Fundamental Pricing of Utility Tokens

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Abstract

We propose a framework for the fundamental valuation of utility tokens. Our model endogenizes the velocity of circulation of tokens and yields a pricing formula that is fully microfounded. According to our model, tokens are valuable because they have to be immediately accessible when the services are needed, a requirement that is reminiscent of the cash-in-advance constraint. The equilibrium price paths of successful projects go through two successive phases: A speculative phase where marginal holders are investors that do not intend to use the services and, later on, a user phase where all tokens are held by clients. Calibrating the model, we find that it helps rationalizing the extreme volatility and significant valuation of tokens early on during the adoption stage.

1 Introduction

The vast majority of startups finance their growth by raising equity from venture capitalists. This market dominance has recently been challenged by a new fundraising method that leverages Blockchain technologies. Following the examples of Ethereum and Ripple, a growing number of startups rely on token sales to raise capital: The company issues a new cryptocurrency, and investors receive its "tokens" in exchange

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for legal tender or other cryptocurrencies. The tokens derive their value from the fact that they will be used to purchase the goods or services offered by the issuer once its project becomes operational. The value of the token is therefore expected to increase with the popularity of the business, thus rewarding early investors.

Although the disruptive potential of token sales is widely recognized, their adoption beyond the crypto-community has been hindered by their controversial reputation. The most common criticism is that Initial Coin Offerings (ICOs hereafter) are used to draw unsophisticated investors towards nonviable projects. Such practice will only be curbed through the creation of a reliable framework for the valuation of tokens, making it possible for investors to identify when they are fairly priced.

This paper takes one of the early steps in that direction by proposing a fully micro-founded pricing model for utility tokens. We show that tokens intrinsically differ from other financial instruments, such as debt or equity, and thus cannot be priced using off-the-shelf valuation techniques. Our model identifies the fundamental value of tokens as a function of two sets of primitives: consumers' preferences and technological constraints. It characterizes their price trajectory, endogenizing the evolution of token velocity whereas reduced-form models currently used by investors arbitrarily specify the speed at which tokens circulate.

Relying on a formal model allows us to clarify the answers to the following three essential questions: Why are utility tokens valuable? How should they be priced? What are the actual benefits of tokenization for the issuer?

Our response to the first question is that tokens are valuable to the extent that, when needed, the application's services have to be accessed immediately. In other words, users cannot delay their consumption until they have been able to acquire the required tokens in the secondary market. This constraint is reminiscent of the cash-in-advance constraint commonly advocated to endow cash with intrinsic value. The parallel is not really surprising since, after all, tokens are means of exchange. An interesting specificity with respect to cash is that the token-in-advance constraint can be hardwired into the technological specification of the application. For instance, the platform can impose a lock-in period to slow down the circulation of its tokens.

Provided that the token-in-advance constraint holds, tokens are valuable and we derive an equilibrium price path that depends on the distribution of preferences among prospective users. The price of the token evolves over time, reflecting changes in the efficiency, and thus adoption, of the application it gives access to.

Early on during the adoption phase, the marginal token holder is not a user but an investor, or speculator, who has no intention to ever use the token. Then its price behaves as a pure asset bubble that appreciates at the required rate of interest. Among the infinity of bubbly paths, the fundamental solution is pinned down by the requirement that its trajectory should eventually transition to a user regime where all tokens are held by agents that intend to use the services of the application.

Knowing that the token eventually enters a user regime enables us to derive a pricing formula which is based on fundamentals only. In a nutshell, the value of holding tokens for users is equal to the *discounted surplus of their next trade.* It is therefore increasing in the rate at which transactions are carried out, as users incur shorter waiting times, and so smaller opportunity costs, for the immobilization of their funds. These opportunity costs also depend on the rate of growth of the token price which incorporates expectations about future technological improvements. When they are positive, token appreciation compensates users, thereby fostering adoption. But this feedback loop between technological prospects and token pricing is a double-edged sword, as pessimistic forecasts entail a depreciation that discourages adoption in the current period.

Concurrent research also highlights that tokens may accelerate adoption by sharing the financial returns of the venture with users (see Cong et al. (2018) or Li and Mann (2018)). A distinctive feature of our model is that, early on during the adoption phase, the price dynamics is solely driven by speculative factors. In order to meet the expectations of investors and deliver their required rate-of-return, the token price is a convex function of the project's productivity. By Jensen's inequality, this convexity raises the expected growth rate of the token price above that of productivity. Hence our model rationalizes the extreme volatility and significant valuation of tokens when adoption is low or even marginal, dispelling the popular belief that these two phenomena are unmistakeable signs of irrational exuberance.

Related literature. ICOs being a very recent phenomenon, the related academic literature is still in its infancy.[1] The first generation of papers focused on the value of privately-issued digital currencies. Athey et al. (2016) analyze the determinants of their exchange rates, demonstrating that investors may hoard currencies in anticipation of future transactional usage. A similar mechanism is at work in the dynamic version of our model where most tokens are initially held by investors. A related strand of research revisits the indeterminacy of exchange rates between two currencies originally

[1] The first documented token sale was held by Mastercoin in 2013.

established by Kareken and Wallace (1981). Garratt and Wallace (2008) distinguish the central bank from the privately issued currency by introducing a storage cost for the former and a disaster risk for the later. Pagnotta (2018) explicitly models how the crash risk is determined by miners' investment, thus giving rise to price–security feedback loops that can amplify or dampen the impact of demand shocks on Bitcoin price. Chiu and Koeppl (2019) demonstrates that seignoriage is more efficient than transaction fees to finance mining while Huberman et al. (2019) derives pricing formulas for Bitcoin transaction fees and studies how limited capacity and priority needs drive them. Uhlig and Schilling (2018) show that indeterminacy can support a speculative equilibrium where the cryptocurrency is held in anticipation of its appreciation. Biais et al. (2018) embed a dual-currency regime into an OLG model and show how their framework can be taken to the data. Our paper differs from this literature in that we are not considering cryptocurrencies whose purpose is to serve as a universal means of payment, but instead utility tokens whose detention is required to consume a particular product. Hence our pricing formula is directly derived from consumers preferences rather than from transactional benefits.

Another range of papers study platform-specific currencies issued at a stable price by digital platforms. Gans and Halaburda (2015) focus on the use of such currencies to increase activity on the platform and the implications for its business model, taking into account consumer preferences. Rogoff and You (2019) demonstrate that platforms will find generally profitable to make such tokens non-tradable. Our paper addresses also the topic of platforms offering services through a dedicated currency but our approach considers tokens that are a tradable on crypto-markets instead of being designed as a platform-specific fiat-equivalent.

Our paper is therefore more closely related to the growing literature studying ICOs. A first branch focuses on corporate finance issues related to the incentives of investors and entrepreneurs. Catalini and Gans (2018) show that ICOs may be more efficient than venture capital when participants in the ICO market are well informed, as token prices reveal the actual quality of the project to a wider set of investors. Chod and Lyandres (2018) explain why token sales lead to underinvestment because they generate an agency conflict between the entrepreneur and investors. In spite of this drawback, they find that ICOs can dominate traditional venture capital when investors are underdiversified. Canidio (2018) also underlines the agency conflicts induced by ICOs since there is a non negligible probability that the entrepreneur will sell all her tokens and halt the development of her project. Moreover, even when this risk is avoided, the entrepreneur will behave myopically by maximizing the project value in the post-ICO period and not over its all lifetime. Going one step further, Canidio (2020) finds that, in presence of developers who can self-finance their project and hence avoid an ICO, only a single platform enters the market leading to a monopolistic equilibrium. A more positive strand of papers outlines the coordination benefits of ICOs in applications with network effects. Bakos and Halaburda (2018), Garratt and Van Oordt (2019), Gryglewicz et al. (2019) and Li and Mann (2018) show that token sales may help overcome coordination failure, since token sales provide a signal about consumers' willingness to use the platform. Similarly, Malinova and Park (2018) propose an optimally-designed token contract that can reach the same pay-off as equity and allow the financing of more projects due to the alleviation of moral hazard induced by ICOs. Gan et al. (2020) reach similar conclusions regarding agency costs, moral hazard and coordination in the specific framework of asset tokenization. From an empirical perspective, Howell et al. (2020) study the conditions for an ICO to lead to a successful project. In accordance with the theoretical literature, they name credible commitment, quality signal and information disclosure as key success factors.

In order to focus on the pricing of tokens, we abstract from issues related to incentives alignment between entrepreneurs and investors. In this respect, the paper most closely related to ours is Cong et al. (2018). They derive a dynamic asset pricing model of tokens, showing that token appreciation can accelerate platform adoption by allowing users to partially internalize network externalities. Mayer (2019) modifies the set-up of Cong et al. (2018) by exogenously adding speculators as an additional type of agent. Focusing on platform developement and conflicts of interest, he highlights the dual role of owner's holdings that drives her incentives but also determines token supply, and he provides insights on token liquidity and speculative investment along the development of the platform. In another study of platform-issued utility tokens, Sockin and Xiong (2020) assess the relationship between token price and demand fundamental and show that the market for a utility token can break down with no equilibrium under conditions that depend on speculator sentiment and user optimism. The main difference between their approaches and our is that we study utility tokens which have to be exchanged in order to access the platform, whereas they assume that tokens give access to a stream of services when staked. As a result, in our model, agents cannot simultaneously benefit from token price appreciation and enjoy the convenience yield offered by the service. Conversely, the specifications in these papers – which recall a money-in-the-utility-function approach – preclude speculation or require to exogenously introduce a speculator type of agents. Consequently, in Cong et al. (2018), the velocity of circulation is not a relevant statistics in their model because tokens are always held by users. By contrast, the share of tokens held by investors is endogenously determined in our model, and its evolution drives changes in the velocity of circulation, thus explaining why this statistics has been the subject of intense scrutiny in the crypto-community. In Mayer (2019), speculators are exogenously introduced with an artificially limited duration in which speculation can occur. Finally, Sockin and Xiong (2020) need a noisy signal unobservable by platform users to explain speculative activity whereas our model proposes a less-constrained framework in which speculators and users have access to the same information regarding the evolution of the token price and simply differ by their will to access the platform services.

Structure of the paper. Section 2 derives the equilibrium price of tokens in steady-state. Section 3 describes how token velocity evolves over time by extending our framework to a setup with gradual adoption. In Section 4, we use adoption data for Maker's token to calibrate the model, showing that it can capture the relationship between token price and user adoption. Proofs of claims and propositions are relegated to the Appendix.

2 Model

2.1 Set-Up

We consider a decentralized platform or application that issues tokens to finance its development. We do not explicitly model the token sale. Instead, we focus on the dynamics of the token price in the secondary market, implicitly assuming that the primary market sale occurred in the past. Tokens are valuable because they allow their owners to purchase the goods and services provided by the platform. The overall supply of tokens, or monetary base, is equal to M. To simplify matters, we assume that, as advertised in most token sales, the mass of tokens remains constant over time.

There are two markets: (i) a trading market where tokens are bought and sold, and (ii) a commodity market where tokens are exchanged against the output of the platform. The price or exchange rate of the token in fiat money is denoted by p_t.

It is determined on the perfectly competitive and frictionless trading market. On the commodity market, the platform is a monopolistic intermediary that connects users with contributors supplying the service. These contributors are typically miners in Blockchain-based environments. We assume that contributors are so numerous compared to the activity required by the platform that their supply can be treated as vertical.[2] Since the costs incurred by contributors, such as hardware or electricity, are paid in fiat, the contributors' supply is decoupled from variations in the token price.[3] Hence the platform can commit to providing a unit of service at a *constant price in fiat currency* by allowing users to acquire n units of service in exchange of n/p_t tokens.

We normalize the mass of users to one. In each period, a *constant share* $\lambda \in (0,1)$ of users are willing to consume the platform's services. Then they derive utility $z * u(c)$ from consuming c units of service, where $u(c)$ is a standard utility function $(u'(c) > 0, u''(c) < 0, \lim_{c\to\infty} u'(c) = 0)$ and z is a demand shifter capturing the technological efficiency of the platform. In this section, we do not specify the law which governs the evolution of the demand shifter. The only requirement at this stage is that z_t is perfectly observable by all agents and that it follows a deterministic process. Hence the per-period utility function of user $i \in [0,1]$ reads

$$\mathcal{U}\left(z, c, d^i\right) = zu(c) * d^i, \text{ where } d^i = \begin{cases} 0 \text{ with probability } 1-\lambda \\ 1 \text{ with probability } \lambda \end{cases}. \tag{1}$$

2.2 Equilibrium Price

Each period is divided into two sub-periods. As summarized in Fig. 1, the commodity market opens first and preference shocks d^i are revealed. Users can buy the service only if they have entered the period with some tokens. Then the commodity market closes and the trading market opens, allowing users to rebalance their token holdings by selling and buying tokens at the market price p_t.

The timing is crucial since it amounts to hardwiring a token-in-advance constraint. Suppose instead that users first observe their willingness to consume and then adjust their token holdings. Since tokens do not bear any interest, users would find it optimal to hold zero tokens at the beginning of the period and the market price p_t would

[2] In Ethereum's blockchain, where most utility tokens are currently issued, this assumption is equivalent to considering that the gas price is not affected by the usage of a single application.

[3] For insights on the interactions between miners and the economic mechanisms driving their overall supply, see Pagnotta (2018), Chiu and Koeppl (2019) or Huberman et al. (2019).

collapse to zero. The only use of tokens is that they enable consumers to satisfy their needs. Thus they are valuable because the service is needed immediately and it is too costly to wait for the next period.

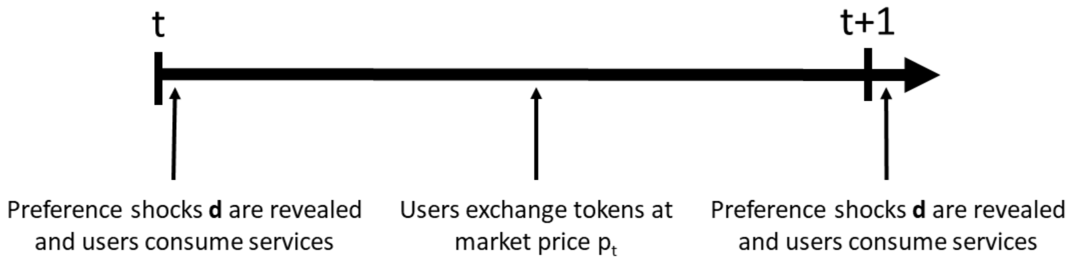

Figure 1: TIMING OF THE MODEL

When the trading market opens, users decide how many tokens m to carry into the next period. Given that users can instead invest their money at the risk-free rate r, their optimal returns read

$$v(p_t, p_{t+1}) \equiv \max_{m \geq 0} \left\{ \lambda \max_{c \in [0, mp_{t+1}]} \left\{ z_{t+1} u(c) + p_{t+1} \left(m - \frac{c}{p_{t+1}} \right) \right\} + (1 - \lambda) m p_{t+1} - (1 + r) m p_t \right\}. \tag{2}$$

The value of the dummy variable d is drawn in $t + 1$ where it will be equal to 0 with probability $1-\lambda$. Then the agent will not need the service and so will enter next period's trading market with the same amount of tokens m_t, earning a reward equal to $p_{t+1}m_t$, as indicated by the penultimate term in (2). With the complementary probability λ, the dummy variable d will be equal to 1 and the agent will use the platform. Then she will choose her optimal level of consumption under the token-in-advance constraint $c_{t+1} \in [0, m_t p_{t+1}]$ because consumption can never be greater than the market value of token holdings. If the agent does not consume all her tokens, she will end next period's trading market with $m_t - c_{t+1}/p_{t+1}$ tokens, thus earning the financial reward $p_{t+1}(m_t - c_{t+1}/p_{t+1})$ on top of the utility benefits $z_{t+1}u(c_{t+1})$. Finally, we ensure that v measures the optimal net returns by subtracting the value that would have been obtained if the funds $m_t p_t$ had been invested at the risk-free rate r.

The growth rate of the token price has to be lower than r for the returns function v to be well defined, as otherwise the agent would find it optimal to hoard an infinite amount of tokens. In other words, the token cannot appreciate at a rate greater than r because the trading market would not clear. Instead, investors would bid up the price of the token until its expected returns are brought down to r. This requirement ensures that the constraint $c_{t+1} \leq m_t p_{t+1}$ always binds for users that wish to access the platform's services.[4] To understand why, remember that consumption is determined after the value of the demand shock d has been revealed, whereas token holdings are decided beforehand. Given that agents take their investment decisions behind the veil of ignorance, they face the risk of not needing the service. This is why users ration their holdings below the level that is optimal when they are hit by a positive demand shock. Their returns function is therefore equivalent to[5]

$$v(p_t, p_{t+1}) = \max_{m_t \geq 0} \left\{ \lambda z_{t+1} u(m_t p_{t+1}) + (1-\lambda) m_t p_{t+1} - (1+r) m_t p_t \right\}, \tag{3}$$

and optimal token holdings, denoted by m_t^*, are such that

$$r p_t = \underbrace{\lambda p_{t+1} \left[z_{t+1} u'(m_t^* p_{t+1}) - 1 \right]}_{\text{Convenience Yield}} + \underbrace{p_{t+1} - p_t}_{\text{Capital Gain}}. \tag{4}$$

The rate of return on tokens can be decomposed into two components: a capital gain and a convenience yield. The capital gain is standard since it corresponds to the appreciation in the price of the token. By contrast, the convenience yield is specific to utility tokens. The marginal token provides a quantity of services p_t whose marginal utility is equal to $z_{t+1} u'(m_t^* p_{t+1})$. But the service is delivered *in exchange* of the token. Thus one also has to take into account the loss of the marginal token and deduct its price from the marginal benefit. From the standpoint of pricing theory, this is the main difference between tokens and shares. Since shares do not have to be exchanged to provide their owners with dividends, their fundamental value is equal to the discounted sum of all future dividends. Utility tokens, on the other hand, do not yield any benefits if they are not traded, so their fundamental value is equal to the *discounted surplus of the next trade.* In our model, a trade occurs with probability λ, which explains why the marginal surplus $p_{t+1} \left[z_{t+1} u'(m_t^* p_{t+1}) - 1 \right]$ is multiplied by λ in the expression of the convenience yield.

All agents being identical, they hoard the same amount of tokens. Given that the

[4]This result immediately follows comparing the FOC for consumption, $z_{t+1} u'(c_{t+1}^*) = 1$, with the one for token holdings (4). Since we focus on cases where $p_{t+1} \leq (1+r) p_t$, (4) can hold only if $z_{t+1} u'(m_t^* p_{t+1}) = z_{t+1} u'(c_{t+1}) > 1$, so that the feasibility constraint, $c_{t+1} \leq m_t p_{t+1}$, binds.

[5]See Appendix A.2 for an explicit derivation of (3).

mass of users is normalized to one, the market for tokens clears when

$$m_t^{i,*} = M \text{ for all } t \text{ and all } i \in [0,1]\,. \tag{5}$$

Replacing the market clearing condition into (4), we find that the token price obeys the following law of motion

$$\frac{p_{t+1}}{p_t} = \frac{1+r}{\lambda z_{t+1} u'(Mp_{t+1}) + 1 - \lambda}. \tag{6}$$

Assuming that the demand shifter converges towards a finite value z_∞ corresponding to the final development of the platform, and setting p_{t+1} equal to p_t, yields the following solution for the steady-state price

$$\hat{p} = \frac{1}{M} u'^{-1}\left(\frac{r+\lambda}{\lambda z_\infty}\right). \tag{7}$$

As expected, our model features money neutrality since the equilibrium price is inversely proportional to the token supply M. The demand $\hat{p}M$ from each user is inferior to the demand that would have prevailed if the services could be bought directly in fiat.[6] This reduction is due to the fact that clients have to be compensated for holding tokens that do not bear any interest. Such incentives are provided at the time of trade: Users that buy less services enjoy a higher marginal utility, and so, extract some surplus from the exchange of their marginal token. Hence, by issuing utility tokens, the company commits to selling less services in the long-run. This insight clarifies the often muddled debate over the trade-off between ICOs and equity financing, most notably by dispelling the belief that ICOs are a free lunch for issuers.[7]

2.3 Endogenous User Base

The equilibrium price ensures that the trading market clears. When users are homogenous, as in the previous subsection, market clearing implies that potential demand is saturated. By contrast, when users are heterogenous, the user base becomes en-

[6]The demand from each user reads $\hat{p}M = u'^{-1}\left((r+\lambda)/(\lambda z_\infty)\right)$. It is easily verified that it is inferior to demand in the tokenless economy $u'^{-1}\left(1/z_\infty\right)$ – see Section 4.

[7]It is also sometimes argued that ICOs are costly because they amount to selling for free the amount of services corresponding to the mass of issued tokens. But this argument is misleading as the company can always sell back its tokens on the secondary market.

dogenous. Network effects can easily be introduced by embedding user base in the utility function of users. Since it is well-known that network effects foster adoption by amplifying early-stage adoption,[8] we shut down this channel in order to focus on the mechanisms that are specific to our model. For simplicity, we assume that users share the same per-period utility function (1) but incur different flow costs of accessing the platform. These flow costs are *inversely proportional* to the level of technological expertise χ_i of user i. The parameter χ_i captures the opportunity cost of the time devoted to using the platform. User i finds it optimal to hold some tokens whenever

$$v(p_t, p_{t+1}) \geq \frac{1}{\chi_i}. \tag{8}$$

Potential users draw their ability from the distribution $G(\chi)$ defined over $[\underline{\chi}, \infty)$, where $\underline{\chi} > 0$. Thus the user base, which we denote by N_t, is equal to

$$N_t = 1 - G\left(\frac{1}{v\left(p_t, p_{t+1}\right)}\right). \tag{9}$$

The equilibrium price $\hat{p}$ is obtained interacting this condition with the law of motion for p_t. First, we have to adjust the market clearing condition by rescaling the overall mass of tokens by the number of users, i.e. $m_t^* = M/N_t$. Then the law of motion (6) generalizes to

$$\frac{p_{t+1}}{p_t} = \frac{1+r}{\lambda z_{t+1} u'\left(\frac{M p_{t+1}}{N_t}\right) + 1 - \lambda}.$$

Hence the price can be stable solely if

$$\hat{p} = \frac{\hat{N}}{M} u'^{-1}\left(\frac{r+\lambda}{\lambda z_\infty}\right), \tag{10}$$

where $\hat{N} \equiv 1 - G\left(v\left(\hat{p}, \hat{p}\right)^{-1}\right)$. Reinserting (10) in (8), we find that the per-period returns – and therefore the user base – are independent from the equilibrium price $\hat{p}$.[9] This is because the price of the service remains constant in fiat. Given that there is no capital gains at the steady state, agents simply choose their optimal quantity of services. As this quantity is the value in fiat of their holdings, the demand for tokens is perfectly price-elastic: Any price change is offset one-for-one so as to keep the purchasing

[8] See, for instance, Bakos and Halaburda (2018) or Gryglewicz et al. (2019).

[9] The steady-state value of returns reads $v(\hat{p}, \hat{p}) = \lambda z_\infty u\left(u'^{-1}\left(\frac{r+\lambda}{\lambda z_\infty}\right)\right) - (r+\lambda) u'^{-1}\left(\frac{r+\lambda}{\lambda z_\infty}\right)$.

power of token holdings constant.

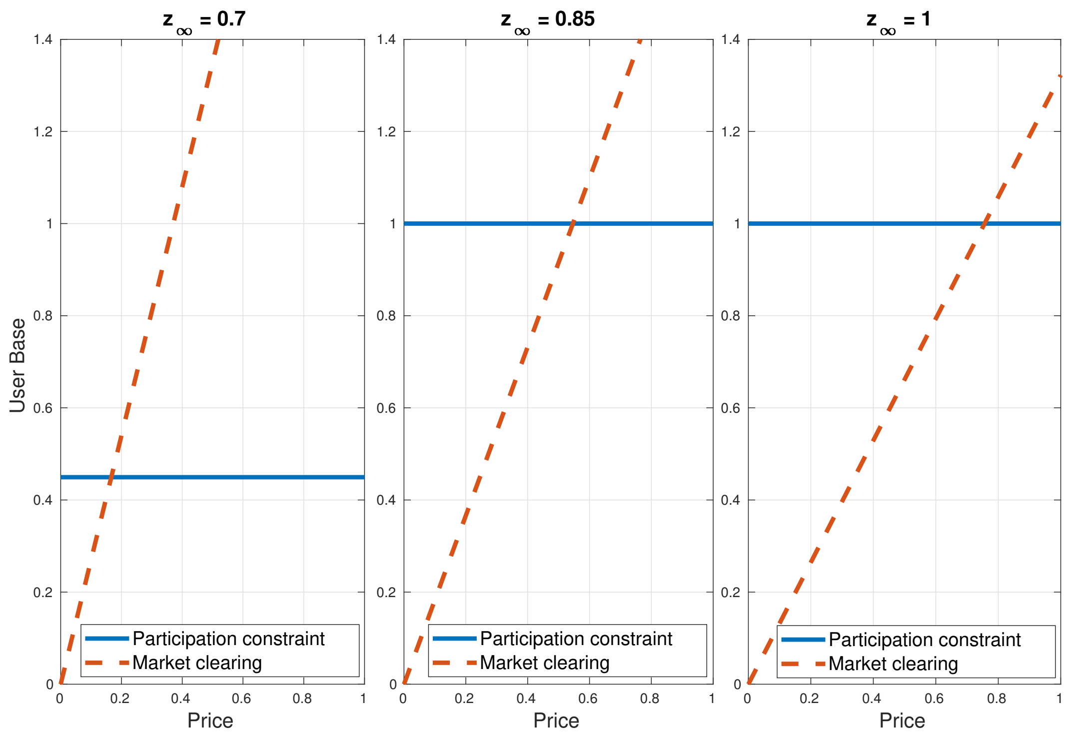

Figure 2: EQUILIBRIUM PRICE AND USER BASE FOR PRODUCTIVITY LEVELS $z_\infty = .7$, $z_\infty = .85$, $z_\infty = 1$. PARAMETERS: $r = .15$, $\eta = .5$, $M = 1$, $\overline{\chi} = 2$, $\alpha = 5$, $\lambda = 1$.

A Closed-Form Solution. The model can be solved analytically when the utility function of users is CRRA and when their abilities are sampled from a Pareto distribution

$$H1 : u(c) = \frac{c^{1-\eta}}{1-\eta}, \text{ with } \eta \in (0,1),$$

$$H2 : G(\chi) = \max\left\{0, 1 - \left(\frac{\underline{\chi}}{\chi}\right)^{\alpha}\right\} \text{ for all } \chi > 0, \text{ with } \underline{\chi} > 0 \text{ and } \alpha > 0.$$

Although restrictive, both hypotheses have some empirical support: CRRA is among the most common specifications; whereas models with heterogeneous agents usually rely on Pareto distributions to capture fat tails in the distribution of abilities. From a formal standpoint, Assumption H1 enables us to explicitly derive net returns as a function of the steady-state price. Combining this solution with H2 yields a closed-form expression for the equilibrium condition (9) and for the mass of users $\hat{N}$ which is weakly decreasing in p. Interacting this condition with the rest point requirement (10) yields a system of two equations for the two unknowns $\hat{N}$ and $\hat{p}$.

Claim 1 *When H1 and H2 hold, the equilibrium mass of users $\hat{N}$ and token price $\hat{p}$ are uniquely determined by the following system of equations*

$$\textit{Participation constraint:} \qquad \hat{N} = \min\left\{1, \left[\frac{\chi(\lambda z_\infty)^{1+\rho}}{\rho\left[(r+\lambda)\right]^{\rho}}\right]^{\alpha}\right\}, \tag{11}$$

$$\textit{Market clearing:} \qquad \hat{p} = \frac{\hat{N}}{M}\left[\frac{\lambda z_\infty}{r+\lambda}\right]^{\frac{1}{\eta}}, \tag{12}$$

where $\rho \equiv (1-\eta)/\eta$.

Proof. See Appendix A.1.1. ■

Figure 2 illustrates how the equilibrium price and market size are pinned down by conditions (11) and (12) for different values of the steady-state productivity. It shows that full adoption is not necessarily achieved at the steady-state if the service is not attractive enough, i.e. if z_∞ is relatively low. The figures also illustrate that, as discussed above, the participation constraint does not depend on the token price. Instead, it directly yields the steady-state level of adoption $\hat{N}$. The price then follows from the market clearing condition as cheaper tokens raise the overall demand from users until it is equal to the available supply.

3 Gradual Adoption

We now turn our attention to the benefits of issuing tokens. We will show in this section that the opportunity costs for users have to be weighted against the financial returns made on the token itself. To incorporate this countervailing channel, we need to consider an environment where the token price evolves over time. Adoption usually takes a while as users gradually migrate to the platform. Slow adoption can be due to a variety of reasons ranging from reputation building and growing awareness about the services provided by the platform, to improvements in the underlying technology. We adopt the last view and focus on cases where user adoption builds up over time because the platform becomes more and more efficient. We capture technological progress through the introduction of the demand shifter z_t.

Building a new platform is a risky business subject to many unforeseen contingencies. In order to take them into account, we generalize our framework and let productivity fluctuate randomly over time. More precisely, we devise our model in continuous time because it greatly simplifies the analysis and assume that log-productivity follows a random walk with constant trend and volatility, so that

$$dz_t = z_t\left(\mu dt + \sigma dB_t\right), \tag{13}$$

where B_t is a standard Brownian motion.[10] We choose a Geometric Brownian Motion specification for z_t because of its widespread use in the asset pricing literature. Our model could easily be modified so as to accommodate alternative stochastic processes, a task that we leave to future research.

The continuous time counterparts to equation (3) and (4) read

$$\begin{aligned} v\left(p_t, \dot{p}_t, z_t\right) &= \lambda\left[z_t u\left(p_t m_t^*\right) - p_t m_t^*\right] + \dot{p}_t m_t^* - r p_t m_t^*, \qquad (14) \\ m_t^* &= \frac{1}{p_t} u'^{-1}\left(\frac{1}{\lambda z_t}\left[r + \lambda - \frac{\dot{p}_t}{p_t}\right]\right), \qquad (15) \end{aligned}$$

where λ denotes the Poisson rate at which users access the platform and $\dot{p}_t \equiv \mathbb{E}_t\left[dp_t/dt\right]$.[11] Reinserting (15) into (14) we see that, due to the neutrality of nominal prices, the flow returns v depend on the rate of growth of the token price, $\dot{p}_t/p_t$, but not on its level p_t. As in Section 2.3, user i draws her ability from the distribution G and buys tokens when the returns exceed her flow costs, i.e. when $v\left(\dot{p}_t/p_t, z_t\right) \geq \chi_i^{-1}$. Hence user adoption N_t is a function of the vector $\left(\dot{p}_t/p_t, z_t\right)$ that satisfies

$$N\left(\frac{\dot{p}_t}{p_t}, z_t\right) = 1 - G\left(\frac{1}{v\left(\frac{\dot{p}_t}{p_t}, z_t\right)}\right). \tag{16}$$

Price dynamics. The appreciation rate of tokens depends on whether the marginal holder is a user or an investor. Assuming a potentially infinitely elastic supply of

[10] The law of motion for productivity is expressed under the risk neutral measure. Assuming that the Stochastic Discount Factor Λ_t follows a GBM under the physical measure, so that

$$d\Lambda_t = -\Lambda_t\left(rdt + \zeta dB_t^{\Lambda}\right),$$

Girsanov's theorem implies that $\mu = \hat{\mu} - \rho\zeta\sigma$ where $\hat{\mu}$ is the deterministic trend of z under the physical measure, while ρ is the correlation between z_t and Λ_t.

[11] See Appendix A.2.2 for a formal derivation.

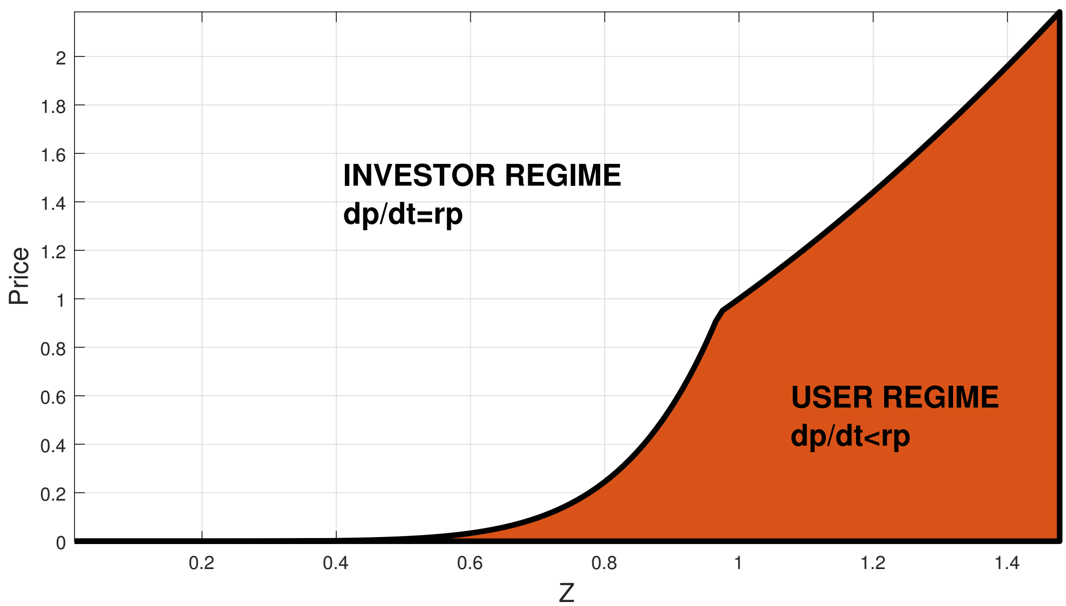

Figure 3: MARGINAL TOKEN HOLDER AS A FUNCTION OF PRICE AND PRODUCTIVITY. PARAMETERS REPORTED IN TABLE 1.

capital from investors, the price cannot grow at a rate higher than the interest rate – i.e. $\dot{p}_t \leq rp_t$. Thus the optimality condition (15) implies that $m_t^* \leq \overline{m}(p_t, z_t) \equiv u'^{-1}(1/z_t)/p_t$. Since the flow returns $v(\dot{p}_t/p_t, z_t)$ are increasing in $\dot{p}_t$, it also follows from the market clearing condition (16) that $N_t \leq \overline{N}(z_t) \equiv N(r, z_t)$. Combining these two upper-bounds, we find that the amount of tokens held by users $m_t^* N_t$ is at most equal to $\overline{M}(p_t, z_t) \equiv \overline{m}(p_t, z_t)\overline{N}(z_t)$. It is easily seen that, for all z, there exists a unique price

$$\overline{p}(z) = u'^{-1}\left(\frac{1}{z}\right)\frac{\overline{N}(z)}{M}, \tag{17}$$

such that $\overline{M}(\overline{p}(z), z)$ is equal to the overall token supply M.

Figure 3 illustrates how $\overline{p}(z)$ splits the (z, p) plane into two non-overlapping regions. If $p_t > \overline{p}(z_t)$, overall demand from users is too low to clear the market. Thus the marginal token is held by agents that only care about its appreciation. Given that we have assumed that the demand from potential investors is infinitely elastic, the market will always clear when the price grows at rate r[12]

$$\dot{p}_t = rp_t, \text{ when } p_t > \overline{p}(z_t). \tag{18}$$

In this regime, the token behaves as an asset bubble because it has no convenience yield for the marginal holder. By contrast, when $p_t \leq \overline{p}(z_t)$, the demand from users is sufficient to clear the market and the rate of appreciation $\dot{p}_t$ adjusts below rp_t until market clearing holds. Then the price dynamics is governed by users' optimality condition[13]

$$\dot{p}_t = -\lambda p_t \left[z_t u' \left(p_t \frac{M}{N_t} \right) - 1 \right] + rp_t, \text{ when } p_t \leq \overline{p}(z_t). \tag{19}$$

Since this equation has the exact same structure as its discrete time counterpart (4), we refer to Section 2.2 for a discussion of the convenience yield. Equations (18) and (19) ensure that the market clears and that token demand is optimal under rational expectations. Using the law-of-motion (13), we can substitute $\dot{p}_t = z_t \mu p'(z_t) + (z_t \sigma)^2 p''(z_t)/2$ into (18) – (19) to express the dynamics of the price as a function of z_t.

Proposition 1 *A Markov equilibrium with state variable z_t is a price function $p(z_t)$ that obeys the law-of-motion*

$$rp(z_t) = \begin{cases} -\lambda p(z_t) + z_t \mu p'(z_t) + \frac{(z_t \sigma)^2}{2} p''(z_t) + \lambda z_t p(z_t) u' \left(p(z_t) \frac{M}{N_t} \right), \textit{ when } p(z_t) \leq \overline{p}(z_t), & \text{(20a)} \\ z_t \mu p'(z_t) + \frac{(z_t \sigma)^2}{2} p''(z_t), \textit{ when } p(z_t) > \overline{p}(z_t), & \text{(20b)} \end{cases}$$

where N_t satisfies the participation constraint (16).

Provided that (i) $\tilde{\mu} \equiv \left[\mu + (1/\eta - 1)\sigma^2/2 \right]/\eta < r$ and (ii) the utility function of users is CRRA; the fundamental solution also satisfies the following two boundary conditions $p(0) = 0$ and

$$\lim_{z_t \to \infty} p(z_t) = \tilde{p}(z_t) = \left[\frac{\lambda z_t}{r + \lambda - \tilde{\mu}} \right]^{\frac{1}{\eta}} \frac{1}{M}. \tag{21}$$

The requirements in Proposition 1 ensure that: (i) agents choose the token holdings that maximize their returns, and (ii) the trading market clears at all time. Hence it identifies an equilibrium path. Its trajectory can be pinned down by two boundary

[12] Equation (18) also arises as a limit case of (19) when $\lambda = 0$, that is when the platform's services will never be needed, as would be the case for pure investors.

[13] Equation (19) is the FOC of (14) where m_t^* has been replaced by M/N_t to take into account the market clearing condition. See also eq. (30) in Appendix A.2.2 for a derivation of (19) when users are homogenous.

conditions since it obeys a second-order ODE. First, given that $z = 0$ is an absorbing state, $p(0) = 0$ should hold. Second, when $\tilde{\mu} < r$ and the utility function of users is CRRA, the limit as z goes to infinity of the price function admits a closed-form solution. To see why $\tilde{p}(z_t)$ satisfies the law-of-motion of the fundamental price, note that $\tilde{\mu} < r$ ensures that $\tilde{p}(z_t) < \overline{p}(z_t)$, hence $\tilde{p}(z_t)$ needs to verify (20a). Moreover, full adoption (i.e. $N(z) = 1$) has to hold for high enough z because we have assumed that the lower-bound $\underline{\chi}$ of the ability distribution $G(\chi)$ is strictly positive. Hence we can reinsert $\tilde{p}(z_t)$ along with $N(z_t) = 1$ into (20a) and verify that it indeed solves the ODE.

4 Illustration

4.1 Calibration

The main empirical prediction of our model is that user adoption and token price should be correlated. To test it, we have to identify a utility token with broad enough adoption. Given the novelty of utility tokens, few candidates are available. We select Maker's token (MK hereafter) because it finances one of the most popular, if not the most popular, application built on Ethereum's blockchain. MakerDAO enables its users to mint stablecoins, named Dai, by locking some collateral in MakerDAO's smart contracts. MK is the utility token native to the platform and is used to pay the fees required to access MakerDAO. Fees are proportional to the amount of stablecoins created. Hence, as assumed in our model, access fees are expressed in fiat and not in token units.

Data. Adoption data are not readily available. In order to infer them, we rely on proxies provided by Nyctale, a startup that focuses on data analytics for crypto-assets.[14] Nyctale has classified wallets into different categories depending on the rate at which their owners rebalance their positions. More precisely, they scanned all the wallets owning MKs and allocated them into one of the following three groups: (i) *Speculators* whose wallets exhibit a significant on-chain exchanged volume and a limited balance evolution, (ii) *holders* whose wallet balances remain fix over time, and (iii) *incoming or outgoing users* whose wallet balances fluctuate moderately over time. By contrast, we have only two types of token owners: users that periodically rebalance their position

[14] For a presentation of Nyctale's services and technology, see https://nyctale.io/.

and investors that never use the service. Because our model does not take into account portfolio management, it does not yield any predictions about trades that are not motivated by usage but instead by portfolio rebalancing strategies. Thus, in the absence of better proxies, we treat owners of wallets within the speculators and holders categories as investors. As for the remaining tokens, we assume that they are held by prospective users.

Calibration strategy. We assume that users draw their technological proficiency χ from a Pareto distribution. Under this premise, our model has eight parameters: the deterministic trend $\tilde{\mu}$ and volatility σ^2 of the log-productivity process z_t, the mass M of issued tokens, the Poisson arrival rate λ at which users need to access the application, the Pareto shape parameter α and lower-bound $\underline{\chi}$ of the ability distribution $G(\cdot)$, the curvature η of users' utility function, and the discount rate r. First, we set the discount rate $r = 5\%$. Then we follow the common practice of normalizing to one the lower-bound $\underline{\chi}$ of the ability distribution since it simply amounts to a rescaling of the ability measure. Finally, in the absence of observable measures, we set the curvature η of users' utility function equal to its mid-value of 0.5.

We are left with five parameters. We choose the value of σ that equalizes the theoretical and empirical volatility of the token price. For the remaining four parameters, we select their values so as to minimize the distance between the predicted and observed adoption-price schedules. More precisely, the empirical moments are summarized by the vector $\hat{\mathbf{p}}_t$ which contains the token price at different time and thus different adoption levels. Using $\mathbf{p}(\hat{A}_t; \mathbf{\Theta})$ to denote the price vector predicted by the model as a function of the observed levels of adoption $\hat{A}_t$ and of the vector of parameters $\mathbf{\Theta} \equiv \{\alpha, \tilde{\mu}, \lambda, M\}$, we compute the quadratic distances $d(\mathbf{\Theta}) \equiv \left(\mathbf{p}(\hat{A}_t; \mathbf{\Theta}) - \hat{\mathbf{p}}_t\right) \cdot \left(\mathbf{p}(\hat{A}_t; \mathbf{\Theta}) - \hat{\mathbf{p}}_t\right)'$. The calibrated vector $\hat{\mathbf{\Theta}}$ solves $\hat{\mathbf{\Theta}} = \arg\min_{\Theta \in \mathbb{R}^4_+} d(\mathbf{\Theta})$.

The values resulting from this calibration strategy are reported in Table 1, while the model's predictions are plotted against the data in Figure 5. It shows that our model is able to capture the overall shape of the price-adoption schedule. In particular, it fits its strong concavity early on during the adoption phase, capturing the high elasticity of the token price with respect to adoption.

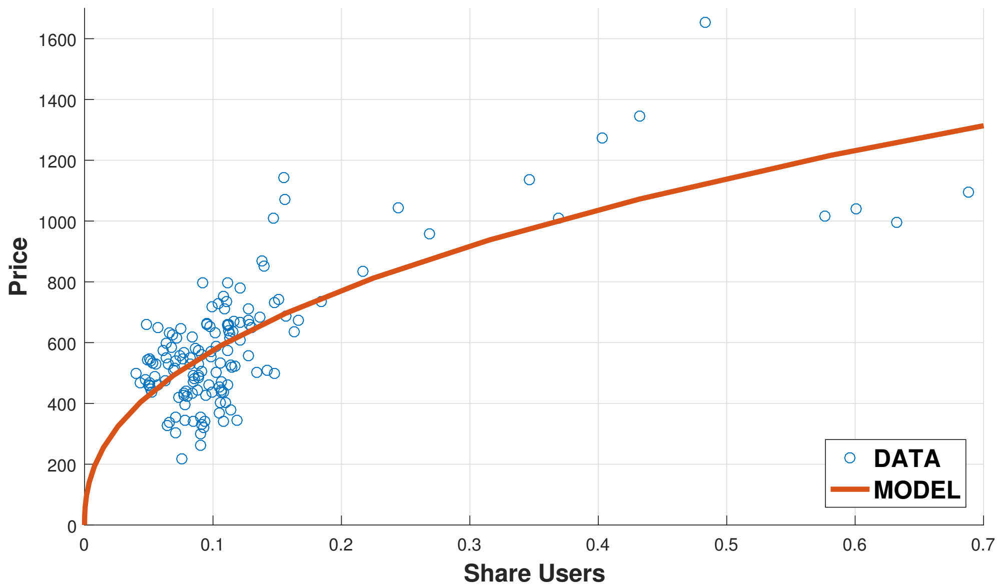

Figure 4: PRICE AS A FUNCTION OF SHARE OF TOKEN HELD BY USERS.
NOTE: DATA ARE COLLECTED FROM NYCTALE DASHBOARD. SIMULATION BASED ON PARAMETERS REPORTED IN TABLE 1.

Table 1: Parameters

Calibrated Parameters	Value	Interpretation	Moment
σ	0.62	Volatility of log-productivity	Volatility of token price
M	$5.4e-4$	Token mass	Price level of token
λ	1.06	Rate of arrival of users	Relation price-adoption
$\tilde{\mu}$	0.018	Mean of log-productivity	Relation price-adoption
α	2.48	Pareto shape parameter	Relation price-adoption
Normalized Parameters			
r	0.05	Risk-free interest rate	
η	0.50	Curvature utility function	
$\underline{\chi}$	1	Lower bound of users' ability	

4.2 Simulations

We now explore the implications of our calibrated model. The token price as a function of productivity is reported in the upper-panel of Figure 5. The blue area is the region

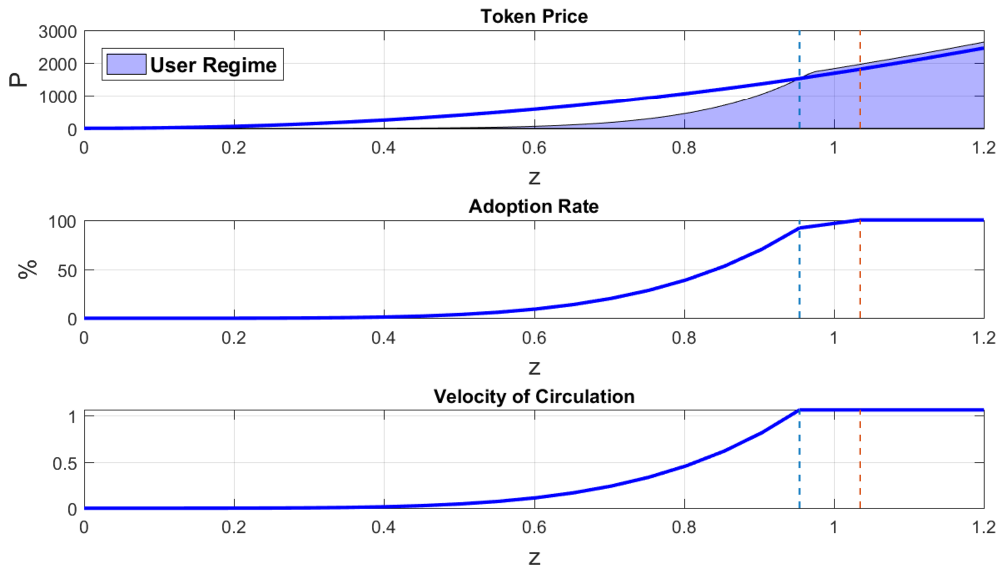

Figure 5: TOKEN PRICE, ADOPTION RATE AND VELOCITY OF CIRCULATION AS A FUNCTION OF PRODUCTIVITY z. PARAMETERS REPORTED IN TABLE 1

where $p(\overline{z}) < \overline{p}(\overline{z})$, so that we are in the user regime whenever $p(z)$ belongs to it. The equilibrium path switches from the investor to the user regime at $\underline{z}$, as indicated by the first vertical line. The second vertical line indicates the value $\overline{z}$ above which full adoption holds.

The middle-panel of Figure 5 reports the adoption rate, that is the share of potential customers who are actually using the platform. As productivity increases, more and more users migrate to the plaform. When the price enters the user regime, the adoption rate has already reached 91%. The second vertical line indicates the productivity level above which full adoption holds. This illustrates that full adoption does not have to hold in the user regime; although only a subset of potential users is using the platform, their demand is sufficient to clear the market for tokens.

The lower-panel of Figure 5 reports the average velocity at which tokens circulate. It depends negatively on the share of tokens that are hoarded by investors and thus never held for transaction purpose. Since this share shrinks over time, velocity increases and converges to λ, i.e. the rate at which users need to access the platform. This limit is reached when the market enters the user regime where all tokes are held by agents who exchange their holdings at the rate λ. Even though the adoption rate may vary in the user regime it has no effect on the velocity of circulation because tokens are simply redistributed among users who exchange them at the same rate. Hence the price and the velocity are correlated solely in the investor regime, as more and more tokens are held with the intention to access the platform's services. This shows that one cannot exogenously set the velocity of circulation in order to determine the equilibrium price because both variables are jointly determined and positively correlated. Our model provides a microfoundation for their relationship, thus supporting the popularity of velocity measures in the crypto community.

Returns dynamics. The upper-panel of Figure 6 reports the expected returns as a function of z. In the investor regime, they are by definition equal to the interest rate. But, as soon as the price enters the user regime, they start to fall to quickly converge to the growth rate $\tilde{\mu}$ of the long-run solution $\tilde{p}(z)$ defined in (21). We observe a similar but more progressive decrease for the volatility of returns depicted in the lower-panel of Figure 6. To understand why volatility is also higher in the investor regime, it is useful to look at the expression of the price function. When $p(z)$ starts in the investor regime, as it is the case in our simulation, we can explicitly derive the function that solves the law-of-motion (20b) as

$$p(z) = \overline{p}(\underline{z}) \left(\frac{z}{\underline{z}} \right)^{\beta}, \text{ for all } z \leq \underline{z} \equiv \min\{z : p(z) \leq \overline{p}(z)\}, \tag{22}$$

where

$$\beta \equiv \frac{\frac{\sigma^2}{2} - \mu + \sqrt{\left(\frac{\sigma^2}{2} - \mu\right)^2 + 2\sigma^2 (\mu + r)}}{\sigma^2} > 1.$$

It is easily verified that, since $\mu < r$, the exponent β is superior to one so that the token price in the investor regime is a *convex function* of productivity. By Ito's lemma, this convexity raises the expected returns above that of productivity,[15] providing investors with the required rate of return r. In other words, higher volatility is required to raise returns above their fundamental level so as to entice investors to hold tokens and clear the market.

Comparison with tokenless economy. We can measure the benefits of introducing tokens by comparing adoption and revenues with and without tokens. The returns of

[15] Remember that expected returns are increasing in the convexity of the price function because $\mathbb{E}\left[dp_t/dt\right] = z_t \mu p'(z_t) + (z_t\sigma)^2 p''(z_t)/2$.

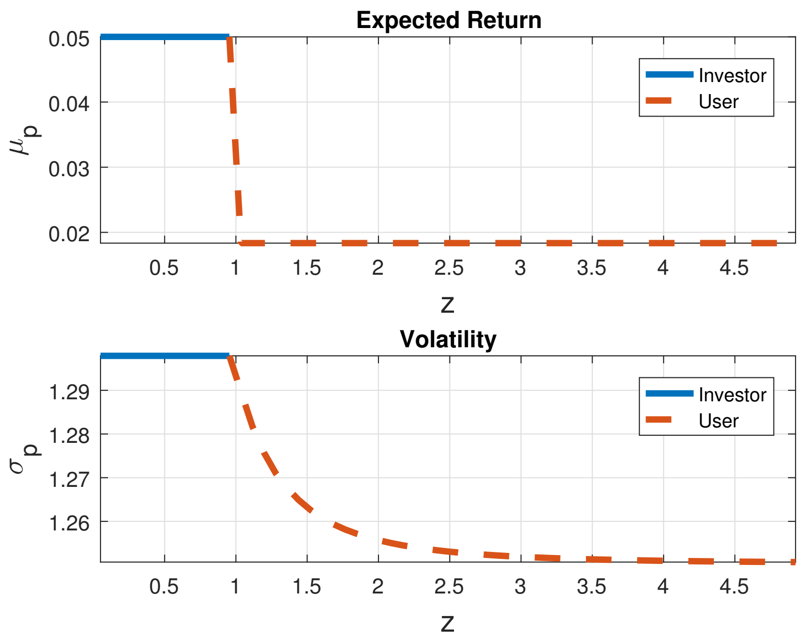

Figure 6: EXPECTED GROWTH RATE AND VOLATILITY OF RETURNS AS A FUNCTION OF PRODUCTIVITY z. PARAMETERS REPORTED IN TABLE 1

users when the platform services can be paid in fiat read

$$v\left(z_t\right) = \max_c \left\{\lambda\left[z_t u\left(c\right) - c\right] - rc\right\}. \tag{23}$$

Figure 7 show that tokenization incentivizes adoption. To understand why, note that, under H2, the token price in the user regime reads

$$p\left(z_t\right) = \left[\frac{\lambda z_t}{r + \lambda - \frac{\dot{p}(z_t)}{p(z_t)}}\right]^{\frac{1}{\eta}} \frac{1}{M}. \tag{24}$$

This equation is similar to the one prevailing in steady-state but for the inclusion of the rate of appreciation $\dot{p}(z_t)/p(z_t)$. Quite intuitively, an expected increase in price renders tokens more attractive since it partially compensates users when they do not need to access the platform in the current period.

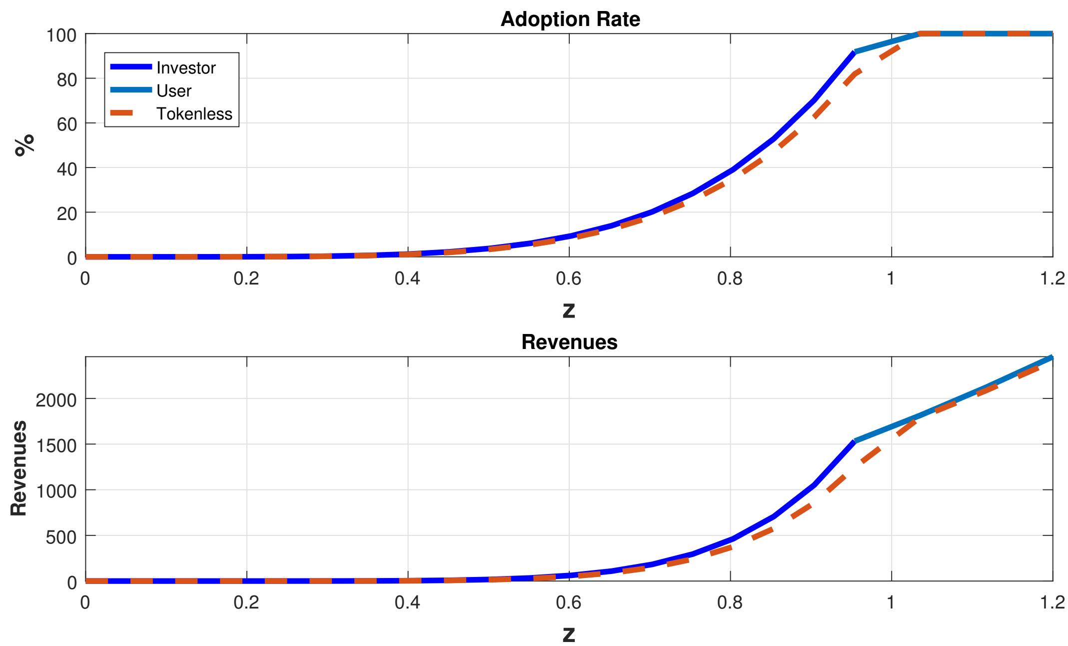

Figure 7: User Adoption and Revenues as a Function of Productivity z with and without Tokens. Parameters reported in Table 1

Tokens in advance vs. Tokens in the utility function. Before concluding, we outline how our approach complements the one proposed by Cong et al. (2018). In a nutshell, we impose a token-in-advance constraint that endows tokens with value, while Cong et al. (2018) assume that users derive a utility flow from holding tokens. Hence our two papers reflect the canonical distinction in monetary economics between models that introduce a cash-in-advance constraint and those that introduce money in the utility function.[16] It is well known that the two approaches are not as different as it may seem since they often yield consistent predictions.[17] However there are instances where the similarity breaks down, and our models is one of them.

When, as in Cong et al. (2018), tokens are staked instead of being exchanged, agents

[16] There are other noticeable differences between our models. For instance, Cong et al. (2018) consider that users have different flow utility but the same participation cost, precisely the opposite of how we model heterogeneity. We do not dwell on these differences because, although they affect the model's algebra, they can easily be accommodated in our setup without modifying its main message.

[17] For instance, Cong et al. (2018) show that their formulation holds when tokens are used as means of payment to save on transaction costs, and transactions are uncertain and lumpy.

solve the following problem

$$v(p_t, \dot{p}_t, z_t) = \max_m \{z_t u(p_t m) + \dot{p}_t m - r p_t m\}. \tag{25}$$

Given that benefits accrue at a constant rate, the utility flow of users does not depend on the frequency λ at which transactions are completed. But this is only a formal distinction since it amounts to a rescaling of the utility function.

There is, however, a more essential difference as tokens are not transferred upon the completion of each transaction, which explains why the term $-\lambda p_t m$ does not appear on the right-hand side of (25). When returns are given by (25) instead of (14), optimal holdings diverge to infinity as the expected returns $\dot{p}_t/p_t$ converge to r. The demand from users is therefore always sufficient to clear the market. Quite intuitively, since users derive a flow utility from their token holdings, they are willing to hoard any arbitrary quantity when expected returns fully compensate foregone interests. This is why there cannot be a speculative regime when token holdings directly enter the utility function.

In our model, by contrast, the demand from users does not always clear the market. It remains bounded even when there are no carry costs because users do not get a free utility flow. They still need to spend their tokens to enjoy the convenience yield. Hence, when $\dot{p}_t/p_t = r$, users hold the amount of tokens corresponding to the optimal quantity of service, that is the quantity they would have bought if the service were accessible with fiat. Then token demand is efficient as users are fully compensated for the cost of holding tokens. They set the marginal utility of the service equals to its marginal cost ($z_t u'(c_t^*) = 1$). For users demand to diverge to infinity, expected returns would have to reach $r + \lambda$. However, this cannot happen as investors steps in and bid up the price of tokens until their expected returns are equal to r. This is why the token-in-advance constraint enables us to generate a speculative regime where a fraction of the token supply is held by investors who do not enjoy any convenience yield. This prediction is in line with the current state of the market for Blockchain technologies, which has seen relatively few adoptions in spite of an ever-growing market size.

5 Conclusion

Our model provides an answer to three of the most fundamental questions regarding token pricing; namely, when are tokens valuable, how should they be priced, and what are the benefits of raising funds through token sales. To the first question, our answer is that tokens are valuable when speed is so central to the delivery of the service that users cannot delay its consumption until they have refilled their token holdings. Provided that this requirement holds, our pricing formula highlights that tokens fundamentally differ from other financial instruments because they do not generate any utility until they are exchanged. Hence, users have to be compensated for the opportunity cost of holding tokens instead of interest-bearing securities. Such compensations are perceived at the time of exchange, where users enjoy a marginal surplus, and during the holding period, where users may benefit from the appreciation of the token. Early on during the adoption stage, the financial returns are strong enough to entice speculators to hoard tokens without having the intention to use them. This speculative demand can foster adoption above the level that would prevail if the platform did not tokenize its services.

Having a microfounded pricing formula opens up many avenues for future research. Embedding network effects and more sophisticated laws of motion for the demand shifter would generate richer price dynamics. A more ambitious extension would also endogenize token supply, studying how commitment to some monetary rule could be used to maximize the expected value of the venture. These lines of investigation are only the first forays into what promises to be a field of research in its own right. Tokenomics is far from providing widely accepted guidelines for the evaluation and design of tokens. As new and more complex tokens are put on the market, the creativity of token issuers is likely to challenge that of researchers for years to come.

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A Appendix

A.1 Proofs of Claims and Propositions

A.1.1 Proof of Claim 1

Proof. Let $\hat{v}(p) \equiv v(p,p)$ denote per-period returns when the price remains constant. Setting $p_t = p_{t+1}$ in (3) yields

$$\hat{v}(p) = \max_{m \geq 0} \{\lambda z_\infty u(pm) - (r+\lambda)pm\}.$$

Setting $p_t = p_{t+1}$ in the FOC (4) and using CRRA-utility (H1), we find that

$$u'(pm^*) = \frac{u(pm^*)}{pm^*}(1-\eta) = \frac{r+\lambda}{\lambda z_\infty}.$$

Hence net returns in steady-state are equal to

$$\begin{aligned}
\hat{v}(p) &= \lambda z_\infty u(pm^*) - (r+\lambda)pm^* \\
&= \lambda z_\infty \eta u(pm^*) \\
&= \lambda z_\infty \eta u\left(u'^{-1}\left(\frac{r+\lambda}{\lambda z_\infty}\right)\right) \\
&= \frac{(\lambda z_\infty)^{1+\rho}}{\rho(r+\lambda)^\rho},
\end{aligned}$$

where $\rho \equiv (1-\eta)/\eta$. As in the general case, we notice that $\hat{v}(p)$ is independent from p. This solution can be reinserted into the first equilibrium condition (9) to obtain

$$\begin{aligned}
\hat{N} &= 1 - G\left(\frac{1}{\hat{v}(p)}\right) \\
&= \min\left\{1, \left[\underline{\chi}\hat{v}(p)\right]^\alpha\right\} \\
&= \min\left\{1, \left[\frac{\underline{\chi}(\lambda z_\infty)^{1+\rho}}{\rho(r+\lambda)^\rho}\right]^\alpha\right\},
\end{aligned}$$

where the second equality follows from H2.

Finally, the equilibrium price is pinned down by the law of motion evaluated at the rest point (10). Rewriting (10) and using H1 leads to (12)

$$\hat{N} = \hat{p}M \left[\frac{r+\lambda}{\lambda z_\infty} \right]^{\frac{1}{\eta}}.$$

■

A.2 Dynamic Programming Approach

A.2.1 Dynamic Programming Solution in Discrete Time

Let $V_t(m)$ and $W_t(m)$ denote the value function of a user with m units of token just before the first and second sub-periods, respectively. Given that the preference shock is not yet revealed when $V_t(m)$ is evaluated, the value function is by definition equal to

$$\begin{aligned} V_t(m) &= \mathbb{E}\left[\max_{c\in[0,mp_t]} \mathcal{U}\left(c,d\right) + W_t(m-c) \right] \\ &= \lambda \left[\max_{c\in[0,mp_t]} u(c) + W_t\left(m - \frac{c}{p_t}\right) \right] + (1-\lambda)W_t(m). \end{aligned} \tag{26}$$

where $\mathbb{E}\left[\cdot\right]$ is the expectation operator. The constraint $c \in [0, mp_t]$ holds because users cannot consume a quantity of services that is greater than the market value of their token holdings. The dummy variable d is equal to 0 with probability $1-\lambda$. Then the agent does not need the service and so she enters the next sub-period with the same amount of tokens, as indicated by the last term in (26). With the complementary probability λ, the dummy variable d is equal to 1 and the agent values the platform's service. To determine her optimal level of consumption, we need to characterize her continuation value W_t.

The value function $W_t(m)$ at the beginning of the second sub-period satisfies the following Bellman equation

$$W_t(m) = p_t m + \max_{m'} \left\{ -p_t m' + \beta V_{t+1}(m') \right\}, \tag{27}$$

where β is the agent's discount factor. The agent can freely rebalance her position at the market price p_t. It follows from (27) that W is linear in m as $W_t'(m) = p_t$. Moreover, the first order condition implies that $V_t'(m_t^*) = p_{t-1}/\beta$, where m_t^* is the optimal amount of tokens by the end of the second sub-period

We are now in a position to differentiate (26) with respect to c. Using the fact that $W_t'(\cdot) = p_t$, we find that optimal consumption is given by

$$c_t^* = \begin{cases} u'^{-1}(1) & \text{if } mp_t \geq u'^{-1}(1), \\ mp_t & \text{otherwise.} \end{cases}$$

Since there is no uncertainty about p_t, users will carry the minimum amount of tokens necessary for the transaction, so that $m_t^* = c_t^* \leq u'^{-1}(1)$. Setting $c = mp_t$ in (26) and differentiating the resulting equation with respect to m, we finally obtain

$$\frac{p_{t-1}}{\beta} = \lambda u'(Mp_t)p_t + (1-\lambda)p_t,$$

where we have replaced the market clearing condition $m = M$. Focussing on the steady-state $\hat{p}$, it must hold true that

$$\hat{p} = \frac{1}{M} u'^{-1}\left(\frac{1-\beta(1-\lambda)}{\beta\lambda}\right). \tag{28}$$

Replacing β with $1/(1+r)$, we recover equation (28) in the main text.

A.2.2 Dynamic Programming Solution in Continuous Time

Devising the model in continuous time alleviates the algebra. We assume that agents are hit by demand shocks that arrive at the Poisson rate λ. As before, conditional on being hit by a demand shock, agents have to buy the service immediately and so cannot go to the trading market to acquire tokens if needed. DISCUSS

Let $\Delta(c)$ denote the adjustment in token holdings as a function of consumption. Then there is only one value function which satisfies the following Bellman equation

$$\begin{aligned} rV_t(m_t) = {} & \lambda \max_{c \in [0, m_t p_t]} \{u(c) - p_t \Delta(c)\} + \lambda \max_{\Delta(c)} \left\{ V_t\left(m_t - \frac{c}{p_t} + \Delta(c)\right) - V_t(m_t) \right\} \\ & + \frac{\partial V_t(m_t)}{\partial t} + \left[V_t'(m_t) - p_t\right] \dot{m}_t. \end{aligned} \tag{29}$$

Given that the cost of marginally increasing the amount of tokens is equal to p_t, holding m_t units can be optimal only if $V_t'(m_t) = p_t$. Hence we can ignore the last term in (29). Moreover, the concavity of the value function implies that it is optimal for agents to restore their token holdings so that $\Delta^*(c) = c/p_t$. Thus the Bellman equation consistent with market clearing ($m_t = M$) boils down to

$$rV_t(M) = \lambda \max_{c \in [0, Mp_t]} \{u(c) - c\} + \frac{\partial V_t(M)}{\partial t}.$$

Setting token holdings equal to potential demand, $Mp_t = c^*$, the first order condition reads

$$rV_t'(M) = \lambda p_t \left(u'(Mp_t) - 1\right) + \frac{\partial^2 V_t(M)}{\partial t \partial m}.$$

Reinserting $V_t^{'}(M) = p_t$ into this condition, we find that

$$rp_t = \lambda p_t \left(u'(Mp_t) - 1\right) + \dot{p}_t. \tag{30}$$