$, which is achieved when all consumers have chosen firm i at least $\overline{n}$ times in the past. Without loss of generality, we assume that $f^A(1) > f^A(0) = 0$, and that f^W is strictly increasing in the last step to reach its maximum threshold (i.e. $f^W(\overline{n}) > f^W(\overline{n}-1)$) with $f^W(0) = 0$.

Here, a coordination problem arises because of the endogenous switching cost created by within-user learning, which provides a reason for each consumer to want to coordinate on the firm that will attract the other consumers, in order to also benefit from across-user learning. For example, if all consumers are expected to choose I in every period, a consumer that chooses E in every period would forgo the benefits of across-user learning, while a consumer that switches between the two firms at least once would not be reaping the maximum benefits of within-user learning. Of course, when both firms have reached maximum learning (i.e. when both have served all consumers $\overline{n}$ times in the past so they offer utility $s_I + f^A(1) + f^W(\overline{n})$ and $s_E + f^A(1) + f^W(\overline{n})$, respectively), there is no coordination problem: the unique equilibrium has consumers choosing I in every period if $s_I > s_E$, and otherwise choosing E in every period. In order to clearly illustrate how network effects create a coordination problem for consumers in this setting, we focus on the case in which both firms start with a blank slate, that is, $N_I = N_E = 0$ and $n_I = n_E = 0$.

It turns out that in this setting there are only two pure-strategy equilibria, i.e. equilibria in which all users who are in the same state choose the same action in every period: (i) all consumers choose I in every period (I-wins), (ii) all consumers choose E in every period (E-wins). The respective conditions for each equilibrium (I-wins or E-wins) are pinned down by ruling out a consumer's two best deviations. These two deviations turn out to be (i) switching to the other firm for all periods, and (ii) starting on the other firm for one period and then switching to the firm that is supposed to win all periods in equilibrium from period 2 onwards (this involves free-riding on the across-user learning generated by all other consumers). In the Appendix, we formally prove this, and determine the ranges of parameter values under which the two equilibria exist, which are as follows:

- if $s_E - s_I < -\delta \min\left\{f^A(1), (1-\delta)\sum_{j=1}^{\overline{n}} \delta^{j-1} f^W(j) + \delta^{\overline{n}} f^W(\overline{n})\right\}$, then I-wins is the unique equilibrium.
- if $s_E - s_I \geq \delta \min\left\{f^A(1), (1-\delta)\sum_{j=1}^{\overline{n}} \delta^{j-1} f^W(j) + \delta^{\overline{n}} f^W(\overline{n})\right\}$, then E-wins is the unique equilibrium.
- in the remaining range of $s_E - s_I$, both I-wins and E-wins equilibria exist.

The multiplicity of equilibria in the third parameter range creates the possibility for consumer beliefs to determine which equilibrium gets selected. Under Pareto beliefs, in case there are multiple equilibria, consumers coordinate on the equilibrium that yields a higher PDV of surplus for them, which means E wins if and only if $s_E - s_I \geq 0$. In contrast, under beliefs that favor I, the only way for E to possibly win is when E-wins is the unique equilibrium (the second case above), since this rules out the equilibrium in which consumers choose I in every period. The next proposition summarizes our findings.

Proposition 13. *Suppose both firms start with no previous learning, i.e.* $N_I = N_E = 0$ *and* $n_I = n_E = 0$ *for every consumer. Under Pareto beliefs, E wins if and only if* $s_E - s_I \geq 0$, *which is the socially optimal cutoff given the two firms are otherwise symmetric. Under favorable beliefs for I, E wins if and only if*

$$s_E - s_I \geq \delta \min \left\{ f^A(1), (1-\delta) \sum_{j=1}^{\overline{n}} \delta^{j-1} f^W(j) + \delta^{\overline{n}} f^W(\overline{n}) \right\},$$

i.e. the cutoff for E to win is too high in this case compared to the socially optimal cutoff.

Like in the two previous sections, beliefs that favor I shift the outcome in I's favor relative to the case with Pareto beliefs, which remains socially optimal. It is important to emphasize that both types of learning are necessary for network effects to matter in this model. If either type of learning is shut down (i.e. if either $f^A(1) = 0$ or $f^W(j) = 0$ for all j), then the condition for E to win under beliefs favorable to I becomes identical to the condition for E to win under Pareto beliefs, so the nature of beliefs and network effects no longer matter.

6 Conclusion

Data-enabled learning is becoming an essential ingredient to the competitive dynamics between providers of an increasing range of products and services. Our analysis provides several key take-aways regarding the determinants of a firm's competitive advantage in the presence of data-enabled learning. First, we have shown that the profits extracted by the winning firm are higher under across-user learning than within-user learning. This suggests that competitive positions built around across-user learning should be more defensible than those built around within-user learning. Second, everything else equal, a firm's advantage (in winning) is greater, the higher is its learning curve, the further along the firm is on its learning curve, and the higher is the remaining learning potential. However, even if an incumbent starts with a data advantage, if the entrant's learning curve is steeper and can ultimately reach a higher threshold of learning value, through dynamic pricing the entrant may be able to seize the market. Third, data-enabled learning creates a user coordination problem due to network effects when across-user learning is combined with either (i) continued product improvement during the period(s) over which consumers derive utility from the product, or

(ii) within-user learning. In these cases, favorable user expectations towards the incumbent can be another important source of competitive advantage.

Even when the equilibrium outcome with data-enabled learning is socially efficient, an unanticipated improvement in the learning of the losing firm helps consumers. This creates a potential role for a data sharing policy. However, if such a policy is anticipated, it creates a tradeoff in terms of consumer surplus: the enhanced competitive position of the losing firm after data is shared tends to lower prices, but the losing firm is now less likely to subsidize in order to win prior to data being shared, which tends to increase prices. Taking into account this tradeoff, we've shown that a key condition for data sharing policy to improve consumer surplus is that the firm that benefits from data sharing is sufficiently far behind the leader.

The analysis in this paper showed how data-enabled learning can give rise to endogenous switching costs and endogenous network effects. Powerful platform businesses like those run by Amazon, Facebook and Google benefit from data-enabled learning, but in addition, they also enjoy traditional direct network effects. An interesting direction for future work, building on our model, is to consider how data-enabled learning works when combined with direct network effects.

7 Appendix

This appendix contains the proofs of results not already proven in the main text.

7.1 Proof of Proposition 1 and Corollary 1

The state $(N_I, N_E) = (\overline{N}_I, \overline{N}_E)$ is already handled in the main text. Recall the value function of profit in this case is given by

$$V^i(\overline{N}_I, \overline{N}_E) = \frac{1}{1-\delta} \max\left\{s_i - s_j + f_i(\overline{N}_i) - f_j(\overline{N}_j), 0\right\}.$$

Consider the case with $N_I = \overline{N}_I$ and $1 \leq N_E \leq \overline{N}_E$. Suppose $\Delta(\overline{N}_I, N_E)$, $V^I(\overline{N}_I, N_E)$ and $V^E(\overline{N}_I, N_E)$ are given by $\Delta(N_I, N_E)$, $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$ as characterized in Proposition 1 and Corollary 1 with $N_I = \overline{N}_I$ (this is the induction hypothesis).

Now consider the case in which $N_I = \overline{N}_I$ and E has previously sold to $N_E - 1$ consumers. Suppose E charges p^E and I charges p^I in the current period. I wins the current period if $s_I + f_I(\overline{N}_I) - p^I > s_E + f_E(N_E - 1) - p^E$. Otherwise, E wins. If E wins the current period, the PDV of its current and future profits will be $p^E - c + \delta V^E(\overline{N}_I, N_E)$. If it doesn't win the current period, the PDV of its current and future profits will be $\delta V^E(\overline{N}_I, N_E - 1)$. So E is willing to price down to the point where $p^E - c + \delta V^E(\overline{N}_I, N_E) = \delta V^E(\overline{N}_I, N_E - 1)$, or in other words

$p^E = c - \delta(V^E(\overline{N}_I, N_E) - V^E(\overline{N}_I, N_E - 1))$. Based on the same logic, I is willing to price down to $p^I = c - \delta(V^I(\overline{N}_I, N_E - 1) - V^I(\overline{N}_I, N_E))$.

Thus, if we denote by $\Omega^i(N_I, N_E)$ the maximum surplus firm i is willing to offer consumers to win in the current period when the current state is (N_I, N_E), then

$$\begin{aligned}
\Omega^I\left(\overline{N}_I, N_E - 1\right) &= s_I - c + f_I(\overline{N}_I) + \delta(V^I(\overline{N}_I, N_E - 1) - V^I(\overline{N}_I, N_E)) \qquad (8)\\
\Omega^E\left(\overline{N}_I, N_E - 1\right) &= s_E - c + f_E(N_E - 1) + \delta(V^E(\overline{N}_I, N_E) - V^E(\overline{N}_I, N_E - 1)). \qquad (9)
\end{aligned}$$

Given Bertrand competition, I wins the current period iff

$$\Omega^I\left(\overline{N}_I, N_E - 1\right) > \Omega^E\left(\overline{N}_I, N_E - 1\right). \qquad (10)$$

Suppose (10) holds. Then in the current period E sets $p^E = c - \delta(V^E(\overline{N}_I, N_E) - V^E(\overline{N}_I, N_E - 1))$ and I wins by setting $p^I = s_I + f_I(\overline{N}_I) - \Omega^E\left(\overline{N}_I, N_E - 1\right)$. Then I's value is

$$V^I(\overline{N}_I, N_E - 1) = \delta V^I(\overline{N}_I, N_E) + \Omega^I\left(\overline{N}_I, N_E - 1\right) - \Omega^E\left(\overline{N}_I, N_E - 1\right),$$

and E's value is $V^E(\overline{N}_I, N_E - 1) = \delta V^E(\overline{N}_I, N_E - 1)$ given it loses and so will be in the same situation next period. Since $\delta < 1$, this implies $V^E(\overline{N}_I, N_E - 1) = 0$.

Conversely, suppose (10) doesn't hold. Then in the current period I sets $p^I = c - \delta(V^I(\overline{N}_I, N_E - 1) - V^I(\overline{N}_I, N_E))$ and E wins by setting $p^E = s_E + f_E(N_E - 1) - \Omega^I\left(\overline{N}_I, N_E - 1\right)$. Then E's value is

$$V^E(\overline{N}_I, N_E - 1) = \delta V^E(\overline{N}_I, N_E - 1) + \Omega^E\left(\overline{N}_I, N_E - 1\right) - \Omega^I\left(\overline{N}_I, N_E - 1\right)$$

and I's value is $V^I(\overline{N}_I, N_E - 1) = \delta V^I(\overline{N}_I, N_E)$ given it loses and will be in the state $(\overline{N}_I, N_E)$ next period.

Combining the above two cases when (10) holds and when it doesn't, we have

$$\begin{aligned}
V^I(\overline{N}_I, N_E - 1) &= \delta V^I(\overline{N}_I, N_E) + \max\left\{\Omega^I\left(\overline{N}_I, N_E - 1\right) - \Omega^E\left(\overline{N}_I, N_E - 1\right), 0\right\} \qquad (11)\\
V^E(\overline{N}_I, N_E - 1) &= \delta V^E(\overline{N}_I, N_E - 1) + \max\left\{\Omega^E\left(\overline{N}_I, N_E - 1\right) - \Omega^I\left(\overline{N}_I, N_E - 1\right), 0\right\}. \qquad (12)
\end{aligned}$$

There are two possibilities: $V^E(\overline{N}_I, N_E - 1) = 0$ and $V^E(\overline{N}_I, N_E - 1) > 0$.

Suppose first $V^E(\overline{N}_I, N_E - 1) = 0$. We can solve (11) for $V^I(\overline{N}_I, N_E - 1)$ using the expressions for $V^I(\overline{N}_I, N_E)$ and $V^E(\overline{N}_I, N_E)$ from the induction hypothesis and the fact that $V^E(\overline{N}_I, N_E - 1) = 0$, which gives

$$V^I(\overline{N}_I, N_E - 1) = \begin{cases} \frac{s_I - s_E + f_I(\overline{N}_I) - f_E(N_E - 1)}{1-\delta} & \text{if } \; s_E - s_I < \Delta(\overline{N}_I, N_E) \\ \frac{s_I - s_E + (1-\delta)(f_I(\overline{N}_I) - f_E(N_E - 1)) + \delta\Delta(\overline{N}_I, N_E)}{(1-\delta)^2} & \text{if } \; s_E - s_I \geq \Delta(\overline{N}_I, N_E) \end{cases},$$

with $V^I(\overline{N}_I, N_E - 1) \geq 0$ iff

$$s_E - s_I \leq (1-\delta)(f_I(\overline{N}_I) - f_E(N_E - 1)) + \delta\Delta(\overline{N}_I, N_E) = \Delta(\overline{N}_I, N_E - 1).$$

Next, suppose $V^E(\overline{N}_I, N_E - 1) > 0$, so $V^I(\overline{N}_I, N_E - 1) = \delta V^I(\overline{N}_I, N_E)$. Since $V^I\left(\overline{N}_I, N_E - 1\right) \geq V^I\left(\overline{N}_I, N_E\right)$, we must have $V^I(\overline{N}_I, N_E - 1) = V^I(\overline{N}_I, N_E) = 0$ in this case. We can solve (12) for $V^E(\overline{N}_I, N_E - 1)$ using the expression for $V^E(\overline{N}_I, N_E)$ from the induction hypothesis and the fact that $V^I(\overline{N}_I, N_E - 1) = V^I(\overline{N}_I, N_E) = 0$, which gives

$$V^E(\overline{N}_I, N_E - 1) = \frac{s_E - s_I + (1-\delta)(f_E(N_E - 1) - f_I(\overline{N}_I)) - \delta\Delta(\overline{N}_I, N_E)}{1-\delta},$$

which is positive iff $s_E - s_I > \Delta(\overline{N}_I, N_E - 1)$.

Thus, combining the results with $V^E(\overline{N}_I, N_E - 1) = 0$ and $V^E(\overline{N}_I, N_E - 1) > 0$, we have proven that $\Delta(\overline{N}_I, N_E - 1)$, $V^I(\overline{N}_I, N_E - 1)$ and $V^E(\overline{N}_I, N_E - 1)$ satisfy the characterization in Proposition 1 and Corollary 1, and by induction, this is true for any $0 \leq N_E \leq \overline{N}_E$ when $N_I = \overline{N}_I$. Symmetry can be used to obtain a parallel result for all $0 \leq N_I \leq \overline{N}_I$ when $N_E = \overline{N}_E$.

Now consider any state (N_I, N_E) with $0 \leq N_I \leq \overline{N}_I - 1$ and $0 \leq N_E \leq \overline{N}_E - 1$, and suppose that the results in Proposition 1 and Corollary 1 hold for the states $(N_I + 1, N_E)$ and $(N_I, N_E + 1)$—this is the induction hypothesis. Using the same logic as above when the state is (N_I, N_E), we have

$$V^I(N_I, N_E) = \delta V^I(N_I, N_E + 1) + \max\left\{\Omega\left(N_I, N_E\right), 0\right\} \tag{13}$$

$$V^E(N_I, N_E) = \delta V^E(N_I + 1, N_E) + \max\left\{-\Omega\left(N_I, N_E\right), 0\right\} \tag{14}$$

where

$$\begin{aligned}\Omega\left(N_I, N_E\right) = {} & s_I + f_I(N_I) + \delta(V^I(N_I + 1, N_E) - V^I(N_I, N_E + 1)) \\ & -s_E - f_E(N_E) - \delta\left(V^E(N_I, N_E + 1) - V^E(N_I + 1, N_E)\right).\end{aligned}$$

Again there are two possibilities, depending on whether $\Omega\left(N_I, N_E\right)$ is positive or not. If $\Omega\left(N_I, N_E\right) \geq 0$, then we must have $V^E(N_I, N_E) = V^E(N_I + 1, N_E) = 0$, while if $\Omega\left(N_I, N_E\right) \leq 0$, then we must have $V^I(N_I, N_E) = V^I(N_I, N_E + 1) = 0$. In each of these two cases, we can use the expressions of $V^I(N_I, N_E + 1)$ and $V^E(N_I + 1, N_E)$ from the induction hypothesis to solve (13) and (14) for $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$. After some careful manipulations, which include rewriting the resulting summation expressions for $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$ in terms of $\Delta(N_I, N_E)$ and $\Delta(N_I, N_E + 1)$, we confirm that the expressions of $\Delta(N_I, N_E)$, $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$ satisfy the characterization in Proposition 1 and Corollary 1 (these additional details are provided in the Online Appendix).

By repeated application of the induction hypothesis, we obtain that the expressions for $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$ in Corollary 1, and the expression for $\Delta(N_I, N_E)$ in Proposition 1 hold for all states (N_I, N_E), where $0 \leq N_I \leq \overline{N}_I - 1$ and $0 \leq N_E \leq \overline{N}_E - 1$.

7.2 Proof of Proposition 2

To show Proposition 1 continues to apply in the limit as $\overline{N}_I \to \infty$ and $\overline{N}_E \to \infty$, we need to prove the expression in (1) continues to be well defined in this limit case. This is equivalent to proving $\lim_{N_i \to \infty} \sum_{j=0}^{N_i} \delta^j f_i(j)$ exists given this also implies $\lim_{N_i \to \infty} \delta^{N_i} f_i(N_i)$ exists since the summation includes the later expression and other positive terms. Consider the power function $f_i(x) = \theta_i x^{\rho_i}$ where $\theta_i > 0$ and $\rho_i > 0$. Using the ratio test, we know that $\lim_{j\to\infty}(\frac{\delta^{j+1}(j+1)^\rho}{\delta^j j^\rho}) = \delta < 1$ which implies the limit exists for any such power function. Then by the limit comparison test, since the relevant terms are all positive, the limits exist for any unbounded learning function f_i provided there exists some $\kappa_i > 0$, such that $\lim_{j\to\infty} \frac{f_i(j)}{j^\rho} \leq \kappa_i$ for $i = \{I, E\}$.

7.3 Proof of Proposition 3

If $N_I = N_E = N$ and $s_E - s_I < 0$, then I wins and E makes zero profits regardless of whether $N < \overline{N}$ or $N = \overline{N}$. If $N = \overline{N}$, then I's PDV of profits is $V^I\left(\overline{N}, \overline{N}\right) = \frac{s_I - s_E}{1-\delta}$. If instead $N < \overline{N}$, then I's PDV of profits is

$$V^I(N, N) = \begin{cases} \frac{s_I - s_E}{1-\delta} - \frac{\delta\Delta(N,N+1)}{(1-\delta)^2} & \text{if} \quad s_E - s_I < \Delta(N, N+1) \\ \frac{s_I - s_E}{(1-\delta)^2} & \text{if} \quad \Delta(N, N+1) \leq s_E - s_I < 0 \end{cases},$$

which is clearly always larger than $\frac{s_I - s_E}{1-\delta}$, given that $\Delta(N, N+1) < 0 = \Delta(N, N)$. By symmetry, the same holds when $s_E - s_I > 0$.

Now suppose there is some asymmetry between the two firms' learning functions, and we have $\overline{N}_I > N_I$, $\overline{N}_E > N_E$ and $s_E - s_I = \Delta(N_I, N_E)$. This means that $V^i(N_I, N_E) = 0$ for $i \in \{I, E\}$. In contrast, if $\overline{N}_I = N_I$, $\overline{N}_E = N_E$, then $V^i(N_I, N_E) = \max\left\{\frac{s_i - s_j + f_i(N_i) - f_j(N_j)}{1-\delta}, 0\right\}$ for $i \neq j \in \{N_I, N_E\}$. Thus, as long as $f_I(N_I) - f_E(N_E) \neq \Delta(N_I, N_E)$, one firm sees its profits strictly increase when the prospect of future learning is eliminated (the other firm's profit remains equal to zero). By continuity, this remains true when $s_E - s_I$ is sufficiently close to $\Delta(N_I, N_E)$.

7.4 Proof of Proposition 4

Consider the socially optimal outcome. If I wins in every period, the value created is

$$\frac{s_I}{1-\delta} + \frac{\delta^{\overline{N}_I - N_I + 1}}{1-\delta} f_I(\overline{N}_I) + \sum_{j=0}^{\overline{N}_I - N_I} \delta^j f_I(N_I + j)$$

while if E wins in every period, the value created is

$$\frac{s_E}{1-\delta}+\frac{\delta^{\overline{N}_E-N_E+1}}{1-\delta}f_E(\overline{N}_E)+\sum_{j=0}^{\overline{N}_E-N_E}\delta^j f_E(N_E+j).$$

The socially optimal cutoff is the level of s_I-s_E for which these two expressions are equal, which corresponds exactly to the equilibrium cutoff $\Delta(N_I,N_E)$.

7.5 Proof of Proposition 5

Given that E moves to the threshold level of data at the end of each period with probability $0\leq\lambda\leq 1$ which is independent of the firm's current position, the maximum surplus firm E and I are willing to offer consumers to win are given by the same expressions (8)-(9) except that δ is replaced by $\delta(1-\lambda)$. We can derive the value functions and the cutoff for E to win following the same steps as in Proposition 1 and Corollary 1. The main difference is that it is now possible for both firms to have positive value. The full details are relegated to the Online Appendix, where we also show that $\Delta(\overline{N},N_E,\lambda)$ is increasing in λ, a property we use below. The resulting value functions are:

$$V^I(\overline{N},N_E,\lambda)=\begin{cases}\frac{s_I-s_E}{1-\delta}+\frac{f(\overline{N})-f(N_E)}{1-\delta(1-\lambda)} & \text{if} \quad s_E-s_I<\Delta(\overline{N},\overline{N},\lambda)=0\\ \frac{s_I-s_E+f(\overline{N})-f(N_E)}{1-\delta(1-\lambda)} & \text{if} \quad \Delta(\overline{N},\overline{N},\lambda)\leq s_E-s_I<\Delta(\overline{N},N_E+1,\lambda)\\ \frac{s_I-s_E+\Delta(\overline{N},N_E,\lambda)}{(1-\delta(1-\lambda))^2} & \text{if} \quad \Delta(\overline{N},N_E+1,\lambda)\leq s_E-s_I<\Delta(\overline{N},N_E,\lambda)\\ 0 & \text{if} \quad s_E-s_I\geq\Delta(\overline{N},N_E,\lambda)\end{cases} \tag{15}$$

$$V^E(\overline{N},N_E,\lambda)=\begin{cases}0 & \text{if} \quad s_E-s_I<\Delta(\overline{N},\overline{N},\lambda)=0\\ \frac{\delta\lambda}{1-\delta(1-\lambda)}\frac{s_E-s_I}{1-\delta} & \text{if} \quad \Delta(\overline{N},\overline{N},\lambda)\leq s_E-s_I<\Delta(\overline{N},N_E,\lambda)\\ \frac{s_E-s_I}{1-\delta}-\frac{\Delta(\overline{N},N_E,\lambda)}{1-\delta(1-\lambda)} & \text{if} \quad s_E-s_I\geq\Delta(\overline{N},N_E,\lambda)\end{cases}. \tag{16}$$

The PDV of consumer surplus is determined by the surplus offered by the losing firm in each period along the equilibrium path taking into account E's chances of shifting to the threshold.

There are four different cases.

1. Suppose $s_E-s_I<\Delta(\overline{N},\overline{N},\lambda)=0$. This means I always wins, E never subsidizes, and consumers get the surplus offered by E which is $s_E+f(N_E)-c$ each period until I shares its data, and $s_E+f\left(\overline{N}\right)-c$ each period after I shares its data. Since $f\left(\overline{N}\right)>f(N_E)$, the PDV of consumer surplus is increasing in λ.

2. Suppose $0=\Delta(\overline{N},\overline{N},\lambda)\leq s_E-s_I<\Delta(\overline{N},N_E+1,\lambda)$. This means I wins every period until E catches up, after which E wins forever. However, in this range, since even if E somehow manages to win one period, it will still not win the following period (without I sharing its data), it will not subsidize. Thus, consumers get $s_E+f(N_E)-c$ each period until I shares its

data and $s_I + f\left(\overline{N}\right) - c$ each period after I shares its data. Since $s_E - s_I < \Delta(\overline{N}, N_E + 1, \lambda) < f\left(\overline{N}\right) - f\left(N_E\right)$, the PDV of consumer surplus is increasing in λ. Note this range only exists if $N_E \leq \overline{N} - 2$.

3. Suppose $\Delta(\overline{N}, N_E + 1, \lambda) \leq s_E - s_I < \Delta(\overline{N}, N_E, \lambda)$. In this range, consumer surplus is the same as in case 2 except now consumer surplus is higher because E offers a subsidy of

$$\delta\left(1-\lambda\right)\left(V^E(\overline{N}, N_E+1) - V^E(\overline{N}, N_E)\right) = \frac{\delta\left(1-\lambda\right)\left(s_E - s_I - \Delta(\overline{N}, N_E+1, \lambda)\right)}{1-\delta\left(1-\lambda\right)} \tag{17}$$

each period until I shares its data. This subsidy reflects that if E manages to win one period, it will thereafter win in all following periods even if I doesn't share its data. Using (15)-(16), the PDV of consumer surplus can be written as

$$\frac{s_E - s_I - \Delta(\overline{N}, N_E, \lambda)}{\left(1-\delta\left(1-\lambda\right)\right)^2} + \frac{s_I + f\left(\overline{N}\right) - c}{1-\delta}$$

and the change in consumer surplus due to data sharing when $s_E - s_I = x$ is therefore

$$\Delta CS\left(x, \lambda\right) \equiv \frac{x - \Delta(\overline{N}, N_E, \lambda)}{\left(1-\delta\left(1-\lambda\right)\right)^2} - \frac{x - \Delta(\overline{N}, N_E, 0)}{\left(1-\delta\right)^2}.$$

Clearly, $\Delta CS\left(x, \lambda\right)$ is decreasing in x. Furthermore, when $x = \Delta(\overline{N}, N_E, \lambda)$, we have

$$\Delta CS\left(\Delta(\overline{N}, N_E, \lambda), \lambda\right) = \frac{\Delta(\overline{N}, N_E, 0) - \Delta(\overline{N}, N_E, \lambda)}{\left(1-\delta\right)^2} < 0.$$

Finally, when $s_E - s_I = \Delta(\overline{N}, N_E + 1, \lambda)$, the subsidy in (17) is exactly zero and the change in consumer surplus due to data sharing is identical to that arising in case 2 for the same value of $s_E - s_I$, which recall is positive. Thus, we have $\Delta CS\left(\Delta(\overline{N}, N_E + 1, \lambda), \lambda\right) > 0$. So we conclude there exists a unique $\Theta^a\left(\overline{N}, N_E, \lambda\right) \in \left(\Delta(\overline{N}, N_E + 1, \lambda), \Delta(\overline{N}, N_E, \lambda)\right)$ such that $\Delta CS\left(\Theta^a\left(\overline{N}, N_E, \lambda\right), \lambda\right) = 0$, which indeed has the solution given by (3).

4. Finally, suppose $s_E - s_I \geq \Delta(\overline{N}, N_E)$. This means E wins in every period. Since we must also have $s_E - s_I > 0$, I will never subsidize in this region. Thus, consumers always get the surplus offered by I when it loses and prices at c. As a result, the PDV of consumer surplus is independent of λ.

7.6 Proof of Proposition 7

To prove the result in (i) we need to show that the term (in large brackets) multiplied by θ_I in (4) is greater than the corresponding term multiplied by θ_E when $\overline{N}_I - N_I \leq \overline{N}_E - N_E$ and $\overline{N}_I - \underline{N}_I \geq \overline{N}_E - \underline{N}_E$, with one inequality strict.

If $N_E \leq \underline{N}_E$ and $N_I \leq \underline{N}_I$, then the inequality we require simplifies to

$$\delta^{\underline{N}_I - N_I + 1}\left(1 - \delta^{\overline{N}_I - \underline{N}_I}\right) > \delta^{\underline{N}_E - N_E + 1}\left(1 - \delta^{\overline{N}_E - \underline{N}_E}\right),$$

which follows by combining the above assumptions. If $N_E > \underline{N}_E$ and $N_I > \underline{N}_I$, then the inequality we require simplifies to

$$N_I - \underline{N}_I + \delta\frac{1 - \delta^{\overline{N}_I - N_I}}{1 - \delta} > N_E - \underline{N}_E + \delta\frac{1 - \delta^{\overline{N}_E - N_E}}{1 - \delta},$$

which once again can be shown to follow from the above assumptions. Finally, given $\overline{N}_I - N_I \leq \overline{N}_E - N_E$ and $\overline{N}_I - \underline{N}_I \geq \overline{N}_E - \underline{N}_E$, the only remaining case is $N_E \leq \underline{N}_E$ and $N_I > \underline{N}_I$. In this case, the inequality we wish to prove becomes

$$N_I - \underline{N}_I + \delta\left(\frac{1 - \delta^{\overline{N}_I - N_I}}{1 - \delta}\right) > \delta^{\underline{N}_E - N_E + 1}\left(\frac{1 - \delta^{\overline{N}_E - \underline{N}_E}}{1 - \delta}\right).$$

The LHS is lowest when $N_I = \underline{N}_I + 1$ while the RHS is highest when $N_E = \underline{N}_E$, so a sufficient condition for the inequality to hold is that

$$1 + \delta\left(\frac{1 - \delta^{\overline{N}_I - \underline{N}_I - 1}}{1 - \delta}\right) > \delta\left(\frac{1 - \delta^{\overline{N}_E - \underline{N}_E}}{1 - \delta}\right)$$

or equivalently $1 - \delta^{\overline{N}_I - \underline{N}_I} > \delta\left(1 - \delta^{\overline{N}_E - \underline{N}_E}\right)$, which holds because $\overline{N}_I - \underline{N}_I \geq \overline{N}_E - \underline{N}_E$.

For (ii), suppose $N_I + k \leq \underline{N}$, which implies $N_E + k < \underline{N}$ because $N_I > N_E$. Then imposing that learning functions are identical on (4), we get

$$\Delta(N_I + k, N_E + k) - \Delta(N_I, N_E) = \frac{(1 - \delta^k)(1 - \delta^{\overline{N} - \underline{N}})(\delta^{\overline{N} - N_I} - \delta^{\overline{N} - N_E})\theta}{(1 - \delta)\delta^{\overline{N} - \underline{N} + k - 1}} > 0.$$

Suppose instead $N_I > \underline{N}$ and $N_E > \underline{N}$. Then

$$\Delta(N_I + k, N_E + k) - \Delta(N_I, N_E) = -\frac{(1 - \delta^k)(\delta^{\overline{N} - N_I + 1} - \delta^{\overline{N} - N_E + 1})\theta}{(1 - \delta)\delta^k},$$

which is negative given $N_I > N_E$.

7.7 Proof of Proposition 8

As discussed in the text, the analysis can be conducted with a representative consumer. A key difference with the proof of Proposition 1 is that we need to construct the value function for the consumer since the consumer needs to take future utility into account when deciding which firm to buy from. We denote this $u(N_I, N_E)$, which measures the PDV of surplus a consumer expects to obtain from making her optimal choices in every stage when firms play their equilibrium strategies and the consumer has previously bought N_I times from firm I and N_E times from firm E.

It is easily verified that when $(N_I, N_E) = (\overline{N}_I, \overline{N}_E)$, we have

$$\begin{aligned} V^I(\overline{N}_I, \overline{N}_E) &= \frac{\max\left\{s_I + f_I(\overline{N}_I) - s_E - f_E(\overline{N}_E), 0\right\}}{1-\delta} \\ V^E(\overline{N}_I, \overline{N}_E) &= \frac{\max\left\{s_E + f_E(\overline{N}_E) - s_I - f_I(\overline{N}_I), 0\right\}}{1-\delta} \\ u(\overline{N}_I, \overline{N}_E) &= \frac{\min\left\{s_I + f_I(\overline{N}_I), s_E + f_E(\overline{N}_E)\right\} - c}{1-\delta}. \end{aligned}$$

Consider the case with $N_I = \overline{N}_I$ and $1 \leq N_E \leq \overline{N}_E$. Suppose that $\Delta(\overline{N}_I, N_E)$, $V^I(\overline{N}_I, N_E)$ and $V^E(\overline{N}_I, N_E)$ are given by $\Delta(N_I, N_E)$, $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$ as characterized in Proposition 8 with $N_I = \overline{N}_I$, and that the corresponding value function for consumers is

$$u(\overline{N}_I, N_E) = \min\left\{ f_I(\overline{N}_I) + \frac{s_I + \delta f_I(\overline{N}_I) - c}{1-\delta}, \sum_{j=0}^{\overline{N}_E - N_E} \delta^j f_E(N_E + j) + \frac{s_E + \delta^{\overline{N}_E - N_E + 1} f_E(\overline{N}_E) - c}{1-\delta} \right\}.$$

This is the induction hypothesis.

Consider now the state $(\overline{N}_I, N_E - 1)$. The consumer chooses I in the current period iff

$$s_I + f_I(\overline{N}_I) - p^I + \delta u(\overline{N}_I, N_E - 1) > s_E + f_E(N_E - 1) - p^E + \delta u(\overline{N}_I, N_E).$$

I is willing to set p^I down to the point where it is indifferent between winning or losing the current period, i.e. $p^I = c - \delta(V^I(\overline{N}_I, N_E - 1) - V^I(\overline{N}_I, N_E))$. Similarly, E is willing to set p^E down to $p^E = c - \delta(V^E(\overline{N}_I, N_E) - V^E(\overline{N}_I, N_E - 1))$.

If we denote by $\Omega^i(N_I, N_E)$ the PDV of the maximum surplus firm i is willing to offer consumers to win in the current period when the current state is (N_I, N_E), then

$$\begin{aligned} \Omega^I\left(\overline{N}_I, N_E - 1\right) &= s_I - c + f_I(\overline{N}_I) + \delta(V^I(\overline{N}_I, N_E - 1) - V^I(\overline{N}_I, N_E) + u(\overline{N}_I, N_E - 1)) \\ \Omega^E\left(\overline{N}_I, N_E - 1\right) &= s_E - c + f_E(N_E - 1) + \delta(V^E(\overline{N}_I, N_E) - V^E(\overline{N}_I, N_E - 1) + u(\overline{N}_I, N_E)). \end{aligned}$$

Given Bertrand competition, I wins the current period iff

$$\Omega^I\left(\overline{N}_I, N_E - 1\right) > \Omega^E\left(\overline{N}_I, N_E - 1\right).$$

Using the same logic as in the Proof of Proposition 1 and Corollary 1, we obtain

$$\begin{aligned} u(\overline{N}_I, N_E - 1) &= \min\left\{\Omega^I\left(\overline{N}_I, N_E - 1\right), \Omega^E\left(\overline{N}_I, N_E - 1\right)\right\} && (18) \\ V^I(\overline{N}_I, N_E - 1) &= \delta V^I(\overline{N}_I, N_E) + \max\left\{\Omega^I\left(\overline{N}_I, N_E - 1\right) - \Omega^E\left(\overline{N}_I, N_E - 1\right), 0\right\} && (19) \\ V^E(\overline{N}_I, N_E - 1) &= \delta V^E(\overline{N}_I, N_E - 1) + \max\left\{\Omega^E\left(\overline{N}_I, N_E - 1\right) - \Omega^I\left(\overline{N}_I, N_E - 1\right), 0\right\}. && (20) \end{aligned}$$

There are two possibilities: $\Omega^I\left(\overline{N}_I, N_E-1\right) \geq \Omega^E\left(\overline{N}_I, N_E-1\right)$ or $\Omega^I\left(\overline{N}_I, N_E-1\right) < \Omega^E\left(\overline{N}_I, N_E-1\right)$.

Suppose first $\Omega^I\left(\overline{N}_I, N_E-1\right) \geq \Omega^E\left(\overline{N}_I, N_E-1\right)$, which immediately implies $V^E(\overline{N}_I, N_E-1) = 0$. Using the expressions of $V^I(\overline{N}_I, N_E)$, $V^E(\overline{N}_I, N_E)$ and $u(\overline{N}_I, N_E)$ from the induction hypothesis, we then solve (18) and (19) for $V^I(\overline{N}_I, N_E-1)$ and $u(\overline{N}_I, N_E-1)$ to obtain

$$\begin{aligned}
V^I(\overline{N}_I, N_E-1) &= \frac{s_I - s_E + \Delta(\overline{N}_I, N_E-1)}{1-\delta} \\
u(\overline{N}_I, N_E-1) &= \frac{s_E + \delta^{\overline{N}_E - N_E + 2} f_E(\overline{N}_E) - c}{1-\delta} + \sum_{j=0}^{\overline{N}_E - N_E + 1} \delta^j f_E(N_E - 1 + j). \qquad (21)
\end{aligned}$$

Now suppose $\Omega^I\left(\overline{N}_I, N_E-1\right) < \Omega^E\left(\overline{N}_I, N_E-1\right)$, which means E wins the current period, so $V^I(\overline{N}_I, N_E-1) = \delta V^I(\overline{N}_I, N_E)$, which implies $V^I(\overline{N}_I, N_E-1) = V^I(\overline{N}_I, N_E) = 0$ because we must have $V^I(\overline{N}_I, N_E-1) \geq V^I(\overline{N}_I, N_E)$. Furthermore, (18) and (20) become

$$\begin{aligned}
u(\overline{N}_I, N_E-1) &= \frac{s_I + f_I(\overline{N}_I) - c}{1-\delta} \qquad (22) \\
V^E(\overline{N}_I, N_E-1) &= \frac{s_E - s_I - \Delta(\overline{N}_I, N_E-1)}{1-\delta}.
\end{aligned}$$

Thus, combining the results with $\Omega^I\left(\overline{N}_I, N_E-1\right) \geq \Omega^E\left(\overline{N}_I, N_E-1\right)$ and $\Omega^I\left(\overline{N}_I, N_E-1\right) < \Omega^E\left(\overline{N}_I, N_E-1\right)$, we have proven that $\Delta(\overline{N}_I, N_E-1)$, $V^I(\overline{N}_I, N_E-1)$ and $V^E(\overline{N}_I, N_E-1)$ satisfy the characterization in Proposition 8, and $u(\overline{N}_I, N_E-1)$ is given by the minimum of the two expressions in (21) and (22). By induction, this is true for any $0 \leq N_E \leq \overline{N}_E$ when $N_I = \overline{N}_I$. Symmetry can then be used to obtain a parallel result for all $0 \leq N_I \leq \overline{N}_I$ when $N_E = \overline{N}_E$.

Now consider any state (N_I, N_E) with $0 \leq N_I \leq \overline{N}_I - 1$ and $0 \leq N_E \leq \overline{N}_E - 1$. The induction hypothesis is that the result in Proposition 8 holds for the states (N_I+1, N_E) and (N_I, N_E+1) and that the value functions for consumers in these two states are

$$u(N_I+1, N_E) = \min\left\{ \begin{array}{c} \sum_{j=0}^{\overline{N}_I - N_I - 1} \delta^j f_I(N_I+1+j) + \frac{s_I + \delta^{\overline{N}_I - N_I} f_I(\overline{N}_I) - c}{1-\delta}, \\ \sum_{j=0}^{\overline{N}_E - N_E} \delta^j f_E(N_E+j) + \frac{s_E + \delta^{\overline{N}_E - N_E + 1} f_E(\overline{N}_E) - c}{1-\delta} \end{array} \right\}$$

$$u(N_I, N_E+1) = \min\left\{ \begin{array}{c} \sum_{j=0}^{\overline{N}_I - N_I} \delta^j f_I(N_I+j) + \frac{s_I + \delta^{\overline{N}_I - N_I + 1} f_I(\overline{N}_I) - c}{1-\delta}, \\ \sum_{j=0}^{\overline{N}_E - N_E + 1} \delta^j f_E(N_E+1+j) + \frac{s_E + \delta^{\overline{N}_E - N_E} f_E(\overline{N}_E) - c}{1-\delta} \end{array} \right\}.$$

Using the same logic as above, we have

$$u(N_I, N_E) = \min\left\{\Omega^I(N_I, N_E), \Omega^E(N_I, N_E)\right\}, \qquad (23)$$

where

$$\Omega^I(N_I,N_E) = s_I - c + f_I(N_I) + \delta(V^I(N_I+1,N_E) - V^I(N_I,N_E+1) + u(N_I+1,N_E))$$
$$\Omega^E(N_I,N_E) = s_E - c + f_E(N_E) + \delta(V^E(N_I,N_E+1) - V^E(N_I+1,N_E) + u(N_I,N_E+1))$$

and

$$V^I(N_I,N_E) = \delta V^I(N_I,N_E+1) + \max\{\Omega(N_I,N_E),0\} \tag{24}$$
$$V^E(N_I,N_E) = \delta V^E(N_I+1,N_E) + \max\{-\Omega(N_I,N_E),0\}, \tag{25}$$

where $\Omega(N_I,N_E) \equiv \Omega^I(N_I,N_E) - \Omega^E(N_I,N_E)$.

Again there are two possibilities, depending on whether $\Omega(N_I,N_E) > 0$ or not. If $\Omega(N_I,N_E) > 0$, then we must have $V^E(N_I,N_E) = V^E(N_I+1,N_E) = 0$, while if $\Omega(N_I,N_E) \leq 0$, then we must have $V^I(N_I,N_E) = V^I(N_I,N_E+1) = 0$. For each of these cases, we can use the expressions of $u(N_I+1,N_E)$, $u(N_I,N_E+1)$, $V^I(N_I,N_E+1)$ and $V^E(N_I+1,N_E)$ from the induction hypothesis to solve (23), (24) and (25) for $V^I(N_I,N_E)$, $V^E(N_I,N_E)$ and $u(N_I,N_E)$. Then we can rewrite the resulting summation expressions in $V^I(N_I,N_E)$ and $V^E(N_I,N_E)$ in terms of $\Delta(N_I,N_E)$ to obtain

$$V^I(N_I,N_E) = \max\left\{\frac{s_I - s_E + \Delta(N_I,N_E)}{1-\delta}, 0\right\}$$
$$V^E(N_I,N_E) = \max\left\{\frac{s_E - s_I - \Delta(N_I,N_E)}{1-\delta}, 0\right\}$$

and

$$u(N_I,N_E) = \min\left\{\begin{array}{c} \sum_{j=0}^{\overline{N}_I - N_I} \delta^j f_I(N_I+j) + \frac{s_I + \delta^{\overline{N}_I - N_I + 1} f_I(\overline{N}_I) - c}{1-\delta}, \\ \sum_{j=0}^{\overline{N}_E - N_E} \delta^j f_E(N_E+j) + \frac{s_E + \delta^{\overline{N}_E - N_E + 1} f_E(\overline{N}_E) - c}{1-\delta} \end{array}\right\}.$$

By repeated application of the induction hypothesis, the result in Proposition 8 holds for all states (N_I,N_E), with $0 \leq N_I \leq \overline{N}_I$ and $0 \leq N_E \leq \overline{N}_E$.

Finally, we can confirm that consumers do not want to switch firms along the equilibrium path. Suppose the current state is (N_I,N_E). If $s_E - s_I < \Delta(N_I,N_E)$, then consumers choose I in the current period, so next period the state will be (N_I+1,N_E). Since $\Delta(N_I,N_E)$ is increasing in N_I, this means in the subsequent period consumers will choose I again, and so on until $N_I = \overline{N}_I$. When the state is $(\overline{N}_I,N_E)$, all subsequent periods will have an identical state and outcome, in which consumers choose I. Conversely, if $s_E - s_I \geq \Delta(N_I,N_E)$, then consumers choose E in the current period, so next period the state will be (N_I,N_E+1). Since $\Delta(N_I,N_E)$ is decreasing in N_E, this means in the subsequent period consumers will choose E again, and so on until $N_E = \overline{N}_E$. At the state $(N_I,\overline{N}_E)$, all subsequent periods will have an identical state and outcome, in which consumers choose E.

7.8 Proof of Proposition 9

There are four different cases, depending on the value of $s_E - s_I$. If $s_E - s_I < \Delta(N_I, N_E+1)$ then $V^I(N_I, N_E)$ is higher under across-user learning since

$$\frac{s_I - s_E}{1-\delta} + \frac{\Delta(N_I, N_E) - \delta\Delta(N_I, N_E+1)}{(1-\delta)^2} > \frac{s_I - s_E + \Delta(N_I, N_E)}{1-\delta} > 0$$

given $\Delta(N_I, N_E) > \Delta(N_I, N_E+1)$. If $\Delta(N_I, N_E+1) \leq s_E - s_I < \Delta(N_I, N_E)$, then $V^I(N_I, N_E)$ is higher under across-user learning since

$$\frac{s_I - s_E + \Delta(N_I, N_E)}{(1-\delta)^2} > \frac{s_I - s_E + \Delta(N_I, N_E)}{1-\delta} > 0.$$

If $\Delta(N_I, N_E) \leq s_E - s_I < \Delta(N_I+1, N_E)$, then $V^E(N_I, N_E)$ is higher under across-user learning since

$$\frac{s_E - s_I - \Delta(N_I, N_E)}{(1-\delta)^2} > \frac{s_E - s_I - \Delta(N_I, N_E)}{1-\delta} > 0.$$

Finally, if $s_E - s_I \geq \Delta(N_I+1, N_E)$, then $V^E(N_I, N_E)$ is higher under across-user learning since

$$\frac{s_E - s_I}{1-\delta} - \frac{\Delta(N_I, N_E) - \delta\Delta(N_I+1, N_E)}{(1-\delta)^2} > \frac{s_E - s_I - \Delta(N_I, N_E)}{1-\delta} > 0$$

given $\Delta(N_I+1, N_E) > \Delta(N_I, N_E)$.

7.9 Data sharing under within-user learning

Here we prove the results regarding data sharing under within-user learning claimed at the end of Section 4. The logic parallels that used in the Proof of Proposition 5 but adjusted to take into account in this case we have pure within-user learning. The details of obtaining the firms' (and in this case, the consumers') value functions are relegated to the Online Appendix. There we show that a representative user's value function is

$$u(\overline{N}, N_E, \lambda) = \begin{cases} \frac{s_E + f(\overline{N}) - c}{1-\delta} - \frac{\Delta(\overline{N}, N_E, \lambda)}{1-\delta(1-\lambda)} & \text{if} \quad s_E - s_I < 0 \\ \frac{s_E + f(\overline{N}) - c}{1-\delta} - \frac{\Delta(\overline{N}, N_E, \lambda)}{1-\delta(1-\lambda)} + \frac{\delta\lambda}{1-\delta(1-\lambda)} \frac{s_I - s_E}{1-\delta} & \text{if} \quad 0 \leq s_E - s_I < \Delta(\overline{N}, N_E, \lambda) \\ \frac{s_I + f(\overline{N}) - c}{1-\delta} & \text{if} \quad s_E - s_I \geq \Delta(\overline{N}, N_E, \lambda) \end{cases}$$

and that (2) still characterizes the cutoff in $s_E - s_I$, above which E wins.

To determine the effect of data sharing on consumer surplus, we simply compare $u(\overline{N}, N_E, \lambda)$ to $u(\overline{N}, N_E, 0)$. There are three cases:

1. If $s_E - s_I < 0$, then $u(\overline{N}, N_E, \lambda)$ is increasing in λ, so data sharing increases consumer surplus.

2. If $0 \le s_E - s_I < \Delta(\overline{N}, N_E, \lambda)$, then

$$u\left(\overline{N}, N_E, \lambda\right) - u\left(\overline{N}, N_E, 0\right) = -\frac{\delta\lambda\left(s_E - s_I\right)}{\left(1-\delta\left(1-\lambda\right)\right)\left(1-\delta\right)} + \frac{\Delta(\overline{N}, N_E, 0)}{1-\delta} - \frac{\Delta(\overline{N}, N_E, \lambda)}{1-\delta\left(1-\lambda\right)},$$

which is clearly decreasing in $s_E - s_I$. Furthermore, when $s_E - s_I = 0$, we have $u\left(\overline{N}, N_E, \lambda\right) - u\left(\overline{N}, N_E, 0\right) > 0$. To see this, note that

$$\frac{\Delta(\overline{N}, N_E, \lambda)}{1-\delta\left(1-\lambda\right)} = \sum_{j=0}^{\overline{N}-N_E-1} \left(\delta\left(1-\lambda\right)\right)^j \left(f(\overline{N}) - f\left(N_E + j\right)\right),$$

so $\frac{\Delta(\overline{N}, N_E, \lambda)}{1-\delta(1-\lambda)}$ is decreasing in λ. And when $s_E - s_I = \Delta(\overline{N}, N_E, \lambda)$, we have

$$u\left(\overline{N}, N_E, \lambda\right) - u\left(\overline{N}, N_E, 0\right) = \frac{\Delta(\overline{N}, N_E, 0) - \Delta(\overline{N}, N_E, \lambda)}{1-\delta} < 0,$$

because recall from the Proof of Proposition 5 that $\Delta(\overline{N}, N_E, \lambda)$ is increasing in λ. Thus, we can conclude that data sharing increases consumer surplus when $0 \le s_E - s_I < \Theta^w\left(\overline{N}, N_E, \lambda\right)$ and decreases consumer surplus when $\Theta^w\left(\overline{N}, N_E, \lambda\right) < s_E - s_I < \Delta(\overline{N}, N_E, \lambda)$, where $\Theta^w\left(\overline{N}, N_E, \lambda\right)$ is given by (6).

3. If $s_E - s_I \ge \Delta(\overline{N}, N_E, \lambda)$, then $u(\overline{N}, N_E, \lambda)$ is constant in λ, so data sharing has no effect on consumer surplus.

7.10 Proof of Proposition 13

The first step to prove the result is to show the only pure-strategy equilibria involve consumers always buying from I or always buying from E. It is easy to rule out equilibria in which consumers switch more than once from one firm to the other. To rule out equilibria with one switch, we show that there is always a profitable deviation for a consumer: not switching at all, or switching one period later than everyone else (i.e. free-riding on other consumers generating the across-user learning), or else switching to the other firm earlier than everyone else because it must offer higher utility when the other two deviations are not profitable. The details of the proof are a bit involved and so relegated to the Online Appendix. Here we characterize the two candidate pure-strategy equilibria.

To keep the notation streamlined, define

$$\overline{f}^W(n) \equiv \begin{cases} f^W(n) & \text{if} \quad n \le \overline{n} \\ f^W(\overline{n}) & \text{if} \quad n > \overline{n} \end{cases}.$$

Consider the equilibrium in which consumers choose I in all periods. In this equilibrium, each consumer gets a surplus of

$$\frac{s_I}{1-\delta}+\frac{\delta}{1-\delta}f^A(1)+\delta\sum_{j=1}^{\infty}\delta^{j-1}\overline{f}^W(j).$$

The optimal deviation by a consumer cannot involve switching more than once between I and E. To see this, suppose the optimal deviation involves choosing E for the first $m_E \geq 1$ periods, then I for the next $m_I \geq 1$ periods, and then switching back to E in period $m_E + m_I + 1$. There are two possible cases. If $s_E + \overline{f}^W(m_E) \geq s_I + f^A(1) + \overline{f}^W(m_I)$, then, regardless of the deviation path from period $m_E + m_I + 2$ onwards, choosing E in every period yields a higher total payoff: it yields a strictly higher payoff in every period from $m_E + 1$ to $m_E + m_I$ and a weakly higher payoff in all periods thereafter. If instead $s_E + \overline{f}^W(m_E) < s_I + f^A(1) + \overline{f}^W(m_I)$, then choosing I from period $m_E + m_I + 1$ onwards yields a strictly higher payoff. Thus, this cannot be an optimal deviation. Similarly, suppose the optimal deviation involves choosing I for the first $m_I \geq 1$ periods, then E for the next $m_E \geq 1$ periods, and then switching back to I in period $m_I + m_E + 1$. If $s_E + \overline{f}^W(m_E) < s_I + \overline{f}^W(m_I) + f^A(1)$, then choosing I in every period would yield a higher total payoff. If instead $s_E + \overline{f}^W(m_E) \geq s_I + \overline{f}^W(m_I) + f^A(1)$, then choosing E from period $m_I + m_E + 1$ onwards yields a strictly higher payoff. So this cannot be an optimal deviation either.

Now suppose the optimal deviation involves choosing E for $m_E \geq 1$ periods and then choosing I from period $m_E + 1$ onwards. This means we must have $s_E + \overline{f}^W(m_E) < s_I + f^A(1)$, otherwise the payoff would be higher by sticking to E forever. This implies that if $m_E > 1$, then in period $m_E - 1$ choosing I yields $s_I + f^A(1)$, whereas choosing E yields $s_E + \overline{f}^W(m_E)$, which is strictly lower. Thus, the deviation payoff could be increased by switching to I from period $m_E - 1$. So for this to be an optimal deviation it must be that $m_E = 1$.

Finally, suppose the optimal deviation involves choosing I for $m_I \geq 0$ periods and then choosing E from period $m_I + 1$ onwards. If $m_I \geq 1$, this means we must have $s_E \geq s_I + f^A(1) + \overline{f}^W(m_I)$, otherwise the payoff would be higher by sticking to I forever. But this implies $s_E \geq s_I + f^A(1) + \overline{f}^W(m_I - 1)$, so the payoff could be improved by switching to E in period m_I. Consequently, for this to be an optimal deviation, we must have $m_I = 0$, i.e. choosing E in every period.

Thus, we have proven there are only two possible deviations that we need to consider. Either a consumer chooses E in every period, or chooses E in period 1, waits for I's product to improve from across-user learning, and switches to I in period 2 and stays with I for all subsequent periods.

Considering these two optimal deviations, for always choosing I to be an equilibrium we need

$$\begin{aligned}&\frac{s_I}{1-\delta}+\frac{\delta}{1-\delta}f^A(1)+\delta\sum_{j=1}^{\infty}\delta^{j-1}\overline{f}^W(j)\\ \geq\ &\max\left\{\frac{s_E}{1-\delta}+\delta\sum_{j=1}^{\infty}\delta^{j-1}\overline{f}^W(j),\, s_E+\frac{\delta}{1-\delta}\left(s_I+f^A(1)\right)+\delta^2\sum_{j=1}^{\infty}\delta^{j-1}\overline{f}^W(j)\right\}\end{aligned}$$

or equivalently,

$$s_E - s_I \leq \delta \min \left\{ f^A (1), (1-\delta) \sum_{j=1}^{\infty} \delta^{j-1} \overline{f}^W (j) \right\}.$$

By symmetry, for always choosing E to be an equilibrium we need

$$s_E - s_I \geq -\delta \min \left\{ f^A (1), (1-\delta) \sum_{j=1}^{\infty} \delta^{j-1} \overline{f}^W (j) \right\}.$$

This leads to the three possible parameter regions for $s_E - s_I$ in Section 5.3. The rest of the Proposition is proven in the main text.

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