Platform governance*

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Abstract

Platforms that intermediate trades such as Amazon, Airbnb, and eBay play a regulatory role in deciding how to govern the "marketplaces" they create. We propose a framework to analyze a platform's non-price governance designs and its incentive to act in a welfare-enhancing manner. We show that the platform's governance designs can be distorted towards inducing insufficient or excessive seller competition, depending on the nature of the fee instrument employed by the platform. These results are illustrated with micro-founded applications to a platform's control over seller entry, information provision and recommendation, quality standards, and search design choices.

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1 Introduction

A growing number of proprietary platforms operate as marketplaces through which buyers and third-party sellers trade. Well-known examples include third-party marketplaces Amazon and eBay, accommodation sharing site Airbnb, software distribution platforms such as Google's Playstore, Steam, and Apple's Appstore, video game console such as Sony's PS4, as well as brick-and-mortar shopping malls. Much like a government regulator that runs an economy, a platform regulates the behavior of platform users through setting its rules or governance designs. Several policy reports have highlighted the importance of the governance role of platforms (Furman et al., 2019; Scott Morton et al., 2019). Most notably, according to Section 4.III of the EC Competition Report:

"Platforms impose rules and institutions that reach beyond the pure matching service and shape the functioning of the marketplace and, potentially, the relationship between the various platform sides, e.g. by regulating access to and exclusion from the platform, by regulating the way in which sellers can present their offers, the data and APIs they can access, setting up grading systems, regulating access to information that is generated on the platform, imposing minimum standards... Such rule-setting and 'market design' determine the way in which competition takes place [on a platform]." — Crémer, de Montjoye, and Schweitzer (2019)

Given the increasingly prominent roles played by dominant platforms in the economy, the EC Report and other observers have pointed out that these platforms have a responsibility to write good rules to ensure that competition on their platforms is fair and pro-users. However, a key question is whether a proprietary platform necessary has an incentive to act in a way that maximizes welfare (or joint value on its ecosystem). In this paper, we provide a framework to examine the incentives and trade-offs that a platform faces in its governance designs, and study how its designs can be distorted away from what is optimal for the entire marketplace or from a total welfare perspective.

In practice, the scope of platform governance is wide. We focus on governance designs that have these two characteristics: (i) they (indirectly) influence the gross transactional value generated on the platform for buyers (V), and (ii) they influence the intensity of on-platform seller competition (M). For instance:

• Example 1: a platform can regulate the effective number of competing sellers by carefully selecting, for each category, how many sellers to admit and display to buyers. Admitting and displaying more sellers improves product variety and intensifies the competition.¹

 $^{^{1}}$ This is a common logic of standard competition models with horizontal product differentiation, e.g. Salop (1979) and Perloff and Salop (1985).

• Example 2: a platform can determine how its recommendation system allocates weights between product price and product match (or relevance). An emphasis on price intensifies competition between sellers but may result in a poor product match.²

Beyond these two examples, the duo-characteristic formulation of platform governance is simple yet rich enough to cover several other design decisions, including those regarding quality control, the extent of advertisement targeting and personalization, and on-platform search friction.

Using our framework, we compare the optimal governance design by a profit-maximizing platform against the governance design maximizing welfare in order to examine the source of welfare distortion. In our framework, there is a platform that facilitates transactions between buyers and price-setting sellers, and it chooses its governance design and the fee(s) charged to sellers. Each buyer wants to buy one unit of a single product and purchases it from the seller of its choice. Our main result identifies a class of models where the sign of the welfare distortion can be related precisely to the fee instrument employed by the platform.

We start by analyzing the case in which the platform fee is purely transaction-based, which can be either a proportional fee or a per-transaction fee. With proportional fees, which are used by many different online platforms, we show that the platform's profit can be interpreted as a weighted sum of seller profit and transaction volume, and so its governance design aims to balance the interest of these two components. When the marginal cost of sellers is small relative to the elasticity of buyer demand (with respect to the net utility offered), the platform's profit — which is proportional to seller revenue approximates seller profit. Therefore, the platform benefits from a governance design that relaxes seller competition and sustains a high markup for sellers. The profit-maximizing design can be distorted towards relaxing on-platform seller competition as compared to the welfare benchmark. However, as the sellers' marginal cost increases, the platform's profit begins to diverge from seller profit given that it does not internalize sellers' marginal cost. Once the marginal cost is sufficiently high, the platform's incentive is reversed and it now prefers to set a governance design that maximizes transaction volume, so that its design becomes distorted towards intensifying seller competition instead. We further show this distortion can lead to insufficient gross transactional value generated for buyers, depending on the correlation between V and M whenever there is a change in the platform's governance design.

With per-transaction fees, the platform's profit increases with total transaction volume so that it sets its governance design to maximize the volume. This over-emphasis on

²See, e.g. Dinerstein et al. (2018) and Armstrong and Zhou (2019) for empirical and theoretical analysis on this trade-off in ranking design.

transaction volume means that the platform can potentially fail to balance the dual roles of governance (i.e. influencing V and M) in a welfare-maximizing manner. Indeed, we find that the profit-maximizing design can be distorted towards intensifying on-platform seller competition and insufficient gross transactional value, as compared to the welfare benchmark. This distortion arises provided the correlation between V and M is sometimes negative, e.g., the ranking design decision mentioned above.

We then consider the case where the platform charges participation fees on sellers, e.g. listing fees that many online marketplaces charge on sellers. With pure participation fees, the platform profit becomes proportional to the joint industry (the platform and sellers) profit, and its governance design is distorted towards relaxing on-platform seller competition to maximize this joint profit. A similar intuition applies when the platform charges seller two-part tariffs (i.e. when both transaction-based fees and participation fees are feasible). Finally, we also consider alternative forms of revenue models such as external advertisement income or buyer participation fees in the form of subscription charges or device sales, and show that the result from the case of per-transaction fees is applicable in these cases.

Our results thus motivate the following taxonomy for platform fee instruments (or business models in general). On one hand, there are seller-aligned fee instruments (e.g. proportional fees when seller marginal cost is high, seller participation fees, and two-part tariffs), in which the platform prefers governance designs that relax seller competition and increase seller surplus instead. For instance, in Example 1, a platform with seller-aligned fee instruments may admit and show too few sellers (relative to the welfare benchmark). On the other hand, there are volume-aligned fee instruments (e.g. per-transaction fee, proportional fee when seller marginal cost is low, buyer participation fee, and external advertisement revenue), in which the platform prefers governance designs that intensify seller competition and increase transaction volume. In Example 2, a platform with volume-aligned fee instruments may over-emphasize the price dimension in its recommendation design.

The results have two main implications. First, we highlight that welfare results on platform models can be sensitive towards different modelling assumptions on fee instruments available. Therefore, it is important to be cautious when using theoretical results obtained under a certain fee instruments to make predictions about real-world markets if in these real-world markets different fee instruments are used. Second, our framework echoes the recent regulatory discussion that emphasizes the need to understand how different "business models" or monetizing methods of digital platforms can lead to different antitrust implications.³ To this end, the taxonomy outlined in the previous paragraph

³For instance, Caffarra et al. (2019) points out that "One major source of differentiation we need to take on board is distinctions in the business models the various ecosystems operate, and how these different strategies for monetising the surplus created by their platforms influence their incentives." See also Caffarra (2019) and Crawford et al. (2020).

provides a simple way to relate a platform's fee instrument (i.e. its monetization method) with potential welfare distortions in a class of governance design decisions captured by the current framework.

The rest of the paper proceeds as follows. Section 1.1 surveys the relevant literature. Section 2 lays out a general framework that nests various models of on-platform competition and governance. Section 3 and Section 4 analyze the general framework under a variety of platform fee instruments. Section 5 applies the insights obtained to discuss specific models of platform governance. Section 6 explores several extensions of our framework: allowing for endogenous choices of fee instrument, exploring what happens when governance design is costly, allowing for buyers with downward-sloping demand, and considering the consumer-surplus benchmark. Finally, Section 7 concludes. All omitted proofs and derivations are relegated to the Appendix.

1.1 Relation to the literature

Most of the existing literature on multi-sided platforms has been focused on pricing aspects (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Armstrong and Wright, 2007; Weyl, 2010; Jullien and Pavan, 2019). Our work contributes to the recent efforts that expand the formal study of multi-sided platforms beyond pricing into the domain of platform governance. Among the platform governance design decisions investigated in the strategy and economics literature are: platform openness and innovation (Boudreau, 2010; Parker and Van Alstyne, 2018), intellectual property sharing (Niculescu, 2018), introduction of platform first-party content (Hagiu and Spulber, 2013), and delegation of control rights (Hagiu and Wright, 2015a; 2015b; 2018). Most of these works focus on how the governance choices can generate additional surplus on platforms by encouraging innovations by third-party developers or coordinating end-user behavior. These works do not focus on the role of governance in influencing on-platform price competition between sellers.

The focus on on-platform seller competition is also at the heart of Hagiu (2009), Nocke, Peitz, and Stahl (2007), Belleflamme and Peitz (2019), and Anderson and Bedre-Defolie (2020). In these papers, platform pricing (in particular, the seller-side membership fee component) endogenously determine the number of participating sellers through the free entry condition of sellers. As such, product variety on the platform is directly tied to platform pricing rather than being a separate governance decision. These papers show that, in equilibrium, different levels of seller competition are induced by platforms' pricing depending on various exogenous factors, such as platform ownership, the strength of crossnetwork effect, and consumers' preference for product variety. Compared to this line of literature, the current paper takes a different modelling approach by treating platform governance design (in this case, variety provision) and platform pricing as two distinct

decisions.⁴ This approach offers two benefits. First, it allows us to explore how the types of fee instruments employed by the platform shape its governance design choices, and how its choices differ from the welfare-maximizing solution. Second, it extends the insight from this literature to other design decisions that may not be directly tied to free entry condition, e.g. search interface design and information provision.

A key theme in our framework is the connection between value generation and the extent of seller competition in governance design. This is closely related to the works by De Cornière (2018) and Karle and Peitz (2017) which look at the degree of targeted advertising implementable on a monopoly search engine (which can be interpreted as a platform design choice). In the model of de Cornière (2018), the platform charges sellers a lump sum advertising fee and it benefits from lowering the accuracy of advertisement targeting. Doing so induces consumers to search through fewer products in equilibrium, which deteriorates product match and relaxes competition between sellers. Karle and Peitz (2017) considers a similar context with loss-averse consumers. In their setup, an industry-profit-maximizing platform can use imprecise targeting to relax seller competition through manipulating the formation of consumers' consideration sets and reference points. Using the terminologies of our framework, both of these setups can be categorized as having seller-aligned fees, whereby the platform is predicted to choose the design that is associated with less seller competition (i.e. less targeting). We complement these two works by offering a framework to extend their insights to other types of fee instruments.

In interpreting our framework, we also have in mind decisions regarding search interface design, information provision, and quality control. Here we provide a non-exhaustive survey on recent works along this line:

□ Search design and information provision. Hagiu and Jullien (2011) and White (2013) consider a platform that can garble the consumer search process in order to divert consumers towards sellers that generate a higher revenue for the platform. In doing so the platform trades off between earning a higher margin per consumer versus less consumer participation. However, such search diversion has no impact on the price-competition among sellers. Casner (2019) analyze a platform's incentive to increase consumer search cost in a search environment based on the random sequential search model of Wolinsky (1986). He independently obtains one of the same findings — a platform with a proportional fee has an incentive to obfuscate search to sustain seller markups. However, Casner's analysis focuses on an exogenously fixed proportional fee on sellers, whereas our framework considers other fee instruments under which the platform may have no incentive to obfuscate search. Our framework is also readily applicable to study other non-random search environments, in particular, the price-directed search

⁴Despite the difference in modelling approaches, some of our results coincide with those obtained by Nocke et al. (2007) and Anderson and Bedre-Defolie (2020). We discuss these connections in Sections 4.3 and 4.4.

environment (e.g. Armstrong and Zhou, 2011; Armstrong, 2017; Choi et al., 2018), whereby the platform's incentive to obfuscate search can be reversed.

Quality control. Jeon and Rochet (2010) analyzes how the quality standard decisions of an academic journal depend on whether it operates as an "open access" journal (charging nothing to readers) or a standard subscription-based journal. They show that the resulting quality standard is too high relative to welfare benchmarks if the journal charges readers for access, while the standard is too low if the journal is open access. Bouvard and Levy (2018) consider a certification intermediary that can invest in the quality of its certification technology, and they show the intermediary may invest too little in quality when its revenue relies on participation by low-quality firms. In contrast to these works, however, we consider a marketplace setting and show that the incentive to manipulate on-platform competition provides another explanation for why platforms may set quality controls that are either too restrictive or too permissive.

Finally, our emphasis on the role of different platform fee instruments (in particular, transaction-based fees) relates to the works by Shy and Wang (2011), Johnson (2017), and Wang and Wright (2017), among others. These works compare constant per-transaction fees against proportional fees, and they show the superiority of the latter in mitigating the double marginalization problem or in facilitating price discrimination across product categories. These works (and ours) do not address the question of the optimal instrument to use, which can reflect other considerations, such as technological limitations (such as the inability to monitor the price and/or quantity of transactions) or coordination issues (participation-based fees may be infeasible when platforms face a chicken-and-egg problem to launch). For this line of inquiry, see Hagiu and Wright (2018).

2 Model setup

The environment consists of a continuum of unit-demand buyers, multiple sellers (can be finite or a continuum), and a platform that enables transactions between buyers and sellers. We first present a general framework that is meant to encompass several different models of platform governance. We then illustrate the framework with four specific micro-foundations in Section 2.2.

Governance design. The platform chooses and announces a governance design, which is parameterized as a continuous variable $a \in \Theta$. Here, $\Theta \subseteq \mathbb{R}$ is a compact set indicating the designs implementable by the platform. To fix idea, it is useful to think of a as a stylized representation of a platform's choices regarding the effective number of sellers on the platform, quality control level, search interface, and information design. The choice of a affects (i) the gross transactional value V(a) that each buyer obtains from visiting the platform and purchasing items from the sellers, and (ii) the intensity of on-platform seller competition, which is parameterized as the markup M(a) that sellers

earn (to be made precise below). Both V(a) and M(a) are continuous functions for all $a \in \Theta$. To highlight our main points in a simple fashion, we assume that the platform faces zero fixed and marginal costs, and both costs are independent of a.

Without loss of generality, we rank a in decreasing order of M(a) so that M(a) is an decreasing function. Hence, a higher a is interpreted as a "pro-competitive" design, i.e. it is associated with more intense seller competition. We allow V(a) to be non-monotone in general, but in various parts of the paper it is useful to take note of two special cases:

- Pro-competitive value generation: V(a) is monotonically increasing.
- Anti-competitive value generation: V(a) is monotonically decreasing.

Seller pricing. For each design a chosen by the platform, sellers engage in price competition to attract buyers. Sellers have a constant marginal cost c > 0. Suppose we ignore any platform fees at the moment. Without imposing any specific micro-foundation, we posit that the seller competition results in a symmetric equilibrium price

$$p = c + M(a). (1)$$

Hence, $M(a) \ge 0$ is exactly the equilibrium markup that sellers earn. With an arbitrary form of M(a), the equilibrium price equation (1) is consistent with those arising from various micro-foundations with unit-demand consumers, e.g. the circular city model of Salop (1979), the discrete choice model of Perloff and Salop (1985), spokes model of Chen and Riordan (2007), the sequential search model of Wolinsky (1986) and Anderson and Renault (1999), and the price-directed search model of Choi et al. (2018), among others. The reduced-form formulation allows us to concisely capture the effect of platform governance on how the seller competition unfolds.

Buyers and volume of transactions. Buyers need to pay an intrinsic cost, $d \ge 0$, to visit the platform. We assume that the market is conditionally ex-post covered, that is, every buyer purchases one product conditioned on visiting the platform. The net surplus that a buyer obtains from visiting platform is V(a) - p, while the buyer's surplus from outside option is normalized to zero. There is heterogeneity in d so that only buyers with $d \le V(a) - p$ would participate on the platform. Given this, the total number of transactions (or aggregate demand) faced by the platform is the total number of participating buyers, denoted as

$$Q(V(a) - p) = \Pr\left(d \le V(a) - p\right),\tag{2}$$

where Q is strictly increasing, continuously differentiable, and weakly log-concave.

Platform fees. The platform levies a fee on sellers for each transaction, which can be a linear per-transaction fee τ , or it can be a proportional fee r (also known as a revenue

sharing contract).⁵ For notational brevity, we assume that the platform does not charge any transaction fee to buyers, which is without loss of generality due to the tax neutrality principle (Weyl and Farbinger, 2013).⁶

Under a per-transaction fee, the fee τ is essentially an additional marginal cost to sellers, so that the equilibrium price equation in (1) becomes

$$p = c + \tau + M(a).$$

Under a proportional fee, for each unit of sales revenue generated, a seller receives its share 1-r while the platform keeps the remaining share $r \in [0,1]$. For any given r, each seller's sales margin can be written as $(1-r)\left(p-\frac{c}{1-r}\right)$. Ignoring the multiplicative factor, the seller sales margin is $p-\frac{c}{1-r} < p-c$, reflecting that a seller keeps only a share of its revenue but bears all of its costs of product, so that the seller acts as if its "effective" marginal cost is $\frac{c}{1-r}$. Hence, the equilibrium price equation in (1) becomes

$$p = \frac{c}{1 - r} + M(a).$$

Timing: (i) Given the fee instrument, the platform sets its fee level and governance design. (ii) Sellers and buyers decide whether to enter the platform. (iii) The on-platform interaction between buyers and sellers unfolds according to the specified microfoundation.

2.1 Discussion of modelling features

The assumption of governance design being a continuous variable simplifies the exposition, but it is not a necessary ingredient to derive our main insights. In practice, platforms may make discrete or even multi-dimensional governance design choices, e.g. deciding whether to ban certain behaviors or choosing a set of rules to govern interactions between buyers and sellers. In Section A of the Online Appendix, we extend the model to the case where each design choice a is a vector from a compact finite set Θ (possibly multi-dimensional). By exploiting the correspondence between the finite design choices and the outcomes (captured through the pair (V(a), M(a))), we show that our main results continue to hold with weak inequalities.⁷

It is useful to think of our framework as focusing on design decisions on user interface and how information are provided to buyers, which do not involve substantial costs in

⁵The possibility of participation-based fees is explored in Sections 4.3 - 4.5.

⁶In the marketplace environments we study, the standard principle of tax-neutrality - whereby sellers take into account the buyer-side fees when they set prices - implies that aggregate demand does not depend directly on the decomposition of platform fees between buyer fees and seller fees. Even if such neutrality does not hold, in many of the platform examples we have in mind, buyers do not face any fees, suggesting our focus on seller fees, is anyway a realistic assumption.

⁷The only exceptions are Propositions 2 and 4, where we need to impose tighter thresholds on c in order to obtain each of the stated cases in the proposition statements.

their implementation. In Section 6.2, we allow for costly design decision by assuming that the platform's fixed cost $K = K(V(a)) \ge 0$ is increasing in V(a), i.e., a design that is associated with a higher gross transactional value involves a higher implementation cost. For example, in order to raise the number of sellers admitted, a platform may need to invest to expand its hosting capacity. We discuss how our results can be extended to this case.

We have assumed that the market is conditionally ex-post covered, meaning that every buyer visiting the platform purchases one product so that the aggregate demand Q depends only on the mass of visiting buyers. This allows the framework to be compatible with microfoundation with ex-post covered market, most notably the spatial competition models, while still allowing for an elastic aggregate demand. An alternative approach is to assume that buyers face no visiting cost (so all buyers visit the market by default), but they have a downward sloping demand for products after visiting the platform. A complication in such a setup is that sellers' equilibrium markup would generally depend on the effective marginal costs faced by sellers, which in turns depend on the fee level set by the platform. We consider this setup in Section 6.3.

Finally, the assumption that transactional value V(a) is the same for every buyer shuts down the well-known Spence (1975) distortion. Spence distortion arises from the fact that a monopolist focuses on the valuation of marginal users in its choice of product quality (or other product attributes) while the social planner focuses on the valuation of average users. Shutting down Spence distortion allows us to isolate the new form of distortion that arises in our platform setting.

2.2 Micro-foundations

In this subsection, we provide three simple micro-foundations that fit the general framework presented above, whereby each example corresponds to a different aspect of platform governance design. To keep the exposition brief, we focus on showing how each example maps into the general framework, and relegate the detailed derivations to Section B of the Online Appendix. We will return to these examples when we discuss the implications from our analysis.

Example 1 Variety choice by platform (based on Salop, 1979)

There is a continuum of unit-demand buyers and multiple ex-ante symmetric and horizontally-differentiated sellers. The platform announces the number of sellers it wants to display to each visiting buyer, which we denote as $a \in \{2, ..., N\}$. Specifically, for each visiting buyer, the platform randomly (and uniformly) selects a number of sellers

 $^{^{8}}$ A mathematically equivalent interpretation is platform directly chooses the number of sellers admitted to the platform. We will ignore the integer constraint on a in the main analysis. The same

to enter the buyer's "consideration set", so that the model is effectively equivalent to an oligopolistic price competition with $a \geq 2$ sellers. After observing a, a buyer chooses whether to incur a joining cost d to join the platform and learn the match values and prices of the available products in her consideration set. The utility from buying a product i is $V_0 - tx_i - p_i$, where V_0 is the homogenous intrinsic product value, tx_i is the linear mismatch cost specific to seller i, and p_i is the price by seller i. Assume that V_0 is large enough so that the market is fully covered ex-post. For each seller, conditional on being selected into consideration sets the effective demand is derived as per the Salop circular city model: $\frac{1}{a} + \frac{p-p_i}{t}$.

Given that the probability of entering consideration set is independent of price, the standard derivation shows that the symmetric equilibrium price is

$$p = c + M(a) \equiv c + \frac{t}{a},$$

where M(a) is decreasing in a. A buyer joins the platform if the expected gain outweights the joining cost, i.e. V(a) - p > d, where $V(a) = V_0 - \frac{t}{4a}$ is increasing in a. Therefore, value generation is pro-competitive: displaying more sellers improves product variety and intensifies seller competition.

Example 2 Information design by platform (based on Lewis and Sappington, 1994)

Consider the same utility specification as in Example 1 with N=2 sellers and both sellers are in buyers' consideration sets by default. Buyers observe both prices p_1 and p_2 but they are initially uncertain about their match values. Before purchasing, buyers observe all prices and a private signal s of her mismatch costs $x_1 \in [0, 1/2]$ and $x_2 = 1/2 - x_1$, and then decide which seller to purchase from. The platform commits to a "truth-or-noise" signal structure parameterized by $a \in [0,1]$ which is implemented through its recommendation algorithm. With probability 1-a the signal s is informative and equals to the true value x_1 and with probability 1-a the signal is an uninformative random draw from a uniform distribution over [0,1/2].

Upon receipt of a signal s, a buyer is unable to distinguish between truth or noise and has to form posterior expectation on the mismatch costs via Bayesian updating: $\mathbf{E}(x_1|s) = (1-a)s + a\mathbf{E}(x_1)$, where $\mathbf{E}(x_1) = 1/4$ is the prior expectation and $\mathbf{E}(x_2|s) = 1/2 - \mathbf{E}(x_1|s)$. A buyer receiving signal s is indifferent between the two sellers if $p_1 + t\mathbf{E}(x_1|s) = p_2 + t\mathbf{E}(x_2|s)$, and the demand for seller 1 can be derived as $\frac{1}{2} + \frac{p_2 - p_1}{t(1-a)}$. The symmetric equilibrium price is

$$p = c + M(a) \equiv c + \frac{t(1-a)}{2}.$$

microfoundation can be derived based on Perloff-Salop (1985) random-utility discrete model. See also Huang (2020) for a similar model where products are homogenous and the platform influences the size of consideration set a through non-uniform sampling.

Meanwhile, a buyer joins the platform if the expected gain outweights the joining cost, i.e. V(a) - p > d, where $V(a) = V_0 - (1+a)\frac{t}{8}$. Value generation is anti-competitive in this setup: an informative signal structure (a=0) has the highest M(a) and V(a) because it maximizes the perceived product differentiation and buyers always purchase their preferred product; an uninformative signal structure (a=1) has the lowest M(a) and V(a) because in equilibrium sellers compete away all their margin while buyers purchase their preferred product only with probability 1/2.9 As such, variable a can be interpreted as the weight that the recommendation algorithm assigns to the price dimension (relative to the product match).

Example 3 Quality control by platform (based on Eliaz and Spiegler, 2011).

There is a continuum of unit-demand buyers and a continuum of sellers. Each seller i has quality $q_i \in [0,1]$, which is distributed according to the cdf H. When a buyer is matched with a seller of type q_i , with probability q_i the seller's product is suitable and provides utility value x_i where $x_i > 0$ is a consumer-product match component; With probability q_i the product is defective and provides zero utility. The values of x_i are i.i.d drawn across buyers and product, with cumulative distribution function (CDF) F and log-concave density f. Upon visiting the platform, buyers search randomly and sequentially with perfect recall with a search cost s > 0 each time they sample a seller. By sampling seller i, a buyer learns the product price p_i , the match value x_i , and whether product i is suitable, but the buyer never observes seller type q_i .

The platform sets a minimum quality standard $a \in [0, \bar{a}] \subseteq [0, 1]$, such that only sellers with quality $q_i \geq a$ are allowed to sell on the platform.¹⁰ Buyers search only within the pool of sellers with $q_i \geq a$ and they infer from a that the average quality of the seller pool is $E(q_i|q_i \geq a)$. Define a buyer's search reservation value V(a) implicitly as

$$\int_{V}^{\bar{\epsilon}} (\epsilon - V) dF(\epsilon) = \frac{s}{\mathbf{E}(q_i | q_i \ge a)}.$$
 (3)

Notably, the "effective search cost" faced by consumers (right hand side of (3)) is decreasing in a, so that V(a) is increasing in a. Following the standard derivation, the demand faced by a seller i is proportional to $1 - F(V(a) - p + p_i) q_i$ The symmetric equilibrium price is

$$p = c + M(a) \equiv c + \frac{1 - F(V(a))}{f(V(a))},$$

⁹See Johnson and Myatt (2006) for a more recent generalization of Lewis and Sappington's (1994) specification. Armstrong and Zhou (2020) consider the more general information design problem and derive the exact "buyer-optimal" and "seller-optimal" and show that the trade-off between intensifying seller competition and ensuring that buyers purchase their preferred product applies much more generally.

¹⁰This can be done by screening out low-quality sellers, or by a commitment to remove problematic listings. In practice, how strictly the platform's ranking algorithm penalizes listings with poor reviews will have a similar effect.

where M(a) is decreasing in a. It can be shown that if a buyer ever visits the platform then she eventually purchase one of the products, so that she visits the platform if and only if V(a) - p > d. Value generation is pro-competitive: a higher quality standard is equivalent to a lower search cost, which improves the match value from search and intensifies seller competition.¹¹

3 Baseline analysis: exogenous platform fees

Given that we rank a in decreasing order of M(a), we can can equivalently reformulate platform's problem as choosing the level of seller markup $m \in M(\Theta)$ directly whereby each m is associated with gross surplus $v(m) \equiv V(M^{-1}(m))$. We will use this reformulation throughout the rest of the paper to simplify the notations. In addition, it is useful to denote the smallest and largest elements of $M(\Theta)$ as \underline{m} and \overline{m} respectively.

To develop initial intuitions, in this section we slightly deviate from the general framework of Section 2 by assuming that the fee levels τ and r are exogenously fixed. By doing so we shut down any distortion introduced by fee-setting decisions of the platform, which allows us to highlight distortions in the platform's governance designs. Moreover, this baseline analysis is also relevant when fee levels are determined through some unmodeled institutional constraints (e.g. binding fee caps) or bargaining process. We first consider the case of per-transaction fees (Section 3.1), and then the case of proportional fees (Section 3.2).

3.1 Per-transaction fees

When the platform charges a per-transaction fee $\tau > 0$ to sellers, the equilibrium price that arises from seller competition is $p_{\tau}(m) = c + \tau + m$. Consider a profit-maximizing platform, with profit function

$$\Pi(m) = \tau Q(v(m) - p_{\tau}(m)).$$

We denote $m^p \equiv \arg \max_m \Pi$, where the superscript refers to "profit-maximization". Using $dp_{\tau}(m)/dm = 1$, the first-order derivative is

$$\frac{d\Pi}{dm} = \left(\frac{dv}{dm} - 1\right)\tau Q',\tag{4}$$

¹¹The feature of a lower search cost intensifying competition is common to the more general random sequential search models, as shown by Wolinsky (1986) and Anderson and Renault (1999).

¹²For example, Parker and Van Alstyne (2017) considers the Nash bargaining solution with equal bargaining power between the platform and third-parties sellers/developers, which gives r = 0.50.

¹³In this section, we do not consider lump-sum participation fees. If such fees were exogenously fixed then they are independent of the design choice and do not affect the analysis below as long as sellers are willing to participate.

so that the profit maximized by maximizing the transaction volume, i.e.

$$m^p = \arg\max \{v(m) - m\}.$$

First, note that if value generation is pro-competitive (i.e. $dv/dm \leq 0$ for all m), then it is the most profitable to decrease m until $m^p = \underline{m}$ because doing so increases the transactional value and decreases price, both of which increases transaction volume. More generally however, if v(m) is increasing or non-monotone, then a trade-off arises: if the platform attempts to increase the transactional value v(m), this would come with an implicit "cost" of increasing the seller markup and hence price.

To identify the source of distortion in the profit-maximizing governance choice, we consider the welfare-maximization benchmark for comparison. Note that welfare-maximization is equivalent to a pure value-creation benchmark (Boudreau and Hagiu, 2009; Scott Morton et al., 2019), i.e., maximizing the total amount of economic value generated from user interactions on the platform. Total welfare is defined as the sum of joint industry profit (the platform and sellers) and buyer surplus:

$$W(m) = (p_{\tau}(m) - c)Q(v(m) - p_{\tau}(m)) + \int_{-\infty}^{v(m) - p_{\tau}(m)} Q(t) dt,$$

where the buyer surplus is obtained by integrating the aggregate demand from Q = 0 up to the marginal demand.¹⁵ Define $m^w \equiv \arg\max_m W$, where the superscript refers to "welfare-maximization".¹⁶

The first-order derivative of the welfare function is

$$\frac{dW}{dm} = \left(\frac{dv}{dm} - 1\right) \left(\left(p_{\tau} - c \right) Q' + Q \right) + Q. \tag{5}$$

Similar to the profit maximization problem, if value generation is pro-competitive then

$$\frac{dW}{dm} < \left(\frac{dv}{dm} - 1\right)Q + Q \le 0,$$

so that it is welfare-maximizing to set $m^w = \underline{m}$. If instead v(m) is increasing or non-monotone, the trade-off between value generation and the implicit cost of a higher seller markup again arises. An important distinction between (4) and (5) is the additional term

¹⁴We consider the consumer surplus benchmark in Section 6.4.

¹⁵This specific reduced-form buyer surplus representation relies on our assumption that the buyers' outside option is normalized to zero. Our results would remain valid even if the outside option is some non-zero constant; we only require that it is not a function of the governance design.

¹⁶This benchmark is welfare-maximizing only in a "partial" sense given that the platform fees are assumed to be fixed. In Section 4, where we endogenize the platform fee-setting decision, we consider the "second-best" welfare benchmark whereby there is a social planner that controls the governance design but does not control the fee decision of the platform.

Q > 0, reflecting that the implicit cost of a higher seller markup is smaller in welfare maximization. This is because the loss in transaction volume (or output) due to a higher markup is partially offset by the corresponding gain in seller surplus (which increases if price increases). This implies that the welfare-maximizing design calls for a higher m and v(m) compared to the profit-maximizing design, as proven formally in the next proposition:

Proposition 1 (Exogenous per-transaction fees) Suppose the platform charges an exogenous per-transaction fee τ , then $m^p \leq m^w$ and $v(m^p) \leq v(m^w)$, with the inequalities strict if m^p or m^w is an interior solution.

Proposition 1 suggests that the profit-maximizing governance design is associated with excessive on-platform competition and insufficient value generation. Notably, this holds even though there is no explicit cost associated with value generation. To illustrate the implications of Proposition 1, we return the first two examples in Section 2.2.

In the variety decision of Example 1, value generation is pro-competitive so that $m^p = m^w = \underline{m}$. Both the profit-maximization and welfare-maximization calls for the highest possible number of variety displayed at a = N, so that there is no distortion in this case.

In the information design decision of Example 2, value generation is anti-competitive in which a more informative information structure (lower a) involves a trade-off between value generation and the implicit cost of a higher seller markup. Assuming that $V_0 = 10$, c = 4, $\tau = 1$, t = 2, and Q(t) is the CDF of uniform distribution with domain $[0, V_0/2]$, then we can solve the profit-maximizing and welfare-maximizing information design as a = 1 and a = 0.6 respectively. The profit-maximizing design is insufficiently informative and leads to excessive product mismatch.

3.2 Proportional fees

With a proportional fee r > 0, the equilibrium price that arises from seller competition is

$$p_r(m) = \frac{c}{1-r} + m. \tag{6}$$

With a slight abuse of notation, we continue to denote platform's profit and total welfare as:

$$\Pi(m) = r p_r(m) Q(v(m) - p_r(m))$$

$$W(m) = (p_r(m) - c) Q(v(m) - p_r(m)) + \int_{-\infty}^{v(m) - p_r(m)} Q(t) dt.$$

After substituting in (6), the platform's profit can be written as

$$\frac{1}{r}\Pi(m) = \underbrace{mQ(v(m) - p_r(m))}_{\text{proportional to seller surplus}} + \frac{c}{1 - r}\underbrace{Q(v(m) - p_r(m))}_{\text{volume of transactions}}.$$
 (7)

Equation (7) essentially decomposes the platform's profit into two components. The first component in (7) is proportional to the volume of transactions. The second component is proportional to seller surplus that is given by $(1-r) mQ(v(m)-p_r(m))$. This means that the platform's profit is fully aligned with the seller surplus if the first component of (7) is absent. Therefore, (7) can be loosely interpreted as a weighted sum between seller surplus and the volume of transactions, in which the relative weight depends on $\frac{c}{1-r}$. This weighted-sum interpretation suggests that the direction of distortion in platform governance depends on the level of seller marginal cost.

More formally, using $dp_r(m)/dm = 1$, the first-order derivative of the profit function can be written as

$$\frac{d\Pi}{dm} = \left(\frac{dv}{dm} - 1\right) r p_r Q' + r Q. \tag{8}$$

Similar to (4), the platform has an incentive to expand the transaction volume, as captured by the first term of (8). However, with proportional fee the platform's margin increases with the price, which in turn increases with seller markup. There is an additional incentive to maintain the markup level, as captured by the last term in (8).

Meanwhile, the first-order derivative of the welfare function is

$$\frac{dW}{dm} = \left(\frac{dv}{dm} - 1\right) \left(\left(p_r - c\right)Q' + Q\right) + Q. \tag{9}$$

Both profit-maximization and welfare-maximization consider the governance design's effect on (i) transaction volume (the first component in both equations) and (ii) seller markup (the second component in both equations). The key difference between profit-maximization and welfare-maximization is on how the relative weight is assigned between the two considerations.

Proposition 2 (Exogenous proportional fee) Suppose the platform charges an exogenous proportional fee r and W(m) is unimodal, r then:

- If $c < \frac{Q(v(m^p) p_r(m^p))}{Q'(v(m^p) p_r(m^p))}$, then $m^p \ge m^w$, with the inequality strict if m^p or m^w is an interior solution.
- If $c = \frac{Q(v(m^p) p_r(m^p))}{Q'(v(m^p) p_r(m^p))}$, then $m^p = m^{sb}$.

 $^{^{17}}$ A function F(x) is unimodal if for some value \bar{x} , it is monotonically increasing for $x \leq \bar{x}$ and monotonically decreasing for $x \geq \bar{x}$. Lemma 1 in the appendix shows that a sufficient conditions for W(m) to be unimodal are (i) v(m) is monotone decreasing, or (ii) v(m) is concave, which is satisfied in Examples 1-3.

• If $c > \frac{Q(v(m^p) - p_r(m^p))}{Q'(v(m^p) - p_r(m^p))}$, then $m^p \leq m^w$, with the inequality strict if m^p or m^w is an interior solution.

In Proposition 2, the condition c < Q/Q' is more likely to hold if c is small or if Q is inelastic (because Q'/Q is the semi-elasticity of the aggregate demand). Intuitively, both a lower c and a less elastic aggregate demand shift the platform's objective function towards seller surplus, so that the profit-maximizing governance design is associated with a competition level no higher than the welfare-maximizing design. Conversely, a higher c and a more elastic aggregate demand shift the platform's objective function towards the volume of transactions, so that its design is associated with a competition level no lower than the welfare benchmark.

To sharpen the results and intuitions, consider first the special case of pro-competitive value generation $(dv/dm \leq 0)$. In this case, we know from (9) that $m^w = \underline{m}$. Meanwhile, from (8), if $c \geq Q'/Q$ so that $p_r > \frac{Q}{Q'}$, we have

$$\frac{d\Pi}{dm} < \left(\frac{dv}{dm} - 1\right)rQ + rQ \le 0,$$

or $m^p = \underline{m}$, meaning that there is no distortion when c is large. Strict distortion $m^p > m^w$ and $v(m^p) < v(m^w)$ occurs only if c < Q'/Q. These observations reflect that: (i) the profit-maximizing platform tends to prefer a less intense seller competition to maintain the markup and price level, and (ii) as c increases, the price level becomes high enough so that the platform has a weaker incentive to maintain markup, in which case distortion disappears.¹⁸

To illustrate this, we return to the variety decision in Example 1, assuming N=10. Figure 1.1 below shows that the profit-maximizing number of variety displayed is strictly lower than the welfare-maximizing one when c<3.8, so that the level of seller competition and product variety are both too low under profit-maximization. As c increases, the distortion becomes smaller and eventually vanishes.

Next, consider anti-competitive value generation $(dv/dm \ge 0)$. Starting from $c \to 0$, so that the platform's profit function approximates seller surplus. The platform faces no trade-off in this case because it can keep increasing m to increase both v(m) and m as long as the resulting price is not above the price that maximizes joint industry profit. In contrast, there is a trade-off from the welfare perspective because increasing v(m) comes at the implicit cost of a higher price set by sellers. Therefore, we have $m^w \le m^p$ and $v(m^w) \le v(m^p)$. As c increases, the platform profit and the seller surplus begin to diverge as can be seen from (7). The divergence reflects that, under proportional fees, the platform internalizes sellers' revenue but does not internalize sellers' marginal

¹⁸It can be formally shown that m^p is monotone decreasing in c (see Lemma 2 in the appendix).

cost. A higher c increasingly skews the platform's objective function away from seller surplus. When c is high enough, the logic of Proposition 1 applies, whereby the profit-maximizing governance design is skewed towards maximizing the volume of transactions instead, where $m^w \ge m^p$ and $v(m^w) \ge v(m^p)$. Figure 1.2 illustrates the result with the information design decision of Example 2, showing that the profit-maximizing design is excessively informative for c < 4.1 and insufficiently informative for c > 4.1.

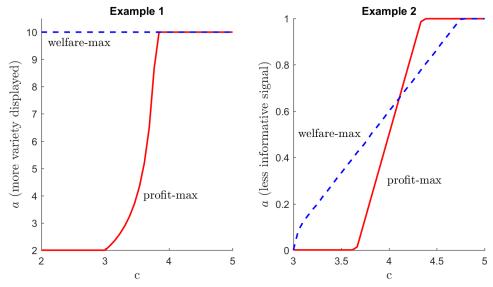


Figure 1: Governance design with exogenous proportional fees: (1) Variety display of Example 1, and (2) Information design of Example 2. Both figures assume r = 0.2, $V_0 = 10$, t = 2 and Q(t) is the uniform cdf with domain $[0, V_0/2]$.

Finally, note that the conditions in Proposition 2 are stated in terms of endogenous variables whereby m^p depends on c. Lemma 2 in the appendix shows that if v(m) is weakly concave or decreasing (which is satisfied in Examples 1-3), then there exists a unique threshold \bar{c} such that $c \leq \bar{c}$ implies $m^p \geq m^w$ while $c \geq \bar{c}$ implies $m^p \leq m^w$ Moreover, in cases where unimodality of W(m) does not holds, we can instead construct exogenous thresholds \bar{c}_l and \bar{c}_h , where $\bar{c}_h \geq \bar{c}_l > 0$, such that $c < \bar{c}_l$ implies $m^p \geq m^w$ while $c > \bar{c}_h$ implies $m^p \leq m^w$ (see Section A of the Online Appendix).

4 Analysis: endogenous platform fees

So far we have assumed that the fee level charged by the platform is exogenously fixed. We now consider the general analysis with endogenous fee levels. Throughout this section, the welfare benchmark we adopt is a "second-best" one, in which there is a social planner that fixes the governance design before the platform sets its fee so that the planner is constrained by the fee-setting response of the profit-maximizing platform. One interpretation is that the social planner regulates only the platform's governance while leaving its fees unregulated.

4.1 Per-transaction fees

When the platform sets endogenous per-transaction fee τ , its profit function is given by $\tau Q(v(m) - m - c - \tau)$. For each given m the profit-maximizing fee $\tilde{\tau} = \tilde{\tau}(m)$ is implicitly defined by first-order condition

$$\tilde{\tau} = \frac{Q\left(v\left(m\right) - m - c - \tilde{\tau}\right)}{Q'\left(v\left(m\right) - m - c - \tilde{\tau}\right)}.$$
(10)

Denote $p_{\tilde{\tau}}(m) = c + \tilde{\tau}(m) + m$. Given this, we can write down the platform's profit function as an indirect function of m:

$$\tilde{\Pi}(m) = \tilde{\tau}(m) Q(v(m) - p_{\tilde{\tau}}(m))$$

By the envelope theorem we can ignore the indirect effect of m on $\tilde{\tau}$ so that

$$\frac{d\tilde{\Pi}\left(m\right)}{dm} = \left(\frac{dv}{dm} - 1\right)\tilde{\tau}Q',$$

which involves the exact same trade-off as the baseline analysis (4). In particular, $m^p \equiv \arg\max \tilde{\Pi} = \arg\max \{v\left(m\right) - m\}$. Nonetheless, compared to the baseline case of exogenous transaction fees, maximizing v-m has a two-fold effect here. First, it allows the platform to raise the number of transactions Q; second, it allows the platform to reoptimize by increasing its fee, which leads to an even higher profit. The latter point can be seen from (10), whereby Q being log-concave means that $\tilde{\tau}$ is increasing in $v\left(m\right) - m$, though with an incomplete pass-through, i.e. $\frac{d\tilde{\tau}}{d(v-m)} \in (0,1)$.

The welfare function is

$$\tilde{W}(m) = (p_{\tilde{\tau}}(m) - c) Q(v(m) - p_{\tilde{\tau}}(m)) + \int_{-\infty}^{v(m) - p_{\tilde{\tau}}(m)} Q(t) dt.$$
(11)

We denote $m^{sb} \equiv \arg \max \tilde{W}(m)$, where the superscript refers to "second-best". Compared to the welfare benchmark in the baseline case, here the social planner needs to take into account the possibility that the platform may increase its fee in response to changes in the governance design:

$$\frac{d\tilde{W}}{dm} = \left(\frac{dv}{dm} - 1 - \frac{d\tilde{\tau}}{dm}\right) \left(\left(p_{\tilde{\tau}} - c\right)Q' + Q\right) + \left(1 + \frac{d\tilde{\tau}}{dm}\right)Q,$$

where we have used $\frac{dp_{\tilde{\tau}}}{dm} = 1 + \frac{d\tilde{\tau}}{dm}$. If value generation is pro-competitive then the incomplete pass-through property of $\tilde{\tau}$ implies $\frac{dv}{dm} - 1 - \frac{d\tilde{\tau}}{dm} \leq 0$, so that $\frac{d\tilde{W}}{dm} \leq \left(\frac{dv}{dm} - 1 - \frac{d\tilde{\tau}}{dm}\right)Q + \left(1 + \frac{d\tilde{\tau}}{dm}\right)Q \leq 0$. Therefore, similar to the analysis of the exogenous fee, if value generation is pro-competitive then $m^w = m^p = \underline{m}$, and there is no distortion. More generally,

the following result show that Proposition 1 continue to hold when we endogenize the setting of the platform's fee:

Proposition 3 (Endogenous per-transaction fees) Suppose the social planner can control the platform's governance design, but cannot control the per-transaction fee τ set by the platform. Then, $m^p \leq m^{sb}$ and $v(m^p) \leq v(m^{sb})$, with the inequalities strict if m^p or m^{sb} is an interior solution.

The key step in the analysis comes from the fact that $\frac{d\tilde{\tau}}{d(v-m)} \in (0,1)$, so that the profit-maximizing governance design m^p also maximizes the transaction volume $Q(v(m) - p_{\tilde{\tau}}(m))$. Proposition 3 then follows from the same logic that establishes Proposition 1: the plat-form focuses on maximizing volume and fails to internalize seller surplus. It is unwilling to set a design that corresponds to a high seller markup, even when doing so increases the transactional value generated.

4.2 Proportional fees

Suppose the platform sets endogenous proportional fee $r \in [0, 1]$ so that its profit function is $r\left(m + \frac{c}{1-r}\right)Q\left(v\left(m\right) - m - \frac{c}{1-r}\right)$. For each given m, setting r = 0 and r = 1 are obviously sub-optimal as they result in a zero profit for the platform. Therefore, the platform necessarily sets some $\tilde{r}\left(m\right) \in (0, 1)$ implicitly pinned down by first-order condition

$$\tilde{r} = \frac{Q\left(v\left(m\right) - m - \frac{c}{1 - \tilde{r}}\right)}{Q'\left(v\left(m\right) - m - \frac{c}{1 - \tilde{r}}\right)} \left(\frac{\left(1 - \tilde{r}\right)^2}{c} + \frac{\tilde{r}}{m + \frac{c}{1 - r}}\right). \tag{12}$$

One can verify that the right hand side of (12) is decreasing in \tilde{r} , so that (12) has a unique solution. Denote $p_{\tilde{r}}(m) = c + m + \frac{c}{1-\tilde{r}(m)}$. Given this, the platform's profit function becomes

$$\tilde{\Pi}(m) = \tilde{r}(m) p_{\tilde{r}}(m) Q(v(m) - p_{\tilde{r}}(m)).$$
(13)

By the envelope theorem we can ignore the indirect effect of m on \tilde{r} , so that

$$\frac{d\tilde{\Pi}(m)}{dm} = \left(\frac{dv}{dm} - 1\right)\tilde{r}p_{\tilde{r}}Q' + \tilde{r}Q,\tag{14}$$

which is the same as (8) except that endogeneity of the fee restricts the possible range of $p_{\tilde{r}}$. Indeed, if we rewrite (12) using the definition of $p_{\tilde{r}}$, we get

$$p_{\tilde{r}} = \frac{Q(v(m) - p_{\tilde{r}})}{Q'(v(m) - p_{\tilde{r}})} \left(1 + \frac{p_{\tilde{r}}c}{(p_{\tilde{r}} - m - c)(p_{\tilde{r}} - m)} \right)$$

$$> \frac{Q(v(m) - p_{\tilde{r}})}{Q'(v(m) - p_{\tilde{r}})}.$$
(15)

Therefore, if value generation is pro-competitive, then (14) implies $\frac{d\tilde{\Pi}(m)}{dm} \leq -\tilde{r}p_{\tilde{r}}Q' + \tilde{r}Q < 0$, so that $m^p = \underline{m}$. Intuitively, when the platform is free to set its fee, it no longer needs to raise the transaction price through increasing the seller markup. With pro-competitive value generation, there is no downside from lowering m so that the lowest m maximizes profit (which is in contrast to the case of exogenous r). However, if v(m) is increasing or non-monotone, the trade-off between value generation and the implicit cost of a higher seller markup again arises and the lowest m is not necessarily optimal.

Meanwhile, with endogenous proportional fee r, the derivative of welfare function with respect to m is

$$\frac{d\tilde{W}}{dm} = \left(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm}\right) \left(\left(p_{\tilde{r}} - c\right)Q' + Q\right) + \frac{dp_{\tilde{r}}}{dm}Q. \tag{16}$$

Comparing m^{sb} and m^p , the following result extends Proposition 2 to endogenous fees.

Proposition 4 (Endogenous proportional fee) Suppose the social planner can control the platform's governance design, but cannot control the proportional fee set by the platform. Suppose W(m) is unimodal, and denote

$$\Psi(m) \equiv \frac{\left(\frac{p_{\tilde{r}}}{\tilde{r}}\right) \frac{d\tilde{r}}{dm}}{\frac{dp_{\tilde{r}}}{dm} - \frac{dv}{dm}}.$$

- If $c < \frac{Q(v(m^p) p_{\tilde{r}}(m^p))}{Q'(v(m^p) p_{\tilde{r}}(m^p))} (1 + \Psi(m^p))$, then $m^p \ge m^{sb}$, with the inequality strict if m^p or m^{sb} is an interior solution.
- If $c = \frac{Q(v(m^p) p_{\tilde{r}}(m^p))}{Q'(v(m^p) p_{\tilde{r}}(m^p))} (1 + \Psi(m^p))$, then $m^p = m^{sb}$.
- If $c > \frac{Q(v(m^p) p_{\bar{r}}(m^p))}{Q'(v(m^p) p_{\bar{r}}(m^p))} (1 + \Psi(m^p))$, then $m^p \leq m^{sb}$, with the inequality strict if m^p or m^{sb} is an interior solution.

To sharpen the results, it is again useful to consider special cases of monotone v(m). With pro-competitive value generation $(dv/dm \le 0)$, applying implicit function theorem on (15) shows that that $\frac{dv}{dm} - \frac{dp_{\bar{r}}}{dm} < 0$, so that $\frac{d\tilde{W}}{dm} \le \left(\frac{dv}{dm} - \frac{dp_{\bar{r}}}{dm}\right)Q + \frac{dp_{\bar{r}}}{dm}Q < 0$ from (16). Therefore, $m^{sb} = m^p = \underline{m}$ and there is no distortion in this case. Returning to Example 1, this implies that the profit-maximizing platform provides the welfare-maximizing level of variety (N = 10), as depicted in Figure 2.1.

With anti-competitive value generation or non-monotone v(m) in general, Proposition 4 reflects a similar intuition as Proposition 2: a small c (relative to the inverse semi-elasticity of Q) shifts the platform's objective function towards seller surplus, while a large c shifts the platform objective function towards volume maximization. However, the key difference in Proposition 4 is the additional coefficient $\Psi(m^p)$, which has the same sign as $\frac{d\tilde{r}}{dm}$. The additional coefficient reflects that the choice of the second-best design

For the knife-edge case of $\frac{d\tilde{r}}{dm} = 0$, we get $\Psi(m^p) = 0$ so that Proposition 4 recovers Proposition 2 as a special case.

has to taken into account how the platform endogenously adjusts its fees in response. In the proof of Proposition 4, we show that $\frac{d\tilde{r}}{dm} \leq 0$, and so does $\Psi(m^p)$. Therefore, the range of parameter for $m^p \geq m^{sb}$ becomes smaller with endogenous fees.

As an illustration, in Figure 2.2 we return to Example 2 with the same set of parameters used in Figure 1.2. The figure shows that the profit-maximizing design is excessively informative for c < 0.7, and insufficiently informative for c > 0.7.

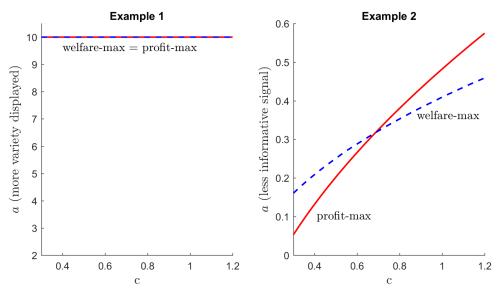


Figure 2: Governance design with endogenous proportional fees: (1) Variety display of Example 1, and (2) Information design of Example 2. Both figures assume $V_0 = 20$, t = 15, and Q(t) is the uniform cdf with domain $[0, V_0/2]$.

4.3 Lump sum seller fees

In some contexts a platform may find it hard to implement transaction-based fees due to the difficulties in monitoring transactions. Suppose instead that the platform charges a lump sum fee T_S on each seller, which can be interpreted as a participation fee or a per-item listing fee. Alternatively, T_S could be any transaction-independent fee that sellers have to incur in order to reliably make sales on the platform, e.g. advertising fees considered by de Cornière (2016).

It is easy to see that the platform does best by choosing a governance design that maximizes total seller surplus and then sets T_S to fully extract the total seller surplus. Its profit is

$$\tilde{\Pi}(m) = (p-c)Q(v(m)-p)$$

$$= mQ(v(m)-m-c).$$
(17)

while the welfare function is

$$\tilde{W}(m) = \tilde{\Pi}(m) + \int_{-\infty}^{v(m)-m-c} Q(t) dt$$

Similar to the logic of Proposition 2 (with small c), the profit-maximizing platform's choice of governance design induces a lower level of seller competition than the level that the planner desires, i.e. $m^p \geq m^{sb}$.

Proposition 5 (Lump sum seller fees) Suppose the social planner can control the platform's governance design, but cannot control the lump sum seller fee set by the platform. Then, $m^p \geq m^{sb}$, with the inequality strict if m^p or m^w is an interior solution.

Proposition 5 is closely related to the analysis by Nocke et al. (2007) that focuses on the platform's variety choice, as in our Example 1. In their model, for each given listing fee, the number of participating sellers is endogenously determined by the free entry condition (rather than being directly chosen as a design choice). Moreover, sellers have heterogenous fixed cost for operating, so that the platform does not fully extract the total seller surplus. Despite the differences, they show that the platform under-provides variety, which is consistent with Proposition 5 given that a higher m corresponds to fewer varieties. Proposition 5 indicates that their result does not specifically rely on the free entry condition.²⁰

Even though Proposition 5 (and Proposition 6 below) has utilized the fact that the platform fully extracts all seller surplus through the participation fee, these results continue to hold even if sellers face some uncertain fixed cost C, such that the platform does not fully extract all seller surplus. To see this, consider a simple example in which there are two possible realizations of seller fixed cost C_l and C_h , where $C_h > C_l$. Each realization occurs with probabilities λ and $1-\lambda$ respectively, and the actual realization of fixed cost is not observed by the platform. When λ is such that the platform is better off setting the participation fee at $mQ(v(m)-p)-C_l$ in which sellers participate regardless of the realized cost, then the platform does not extract all the surplus, and the only amendment to the analysis is an extra fixed cost term in the platform's profit expression, which does not affect the determination of the optimal governance design.

4.4 Two-part tariffs

Suppose the platform charges sellers a two-part tariff consisting of a participation fee T_S and a combination of per-transaction and proportional fees. The latter component means that the platform charges sellers a total of $rp + \tau$ for each facilitated transaction that is

²⁰Nocke et al. also allow for platform costs to depend on the number of sellers on the platform, We consider this extension in Section 6.2 and show that Proposition 5 continues to hold provided that value generation is pro-competitive (which includes Example 1).

priced at p. We rule out the possibility of platform providing net subsidy, i.e. we require $rp+\tau \geq 0$ (this does not rule out r < 0 and $\tau < 0$). Such net subsidies rare in practice, and they are difficult to implement because each seller can potentially fabricate transactions by fraudulently purchasing from itself in order to exploit the subsidy scheme.²¹

The platform sets T_S to fully extract the total seller surplus, so that its profit function equals the joint industry profit, as in (17). The key difference is that the platform can influence the price level through its variable fee component because $p = m + \frac{\tau + c}{1 - r}$. To state the result, define $p^*(m)$ as the monopoly price that maximizes the joint industry profit, i.e. it solves $p^* = c + \frac{Q(v(m) - p^*)}{Q'(v(m) - p^*)}$, and denote the value-maximizing design as

$$m^* = \arg\max_{m \in M(\Theta)} v(m).$$

Then:

Proposition 6 (Two-part tariff) Suppose the social planner can control the platform's governance design, but cannot control the two-part tariff set by the platform. Suppose m* is unique, then:

- If $c \le p^*(m^*) m^*$, then $m^p = m^{sb} = m^*$.
- If $c > p^*(m^*) m^*$ and v(m) is unimodal, then $m^p \ge m^{sb}$, with the inequality strict if m^p or m^w is an interior solution.

Absence of any constraint, the platform optimally sets its governance design at $m^p = m^*$ to maximize v(m), and then adjusts τ and r accordingly to implement the corresponding monopoly price $p^*(m^*)$. The optimal monopoly price $p^*(m^*)$ is implementable with non-negative fees as long as the corresponding seller markup is higher than the monopoly markup, i.e. $m^* \leq p^*(m^*) - c$. In addition, Q being log-concave means that p^* is increasing in v but with an incomplete pass-through, implying that $v(m) - p^*(m)$ and transaction volume are also maximized at m^* . It follows that $m^{sb} = m^*$ in this case.

However, when $m^* > p^*(m^*) - c$, 22 the optimal monopoly price $p^*(m^*)$ is longer implementable. Intuitively, the price set by sellers is determined by the extent of competition, as captured by m^* . When the competition is weak, the resulting price $m^* + c$ is higher the platform's target monopoly price $p^*(m^*)$, and so any attempt to induce $p^*(m^*)$ would violate the constraint of no net subsidy. With the constraint binding,

 $^{^{21}}$ Allowing the platform to charge $rp+\tau$ does not affect the analysis in Section 4.1. because the platform would optimally set $\tau=0$ and r given by (12). To see this, we can use the relation $p=m+\frac{c+\tau}{1-r}$ to substitute away r in the platform's fee-setting problem. Then, the platform's margin becomes $p(1-\frac{c+\tau}{p-m})+\tau,$ which is decreasing in τ for each given p.

²²It is easy to check that $p^*(m^*)$ is increasing in c with gradient less than 1, and so there exists a unique cost threshold such that c being smaller than the threshold is equivalent to $c \le p^*(m^*) - m^*$.

²³One way to work around the constraint is to extend the model by allowing the platform to subsidize buyer participation directly. See Section 6.1 for further details.

the platform's profit function becomes as if it charges only participation fees. In this case, the profit-maximizing governance design maximizes seller surplus, and we arrive at a result similar to Proposition 5.

If we apply Proposition 6 to Example 1, then it says that the platform (weakly) under-provides variety under a two-part tariff. Anderson and Bedre-Defolie (2020) independently derive a similar result, but with a mechanism that is slightly different from ours. In their model, the platform's fixed cost increases with the number of sellers on the platform. The platform does not extract the entire consumer surplus, meaning that it does not fully internalize the benefit from a higher variety and so it "under-invests" in variety. In Section D, we allow for costly governance design and show that the mechanism pointed out by Anderson and Bedre-Defolie reinforces the existing mechanism in Proposition 6, provided that value generation is pro-competitive (which includes Example 1).

4.5 Other revenue models

Apart from the fee instruments analyzed above, platforms may use other forms of revenue models such as external advertisement or buyer participation fees in the form of subscription charges or device sales. We briefly discuss how these revenue models can be mapped onto our existing analysis.

 \square Ad-funded platform. Suppose the platform generates revenue through displaying external ads, which are viewed as nuisance by consumers. For simplicity, we assume the platform charges no other fees, so p(m) = m + c. Let T_A be the amount of ads that the platform carries, and the net surplus that a buyer obtains from visiting platform is $v(m) - p(m) - T_A$. Following Casadeus-Masanell and Zhu (2010), we assume that (i) the revenue rate for each piece of ads, R = R(Q), is an increasing and concave function of the aggregate demand faced by the platform; and (ii) the revenue rate is determined by a competitive advertisement sector that earns zero profit. Thus, platform's profit is

$$\widetilde{\Pi}(T_A, m) = T_A R \left(Q(v(m) - p(m) - T_A) \right).$$

By the envelope theorem, the ad-funded platform platform chooses its design to maximize aggregate demand, so $m^p = \arg \max \{v(m) - m\}$. Given the advertisement sector earns zero profit so that it does not enter the welfare function, we can apply the same analysis in Section 4.1 to yield the same result as Proposition 5.

□ Buyer participation fee. In practice, most online marketplaces focus on levying fees on the seller side. An important reason is that buyers are often uncertain about whether or not they want to buy a product, and they first inform themselves on the platform about characteristics of the available products. Thus, charging a participation fee on buyers will deter many buyers. Alternatively, buyer participation fee may be

infeasible if a platform cannot monitor participation decision by buyers. Nonetheless, in contexts where buyer participation fee is feasible, we can amend the framework by supposing that the platform charges buyers a lump sum participation fee T_B . Following the unit-demand assumption, the aggregate demand function Q quantifies the number of participating consumers. Thus, platform's profit is

$$\tilde{\Pi}(T_B, m) = T_B R \left(Q(v(m) - p(m) - T_B) \right).$$

If the platform charges no other fees, then p(m) = m + c so that Proposition 5 applies. If instead the platform mixes participation fee with seller participation fee and transaction fees, then Proposition 6 applies.

4.6 Summary

Our analysis highlights that the platform's incentives in choosing its governance design are strongly tied to (i) the chosen fee instrument and (ii) market characteristics in terms of the seller marginal cost and elasticity of the aggregate demand.

To summarize the main insights, in Table 1 we categorize each of the analyzed platform fee instruments according to the direction of the welfare distortion in platform governance design. For *volume-aligned* fee instruments the profit-maximizing governance design maximizes the volume of transactions. It can be distorted towards intensifying seller competition and generating low transactional value. For *seller-aligned* fee instruments, the profit-maximizing governance design is skewed towards increasing the seller surplus hence tends to induce too little seller competition.

Fee instrument:	Per-transaction fees / Proportional fees (high c) / Advertisement revenue / Buyer participation fees	Seller participation fees $/$ Proportional fees (low c) $/$ Two-part tariffs
Platform's incentive in setting governance design:	Volume-aligned: to intensify on-platform competition $(m^p \leq m^{sb})$	Seller-aligned: to $relax$ on-platform competition $(m^p \ge m^{sb})$

Table 1: Summary

5 Discussion and stylized facts

We apply the insights developed in Sections 3 and 4 to several platform governance design decisions.

 \Box Variety choice by platform. In Example 1, displaying more sellers increases product variety and reduces sellers' markup. Both effects increase welfare and aggregate demand, so that with volume-aligned fee instruments, both welfare-maximization and profit-maximization call for admitting the highest possible number of sellers, N. In contrast, with seller-aligned fee instruments, intensifying seller competition reduces the platform's revenue. Therefore, if the markup-reducing effect dominates, the platform will admit strictly less than N sellers.

To illustrate the implications, consider the example of video game platforms, e.g. Sony PS4, Microsoft Xbox One, and Nintendo. These platforms price their game consoles to buyers at approximately cost, generating most of the revenue through charging game developers/publishers a two-part tariff consisting of developer kit fees (i.e. participation fees) and fixed per-game licensing payment (i.e. constant per-transaction fees). Proposition 6 suggests that these platforms tend to restrict the number of competing video game titles too much in order to sustain the profit of major game developers. Consistent with this prediction, Sony, Microsoft, and Nintendo indeed restrict access to a selected set of game developers and exclude many others, as documented by Evans et al. (2008).

□ Information design by platform. Consumers typically rely on online platforms to obtain product information and their preference for these products. The platform can choose how it wants to disclose these information to consumers, such as how detailed the product information is, how it wants to aggregate customer reviews, and the extent to which consumers can filter the information displayed. Example 2 provides a stylized model to capture this, highlighting a fundamental trade off between improving product match (which involves disclosing more product match information) and intensifying competition (which involves disclosing less product match information to reduce the perceived product differentiation).

The model in Example 2 can also be interpreted as a platform choosing its recommendation algorithm. An algorithm that emphasizes recommending products with low prices intensifies seller competition and so it is analogous to disclosing less product information. Likewise, an algorithm that emphasizes facilitating product match (i.e. by taking into account product differentiation) would be analogous to disclosing more product information. Dinerstein et al. (2018) found evidences that are consistent with this trade off in the context of search interface redesign by eBay. In May 2011, ebay switched its search algorithm from the so-called best-match algorithm (which has a stronger emphasis on facilitating product match, especially when there is high degree of differentiation between products) to a two-stage algorithm that gives a greater emphasis on pricing but requires sellers and the platform to accurately classify listings according to the products being offered. However, eBay reverted back to the best-match algorithm a year later. Dinerstein et al. (2018) explain this design reversion by pointing out that best-match algorithm is

easier for sellers to use, especially those that are less professional in categorizing their listings. The analysis in the current paper suggests that another possible explanation for the design reversion is the platform's incentive to tune the extent of seller competition and maintain markup.

Quality control by platform. In online markets, the prevalence of information asymmetry between buyers and sellers means that platforms need to carefully regulate the quality of the listed sellers. In the quality control model of Example 3, a search pool with a higher expected quality is analogous to a lower "effective search cost" for buyers. This reflects that each buyer searches less and consequently incurs a lower total expected search cost before reaching a positive-valued match. Raising the quality standard improves the expected product match and reduces seller markup.²⁴ If the markup-reducing effect dominates, with seller-aligned fee instruments, a platform's choice of quality standard will be strictly less than the welfare-maximizing one.

To illustrate the implications, consider the example of PC games distribution platforms such as Steam. With the extensive online user reviews available, prospective consumers can learn the performance of each game if they carefully go through the review system. Steam charges game developers a fixed proportional fee 30% for each transaction on the platform, while developers typically have a marginal cost that is close to zero. Our results suggest that the platform does not have an incentive to set a very strict quality control. Reportedly, Steam has shown reluctance in imposing simple low cost measures that could significantly increase the average quality of product pool on offer.²⁵ One explanation for the reluctance to engage in stricter quality control could be its attempt to allow PC developers to maintain higher prices and revenues, which the platform can extract through its proportional fee.

Even though our analysis has focused on the role of a high quality standard in facilitating search (which intensifies seller competition), in some contexts a high quality standard can plausibly generate countervailing effects that relax seller competition instead. For instance, if the number of sellers is finite, excluding low-quality sellers reduces the total number of sellers akin to entry restriction, which relaxes seller competition, so that quality control may involve anti-competitive value generation as well. The overall effect of a higher quality standard on the extent of seller competition generally depends on whether the search-facilitating effect or the entry-restriction effect dominates.

□ On-platform search friction. Online platforms often make design decisions that influence the ease of buyer search on the platform (i.e. search cost incurred by buyers to browse and inspect products). When platforms can costlessly manipulate buyers'

²⁴Consistent with our formulation, Hui et al. (2019) provide empirical evidence showing that a more stringent quality certification policy on ebay has increased the extent of on-platform seller competition while at the same time increasing the average quality of the seller pool.

²⁵See https://www.rockpapershotgun.com/2017/02/14/steam-curation-user-reviews- xes/

search cost, the question is: would a platform profit from obfuscating search, that is, not minimizing buyers' search cost?

A natural starting point to analyze search cost manipulations is Wolinsky's (1986) random search model. In Wolinsky's model, lowering search cost improves the expected product match and intensifies seller competition because the demand faced by each seller becomes more elastic. Value generation is pro-competitive in this context. Welfare-maximization calls for the lowest possible search cost. As for the profit-maximizing platform, the results in Sections 3 and 4 imply that the platform has no incentive to obfuscate search if its fee instrument is volume-aligned, consistent with the conventional wisdom (Dinerstein et al., 2019) that platforms want to limit search frictions and provide buyers with transparent and low prices. However, the results also imply that a platform would want to obfuscate search if its fee instrument is seller-aligned instead, consistent with Hagiu and Jullien's (2011) point that platforms do not always want to eliminate search frictions.²⁶ Thus, our framework offers a reconciliation between Hagiu and Jullien (2011)'s point and the conventional wisdom by showing that the platform's incentive to reduce search frictions can go in either direction depending on the fee instrument employed by the platform.

Departing from Wolinsky's random search model, another interesting setting to analyze search cost manipulations is to allow buyer search to be price-directed, e.g. the model of Choi et al. (2018).²⁷ In a price-directed search setting, buyers can observe prices before sampling for product match values. This feature is particularly relevant in the context of price-comparisons websites, whereby a buyer first looks at a list of product-price offers before clicking on offers that she wants to spend time investigating further.

Following our terminology, value generation is anti-competitive with price-directed search: lowering search cost improves the expected product match and, somewhat counter-intuitively, relaxes seller competition. To understand the latter point, note that a higher search cost means buyers become less likely to visit another seller after having visited the first seller. This makes it worthwhile for each seller to set a low price and attract buyers to visit it first (recall that buyers' search sequence is influenced by the prices they observe). Due to this mechanism, a lower search cost essentially makes demand less price-elastic in a price-directed search environment as opposed to the random search environment of Wolinsky (1986).²⁸

²⁶In the current paper the exact mechanism for this result differs from those in Hagiu and Jullien (2011). In our setup, search diversion or obfuscation relaxes seller competition, increases seller pricing, and increases the revenue for the intermediary (through a proportional fee) from each consumer. Hagiu and Jullien shut down this channel by assuming that the sellers are either (i) independent or (ii) interdependent but unable to adjust their prices in response to any diversion. In their model, the intermediary obtains revenue for each store visit by consumers, and search diversion increases the number of store visit for each consumer that goes to the intermediary.

²⁷See also Armstrong and Zhou (2011) and Armstrong (2017).

²⁸In the random search model, prices are unobservable before search so that a lower search cost makes demand more price-elastic instead.

Given that lowering search costs relaxes competition, the welfare-maximizing search quality optimally balances between higher search quality and avoiding high seller markups. Meanwhile, a profit-maximizing platform with volume-aligned fee instruments prefers an even lower seller markup than a social planner hence its design may correspond to search costs that are too high, while a platform with seller-aligned fee instruments may choose a design that corresponds to search costs that are too low. The results suggest that price comparison websites, which typically charge sellers a listing fee for displaying their offers on the platforms, have a strong incentive to continuously improve the layout of information in order to facilitate search on the platforms.

6 Extensions

This section explores several extensions of our baseline model. To keep the exposition brief, we focus on presenting the main insights in this section and relegate further details and formal proofs of the propositions to Sections C to F of the Online Appendix.

6.1 Endogeneity of fee instrument

In Section 4 we have treated the choice of fee instruments as being exogenous. There are two reasonings behind this approach. First, by comparing exogenously imposed instruments, we show that different modelling assumptions on platform fee instruments can generate substantially different welfare results. Second, the choice of fee instruments may reflect considerations not captured in the current framework. For example, if there are technological limitations such that the platform cannot reliably monitor transactions, then any transaction-based fee component may not be feasible.²⁹ On the other hand, a participation-based fee component may give rise to the possibility of a chicken-and-egg coordination problem and leads to a no-participation outcome. Another possible reason is that the sellers may face liquidity constraints in which they could not pay up-front participation fees before getting their products sold. Finally, in richer environments, the choice of fee instrument may take into account asymmetric information, moral hazard, or ongoing investments by the platform and sellers. The resulting For this line of inquiry, see, e.g., Foros, Hagen, and Kind (2009), and Hagiu and Wright (2018).

Nonetheless, if we endogenize the platform's choice of fee instrument in our framework, then for each given m, the two-part tariff is optimal. It allows the platform to obtained the highest possible profit equivalent to a (multiproduct) monopolist's profit, as have been

²⁹A relevant example is price comparison websites in housing or rental markets, in which deals are typically conducted outside the platform and are difficult to be monitored. Instead of transaction-based fees, these platforms typically charge sellers an up-front listing fee. Notable examples include rental market websites such as Rightmove and Zoopla in the United Kingdom, or Immobilienscout24 and Immowelt in Germany (Karle, Peitz, and Reisinger, 2020).

noted by Anderson and Bedre-Defolie (2020). The platform (i) uses the lump sum fee to fully extract seller surplus, (ii) sets variable fees to implement the optimal monopoly price $p^*(m^*)$, and (iii) chooses design m^* to achieve the maximum transactional value $v(m^*)$. However, in our model the equivalence disappears when the constraint of no net subsidy (on transactions) binds. To restore the equivalence, one way is to extend the model by allowing the platform to subsidize buyer participation directly. Provided that we interpret Q as the mass of participating buyers, the per-participation subsidy would be mathematically equivalent to a negative τ . With this extension, then for any m, the platform can always feasibly induce $p^*(m)$ without subsidizing transactions.

Remark 1 Suppose the fee instrument choice is endogenously chosen by the platform. For any given design, the profit-maximizing fee instrument is a two-part tariff with a subsidy on buyer participation.

Once we endogenize fee instruments, it becomes natural to consider a "stronger" second-best benchmark whereby the regulator may not only intervene in platform design but also by limiting the use of certain fee instruments (but the exact fee levels are still chosen by the platform). In Section C of the Online Appendix, we compare this stronger second-best design benchmark, m^{sb+} , and the profit-maximizing design (without any intervention), m^{p+} , and obtain the following result:

Remark 2 (Endogenous fee instrument) Suppose the social planner can intervene the governance design and the choice of fee instrument. The second-best intervention involves requiring: (i) the platform to charge lump sum listing fee to exactly recover its fixed cost (if any), and (ii) a design choice of $m^{sb+} \leq m^{p+}$, with the inequality strict if m^{p+} or m^{sb+} is an interior solution.

Part (i) of the proposition is straightforward because pure listing fees eliminate the pricing distortions resulting from transaction-based fee components (given that the platform would attempt to utilize such components to induce the monopoly price level). Part (ii) follows from the fact that m^{p+} maximizes v(m), while m^{sb+} has to take into account how design affects the final price level.³⁰

In cases where the lump sum fee component of the two-part tariff is infeasible, then we have to focus on pure transaction-based fees as in Shy and Wang (2011), Johnson (2017), and Wang and Wright (2017). In our framework, for each given m it can be shown that the platform's profit is higher with proportional fee as compared

 $^{^{30}}$ The welfare-optimality of the listing fee relies on seller participation being inelastic in the current framework. If seller participation is elastic, listing fees could downward distort seller participation. Still, provided that seller participation is not too elastic relative to Q, the downside from the participation distortion would be dominated by the benefit of a smaller price distortion, so that pure listing fee remains welfare-optimal.

to per-transaction fee. The proof and reasoning follow directly from the more general result by Johnson (2017), whereby proportional fee mitigates double marginalization. To see this, notice that if we transform the platform's decision in Section 4.1 as choosing the final price level directly, then for each given m, the platform's profit function is $\Pi_{\tau} = \max_{p} \left\{ (p-c-m)Q(v(m)-p) \right\}$ under a per-transaction fee, and $\Pi_{r} = \max_{p} \left\{ (p-c-\frac{mc}{p-m})Q(v(m)-p) \right\}$ under a proportional fee. We know $p-m \geq c$, and so a standard monotone comparative statics argument shows $\Pi_{r} > \Pi_{\tau}$. Meanwhile, the second-best intervention call for the lowest possible fee subject to platform recovering its cost. The same logic of mitigating double marginalization means that charging proportional fee is the more efficient approach to recover platform's cost.

6.2 Costly governance design

In this section, we briefly discuss how our results in Section 4 can be extended to the case where platform governance design is costly. Suppose that the platform's fixed cost K = K(v(m)) is increasing and convex in its argument. For simplicity, we focus on the case where v(m) is monotone. The details of analysis are available in Section D of the Online Appendix.

For all instruments analyzed in Section 4, we know that the platform does not extract the entire total welfare. This implies that it does not internalize the entire benefit from a higher transactional value while at the same time it bears the entire cost of governance design. This cost consideration creates an additional distortion that shifts the profit-maximizing governance design towards a lower v relative to the welfare-maximizing design.

Suppose that value creation is pro-competitive (e.g. Example 1) so that, all else being equal, the cost consideration induces the platform to set design $m^p \geq m^{sb}$. Recall from Table 1 that, absent of design cost, we have $m^p \geq m^{sb}$ when the platform uses seller-aligned fee instruments, meaning that the cost consideration reinforces the existing distortion. Therefore, the result from seller-aligned fee instruments continue to hold. In contrast, when the platform uses volume-aligned fee instruments, the distortion identified in Table 1 acts in the opposite direction of the cost consideration. In other words, the cost consideration mitigates the existing distortion (and possibly overturns it).

Following the same logic as above, the reverse is true when value creation is anticompetitive (e.g. Example 2). All else being equal, the cost consideration makes $m^p \leq m^{sb}$ more likely, thus reinforcing the distortion identified in the cases of volume-aligned fee instruments, so that the existing result continue to hold in this case. Meanwhile, the cost consideration existing mitigates the distortion identified in the cases of seller-aligned fee instruments.

6.3 Alternative notion of market coverage

Our framework has focused on micro-foundations with unit-demand consumers and conditional ex-post covered model, which implies a full pass-through of marginal cost to price. In this section, we consider an alternative notion of market coverage. Suppose that all buyers (costlessly) visit the market by default, and each of them has a downward sloping demand for products after visiting the platform. To make things concrete, consider the following amended version of the representative consumer model by Shubik and Leviatan (1980) and Singh and Vives (1984):

Example 4 Variety choice with quasilinear quadratic utility model

Suppose there is a finite number of $N \ge 1$ ex-ante symmetric unit-product sellers. Let $a \in \{1, ..., N\}$ represents the number of sellers admitted to the platform. A representative consumer, having access to the admitted sellers, has utility

$$U = V \sum_{i=1}^{a} Q_i - \frac{a}{2} \left((1 - \gamma) \sum_{i=1}^{a} Q_i^2 + \frac{\gamma}{a} \left(\sum_{i=1}^{a} Q_i \right)^2 \right) + Y - \sum_{i=1}^{a} p_i Q_i.$$

Here, V is the direct utility from product consumption; $\gamma \in (0,1)$ is a measure of product differentiation; Q_i and p_i are the quantity of product i consumed and its price; and $Y - \sum_{i=1}^{a} p_i Q_i$ is the residual income of the consumer after the total expenditure on the products. We extend Shubik and Leviatan's model by allowing V = V(a) to be a function of a, so that the consumer's utility depends on the product variety available on the platform. The consumer's demand for each product i is

$$Q_{i} = \frac{1}{a} \left(V\left(a\right) - \frac{p_{i}}{1 - \gamma} + \frac{\gamma}{1 - \gamma} \sum_{i=1}^{a} \frac{p_{i}}{a} \right).$$

Each seller faces a constant marginal cost c and solves $\max_{p_i} (p_i - c) Q_i$, implying symmetric equilibrium price p = c + M(a, c), where

$$M(a,c) = (V(a) - c)\frac{(1-\gamma)a}{(2-\gamma)a - \gamma}.$$

We can allow V(a) to be non-monotone as long as M(a,c) is monotone decreasing in a. The aggregate demand faced by platform (i.e. the total demand from the representative consumer) is $Q = \sum_{i=1}^{a} Q_{i} = V(a) - p$.

Motivated by Example 4, we formulate our reduced-form framework in this section as follow. We posit that the seller competition results in a symmetric equilibrium price p = c + M(a, c), where M is decreasing in its first argument (i.e. designs are sorted

in the order of the induced competition intensity) and linearly decreasing in its second argument with slope $-\beta$ where $\beta \in (0,1)$ is independent of c. Note that the latter implies a constant and incomplete cost pass through rate $1 - \beta \in (0,1)$. The aggregate demand remains denoted as Q(V(a) - P).

Given that M is a bivariate function in this alternative framework, it is more straightforward to compare the profit-maximizing and welfare-maximizing designs directly (instead of comparing in term of the equilibrium markup induced). Denote the the profit-maximizing and the (second-best) welfare-maximizing designs as a^p and a^{sb} respectively. In Section E of the Online Appendix, we obtain formal results that are analogous to Proposition 5 - 6, whereby $a^p \leq a^{sb}$ when the platform uses lump sum fees or two-part tariffs. The analysis of pure transaction-based fee is more complicated because it needs to take into account how governance design affects the fee pass through rate $1 - \beta$. In the special case of β being independent of a, then results similar to Propositions 3 - 4 continue to hold as well. These results are consistent with the results in Section 4.

6.4 Consumer surplus benchmark

In several countries, antitrust authorities have adopted a consumer welfare (rather than a total welfare) standard. Recall that consumer surplus is $CS = \int_{-\infty}^{v(m)-p(m)} Q(t) dt$. Similar to the second-best welfare benchmark, we let m^{CS} be the design that maximizes CS, subject to the endogenous fee response of the platform.

Note that CS is increasing with the aggregate transaction volume, so m^{CS} also maximizes transaction volume. Given the intuition and categorization in Table 1, we would expect that profit-maximizing design involves no distortions for volume-aligned fee instruments, but corresponds to insufficient seller competition for seller-aligned fee instrument. We formally confirm this intuition in Section F of the Online Appendix. The following remark corresponds to the propositions in Section 4:

Remark 3 Comparing profit-maximizing design and consumer surplus-maximizing design:

- If the platform uses per-transaction fees, advertisement revenues, or pure buyer participation fee then $m^p = m^{CS}$.
- If the platform uses proportional fees (for all c), lump sum seller participation fees, or two-part tariffs, then $m^p \geq m^{CS}$.

With the consumer surplus benchmark, the only possible direction of distortion is $m^p > m^{CS}$. The sole concern is that the platform's design induces insufficient competition. This is in contrast to the total welfare benchmark, in which the opposite concern

of $m^p < m^{CS}$ is possible: the platform (with volume-aligned fee instruments) may focus on intensifying on-platform competition at the expense of value generation and seller surplus. As such, Remark 3 highlights that an over-emphasis on the consumer welfare standard in regulating platforms' behaviors may sometimes lead to a different conclusion relative to the total welfare standard.

7 Conclusion

An important distinction between a platform (marketplace) business and a traditional retailer is that the platform hosts groups of sellers that make independent pricing decisions, whereas the retailer sells and prices all its products directly. As such, a platform's governance design decisions affect not just the gross surplus generated from on-platform transactions but also how the seller competition unfolds. The current paper investigate what drives the difference between the profit-maximizing and the socially-optimal governance designs.

As summarized in Table 1, the sign of welfare distortion in platform governance crucially depends on the fee instrument employed. Given that prices enter transaction volume and seller surplus in opposite directions, each platform business can be seen as positioning itself on a continuum of business models. At one end, there is a pure volume-aligned model in which the platform prefers governance designs that induce more intense seller competition than the socially optimal design. At the other end, there is a pure seller-aligned model in which the platform prefers governance designs that induce less intense seller competition than the socially optimal design.

To extend our framework, an obvious direction is to investigate how competition between platforms affects platform governance design. With two rival platforms, it is natural for each seller to join both intermediaries and for each buyer to join only one. This leads to a competitive bottleneck equilibrium similar to that analyzed in Armstrong and Wright (2007). Inter-platform competition implies a transaction volume that is more elastic (with respect to the net utility offered to buyers) than the case of a monopoly platform, which induces the platforms to adjust their governance design towards inducing more on-platform competition, so as to achieve a more competitive price on their respective platforms. Following this intuition, for each given fee instrument, we expect that the introduction of inter-platform competition shifts each platform business to be more volume-aligned.

The paper focuses on the case where the platform operates purely as a marketplace. One of the issues that has come to forefront in recent policy discussion is when the platforms play a dual role of "umpire and player": operating marketplaces while at the same time offering their own products on these marketplaces. Considering vertically integrated platforms can give rise to other design distortion factors that the current

framework does not capture. For example, the platform may engage in abusive design decisions such as preferencing its own product in ranking algorithms or imitating third-parties' innovative product, both of which effectively shun away rival sellers (Hagiu, Teh, and Wright, 2020). Alternatively, if sellers are asymmetric, the platform can distort its search ranking mechanism to "steer" buyers to low-quality sellers if such sellers offer higher per-transaction revenue to the platform (Teh and Wright, forthcoming). The current paper shows that distortion can arise even in the absence of vertical integration or asymmetry among sellers. An important direction for future studies is to extend the current framework to incorporate these additional features.

8 Appendix: Proofs

8.1 Proposition 1

For all $m < m^p$, we have

$$W(m) \leq \Pi(m^{p}) + mQ(v(m^{p}) - p(m^{p})) + \int_{-\infty}^{v(m^{p}) - p(m^{p})} Q(t) dt$$

$$< \Pi(m^{p}) + m^{p}Q(v(m^{p}) - p(m^{p})) + \int_{-\infty}^{v(m^{p}) - p(m^{p})} Q(t) dt$$

$$= W(m^{p}),$$

where the first inequality follows from the definition of $m^p \equiv \arg\max\{v(m) - p(m)\}$, while the second inequality follows from $m < m^p$. Therefore, $m^w \ge m^p$, and it follows that $v(m^w) \ge v(m^p)$ as otherwise m^w cannot be welfare-maximizing. To complete the proof, we need to rule out the existence of an interior m^p that simultaneously satisfies first order conditions (FOCs) $\frac{dW}{dm} = 0$ and $\frac{d\Pi}{dm} = 0$. From (4), we know $\frac{d\Pi}{dm}|_{m=m^p} = 0$ implies $\frac{dv}{dm}|_{m=m^p} = 1$. Substituting this into (5), we get $\frac{dW}{dm}|_{m=m^p} = Q > 0$, so there is no interior solution satisfying both FOCs simultaneously.

8.2 Proposition 2

Suppose $\frac{dv}{dm}|_{m=m^p} \ge 1$, then from (8) this implies $\frac{d\Pi}{dm}|_{m=m^p} > 0$, so that $m^p = \bar{m}$. Moreover, from (5), $\frac{dW}{dm}|_{m=m^p} > 0$ and so unimodality of W implies $m^w = m^p = \bar{m}$.

Suppose instead $\frac{dv}{dm}|_{m=m^p} < 1$. Rearrange (5) as

$$\begin{split} \frac{dW}{dm} &= \left(\frac{dv}{dm} - 1\right) p(m)Q' + Q + \left(\frac{dv}{dm} - 1\right) (Q - cQ') \\ &= \frac{1}{r} \frac{d\Pi}{dm} + \left(\frac{Q}{Q'} - c\right) \left(\frac{dv}{dm} - 1\right) Q'. \end{split}$$

Then:

- When $c = \frac{Q(v(m^p) p(m^p))}{Q'(v(m^p) p(m^p))}$, then $\frac{dW}{dm}|_{m=m^p} = \frac{1}{r} \frac{d\Pi}{dm}|_{m=m^p}$, so $m^w = m^p$.
- When $c < \frac{Q(v(m^p) p(m^p))}{Q'(v(m^p) p(m^p))}$, then $\frac{dW}{dm}|_{m=m^p} < \frac{1}{r}\frac{d\Pi}{dm}|_{m=m^p}$. If m^p is a boundary solution then $m^w \le m^p$ trivially. If instead m^p is interior with $\frac{d\Pi}{dm}|_{m=m^p} = 0$, then unimodality of W implies $\frac{dW}{dm} < 0$ for all $m \ge m^p$. It follows that $m^w \le m^p$, with the inequality strict if either m^p or m^w is interior.

• When $c > \frac{Q(v(m^p) - p(m^p))}{Q'(v(m^p) - p(m^p))}$, the same analysis shows $m^w \ge m^p$, with inequality strict if either m^p or m^w is interior.

Finally, we prove the following claims stated in the main text:

Lemma 1 If v(m) is monotone decreasing or weakly concave, then W defined in Section 3.2 is unimodal.

Proof. If v(m) is monotone decreasing, then

$$\frac{1}{Q}\frac{dW}{dm} = \left(\frac{dv}{dm} - 1\right)\left(p_r(m) - c\right)\frac{Q'}{Q} + \frac{dv}{dm} < 0,$$

so unimodality is trivial. Consider non-monotone but weakly concave v(m), i.e. $\frac{dv}{dm}$ is decreasing. For all m such that $\frac{dv}{dm} \ge 1$, $\frac{dW}{dm} > 0$. For all m such that $\frac{dv}{dm} < 1$, we have

$$\frac{1}{Q}\frac{dW}{dm} = \left(\frac{dv}{dm} - 1\right)\left(\left(p_r(m) - c\right)\frac{Q'}{Q} + 1\right) + 1$$

As m increases, $\frac{dv}{dm}-1$ becomes more negative. We also know $(p(m)-c)\frac{Q'}{Q}$ is increasing in m (using log-concavity and $\frac{dv}{dm}<1$), so that the first term in decreasing in overall. It follows that if there exists some \bar{m} such that $\frac{dW}{dm}|_{m=\bar{m}}=0$, then $\frac{dW}{dm}<0$ for all $m>\bar{m}$, which proves the unimodality.

Lemma 2 m^p is monotone decreasing in c.

Proof. From (8), whenever m^p is interior it is implicitly pinned down by FOC:

$$\left(\frac{dv}{dm} - 1\right) \left(m + \frac{c}{1 - r}\right) + \frac{Q(v(m) - m - \frac{c}{1 - r})}{Q'(v(m) - m - \frac{c}{1 - r})} = 0.$$
(18)

For the FOC to hold, m^p must be such that $\frac{dv}{dm}|_{m=m^p} < 1$. Moreover, interiority ensures that (18) is locally decreasing in m when evaluated at $m=m^p$. By implicit function theorem, the sign of $\frac{dm^p}{dc}$ is the same as the sign of the derivative of the left hand side of (18) with respect to c, which is strictly negative because Q is log-concave (i.e., Q/Q' is increasing in its argument).

Lemma 3 Suppose v(m) is monotone decreasing or weakly concave. There exists a cutoff \bar{c} such that $c < \bar{c}$ implies $m^p \ge m^w$ and $c > \bar{c}$ implies $m^p \le m^w$.

Proof. Suppose $\frac{dv}{dm}|_{m=m^p} > 0$, then it suffices to show that $v\left(m^p\right) - p_r(m) = v\left(m^p\right) - m^p - \frac{c}{1-r}$ is decreasing in c so that the lemma follows from Proposition 2. If m^p is a corner solution then $v\left(m^p\right) - m^p - \frac{c}{1-r}$ is obviously decreasing in c. If m^p is interior, we use notation $\psi \equiv v\left(m\right) - m - \frac{c}{1-r}$ to rewrite (18) as

$$\left(\frac{dv}{dm}|_{m=m^p}-1\right)\left(v\left(m^p\right)-\psi\right)+\frac{Q(\psi)}{Q'(\psi)}=0,$$

where $\frac{dv}{dm}|_{m=m^p} \in (0,1)$. By implicit function theorem,

$$\frac{d\psi}{dc} = \left(\frac{\left(\frac{dv}{dm} - 1\right)\frac{dv}{dm} + \frac{d^2v}{dm^2}\left(v\left(m^p\right) - \psi\right)}{\left(\frac{dv}{dm} - 1\right) - \frac{dQ/Q'}{dL}}\right)\frac{dm^p}{dc} < 0,$$

where everything is evaluated at $m=m^p$. The denominator is negative given $\frac{dv}{dm}\in(0,1)$, while the numerator is positive given v(m) is weakly concave and m^p is decreasing (Lemma 2). If $\frac{dv}{dm}|_{m=m^p}\leq 0$ (which includes the case of v(m) being monotone decreasing), then from the main text we know $m^w=\underline{m}$, so $m^p\geq m^w$ for all c and the lemma is trivially satisfied.

8.3 Proposition 3

Given the incomplete pass-through property $\frac{d\tilde{\tau}}{d(v-m)} \in (0,1)$, we know $v(m) - p_{\tilde{\tau}}(m)$ is increasing if and only if v(m) - m is increasing. Since $m^p \equiv \arg\max \tilde{\Pi} = \arg\max \{v(m) - m\}$, it follows that $v(m) - p_{\tilde{\tau}}(m)$ is maximized at m^p . Proposition 3 then follows from the same proof as Proposition 1.

8.4 Proposition 4

We first establish the following properties of $p_{\tilde{r}}(m)$ given in (15).

Lemma 4 At each given m:

- If $\frac{dv}{dm} \geq 0$ then $\frac{dp_{\tilde{r}}}{dm} > 0$.
- If $\frac{dv}{dm} \leq 1$ then $\frac{dp_{\tilde{r}}}{dm} < 1$.
- If $\frac{dv}{dm} \leq 0$ or $\frac{dv}{dm} \leq 1 \frac{Q}{p_{\bar{r}}Q'}$, then $\frac{dv}{dm} < \frac{dp_{\bar{r}}}{dm}$.
- If $\frac{dv}{dm} \leq \frac{dp_{\tilde{r}}}{dm}$, then $\frac{dp_{\tilde{r}}}{dm} < 1$ and $\frac{d\tilde{r}}{dm} < 0$.

Proof. Implicit differentiation on (15) shows

$$\frac{dp_{\tilde{r}}}{dm} = \frac{\left(1 + \frac{p_{\tilde{r}}c}{(p_{\tilde{r}} - m - c)(p_{\tilde{r}} - m)}\right) \frac{dQ/Q'}{dx} \frac{dv}{dm} + \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \frac{p_{\tilde{r}}(2p_{\tilde{r}} - 2m - c)}{(p_{\tilde{r}} - m - c)(p_{\tilde{r}} - m)}}{\frac{Q}{p_{\tilde{r}}Q'} + \left(1 + \frac{p_{\tilde{r}}c}{(p_{\tilde{r}} - m - c)(p_{\tilde{r}} - m)}\right) \frac{dQ/Q'}{dx} + \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \frac{p_{\tilde{r}}(2p_{\tilde{r}} - 2m - c)}{(p_{\tilde{r}} - m - c)(p_{\tilde{r}} - m)}}.$$

where $\frac{dQ/Q'}{dx} = \frac{d}{dx} \left(\frac{Q(x)}{Q'(x)} \right) |_{x=v(m)-p_{\bar{r}}(m)} \ge 0$ by log-concavity of Q and we have used $1 - \frac{Q}{Q'} \frac{c}{(p_{\bar{r}}-m-c)(p_{\bar{r}}-m)} = \frac{Q}{p_{\bar{r}}Q'} > 0$ (from (15)) to simplify the denominator. Clearly, $\frac{dv}{dm} > 0$ implies $\frac{dp_{\bar{r}}}{dm} > 0$; $\frac{dv}{dm} \le 1$ implies $\frac{dp_{\bar{r}}}{dm} < 1$. Next

$$\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm} = \frac{\frac{dv}{dm} \frac{Q}{p_{\tilde{r}}Q'} - \left(1 - \frac{dv}{dm}\right) \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \frac{p_{\tilde{r}}(2p_{\tilde{r}} - 2m - c)}{(p_{\tilde{r}} - m - c)(p_{\tilde{r}} - m)}}{\frac{Q}{p_{\tilde{r}}Q'} + \left(1 + \frac{p_{\tilde{r}}c}{(p_{\tilde{r}} - m - c)(p_{\tilde{r}} - m)}\right) \frac{dQ/Q'}{dx} + \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \frac{p_{\tilde{r}}(2p_{\tilde{r}} - 2m - c)}{(p_{\tilde{r}} - m - c)(p_{\tilde{r}} - m)}}.$$
(19)

If $\frac{dv}{dm} \leq 0$, then the numerator of (19) is obviously negative given (15) implies $1 - \frac{Q}{p_{\bar{r}}Q'} > 0$. Meanwhile, if $\frac{dv}{dm} \leq 1 - \frac{Q}{p_{\bar{r}}Q'}$, the numerator of (19) is

$$\leq \frac{dv}{dm} \frac{Q}{p_{\tilde{r}}Q'} - \frac{Q}{p_{\tilde{r}}Q'} \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \frac{p_{\tilde{r}} \left(2p_{\tilde{r}} - 2m - c\right)}{(p_{\tilde{r}} - m - c)(p_{\tilde{r}} - m)} \\
< \frac{dv}{dm} \frac{Q}{p_{\tilde{r}}Q'} - \frac{Q}{p_{\tilde{r}}Q'} \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \leq 0.$$

Finally, to prove the last part of the claim we prove its contrapositive statement, i.e. $\frac{dp_{\bar{r}}}{dm} \geq 1$ implies $\frac{dv}{dm} > \frac{dp_{\bar{r}}}{dm}$. First, notice $\frac{dp_{\bar{r}}}{dm} \geq 1$ implies $\frac{dv}{dm} > 1$ by the second part of Lemma 4 that has been proven above. From (19), the numerator is then greater than $\frac{Q}{p_{\bar{r}}Q'} > 0$, implying $\frac{dv}{dm} > \frac{dp_{\bar{r}}}{dm}$ which proves the contrapositive statement. Finally, $\frac{dp_{\bar{r}}}{dm} < 1$ implies $\tilde{r}(m) = 1 - \frac{c}{p_{\bar{r}} - m}$ is decreasing in m.

To prove Proposition 2, rewrite welfare function as

$$\tilde{W}(m) = \frac{1}{\tilde{r}(m)} \tilde{\Pi}(m) + \int_{-\infty}^{v(m) - p_{\tilde{r}}(m)} Q(t) dt - cQ(v(m) - p_{\tilde{r}}(m)),$$

and

$$\begin{split} \frac{d\tilde{W}}{dm} &= \frac{1}{\tilde{r}}\frac{d\tilde{\Pi}}{dm} - \frac{1}{\tilde{r}^2}\tilde{\Pi}\frac{d\tilde{r}}{dm} + \left(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm}\right)\left(\frac{Q}{Q'} - c\right)Q' \\ &= \frac{1}{\tilde{r}}\frac{d\tilde{\Pi}}{dm} - \frac{p_{\tilde{r}}}{\tilde{r}}Q\frac{d\tilde{r}}{dm} + \left(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm}\right)\left(\frac{Q}{Q'} - c\right)Q' \\ &= \frac{1}{\tilde{r}}\frac{d\tilde{\Pi}}{dm} + \left(\frac{Q}{Q'}(1 + \Psi(m)) - c\right)\left(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm}\right)Q' \end{split}$$

Case 1: Suppose m^p is interior so that $\frac{d\Pi}{dm}|_{m=m^p}=0$. From the main text and (14), we know that $\frac{dv}{dm}|_{m=m^p} \geq 1$ implies $m^p = \bar{m}$ and $\frac{dv}{dm}|_{m=m^p} \leq 0$ implies $m^p = \underline{m}$. Therefore, for m^p to be interior, we must have $\frac{dv}{dm}|_{m=m^p} \in (0,1)$. From (14), m^p is pinned down by FOC

$$\frac{dv}{dm} = 1 - \frac{Q}{p_{\tilde{r}}Q'}. (20)$$

By Lemma 4, (20) implies $\frac{dv}{dm} < \frac{dp_{\tilde{r}}}{dm}$ and $\frac{d\tilde{r}}{dm} > 0$. Hence, $\frac{d\tilde{W}}{dm}|_{m=m^p}$ has the exact opposite sign as $\frac{Q(v(m^p)-p_r(m^p))}{Q'(v(m^p)-p_r(m^p))} - \Psi(m^p) - c$. Then, unimodality of W(m) implies $m^{sb} < (>)m^p$ if $c < (>) \frac{Q(v(m^p)-p_r(m^p))}{Q'(v(m^p)-p_r(m^p))} - \Psi(m^p)$.

Case 2: Suppose $m^p = \underline{m}$, so obviously $m^{sb} \geq m^p$. It remains to rule out the possibility of $m^{sb} > m^p$ when $c < \frac{Q(v(m^p) - p_r(m^p))}{Q'(v(m^p) - p_r(m^p))} - \Psi(m^p)$. Notice $m^p = \underline{m}$ implies $\frac{d\Pi}{dm}|_{m=m^p} < 0$, or

$$\frac{dv}{dm}|_{m=m^p} < 1 - \frac{Q}{p_{\tilde{r}}Q'}|_{m=m^p}. \tag{21}$$

By Lemma 4, (21) implies $\frac{dv}{dm} < \frac{dp_{\bar{r}}}{dm}$ and $\frac{d\tilde{r}}{dm} > 0$. So, if $c \leq \frac{Q(v(m^p) - p_r(m^p))}{Q'(v(m^p) - p_r(m^p))} - \Psi(m^p)$, then $\frac{d\tilde{W}}{dm}|_{m=m^p} < 0$. Unimodality of W(m) implies $m^{sb} = \underline{m}$ as required.

Case 3: Suppose $m^p = \bar{m}$, so obviously $m^{sb} \leq m^p$. It remains to rule out the possibility of $m^{sb} < m^p$ when $c > \frac{Q(v(m^p) - p_r(m^p))}{Q'(v(m^p) - p_r(m^p))} - \Psi(m^p)$. Notice $m^p = \bar{m}$ implies $\frac{d\Pi}{dm}|_{m=m^p} > 0$, which implies $\frac{dv}{dm}|_{m=m^p} > 0$. There are two subcases to consider. If $(\frac{dv}{dm} - \frac{dp_{\bar{r}}}{dm})|_{m=m^p} \geq 0$, then (16) immediately implies $m^{sb} = \bar{m}$ regardless of c and we are done. If $(\frac{dv}{dm} - \frac{dp_{\bar{r}}}{dm})|_{m=m^p} < 0$, then $c > \frac{Q(v(m^p) - p_r(m^p))}{Q'(v(m^p) - p_r(m^p))} - \Psi(m^p)$ implies $\frac{d\bar{W}}{dm}|_{m=m^p} > 0$. Unimodality of W(m) implies $m^{sb} = \bar{m}$ as required.

8.5 Proposition 5

From $\tilde{\Pi}(m) = mQ(v(m) - m - c)$, we know that for all $m > m^p$ we have $v(m) - m < v(m^p) - m^p$ because otherwise if there is some $m' > m^p$ such that $v(m') - m' \ge v(m^p) - m^p$, then $\tilde{\Pi}(m') > \tilde{\Pi}(m^p)$, which contradicts the definition of m^p being a profit maximizer. Next, from the welfare function

$$\tilde{W}(m) = \tilde{\Pi}(m) + \int_{-\infty}^{v(m)-m-c} Q(t)dt, \qquad (22)$$

it follows that $\tilde{W}(m) < \tilde{W}(m^p)$ for all $m > m^{sb}$. Hence we conclude $m^{sb} \leq m^p$. To complete the proof, we know

$$\frac{d\tilde{\Pi}}{dm} = Q + \left(\frac{dv}{dm} - 1\right)Q'$$

 $so \frac{d\tilde{\Pi}}{dm}|_{m=m^p} = 0$ implies $\frac{dv}{dm}|_{m=m^p} < 1$. Substituting this into the derivative of (22) to get

$$\frac{d\tilde{W}\left(m\right)}{dm}|_{m=m^{p}} = \frac{d\tilde{\Pi}\left(m\right)}{dm}|_{m=m^{p}} + Q\left(\frac{dv}{dm} - 1\right)|_{m=m^{p}} < 0.$$

8.6 Proposition 6

For each given m, the platform can induce the joint monopoly price $p^*(m)$ by setting r and τ such that $\frac{rc+\tau}{1-r}=p^*(m)-c-m$, or equivalently, $rp^*(m)+\tau=p^*(m)-c-m$. Such fees imply that the price resulting from seller competition is $p=m+\frac{c+\tau}{1-r}=m+c+\frac{rc+\tau}{1-r}=p^*(m)$. It is optimal to induce $p^*(m)$ as long as the no-subsidy constraint is satisfied, which holds if and only if $p^*(m)-c-m\geq 0$. When $p^*(m)-c-m<0$, quasi-concavity of the profit function (p-c)Q(v(m)-p) with respect to p implies that the no-subsidy constraint in binding, so that the optimal induced price is m+c. Combining both cases, for each given m we denote the profit-maximizing induced price as

$$\tilde{p}(m) = \max \left\{ m + c, p^*(m) \right\}.$$

Case 1. If $c \leq p^* (m^*) - m^*$, then $p^* (m^*)$ is implementable. We know by envelope theorem that $\tilde{\Pi}(m) = (\tilde{p}(m) - c)Q(v(m) - \tilde{p}(m))$ is maximized at $m^p = m^*$. Moreover, log-concavity of Q implies that $v(m) - p^*(m)$ is maximized when $m^* \equiv \arg\max_m v(m)$. Therefore, $v(m) - \tilde{p}(m) \leq v(m) - p^*(m) \leq v(m^*) - p^*(m^*)$, meaning that $\tilde{W}(m) = \tilde{\Pi}(m) + \int_{-\infty}^{v(m) - \tilde{p}(m)} Q(t) dt$ is maximized at $m^{sb} = m^*$.

Case 2. Suppose $c > p^*(m^*) - m^*$, then from the preceding case we know $p^*(m^*)$ is no longer implementable. To proceed, denote $\hat{m} > m^*$ as the solution to $\hat{m} + c = p^*(\hat{m})$. Its existence and uniqueness follow from unimodality of v(m), whereby v(m) and $p^*(m)$ are monotonically decreasing for all $m > m^*$. By construction, $\tilde{p}(m) = p^*(m)$ for all $m > \hat{m}$. Envelope theorem and unimodality of v(m) implies all $m > \hat{m}$ delivers a lower profit than $m = \hat{m}$. Hence, we conclude $\tilde{p}(m^p) = c + m^p$.

To prove $m^p \ge m^{sb}$, suppose by contradiction $m^p < m^{sb}$. If $v\left(m^p\right) - m^p - c > v\left(m^{sb}\right) - \tilde{p}(m^{sb})$ then

$$\tilde{W}\left(m^{sb}\right) < \tilde{\Pi}\left(m^{sb}\right) + \int_{-\infty}^{v(m^p) - m^p - c} Q(t)dt \le \tilde{W}\left(m^p\right),$$

contradicting the definition of m^{sb} . If $v\left(m^{p}\right)-m^{p}-c\leq v\left(m^{sb}\right)-\tilde{p}(m^{sb})$ then

$$\begin{split} \tilde{\Pi}\left(m^{sb}\right) &= (\tilde{p}(m^{sb}) - c)Q\left(v\left(m^{sb}\right) - \tilde{p}(m^{sb})\right) \\ &\geq (\tilde{p}(m^{sb}) - c)Q\left(v\left(m^{p}\right) - m^{p} - c\right) \\ &> m^{p}Q\left(v\left(m^{p}\right) - m^{p} - c\right) = \tilde{\Pi}\left(m^{p}\right), \end{split}$$

where the last inequality is due to $\tilde{p}(m^{sb}) - c \ge m^{sb} > m^p$. This contradicts the definition of m^p . Hence, we conclude $m^p \ge m^{sb}$. To complete the proof, we need to rule out the existence of an interior m^p that simultaneously satisfies $\frac{dW}{dm} = 0$ and $\frac{d\Pi}{dm} = 0$. We know $\frac{d\Pi(m)}{dm}|_{m=m^p} = 0$ implies $\left(\frac{dv}{dm} - 1\right)|_{m=m^p} < 0$, and

$$\frac{d\tilde{W}}{dm}|_{m=m^{p}} = \frac{d\Pi\left(m\right)}{dm}|_{m=m^{p}} + Q\left(\frac{dv}{dm} - 1\right)|_{m=m^{p}},$$

so that $\frac{d\Pi}{dm}|_{m=m^p}=0$ implies $\frac{dW}{dm}|_{m=m^p}>0$.

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Online appendix: Platform governance

Tat-How Teh*

This online appendix contains omitted details from the main paper and the details of extensions in Section 6.

A Discrete governance

In this section, we consider the case where design choice a is not a continuous variable. In what follows, we replicate the results in Sections 3 - 4.

Let $\Theta \subseteq \mathbb{R}^n$ be a finite and compact subset of n-dimensional real vector space. Each design choice is denoted as a vector $a \in \Theta$ and it corresponds to a given level of gross transaction value V(a) and markup M(a). Given that the choice set is finite, we can equivalently reframe the design problem as directly choosing a pair of markup level and transaction value (M(a), V(a)), or (m, v(m)) as in the analysis in the main text. In cases where a given M(a) corresponds to multiple possible levels of V(a), we can select the highest V(a) among them without loss of generality.

Given the reformulation, we note that the proofs of Proposition 1, 3, 5, 6 in the main text does not rely on a being a continuous variable, except the part on strict inequality where we have explicitly used the first-order condition. Therefore, these results carry over immediately. As for propositions 2 and 4, we have the following counterparts that give a similar insight:

Proposition A.1 (Exogenous proportional fee) Suppose the platform charges an exogenous proportional fee r. There exist thresholds \bar{c}^l and \bar{c}^h , where $0 < \bar{c}^l \le \bar{c}^h$, such that:

- If $c < \bar{c}^l$, then $m^p \ge m^w$.
- If $c > \bar{c}^h$, then $m^p \le m^w$.

Proof. From

$$\begin{split} &\Pi\left(m\right) &= r\left(\frac{c}{1-r}+m\right)Q\left(v\left(m\right)-m-\frac{c}{1-r}\right)\\ &W\left(m\right) &= \left(\frac{c}{1-r}+m-c\right)Q\left(v\left(m\right)-m-\frac{c}{1-r}\right)+\int_{-\infty}^{v\left(m\right)-m-\frac{c}{1-r}}Q\left(t\right)dt, \end{split}$$

we make the following two observations using the definitions of m^p and m^{sb} being maximizers.

- $v(m) m < v(m^p) m^p$ for all $m > m^p$.
- $v(m) m < v(m^w) m^w$ for all $m > m^w$.

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Consider the following expression:

$$\psi(x) \equiv -cQ(x) + \int_{-\infty}^{x} Q(t) dt, \tag{A.1}$$

the derivative of which is $\left(\frac{Q(x)}{Q'(x)}-c\right)Q'(x)$. Rewrite the welfare function as

$$W(m) = \frac{1}{r}\Pi(m) + \psi\left(v(m) - m - \frac{c}{1-r}\right).$$

Let $m^h \equiv \arg \max_m \{v(m) - m\}$ and $m^l \equiv \arg \min_m \{v(m) - m\}$, both of which are well-defined by compactness of the domain and continuity of v(m) - m. Let \bar{c}^i , $i \in \{l, h\}$ be the solution to

$$\bar{c}^i - rac{Q(v(m^i) - m^i - rac{\bar{c}^i}{1-r})}{Q(v(m^i) - m^i - rac{\bar{c}^i}{1-r})}.$$

The existence and uniqueness of \bar{c}^i follows from the intermediate value theorem given log-concavity of Q. By construction, $c < \bar{c}^l$ implies $\psi(.)$ is increasing in its argument, and $c > \bar{c}^h$ implies $\psi(.)$ is decreasing in its argument.

Suppose $c < \bar{c}^l$. For all $m > m^p$, we have $v(m) - m < v(m^p) - m^p$, so ψ being increasing means

$$W(m) < \frac{1}{r}\Pi(m) + \psi\left(v(m^p) - m^p - \frac{c}{1-r}\right) \le W(m^p).$$

Hence, all $m > m^p$ yields strictly lower welfare, and we conclude $m^w \leq m^p$.

Suppose $c > \bar{c}^h$. For all $m > m^w$, we have $v\left(m\right) - m < v\left(m^w\right) - m^w$, so ψ being increasing means

$$\Pi(m) = rW(m) - r\psi\left(v(m) - m - \frac{c}{1-r}\right)$$

$$< rW(m^w) - r\psi\left(v(m^w) - m^w - \frac{c}{1-r}\right) = \Pi(m^w).$$

Hence, all $m > m^w$ yields strictly lower profit, and we conclude $m^p \leq m^w$.

Proposition A.2 (Exogenous proportional fee) Suppose the platform charges an exogenous proportional fee r. There exist thresholds \tilde{c}^l and \tilde{c}^h , where $0 < \tilde{c}^l \le \tilde{c}^h$, such that:

- If $c < \tilde{c}^l$, then $m^p \ge m^w$.
- If $c > \tilde{c}^h$, then $m^p < m^w$.

Proof. We first make the following two observations regarding m^{sb} :

Claim 1: For all $m > m^{sb}$, we have $\tilde{r}(m) < \tilde{r}(m^{sb})$. By contradiction, suppose there exists some $m' > m^{sb}$ such that $\tilde{r}(m') \geq \tilde{r}(m^{sb})$. Using these two inequalities, (12) implies

 $v\left(m'\right) - p_{\tilde{r}}(m') \geq v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb})$. From the welfare function, this implies

$$\tilde{W}(m') \geq \left(\frac{c}{1-\tilde{r}(m')} + m' - c\right) Q\left(v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb})\right) + \int_{-\infty}^{v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb})} Q\left(t\right) dt
> \tilde{W}(m^{sb}),$$

a contradiction. Hence, the claim is proven.

Claim 2: For all $m > m^{sb}$, we have $v\left(m\right) - p_{\tilde{r}}(m) < v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb})$. suppose there is some $m' > m^{sb}$ such that $v\left(m'\right) - p_{\tilde{r}}(m') \ge v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb})$. Then, (15) implies $p_{\tilde{r}}\left(m'\right) > p_{\tilde{r}}(m^{sb})$. From the welfare function, this implies $W(m') > W(m^{sb})$, a contradiction. Hence, the claim is proven.

Suppose $c > \tilde{c}^h$, where

$$\tilde{c}^h \equiv \frac{Q(v(m^h) - m^h)}{Q'(v(m^h) - m^h)} \tag{A.2}$$

which implies $\psi(x)$ is decreasing for all $x \leq v(m^h) - m^h$. For all $m > m^{sb}$, we have

$$\begin{split} \tilde{\Pi}(m) &= \tilde{r}\left(m\right)\tilde{W}\left(m\right) - \tilde{r}\left(m\right)\psi\left(v\left(m\right) - p_{\tilde{r}}(m)\right) \\ &< \tilde{r}\left(m^{sb}\right)\tilde{W}\left(m\right) - \tilde{r}\left(m^{sb}\right)\psi\left(v\left(m\right) - p_{\tilde{r}}(m)\right) \\ &< \tilde{r}\left(m^{sb}\right)\tilde{W}\left(m\right) - \tilde{r}\left(m^{sb}\right)\psi\left(v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb})\right) \\ &= \tilde{\Pi}(m^w). \end{split}$$

where the first inequality is due to Claim 1 and the second inequality is due to $v(m) - p_{\tilde{r}}(m) < v(m^{sb}) - p_{\tilde{r}}(m^{sb}) \le v(m^h) - m^h$ (by Claim 2). All $m > m^{sb}$ yields strictly lower profit, and we conclude $m^p \le m^{sb}$.

To establish the existence of the lower threshold \tilde{c}^l , it suffices to consider the case of $c \to 0$. When $c \to 0$, we first note from (12) and (15) that $\tilde{r}(m) \to 1$ and $p_{\tilde{r}}(m) \to m > 0$ and so, $\psi(x)$ is increasing for all x. Moreover, these implies $\tilde{\Pi}(m) \to mQ(v(m) - p_{\tilde{r}}(m))$, meaning that $v(m) - m < v(m^p) - m^p$ for all $m > m^p$ (otherwise m^p is not a maximizer). It follows that

$$W(m) \rightarrow \Pi(m) + \psi \left(v(m) - m - \frac{c}{1 - r} \right)$$

$$\leq \Pi(m) + \psi \left(v(m^p) - m^p - \frac{c}{1 - r} \right) \leq W(m^p).$$

for all $m > m^p$. All $m > m^p$ yields strictly lower profit, and we conclude $m^{sb} \leq m^p$.

B Derivations of Examples 1-3

This section provides the derivations for the micro-founded examples in Section 2. In what follows we do not specify the exact fee instrument used by the platform. Instead, we focus on deriving how the platform's governance design influences the buyer-seller interactions in each of the applications.

B.1 Example 1: Variety choice by platform

Example 1 can be summarized by the following timing: (i) The platform announces the number of sellers displayed sellers a; (ii) Sellers and buyers decide whether to enter the platform; (iii) Sellers set their prices; (iii) a sellers are selected into each buyer's consideration set, and buyers observe the prices and match values of these sellers and purchase accordingly. We focus on symmetric pure strategy Nash equilibrium where all admitted sellers set the same price p (for any a chosen by the platform).

Let Q be the number of participating buyers and $\binom{N}{a}$ be the probability of a given seller being in the consideration set (N choose a), both of which are exogenous from sellers' point of view. To derive seller pricing, consider a deviating seller i who sets price $p_i \neq p$. For each seller, conditional on being selected into consideration sets the effective demand is derived as per the Salop circular city model: $\frac{1}{a} + \frac{p-p_i}{t}$. The demand for seller i's product is

$$Q_i(p_i) = \binom{N}{a} \times Q \times \left(\frac{1}{a} + \frac{p - p_i}{t}\right).$$

and profit function $(p-c) Q_i(p_i)$. The symmetric equilibrium price is $p=c+\frac{t}{a}$. In equilibrium, the maximum buyer mismatch cost is $\frac{t}{2a}$, for $a \geq 2$. To ensure the market is fully covered, we assume $V_0 > c + \frac{3t}{4}$. The ex-ante expected mismatch cost is $\frac{t}{4a}$, and so the expected gain from joining, is $V(a) = V_0 - \frac{t}{4a}$. Finally, we can write $V(a) = V_0 - \frac{M(a)}{4}$, so $v(m) = V(M^{-1}(m))$ is linear.

B.2 Example 2: Information design by platform

Example 2 can be summarized by the following timing: (i) The platform announces the information structure parameterized by $a \in [0,1]$ (ii) Sellers and buyers decide whether to enter the platform; (iii) Sellers set their prices; (iii) Each buyer observes the realized signal and prices and purchase accordingly.

Let \bar{s} be the signal received by the buyer who is indifferent between seller 1 and 2, i.e. $p_1 + t\mathbf{E}(x_1|\bar{s}) = p_2 + t\mathbf{E}(x_2|\bar{s})$, or $\bar{s} = \frac{1}{4} + \frac{p_2 - p_1}{2t(1-a)}$. Given s is drawn from uniform distribution over [0, 1/2], seller 1's demand is $\Pr(s < \bar{s}) = \frac{1}{2} + \frac{p_2 - p_1}{t(1-a)}$. Therefore, the price competition is the same as Example 1 except that transportation cost parameter is replaced by t(1-a). It follows that $p = c + \frac{t(1-a)}{2}$. To ensure the market is fully covered, we assume $V_0 - t\mathbf{E}(x_1|s = 1/4) - p > 0$ for all a, or $V_0 > c + \frac{3t}{4}$.

From an ex-ante perspective, with probability a the signal is uninformative so that the buyer essentially gets a randomly chosen product, with expected mismatch cost t/4; with probability 1-a the signal is informative and the buyer gets the preferred product, with expected mismatch cost t/8. Hence, the ex-ante expected mismatch cost is $(1+a)\frac{t}{8}$, as stated in the main text. Moreover, both M(a) and V(a) are linear in a, so $v(m) = V(M^{-1}(m))$ is linear.

B.3 Example 3: Quality control by platform

Example 3 can be summarized by the following sequence of events: (i) The platform announces its quality standard a; (ii) Buyers and sellers (with $q_i \geq a$) decide whether to enter the platform; (iii) Sellers set their prices, and buyers that enter the platform carry out sequential search. We focus on symmetric Perfect Bayesian Equilibria (PBE) where all sellers set the same price p. As is standard in the search literature, buyers keep the same (passive) beliefs about the distribution of future prices on and off the equilibrium path.

The following derivation follows from Eliaz and Spielger (2011). We first derive buyers' search strategy for a given a set by the platform. Define the reservation value V(a) as the solution to

$$\mathbf{E}\left(q_{i}|q_{i}\geq a\right)\int_{V}^{\bar{\epsilon}}\left(\epsilon-V\right)dF\left(\epsilon\right)=s.\tag{B.1}$$

The left-hand side of (B.1) represents the incremental expected benefit from one more search, while the right-hand side represents the incremental search cost. There is, at most, one solution to (B.1) since the left-hand side is strictly decreasing in v.

It is well known from Weitzman (1979) that buyers' optimal search rule in this environment is stationary and described by the standard cutoff rule. When searching, each buyer employs the following strategy: (i) she stops and buys form seller i if the product is non-defective and $\epsilon_i - p_i \geq V(a) - p$; (ii) she continues to search the next seller otherwise. Following the standard result, the buyer's expected surplus from initiating a search is V(a) - p. Then, a buyer with intrinsic participation cost d enters the platform if and only if d . Provided that search cost is not too large, there is a symmetric price equilibrium where a strictly positive measure of buyers join the platform.

Compared to a standard search model, notice from (3) that a search pool with a higher expected quality $\mathbf{E}(q_i|q_i \geq a)$ is analogous to a lower "effective search cost" for buyers. This reflects that each buyer searches less and consequently incurs a lower total expected search cost of $s/\mathbf{E}(q_i|q_i \geq a)$ before reaching a non-defective match. Given that $\mathbf{E}(q_i|q_i > a)$ increases with a, a higher quality standard set by the platform is analogous to reducing the effective search cost of buyers. Thus, it follows that V(a) is an increasing function of a.

From the buyer search rule above, derivation of demand is straightforward. The mass of buyers initiating search is Q(V(a) - p), which is exogenous from each firm's point of view. Conditional on these buyers, the demand of a deviating firm i with type q_i follows the standard search model and it is given by

$$q_i (1 - F(V(a) - p + p_i)) \sum_{z=0}^{\infty} F(V(a))^z = \left(\frac{1 - F(V(a) - p + p_i)}{1 - F(V(a))}\right) q_i.$$

The log-concavity assumption on 1-F ensures that the usual first-order condition determines a unique optimal price. The symmetric equilibrium price is given by $p = c + \frac{1-F(V(a))}{f(V(a))}$. Note 1-F being log-concavity implies that M(a) is decreasing in a. This reflects that a higher quality standard reduces the effective search cost of buyers, which leads to a more elastic product demand.

C Endogeneity of fee instrument

We prove Remark 2 stated in the main text. Recall that the platform's optimal fee instrument is a two-part tariff with a subsidy on buyer participation. It optimally sets its governance design at $m^{p+} = m^*$ to maximize v(m), and then adjusts transaction-based fee components and the buyer-side participation subsidy accordingly to achieve the monopoly profit denoted as $\Pi^* \equiv (p^*(m^*) - c)Q(v(m) - p^*(m))$.

Meanwhile, the social planner optimally restricts the platform to charge a pure lump-sum participation fee on sellers and disallow the transaction-based fee component. Notice that a combination of participation fees on both sellers and buyers is ineffective as the platform would see a positive participation fee in an attempt to replicate Π^* , which results in deadweight losses. As such, welfare function under the restriction of seller participation fee is

$$\tilde{W}(m) = mQ(v(m) - m - c) + \int_{-\infty}^{v(m) - m - c} Q(t) dt.$$

For all $m > m^*$, we have

$$\tilde{W}(m) \leq mQ(v(m^*) - m - c) + \int_{-\infty}^{v(m^*) - m - c} Q(t) dt$$

$$< m^*Q(v(m^*) - m^* - c) + \int_{-\infty}^{v(m^*) - m^* - c} Q(t) dt$$

$$= \tilde{W}(m^*),$$

which implies $m^{sb+} \leq m^* = m^{p+}$.

D Costly governance

This online appendix corresponds to Section 6.1 of the main text. We focus on extending the results in Section 4 when the platform's fixed cost is an increasing and convex function $K = K(v(m)) \ge 0$. Denote $k \ge 0$ as the derivative of K, where k is increasing in its argument following the convexity assumption. We assume $Q \ge k$

Proposition D.1 (Endogenous per-transaction fees) If value generation is anti-competitive, then Proposition 3 continues to hold.

Proof. If we rewrite the profit function in terms of price, we have

$$\tilde{\Pi}(m) = (p_{\tilde{\tau}}(m) - c - m)Q(v(m) - p_{\tilde{\tau}}(m)) - K(v(m)),$$

where $p_{\tilde{\tau}}(m)$ is implicitly defined by $p_{\tilde{\tau}}(m) = c + m + \frac{Q(v(m) - m - c - \tilde{\tau})}{Q'(v(m) - m - c - \tilde{\tau})}$. Notice that log-concavity of Q implies that $v(m) - p_{\tilde{\tau}}(m)$ is increasing in m if and only if v(m) - m is increasing in m. We first claim that $v(m) - p_{\tilde{\tau}}(m) \leq v(m^p) - p_{\tilde{\tau}}(m^p)$ for all $m < m^p$ if value generation is anticompetitive (v(m)) is increasing). By contradiction, suppose there is some $m' < m^p$ such that

 $v(m') - p_{\tilde{\tau}}(m') > v(m^p) - p_{\tilde{\tau}}(m^p)$, then the definition of $p_{\tilde{\tau}}$ implies $p_{\tilde{\tau}}(m') - m' > p_{\tilde{\tau}}(m^p) - m^p$. If v(m) is increasing, then $K(v(m')) \leq K(v(m^p))$, and so

$$\tilde{\Pi}\left(m'\right) > (p_{\tilde{\tau}}(m^p) - c - m^p)Q\left(v(m^p) - p_{\tilde{\tau}}(m^p)\right) - K(v\left(m'\right))$$

$$\geq \tilde{\Pi}\left(m^p\right),$$

which contradicts the definition of m^p being a maximizer. Hence, the claim is proven.

From the welfare function,

$$\tilde{W}(m) = \tilde{\Pi}(m) + mQ(v(m) - p_{\tilde{\tau}}(m)) + \int_{-\infty}^{v(m) - p_{\tilde{\tau}}(m)} Q(t) dt.$$

the proven claim implies $W(m) < W(m^p)$ for all $m < m^p$, and so we have implying $m^{sb} \ge m^p$, as required. The final step of ruling out equalities follows the same step as the proof of Proposition 3, hence omitted here.

Proposition D.2 (Endogenous proportional fee) Suppose the social planner can control the platform's governance design, but cannot control the proportional fee set by the platform. Suppose W(m) is unimodal, and denote

$$\Psi(m) \equiv \frac{\left(\frac{p_{\tilde{r}}}{\tilde{r}} + \frac{K}{\tilde{r}^2}\right) \frac{d\tilde{r}}{dm}}{\frac{dp}{dm} - \frac{dv}{dm}}.$$

- Suppose $c \leq \frac{Q(v(m^p) p_{\tilde{r}}(m^p))}{Q'(v(m^p) p_{\tilde{r}}(m^p))} (1 + \tilde{\Psi}(m^p))$. If value generation is pro-competitive, then $m^p \geq m^{sb}$.
- Suppose $c \geq \frac{Q(v(m^p) p_{\tilde{r}}(m^p))}{Q'(v(m^p) p_{\tilde{r}}(m^p))} (1 + \tilde{\Psi}(m^p))$. If value generation is anti-competitive, then $m^p \leq m^{sb}$.

Proof. Recall

$$\widetilde{\Pi}(m) = \widetilde{r}(m) p_{\widetilde{r}}(m) Q(v(m) - p_{\widetilde{r}}(m)) - K(v(m)),$$

where $p_{\tilde{r}}(m)$ is defined in (15). Write welfare function as

$$\tilde{W}(m) = (p_{\tilde{r}}(m) - c)Q(v(m) - p_{\tilde{r}}(m)) - K(v(m)) + \int_{-\infty}^{v(m) - p_{\tilde{r}}(m)} Q(t)dt \qquad (D.1)$$

$$= \frac{\tilde{\Pi}(m) + K(v(m))}{\tilde{r}(m)} - K(v(m)) - cQ(v(m) - p_{\tilde{r}}(m)) + \int_{-\infty}^{v(m) - p_{\tilde{r}}(m)} Q(t)dt.$$

and

$$\frac{d\tilde{W}}{dm} = \frac{1}{\tilde{r}}\frac{d\tilde{\Pi}}{dm} + \left(\frac{1-\tilde{r}}{\tilde{r}}\right)k\frac{dv}{dm} + \left(\frac{Q}{Q'}(1+\tilde{\Psi}(m^p)) - c\right)\left(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm}\right)Q'$$

Suppose $c \leq \frac{Q(v(m^p) - p_r(m^p))}{Q'(v(m^p) - p_r(m^p))}(1 + \tilde{\Psi}(m^p))$. If value generation is pro-competitive $(\frac{dv}{dm} \leq 0)$, Lemma 4 implies $\frac{dv}{dm} - \frac{dp_{\tilde{t}}}{dm} \leq 0$. Hence, $\frac{d\tilde{W}}{dm}|_{m=m^p} \leq 0$ whenever $\frac{d\tilde{\Pi}}{dm}|_{m=m^p} = 0$ and $\frac{d\tilde{W}}{dm}|_{m=m^p} < 0$ whenever $\frac{d\tilde{\Pi}}{dm}|_{m=m^p} < 0$. Unimodality of W(m) implies $m^{sb} \leq m^p$ as required.

Suppose $c \geq \frac{Q(v(m^p)-p_r(m^p))}{Q'(v(m^p)-p_r(m^p))}(1+\tilde{\Psi}(m^p))$. Suppose value generation is anti-competitive $(\frac{dv}{dm}\geq 0)$. If $(\frac{dv}{dm}-\frac{dp_{\tilde{r}}}{dm})|_{m=m^p}\leq 0$, $\frac{d\tilde{W}}{dm}|_{m=m^p}\geq 0$ whenever $\frac{d\tilde{\Pi}}{dm}|_{m=m^p}=0$, and $\frac{d\tilde{W}}{dm}|_{m=m^p}>0$ whenever $\frac{d\tilde{\Pi}}{dm}|_{m=m^p}>0$. These imply $m^{sb}\geq m^p$ as required. Suppose instead $(\frac{dv}{dm}-\frac{dp_{\tilde{r}}}{dm})|_{m=m^p}>0$, then we get from (D.1):

$$\frac{d\tilde{W}}{dm}|_{m=m^{p}} = (p_{\tilde{r}}(m) - c)Q'\left(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm}\right)|_{m=m^{p}} + (Q - k)\frac{dv}{dm}|_{m=m^{p}} > 0,$$

Unimodality of W(m) implies $m^{sb} \geq m^p$.

Proposition D.3 (Lump sum fees) If value generation is pro-competitive, then Proposition 5 continues to hold.

Proof. Platform's profit is $\tilde{\Pi}(m) = mQ(v(m) - m - c) - K(v(m))$, while the welfare function is

$$\tilde{W}(m) = \tilde{\Pi}(m) + \int_{-\infty}^{v(m)-m-c} Q(t)dt.$$

We first claim that $v(m) - m \leq v(m^p) - m^p$ for all $m > m^p$ if value generation is procompetitive (v(m)) is decreasing). By contradiction, suppose there is some $m' > m^p$ such that $v(m') - m' > v(m^p) - m^p$. If v(m) is decreasing, then $K(v(m')) \leq K(v(m^p))$, and so $\tilde{\Pi}(m') > m^p Q(v(m^p) - m^p) - K(v(m')) \geq \tilde{\Pi}(m^p)$, which contradicts the definition of m^p being a maximizer. Hence, the claim is proven.

From the welfare function, the proven claim implies $W(m) < W(m^p)$ for all $m > m^p$, and so we have $m^{sb} \le m^p$, as required. The final step of ruling out equalities follows the same step as the proof of Proposition 5, hence omitted here.

Proposition D.4 (Two-part tariff) If value generation is pro-competitive, then Proposition 6 continues to hold.

Proof. Recall that we denote the profit-maximizing induced price as $\tilde{p}(m) = \max\{m + c, p^*(m)\}$ (the consideration of fixed cost does not affect the optimal pricing). Denote

$$\begin{array}{ll} m_k^* & \equiv & \arg\max_{m} \tilde{\Pi}\left(m\right) \\ & = & \arg\max_{m} \left\{ (p^*\left(m\right) - c)Q\left(v(m) - p^*\left(m\right)\right) - K(v(m)) \right\}. \end{array}$$

Recall we focus on the case where v(m) is monotone, which implies unimodality. Similar to the proof of Proposition 6, we denote \hat{m} as the solution to $\hat{m} + c = p^*(\hat{m})$. By construction, $\tilde{p}(m) = p^*(m)$ for all $m > \hat{m}$

Case 1. If $c \leq p^*$ $(m_k^*) - m_k^*$, then p^* (m_k^*) is implementable. We know by envelope theorem that $\tilde{\Pi}(m)$ is maximized at $m^p = m_k^*$. For all $m > m_k^*$, we know $\tilde{p}(m) = p^*$ (m) because $m^p > \hat{m}$. We first claim that $v(m) - p^*$ $(m) \leq v(m^p) - p^*$ (m^p) for all $m > m^p$ if value generation is procompetitive (v(m) is decreasing). By contradiction, suppose there is some $m' > m^p$ such that

 $v(m') - p^*(m') > v(m^p) - p^*(m^p)$, then the definition of p^* implies $p^*(m') - c > p^*(m^p) - c$. If v(m) is decreasing, then $K(v(m')) \le K(v(m^p))$, and so

$$\tilde{\Pi}(m') > (p^*(m^p) - c)Q(v(m^p) - p^*(m^p)) - kv(m') \ge \tilde{\Pi}(m^p),$$

which contradicts the definition of m^p being a maximizer. Hence, the claim is proven. Then, from the welfare function, the proven claim implies $W(m) < W(m^p)$ for all $m > m^p$, and so we have $m^{sb} \le m^p$, as required.

Case 2. Suppose $c > p^*(m^*) - m^*$, then from the preceding case we know $p^*(m_k^*)$ is no longer implementable. Similar to the proof of Proposition 6, we know $m^p < \hat{m}$ and $\tilde{p}(m^p) = c + m^p$. We first claim that $v(m) - \tilde{p}(m) \le v(m^p) - m^p - c$ for all $m > m^p$ if value generation is pro-competitive (v(m)) is decreasing). By contradiction, suppose there is some $m' > m^p$ such that $v(m') - \tilde{p}(m') > v(m^p) - m^p - c$. The definition of \tilde{p} implies $\tilde{p}(m') \ge m' + c > m^p + c$. If v(m) is decreasing, then $K(v(m')) \le K(v(m^p))$, and so

$$\tilde{\Pi}(m') > (m^p - c)Q(v(m^p) - m^p) - K(v(m')) \ge \tilde{\Pi}(m^p),$$

which contradicts the definition of m^p being a maximizer. Hence, the claim is proven. Then, from the welfare function, the proven claim implies $W(m) < W(m^p)$ for all $m > m^p$, and so we have $m^{sb} \le m^p$, as required.

The final step of ruling out equalities follows the same step as the proof of Proposition 6. ■

E Alternative form of market coverage

This online appendix corresponds to Section 6.3 of the main text. It is useful to denote the smallest and largest elements of Θ as \underline{a} and \overline{a} respectively. Let the derivatives of M(a,c) with respect to its first and second argument be M_1 and M_2 . As stated in the main text, we assume that $M_1 < 0$ and $M_2 = -\beta$, where $\beta \in (0,1)$.

To state the result for endogenous per-transaction fees, we first note that equilibrium price for each given a and τ is given by $p(a, c+\tau) \equiv c+\tau+M(a, c+\tau)$. The platform chooses a and τ to maximize its profit $\tau Q(V(a)-c-\tau-M(a,c+\tau))$. Let $\tilde{\tau}=\tilde{\tau}(a)$ be the profit-maximizing fee for each give a:

$$\tilde{\tau} = \frac{Q\left(V(a) - c - \tilde{\tau} - M(a, c + \tilde{\tau})\right)}{Q'\left(V(a) - c - \tilde{\tau} - M(a, c + \tilde{\tau})\right)} \frac{1}{1 - \beta}.$$

Then:

Proposition E.1 (Endogenous per-transaction fees) Suppose the social planner can control the platform's governance design, but cannot control the per-transaction fee τ set by the platform. If β is independent of a, then $a^p \geq a^{sb}$ and $v(a^p) \leq v(a^{sb})$.

Proof. By envelope theorem, we know that the profit-maximizing design

$$a^p = \arg\max_{x} \left\{ V(x) - c - \tilde{\tau}(a^p) - M(x, c + \tilde{\tau}(a^p)) \right\},\,$$

which then implies three useful properties of a^p :

- By implicit function theorem, a^p maximizes $\tilde{\tau}(a)$.
- If we denote $\chi(a) = V(a) c \tilde{\tau}(a) M(a, c + \tilde{\tau}(a))$, and use the definition of $\tilde{\tau}$, we get

$$\chi(a) = V(a) - c - M(a, c + \tilde{\tau}(a)) - \frac{Q(\chi(a))}{Q'(\chi(a))} \frac{1}{1 - \beta}.$$

It follows that a^p maximizes $\chi(a)$ as well because $V(a) - c - M(a, c + \tilde{\tau}(a)) \leq V(a) - c - M(a, c + \tilde{\tau}(a^p)) \leq V(a^p) - c - M(a^p, c + \tilde{\tau}(a^p))$, where the first inequality follows from $M_2 < 0$ and a^p being a maximizer of $\tilde{\tau}(a)$, while the last inequality follows from the definition of a^p .

• For all $a > a^p$, $p(a, c + \tilde{\tau}(a)) < p(a^p, c + \tilde{\tau}(a^p))$. Given $p(a, c + \tau) \equiv c + \tau + M(a, c + \tau)$, this follows from the fact that (i) $M_1 < 0$, and that (ii) a^p maximizes $\tilde{\tau}(a)$ and $M_2 \in (-1, 0)$.

To complete the proof, welfare function is

$$\tilde{W}(a) = p(a, c + \tilde{\tau}(a))Q(\chi(a)) + \int_{-\infty}^{\chi(a)} Q(t) dt.$$

For all $a > a^p$, we have

$$\tilde{W}(a) \leq p(a, c + \tilde{\tau}(a))Q(\chi(a^p)) + \int_{-\infty}^{\chi(a^p)} Q(t) dt$$

$$< p(a^p, c + \tilde{\tau}(a))Q(\chi(a^p)) + \int_{-\infty}^{\chi(a^p)} Q(t) dt$$

$$= \tilde{W}(a^p).$$

Therefore, $a^{sb} \leq a^p$. In addition, it follows that $V(a^w) \geq V(a^p)$ as otherwise a^w cannot be welfare-maximizing.

To state the result for endogenous per-transaction fees, we first note that equilibrium price for each given a and r is given by $p(a, \frac{c}{1-r}) \equiv \frac{c}{1-r} + M\left(a, \frac{c}{1-r}\right)$. The platform chooses a and r to maximize its profit $rp(a, \frac{c}{1-r})Q\left(V\left(a\right) - p(a, \frac{c}{1-r})\right)$. Let $\tilde{r} = \tilde{r}(a)$ be the profit-maximizing fee for each give a, and denote the induced price as $p_{\tilde{r}}(a) = \frac{c}{1-\tilde{r}} + M\left(a, \frac{c}{1-\tilde{r}}\right)$, then:

$$\tilde{r} = \frac{Q\left(V\left(a\right) - p_{\tilde{r}}\right)}{Q'\left(V\left(a\right) - p_{\tilde{r}}\right)} \left(\frac{\left(1 - \tilde{r}\right)^2}{\left(1 - \beta\right)c} + \frac{\tilde{r}}{p_{\tilde{r}}}\right). \tag{E.1}$$

We first prove the following properties of $p_{\bar{r}}(m)$, which is analogous to Lemma 4 in the Appendix:

Lemma 5 Suppose β is independent of a. At each given a:

- If $\frac{dV}{da} < 0$ then $\frac{dp_{\tilde{r}}}{da} < 0$.
- If $\frac{dV}{da} \ge M_1$ then $\frac{dp_{\tilde{r}}}{dm} > M_1$.

- If $\frac{dV}{da} \ge 0$ or $\frac{dV}{da} \ge M_1 \left(1 \frac{Q}{p_{\bar{r}}Q'}\right)$, then $\frac{dV}{da} > \frac{dp_{\bar{r}}}{da}$.
- If $\frac{dV}{da} \ge \frac{dp_{\tilde{r}}}{da}$, then $\frac{dp_{\tilde{r}}}{da} > M_1$ and $\frac{d\tilde{r}}{da} > 0$.

Proof. For each given a, we have stated the pair of $(\tilde{r}, p_{\tilde{r}})$ in terms of (E.1) and $p_{\tilde{r}}(a) = \frac{c}{1-\tilde{r}} + M\left(a, \frac{c}{1-\tilde{r}}\right)$. An equivalent characterization, which is more convenient to work with in this proof, is to replace (E.1) with

$$p_{\tilde{r}} = \frac{Q(V(a) - p_{\tilde{r}})}{Q'(V(a) - p_{\tilde{r}})} \left[1 + \frac{p_{\tilde{r}}c}{(1 - \beta)(p_{\tilde{r}} - M\left(a, \frac{c}{1 - \tilde{r}}\right) - c)(p_{\tilde{r}} - M\left(a, \frac{c}{1 - \tilde{r}}\right))} \right].$$
 (E.2)

Then, from $p_{\tilde{r}}(a) = \frac{c}{1-\tilde{r}} + M\left(a, \frac{c}{1-\tilde{r}}\right)$, implicit function theorem gives $\frac{d\bar{r}}{da} = \frac{\frac{dp_{\tilde{r}}}{da} - M_1}{\frac{c}{(1-\tilde{r})^2}(1-\beta)}$. Given this, we have

$$\frac{dp_{\tilde{r}}}{da} \ = \ \frac{\left(1 + \frac{p_{\tilde{r}}c}{(1-\beta)(p_{\tilde{r}}-M-c)(p_{\tilde{r}}-M)}\right) \frac{dQ/Q'}{dx} \frac{dV}{da} + \frac{Q}{Q'} \frac{cp_{\tilde{r}}(2p_{\tilde{r}}-2M-c)}{(1-\beta)(p_{\tilde{r}}-M-c)^2(p_{\tilde{r}}-M)^2} M_1 \left(1 + \frac{\beta}{1-\beta}\right)}{\frac{Q}{p_{\tilde{r}}Q'} + \left(1 + \frac{p_{\tilde{r}}c}{(1-\beta)(p_{\tilde{r}}-M-c)(p_{\tilde{r}}-M)}\right) \frac{dQ/Q'}{dx} + \frac{Q}{Q'} \frac{cp_{\tilde{r}}(2p_{\tilde{r}}-2m-c)}{(1-\beta)(p_{\tilde{r}}-M-c)^2(p_{\tilde{r}}-M)^2} \left(1 + \frac{\beta}{1-\beta}\right)}{\frac{Q}{p_{\tilde{r}}Q'} + \left(1 + \frac{p_{\tilde{r}}c}{(1-\beta)(p_{\tilde{r}}-M-c)(p_{\tilde{r}}-M)}\right) \frac{dQ/Q'}{dx} \frac{dV}{da} + \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \frac{p_{\tilde{r}}(2p_{\tilde{r}}-2M-c)}{(p_{\tilde{r}}-M-c)(p_{\tilde{r}}-M)} M_1 \left(1 + \frac{\beta}{1-\beta}\right)}{\frac{Q}{p_{\tilde{r}}Q'} + \left(1 + \frac{p_{\tilde{r}}c}{(1-\beta)(p_{\tilde{r}}-M-c)(p_{\tilde{r}}-M)}\right) \frac{dQ/Q'}{dx} + \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \frac{p_{\tilde{r}}(2p_{\tilde{r}}-2M-c)}{(p_{\tilde{r}}-M-c)(p_{\tilde{r}}-M)} \left(1 + \frac{\beta}{1-\beta}\right)}{\frac{Q}{p_{\tilde{r}}Q'} + \left(1 + \frac{p_{\tilde{r}}c}{(1-\beta)(p_{\tilde{r}}-M-c)(p_{\tilde{r}}-M)}\right) \frac{dQ/Q'}{dx} + \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \frac{p_{\tilde{r}}(2p_{\tilde{r}}-2M-c)}{(p_{\tilde{r}}-M-c)(p_{\tilde{r}}-M)} \left(1 + \frac{\beta}{1-\beta}\right)}{\frac{Q}{p_{\tilde{r}}Q'} + \frac{Q}{p_{\tilde{r}}Q'}} \frac{p_{\tilde{r}}(2p_{\tilde{r}}-2M-c)}{(p_{\tilde{r}}-M-c)(p_{\tilde{r}}-M)} \left(1 + \frac{\beta}{1-\beta}\right)}{\frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} + \frac{Q}{p_{\tilde{r}}Q'}} \frac{p_{\tilde{r}}(2p_{\tilde{r}}-2M-c)}{(p_{\tilde{r}}-M-c)(p_{\tilde{r}}-M)} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'} \frac{Q}{p_{\tilde{r}}Q'}} \frac{Q}{p_{\tilde$$

where the second equality uses (E.2). Clearly, $\frac{dV}{da} < 0$ implies $\frac{dp_{\tilde{r}}}{da} < 0$; $\frac{dV}{da} \ge M_1$ implies $\frac{dp_{\tilde{r}}}{da} > M_1$. Next

$$\frac{dV}{da} - \frac{dp_{\tilde{r}}}{da} = \frac{\frac{dV}{da} \frac{Q}{p_{\tilde{r}}Q'} - \left(M_1 - \frac{dV}{da}\right) \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \frac{p_{\tilde{r}}(2p_{\tilde{r}} - 2M - c)}{(p_{\tilde{r}} - M - c)(p_{\tilde{r}} - M)} \left(1 + \frac{\beta}{1 - \beta}\right)}{\frac{Q}{p_{\tilde{r}}Q'} + \left(1 + \frac{p_{\tilde{r}}c}{(1 - \beta)(p_{\tilde{r}} - M - c)(p_{\tilde{r}} - M)}\right) \frac{dQ/Q'}{dx} + \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \frac{p_{\tilde{r}}(2p_{\tilde{r}} - 2M - c)}{(p_{\tilde{r}} - M - c)(p_{\tilde{r}} - M)} \left(1 + \frac{\beta}{1 - \beta}\right)}{(E.3)}$$

If $\frac{dV}{da} \ge 0$, then the numerator of (19) is obviously negative given (15) implies $1 - \frac{Q}{p_{\bar{r}}Q'} > 0$. Meanwhile, if $\frac{dV}{da} \ge M_1 \left(1 - \frac{Q}{p_{\bar{r}}Q'}\right)$, the numerator of (19) is

$$\geq \frac{dV}{da} \frac{Q}{p_{\tilde{r}}Q'} - M_1 \frac{Q}{p_{\tilde{r}}Q'} \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \frac{p_{\tilde{r}} \left(2p_{\tilde{r}} - 2m - c\right)}{(p_{\tilde{r}} - m - c)(p_{\tilde{r}} - m)} \left(1 + \frac{\beta}{1 - \beta}\right)$$

$$> \frac{dV}{da} \frac{Q}{p_{\tilde{r}}Q'} - M_1 \frac{Q}{p_{\tilde{r}}Q'} \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) \geq 0.$$

Finally, to prove the last part of the claim we prove its contrapositive statement, i.e. $\frac{dp_{\tilde{r}}}{da} \leq M_1$ implies $\frac{dV}{da} < \frac{dp_{\tilde{r}}}{da}$. First, notice $\frac{dp_{\tilde{r}}}{da} \leq M_1$ implies $\frac{dV}{da} < M_1$ by the second part of Lemma 5 that has been proven above. From (E.3), the numerator is then smaller than $M_1 \frac{Q}{p_{\tilde{r}}Q'} < 0$, implying $\frac{dV}{da} < \frac{dp_{\tilde{r}}}{da}$ which proves the contrapositive statement. Finally, $\frac{dp_{\tilde{r}}}{da} > M_1$ implies $\frac{d\tilde{r}}{da} = \frac{\frac{dp_{\tilde{r}}}{da} - M_1}{\frac{da}{(1-\tilde{r})^2}(1-\beta)} > 0$.

We write the profit function and welfare function with endogenous per-transaction fees as

$$\tilde{\Pi}(a) = \tilde{r} p_{\tilde{r}} Q \left(V(a) - p_{\tilde{r}} \right)$$

and

$$\tilde{W}(a) = \frac{1}{\tilde{r}}\tilde{\Pi}(a) + \int_{-\infty}^{V(a)-p_{\tilde{r}}} Q(t)dt - cQ\left(V(a) - p_{\tilde{r}}\right).$$

Then:

Proposition E.2 (Endogenous proportional fee) Suppose the social planner can control the platform's governance design, but cannot control the proportional fee set by the platform. Suppose $\tilde{W}(a)$ is unimodal and β is independent of a, and denote

$$\Psi(a) \equiv \frac{\left(\frac{p_{\tilde{r}}}{\tilde{r}}\right) \frac{d\tilde{r}}{da}}{\frac{dp_{\tilde{r}}}{da} - \frac{dV}{da}}.$$

- If $c < \frac{Q(V(a^p) p_{\bar{r}}(a^p))}{Q'(V(a^p) p_{\bar{r}}(a^p))} (1 + \Psi(a^p))$, then $a^p \le a^{sb}$.
- If $c = \frac{Q(V(a^p) p_{\bar{r}}(a^p))}{Q'(V(a^p) p_{\bar{r}}(a^p))} (1 + \Psi(a^p))$, then $a^p = a^{sb}$.
- If $c > \frac{Q(V(a^p) p_{\bar{r}}(a^p))}{Q'(V(a^p) p_{\bar{r}}(a^p))} (1 + \Psi(a^p))$, then $a^p \ge a^{sb}$.

Proof. By envelope theorem,

$$\frac{d\tilde{\Pi}}{da} = \tilde{r}M_1Q + \tilde{r}p_{\tilde{r}}Q'\left(\frac{dV}{da} - M_1\right),\tag{E.4}$$

while

$$\begin{split} \frac{d\tilde{W}}{da} &= \frac{1}{\tilde{r}}\frac{d\tilde{\Pi}}{da} - \frac{1}{\tilde{r}^2}\tilde{\Pi}\frac{d\tilde{r}}{da} + \left(\frac{Q}{Q'} - c\right)\left(\frac{dV}{da} - \frac{dp_{\tilde{r}}}{da}\right)Q' \\ &= \frac{1}{\tilde{r}}\frac{d\tilde{\Pi}}{da} - \frac{p_{\tilde{r}}Q}{\tilde{r}}\frac{d\tilde{r}}{da} + \left(\frac{Q}{Q'} - c\right)\left(\frac{dV}{da} - \frac{dp_{\tilde{r}}}{da}\right)Q' \\ &= \frac{1}{\tilde{r}}\frac{d\tilde{\Pi}}{da} + \left(\frac{Q}{Q'}\left(1 + \Psi\right) - c\right)\left(\frac{dV}{da} - \frac{dp_{\tilde{r}}}{da}\right)Q' \end{split}$$

Case 1: Suppose a^p is interior so that $\frac{d\tilde{\Pi}}{da} = 0$. From (E.4), for a^p to be interior we must have $\frac{dV}{da}|_{a=a^p} > M_1$ and it is pinned down by

$$\frac{dV}{da} = M_1 \left(1 - \frac{Q}{p_{\tilde{r}}Q'} \right). \tag{E.5}$$

By Lemma 5, (E.5) implies $\frac{dV}{da} > \frac{dp_{\tilde{r}}}{da}$ and $\frac{d\tilde{r}}{da} > 0$. Hence, $\frac{d\tilde{W}}{da}|_{a=a^p}$ has the same sign as $\frac{Q}{Q'}(1+\Psi)-c$. Then, unimodality of $\tilde{W}(a)$ implies $a^p < (>)a^{sb}$ if $c < (>)\frac{Q(V(a^p)-p_{\tilde{r}}(a^p))}{Q'(V(a^p)-p_{\tilde{r}}(a^p))}(1+\Psi(a^p))$.

Case 2: Suppose $a^p = \bar{a}$, so obviously $a^{sb} \leq a^p$. It remains to rule out the possibility of $a^{sb} < a^p$ when $c < \frac{Q(V(a^p) - p_{\bar{r}}(a^p))}{Q'(V(a^p) - p_{\bar{r}}(a^p))} (1 + \Psi(a^p))$. Notice $a^p = \bar{a}$ implies $\frac{d\tilde{\Pi}}{da}|_{a=a^p} > 0$, or

$$\frac{dV}{da}|_{a=a^p} > M_1 \left(1 - \frac{Q}{p_{\tilde{r}}Q'} \right). \tag{E.6}$$

By Lemma 5, (E.6) implies $\frac{dV}{da} > \frac{dp_{\tilde{r}}}{da}$ and $\frac{d\tilde{r}}{da} > 0$. So, if $c \leq \frac{Q(V(a^p) - p_{\tilde{r}}(a^p))}{Q'(V(a^p) - p_{\tilde{r}}(a^p))} (1 + \Psi(a^p))$, then $\frac{d\tilde{W}}{da}|_{a=a^p} > 0$. Unimodality of $\tilde{W}(a)$ implies $a^{sb} = \bar{a}$ as required.

Case 3: Suppose $a^p = \underline{a}$, so obviously $a^{sb} \geq m^p$. It remains to rule out the possibility of $a^{sb} > a^p$ when $c > \frac{Q(V(a^p) - p_{\tilde{r}}(a^p))}{Q'(V(a^p) - p_{\tilde{r}}(a^p))} (1 + \Psi(a^p))$. Notice $a^p = \underline{a}$ implies $\frac{d\tilde{\Pi}}{da}|_{a=a^p} < 0$, or

$$\frac{dV}{da}|_{a=a^p} < M_1 \left(1 - \frac{Q}{p_{\tilde{r}}Q'}\right) < 0.$$

There are two subcases to consider. If $(\frac{dV}{da} - \frac{dp_{\bar{r}}}{da})|_{a=a^p} \leq 0$, then we write the derivative of welfare function as

$$\frac{d\tilde{W}}{da}|_{a=a^{p}} = \left(\frac{dV}{da} - \frac{dp_{\tilde{r}}}{da}\right) \left((p_{\tilde{r}} - c) Q' + Q\right) + \frac{dp_{\tilde{r}}}{da}Q$$

$$< \left(\frac{dV}{da} - \frac{dp_{\tilde{r}}}{da}\right) Q + \frac{dp_{\tilde{r}}}{da}Q < 0,$$

which immediately implies $a^{sb} = \underline{a}$ regardless of c. If $(\frac{dV}{da} - \frac{dp_{\tilde{r}}}{da})|_{a=a^p} > 0$, then $c > \frac{Q(V(a^p) - p_{\tilde{r}}(a^p))}{Q'(V(a^p) - p_{\tilde{r}}(a^p))} (1 + \Psi(a^p))$ implies $\frac{d\tilde{W}}{da}|_{a=a^p} < 0$. Unimodality of $\tilde{W}(a)$ implies $a^{sb} = \underline{a}$ as required.

Proposition E.3 (Lump sum seller fees) Suppose the social planner can control the platform's governance design, but cannot control the lump sum seller fee set by the platform. Then, $a^p \leq a^{sb}$.

Proof. From $\Pi(a) = M(a,c)Q(V(a) - c - M(a,c))$, we know that for all $a < a^p$ we have $V(a) - c - M(a,c) < V(a^p) - c - M(a^p,c)$ because otherwise if there is some $a' < a^p$ such that $V(a') - c - M(a',c) \ge V(a^p) - c - M(a^p,c)$, then $\tilde{\Pi}(a') > \tilde{\Pi}(a^p)$, which contradicts the definition of a^p being a profit maximizer. Next, from the welfare function

$$\tilde{W}(a) = \tilde{\Pi}(a) + \int_{-\infty}^{V(a)-c-M(a,c)} Q(t)dt,$$

it follows that $\tilde{W}(a) < \tilde{W}(a^p)$ for all $a < a^p$. Hence we conclude $a^{sb} \ge a^p$.

Proposition E.4 (Two-part tariff) Suppose the social planner can control the platform's governance design, but cannot control the two-part tariff set by the platform. Define $p^*(a) = \frac{Q'(V(a)-p(a))}{Q(V(a)-p(a))}$ and $a^* \equiv \arg\max V(a)$.

- If $c \le p^*(a^*) M(a^*, c)$, then $a^p = a^{sb}$.
- If $c > p^*(a^*) M(a^*, c)$ and V(a) is unimodal, then $a^p \le a^{sb}$.

Proof. With two-part tariff, the equilibrium price is given $p(a) = c + \frac{rc+\tau}{1-r} + M(a, c + \frac{rc+\tau}{1-r})$. For each given a, the platform can induce the joint monopoly price $p^*(a)$ by setting r and τ such that $\frac{rc+\tau}{1-r} = p^*(a) - c - M(a, c + \frac{rc+\tau}{1-r})$. It is optimal to induce $p^*(a)$ as long as the no-subsidy constraint is satisfied, which holds if and only if $p^*(a) - c - M(a, c) \geq 0$. When

 $p^*(a) - c - M(a, c) < 0$, quasi-concavity of the profit function (p - c)Q(V(a) - p) with respect to p implies that the no-subsidy constraint is binding, so that the optimal induced price is c + M(a, c). Combining both cases, for each given a we denote the profit-maximizing induced price as

$$\tilde{p}(a) = \max \left\{ c + M(a, c), p^*(a) \right\},\,$$

and the profit function can be written as a univariate function: $\tilde{\Pi}(a) = (\tilde{p}(a) - c)Q(V(a) - \tilde{p}(a)).$

Case 1. If $c \leq p^*(a^*) - M(a^*,c)$, then $p^*(a^*)$ is implementable. We know by envelope theorem that $\tilde{\Pi}(a) = (\tilde{p}(a) - c)Q(V(a) - \tilde{p}(a))$ is maximized at $a^p = a^*$. Moreover, log-concavity of Q and the definition of $p^*(a)$ implies that $V(a) - p^*(a)$ is maximized when $a^* \equiv \arg\max_a V(a)$. Therefore, $V(a) - \tilde{p}(a) \leq V(a) - p^*(a) \leq V(a^*) - p^*(a^*)$, meaning that $\tilde{W}(a) = \tilde{\Pi}(a) + \int_{-\infty}^{V(a) - p^*(a)} Q(t) dt$ is maximized at $a^{sb} = a^*$.

Case 2. Suppose $c > p^*(a^*) - M(a^*, c)$, then from the preceding case we know $p^*(a^*)$ is no longer implementable. To proceed, denote $\hat{a} < a^*$ as the solution to $p^*(a) - c = M(a, c)$. Its existence and uniqueness follow from unimodality of V(a), whereby V(a) and $p^*(a)$ are monotonically decreasing for all $a > a^*$. By construction, $\tilde{p}(a) = p^*(a)$ for all $a < \hat{a}$ and envelope theorem implies all $a < \hat{a}$ delivers a lower profit than $a = \hat{a}$. Hence, we conclude $\tilde{p}(a^p) = c + M(a^p, c)$.

To prove $a^p \le a^{sb}$, suppose by contradiction $a^p > a^{sb}$. If $V(a^p) - M(a^p, c) - c > V(a^{sb}) - \tilde{p}(a^{sb})$ then

$$\tilde{W}\left(a^{sb}\right) < \tilde{\Pi}\left(a^{sb}\right) + \int_{-\infty}^{V(a^p) - M(a^p, c) - c} Q(t)dt \le \tilde{W}\left(m^p\right),$$

contradicting the definition of a^{sb} . If $V(a^p) - M(a^p, c) - c \leq V(a^{sb}) - \tilde{p}(a^{sb})$ then

$$\begin{split} \tilde{\Pi}\left(a^{sb}\right) &= (\tilde{p}(a^{sb}) - c)Q\left(V\left(a^{sb}\right) - \tilde{p}(a^{sb})\right) \\ &\geq (\tilde{p}(a^{sb}) - c)Q\left(V\left(a^{p}\right) - M(a^{p}, c) - c\right) \\ &> M(a^{p}, c)Q\left(V\left(a^{p}\right) - M(a^{p}, c) - c\right) = \tilde{\Pi}\left(m^{p}\right), \end{split}$$

where the last inequality is due to $\tilde{p}(a^{sb}) - c \ge M(a^{sb}, c) > M(a^p, c)$. This contradicts the definition of a^p . Hence, we conclude $a^p \le a^{sb}$.

F Consumer surplus benchmark

For each given fee instrument, we know $m^{CS} = \arg\max\{v(m) - p(m)\}$. Following the existing analysis, when the platform uses per-transaction fees (regardless of whether it is exogenous or endogenous), $m^p = \arg\max\{v(m) - p(m)\}$, so $m^p = m^{CS}$. Likewise, if the platform uses lump sum fees, two-part tariffs, or exogenous proportional fee, from the profit function expressions it is obvious that $m^p \ge m^{CS}$ as a higher m increases platform's margin.

The only non-obvious case is when the platform uses endogenous proportional fee. Let $m^{CS} = \arg\max\{v(m) - p_{\bar{r}}(m)\}$. If $m^p = \bar{m}$, then $m^p \geq m^{CS}$ trivially holds. If $m^p < \bar{m}$, then either it is an interior solution or $m^p = \underline{m}$. In both cases, we have $\frac{dv}{dm}|_{m=m^p} \leq 1 - \frac{Q}{p_{\bar{r}}Q'}|_{m=m^p}$. Recall from Lemma 4 that $\frac{dv}{dm} - \frac{dp_{\bar{r}}}{dm} < 0$ whenever $\frac{dv}{dm} \leq 1 - \frac{Q}{p_{\bar{r}}Q'}$. Hence, we have

 $\left(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm}\right)|_{m=m^p} < 0$, implying $m^p \ge m^{CS}$, and strictly so if m^p is interior.