

Digital Currency Runs

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Abstract

Digital currency created by the private sector, such as bitcoin, is designed to have a determined supply and enable payments with the premise of competing with and supplanting central bank fiat money and the banking system. Central banks are developing fiat public digital currency and banks are innovating in response. This paper shows that private digital currency may be preferred over fiat money in countries with high inflation, but using it outside of the banking system reduces investment. Banks can re-emerge by taking deposits and lending in private digital currency to increase investment while avoiding inflationary fiat money, but these banks risk having withdrawal runs into the digital currency. A globally used private digital currency acts similar to a traditional hard currency within developing countries by eliminating fiat inflation but exacerbating bank fragility. A regionally used altcoin is superior to a global digital currency and other hard currencies by limiting inflation while alleviating macroeconomic shocks and bank runs.

Keywords: Digital currency, cryptocurrency, central bank digital currency, fiat money, bank runs, inflation, investment

JEL Classification: G21, G01, E42, E58

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1 Introduction

The rapid development of digital currency has renewed traditional questions about whether money creation should be handled by the private sector or public sector. Two main motivations for privately created digital currency, such as bitcoin and Facebook's Libra, are to act as a replacement for fiat money, which can be inflationary, and banks, which provide payments, as emphasized by Raskin and Yermack (2018). They also highlight that with central banks worldwide developing public digital currency, there are widespread expectations that private or public digital currency will eventually replace fractional reserve banking.² However, the role of digital currency for supporting lending and investment in the economy without banks has been little considered. Concerns about digital currency creating fragility in the financial system have also been little formulated or studied.³

This paper studies the potential competition of privately created digital currency, as well as publicly created central bank digital currency (CBDC), to the traditional roles of fiat money and the banking system. While cryptocurrency has a variety of features, two key elements are considered here. It is typically created with an ultimate fixed supply to avoid inflation, and it provides a means of payment without banks. Two recent developments for digital currency are analyzed. Banks are looking to innovate, and new forms of banks that take deposits and lend in digital currency are emerging.⁴ A number of altcoins, which refer to private digital currencies created since bitcoin, are being created for use in specific national and regional economies.

The model features consumers with an endowment, firms that need to borrow to invest, and banks that can intermediate payments and loans using fiat money created by the central bank. With optimal fiat inflation, banks can provide maturity and risk transformation to implement optimal investment, economic output, and risk sharing for consumers. However, excessive inflation reduces investment and economic output.

Private digital currency that is not inflationary can be created to allow for pay-

²For example, see Vigna and Casey (2015).

³Posner (2015a,b) is one of the few to point out that if bitcoin, for example, were to become widely adopted, fractional reserve banking denominated in bitcoin would be a natural outcome because of the value creation that banks provide, but that financial crises in a bitcoin-based banking system would also occur. See also Winkler (2015) and Nelson (2017).

⁴See Bech and Rodney (2017), Bordo and Levin (2017), Broadbent (2016), Cawrey (2014), Jack and Suri (2011), Popper (2015), and Rosenfeld (2015).

ments and lending by consumers directly to firms without using fiat money or banks, but this lending does not provide for maturity transformation and risk sharing. Banks that take deposits and lend in private digital currency may provide for greater investment and risk sharing, with private digital currency held as a form of private reserves to enable standard fractional reserve banking. Public digital currency also allows for payments and lending without banks, but again with a lower amount of investment and risk sharing. Public digital currency incurs the same inflation as standard fiat money, and hence consumers prefer to deposit it at banks rather than hold or lend it directly.

Banks face the threat of inefficient runs when lending and investment returns are low. Central bank fiat inflation prevents such insolvency-based runs for banks with deposits of fiat money and public digital currency but not private digital currency. Banks with private digital currency deposits have runs into the digital currency precisely because it is not inflationary and can be stored and used for payments outside of the banking system. There is a trade-off for using private digital currency to avoid fiat inflation. If consumers hold it or lend it to firms directly, it does not permit as much credit to firms for investment as banks can provide. If instead it is held in the form of bank deposits, it is subject to fragility in the form of digital currency runs.

In addition, if a globally used private digital currency were to be used to avoid high fiat inflation in a developing country, the global digital currency acts as a traditional hard currency that prevents inflation but also exacerbates insolvency-based bank runs and leads to liquidity-based runs. A regionally used altcoin is superior to a global digital currency and other hard currencies by preventing inflation while having a partially flexible value to buffer banks against the region's local macroeconomic asset and liquidity risk.

An economy with a monetary system based on a private digital currency instead of central bank fiat money is a viable possibility, as argued by Raskin and Yermack (2018). Bitcoin has had increasing use at times in several countries with high inflation problems including Venezuela, Iran, Argentina, Ukraine, Zimbabwe, and other countries.⁵ Regional altcoins have been created with the goal of being used in national and regional economies including Russia/CIS, Poland, Philippines, Greece, Spain, Iceland, Scotland, and Catalonia.

⁵For example, see Raskin (2012) and Urban (2017).

Indeed, the Federal Reserve was originally created for the primary purpose of being able to provide an “elastic supply of currency” in order to help banks and the economy weather aggregate liquidity and recessionary risks. But, as with other central banks in more extreme circumstances, the Fed’s discretion over the money supply has often come under pressure following episodes of high inflation. The earliest call for a privately created digital currency to constrain the money supply is likely by Milton Friedman. In 1999, Friedman foresaw and welcomed the opportunities for an internet-based digital currency to be supplied inelastically without discretion, as described by Raskin and Yermack (2018).

Raskin and Yermack (2018) also argue that either public or private digital currency will ultimately displace the banking system. However, fractional reserve banking based on paying a return on deposits and making loans denominated in bitcoin is emerging. Mastercard has recently won patents, and is applying for additional ones, for methods and systems for a fractional reserve digital currency bank.⁶ Several platforms already provide bitcoin savings accounts that pay interest generated by returns from lending bitcoin for leveraged trading.⁷

Traditional banks have also begun issuing their own digital currency for payments, deposits and withdrawals, including by JP Morgan, a consortium of Japanese banks, and a consortium of HSBC with Barclays, UBS, and Santander. A wider bitcoin-tied financial system is also developing with corporate bonds denominated in bitcoin issued by Japan’s largest financial services provider, Fisco, and bitcoin derivatives including futures, forward contracts and swaps developed by the CME, Goldman Sachs, Morgan Stanley, and other financial institutions. In addition, empirical evidence demonstrates that despite the ability for the growing fintech economy to operate outside of financial intermediation, banking in effect reemerges.⁸

In order to focus on the basic premise of private digital currency created with a fixed supply to prevent the type of inflation that central banks have the discretion to permit, I use a simple model of fiat inflation and private digital currency. The

⁶See Madore (2018), Suberg (2018), and U.S. Patent and Trademark Office (2018).

⁷See See Kharif (2019), Lielacher (2017), Tomasicchio (2016), Cruz (2015), Johnson (2015), and for a list of such platforms, <https://cryptalker.com/bitcoin-interest> and <https://cryptalker.com/bitcoin-savings>.

⁸Balyuk and Davydenko (2018) show that fintech platforms designed for direct peer-to-peer lending are evolving toward becoming essentially online intermediaries in the form of banks that take investment from passive lenders and make active investment decisions for lending to borrowers.

central bank uses a simple form of monetary policy to maximize welfare but may have a preference for shorter term than longer term economic output, which leads to excessive inflation, and which captures the basic time-inconsistency problem of monetary policy. Private digital currency can be created with a fixed supply and is a simple technology that allows for all agents to store and make payments with it.

I also make the simplifying assumption of no transactions costs for payments in the economy and financial system made using either digital currency or bank deposits.⁹ In practice, private digital currency utilizes a decentralized distributed ledger with blockchain technology and requires a protocol to achieve consensus for payments transactions in such a ‘trustless’ environment.¹⁰ Most public digital currency payments under current consideration would likely utilize a ‘trusted’ centralized central bank ledger.¹¹ With developments in methods for private digital currency payments to support consensus for transactions in a more cost effective manner, such as with proof-of-stake rather than proof-of-work protocols,¹² or with second-layer protocols such as the Lightning Network to increase scalability,¹³ private digital currency has the potential to be used as an efficient means of digital payments similar to or even more advanced than electronic payments that are cleared and settled within the banking

⁹Payments Canada et al. (2018) demonstrate the potential for widespread banking payments without reliance on a central bank that would be required for banks to take private digital currency deposits. They describe the development and testing in Canada for efficiently settling large-scale wholesale interbank payments with distributed ledger technology. A “notary node” consensus model shows promise for settlement finality, which is required but not achieved with a proof-of-work protocol. Parlour et al. (2017) show that fintech innovation in the bank payment system can reduce banks’ need for intermediate liquidity in the interbank market, which results in an increase in bank lending and productive efficiency.

¹⁰Kroll et al. (2013) examine bitcoin as a consensus game using costly computational mining as proof-of-work for transaction consensus, and which also requires a separate governing consensus for the rules of the bitcoin protocol. Biais et al. (2018a) show that bitcoin transaction consensus using the mining proof-of-work protocol is a Markov perfect equilibrium but that consensus over the protocol is a coordination game with multiple equilibria. Cong et al. (2018) examine methods for moderating the natural concentration of mining pools, and Easley et al. (2017) explain market-based transaction fees charged in addition to mining rewards.

¹¹Raskin and Yermack (2018) describe how central bank digital currency (CBDC) would enable households to hold such public digital currency directly in accounts at the central bank instead of in deposit accounts at commercial banks.

¹²Saleh (2018a,b) shows that protocols such as proof-of-stake or proof-of-burn can overcome the large computing resources costs required for proof-of-work consensus protocols, such as for bitcoin, which Parham (2017) demonstrates are prohibitive on a large scale.

¹³Poon and Dryja (2016) describe how the Lightning Network, which has reached increasing success in recent small-value tests, acts as a decentralized network off of the bitcoin blockchain for micropayments in bitcoin, with net payments then transacted on the bitcoin blockchain.

system.

Digital currency has been recently studied, along with blockchain technology utilized with distributed decentralized ledgers more broadly, in the rapidly growing finance and economics literature on fintech.¹⁴ Current papers on digital currency, banking, and central bank policy highlight several potential benefits and costs of private and public digital currency. These papers focus on private digital currency competing against monopolist central bank money,¹⁵ public digital currency competing against bank deposits,¹⁶ and competition among private digital currencies,¹⁷ but they do not examine financial stability concerns.

In order to focus on the risks of digital currency runs, I shut down other channels affecting digital currency as money that have been studied. For example, the potential for private digital currency to be widely adopted as money is viewed in part as an economic coordination problem. Bitcoin and other private digital currencies have displayed extreme price volatility, which limits their acceptance and use. However, several studies argue that the increasing acceptance and use of private digital currency will lead to a more stable value, further supporting its use.¹⁸ Several papers also

¹⁴Raskin and Yermack (2018) highlight that debates over private digital currency as competition to fiat money is demanding a resurgence in classical monetary economic theory based on von Mises (1912), Hayek (1976) and Mundell (1998).

¹⁵Abadi and Brunnermeier (2018) show that because of free entry and distributed ledger fork competition, private digital currencies do not produce profits for the issuer or miners but provide competition that only partially constrains central bank profits arising through monopoly power as a centralized intermediary of fiat money and payments. Schilling and Uhlig (2018) show that for a central bank with commitment to maintain the real value of fiat money, there is exchange-rate indeterminacy for the price of private digital currency.

¹⁶Andolfatto (2018) finds that interest-bearing central bank digital currency constrains the profit but does not disintermediate monopolistic banks and may even lead to their expansion by providing competition for banks to increase deposit rates. In contrast, Keister and Sanches (2018) find that central bank digital currency increases exchange efficiency in a search economy but crowds out investment by banks that rely on real deposits.

¹⁷Fernandez-Villaverde and Sanches (2017a,b) find that competition among private digital currencies may implement efficient allocations in a search economy with productive capital but otherwise require unconventional methods for central bank monetary policy.

¹⁸Bolt and van Oordt (2016) show how the price volatility of private digital currency is driven by speculators but decreases as it becomes more widely adopted by consumers and accepted by merchants. Cong et al. (2018) explain the volatility of private digital currency based on the feedback-loop dynamics of it being adopted for transactions. Li and Mann (2018) point to initial coin offerings (ICOs) for investment in private digital currency platforms that can solve the adoption coordination problem. Sockin and Xiong (2018) show that the price and volume of private digital currency transactions act as coordination devices that determine whether there is high, low, or no transactions with the digital currency. Kim (2015) uses empirical evidence based on pre-blockchain based virtual currencies to argue that bitcoin volatility driven by speculators will significantly decrease over time.

tie the extreme volatility of bitcoin to the proof-of-work protocol,¹⁹ which may be fundamentally overcome through alternative protocols, as demonstrated by Saleh (2018b).

Additionally, whether private digital currency can displace central bank fiat money is in part a political and technological question. While bitcoin gained early notoriety for use in black markets, recent evidence demonstrates that such illicit use is diminishing because of its public blockchain transaction history, which allows authorities to track down illegal users.²⁰ Meanwhile, bitcoin is increasingly being used for legitimate transactions.²¹ Several less developed countries have struggled between the extremes of officially supporting the adoption of bitcoin and banning its use.²² However, the development of broader applications of blockchain technology may become so widespread and ubiquitous in the financial system and economy that platforms embedded with digital currency may require its use.²³

The model builds on the theory of nominal bank contracts with fiat money as developed by Allen et al. (2014), Skeie (2004, 2008), and Allen and Skeie (2018). They show how nominal bank contracts with fiat central bank money, and without consideration of a short-term central bank bias, can provide depositors with optimal consumption and financial stability against liquidity and asset risk.²⁴

The model also builds on the provision of liquidity provided by banks that enable runs (Diamond and Dybvig, 1983)²⁵ and relates to the theory of banking liquidity

¹⁹Biais et al. (2018b) provide an OLG model and empirical evidence that costly mining determines bitcoin's fundamental value based on the net present value of transactional benefits but also drives large volatility. Pagnotta and Buraschi (2018) show that mining costs being paid in bitcoin amplifies the impact of supply and demand shocks on its price volatility.

²⁰See Jawaheri et al. (2018), Meiklejohn (2016), and Bohannon (2016).

²¹See Tasca et al. (2018).

²²See Hosain (2018).

²³Applications of blockchain include more efficient smart financial contracting (Cong and He, 2018), managing trading transparency in financial markets to increase investor welfare (Malinova and Park, 2017), and the market for, and regulation of, financial reporting and auditing (Cao et al., 2018).

²⁴Conditions for bank runs and contagion with nominal bank contracts are shown by Skeie (2004) as arising from interbank market liquidity freezes, and by Diamond and Rajan (2006) and Champ et al. (1996) as arising to due to withdrawals of currency out of the banking system based on consumer purchases of goods that must be made with traditional paper currency. Diamond and Rajan (2006) further show that nominal contracts do not protect from bank runs caused by heterogeneous shocks in asset returns.

²⁵Bank liquidity creation and fragility is further developed based on the insensitivity to information of deposits (Gorton and Pennachi, 1990, and Dang et al., 2013), contracts relative to markets (Allen

and fragility in the context of interbank lending;²⁶ the role of lending money between banks, central bank lending and injections of money, and demand deposits paid in money in models of real bank deposits;²⁷ liquidity runs and bank insolvency tied to bank lending contracts;²⁸ systemic risk triggered from idiosyncratic bank losses;²⁹ and interbank payments and lending operating through clearinghouse systems for transferring and settling payments between banks.³⁰

The paper proceeds in Section 2 with the model of the real economy, banking, fiat money, and digital currency. Section 3 presents the baseline equilibrium with only fiat reserves, followed by the analysis of private and public digital currency in section 4. In section 5, digital currency runs are examined. Section 5 shows that digital currency runs are ameliorated with use of a regional altcoin rather than a global private digital currency that acts as a traditional hard currency, and Section 7 concludes. An extension to strategic-based digital currency runs and proofs are contained in the appendix.

2 Model

I first present the model of the real economy and then introduce money and nominal contracts, followed by the details of the model with the agent optimizations and equilibrium specification.

and Gale, 2004), global games informational structures (Goldstein and Pauzner, 2005), central bank interest rate policy (Freixas et al., 2011), efficient risk management (DeAngelo and Stulz, 2015), bailout policy (Shapiro and Skeie, 2015), and central bank balance sheet policy (Martin et al., 2016, 2018). Dynamic models of bank runs include Brunnermeier and Samikov (2016), Gertler and Kiyotaki (2015), Martin et al. (2014a, 2014b), Brunnermeier and Oehmke (2013), and He and Xiong (2012).

²⁶Bhattacharya and Gale (1987), Bhattacharya and Fulghieri (1994), Allen and Gale (2000a), Diamond and Rajan (2005), Acharya and Skeie (2011), and Ashcraft et al. (2011).

²⁷Allen and Gale (1998), Gale and Vives (2002), Freixas et al. (2000, 2003), Freixas and Holthausen (2001), Rochet and Vives (2004), and Chang and Velasco (2000).

²⁸Holmström and Tirole (1998) and Diamond and Rajan (2005).

²⁹Rochet and Tirole (1996) and Aghion et al. (2000).

³⁰Furfine (1999), Henckel (1999), Flannery (1996) and Hancock and Wilcox (1996).

2.1 Real economy

The real economy is set up with consumers that have an endowment of goods but require firms for investment and storage.

There are three periods $t = 0, 1, 2$. Consumers have a unit mass, are ex-ante identical with goods at $t = 0$, and live for one or two periods. A random fraction λ of the consumers realize they are ‘early’ types with a need to consume at $t = 1$, while the remaining fraction $1 - \lambda$ are ‘late’ types who can consume at $t = 1$ and $t = 2$.

Firms have no endowment but have technologies to store and invest goods. Investments at $t = 0$ have a random return r_2 at $t = 2$, or can alternatively be liquidated for a salvage return of $r_1 \in (0, 1)$ at $t = 1$.

The random variables λ and r_2 have a joint distribution with support in the interval $[0, 1] \times (r_1, r_2^{\max})$, have means $\bar{\lambda} \in (0, 1)$ and $\bar{r}_2 > 1$, respectively, and are realized at $t = 1$ as the macro state (λ, r_2) .

2.2 Money and contracts

The model is developed to allow for a parsimonious representation of outside money in the form of traditional fiat reserves, public digital currency, and private digital currency.

Money At $t = 0$, fiat money M^s is held by banks and private digital currency M^v is held by a private issuer. Any agent can hold and make transactions with private digital currency. The central bank can limit its fiat money to only be directly held by and transacted among banks, in which case it is referred to as fiat bank reserves. The central bank can also allow its fiat money to be directly held by and transacted among all agents, in which case it is referred to as public digital currency.

P_0^ι , $P_1^\iota(\lambda, r_2)$ and $P_2^\iota(\lambda, r_2)$ are the prices of goods at periods $t = 0, 1, 2$, respectively, in terms of currency $\iota \in \{s, v\}$ as numeraire, where $\iota = s$ denotes fiat (i.e., sovereign) money and $\iota = v$ denotes private digital currency (e.g., bitcoin). Inflation at periods 1 and 2 for each currency $\iota \in \{s, v\}$ is:

$$\begin{aligned}\Pi_1^\iota(\lambda, r_2) &\equiv \frac{P_1^\iota(\lambda, r_2)}{P_0^\iota} \\ \Pi_2^\iota(\lambda, r_2) &\equiv \frac{P_2^\iota(\lambda, r_2)}{P_1^\iota(\lambda, r_2)}.\end{aligned}$$

Interpretation The interpretation of fiat money and digital currency is that the central bank and private issuer each have a technology to create a mutually distinguishable currency. Each currency is a technology which agents can use to make verifiable payment transactions without double-spending. The distinction is that the central bank technology allows it to create new quantities of fiat money at any period and date. The private issuer technology only allows the issuer to create a single fixed quantity of private digital currency at the initial period and date.

The distinction between the types of fiat money is made simply to distinguish an economy with public digital currency in which banks are not required for payments. Whereas, without digital currency, bank payment systems rather than paper currency, for example, are typically required in practice for holding and making transactions in fiat money on the scale of economic and financial transactions in the economy.

In order to focus on the potential role for digital currency to compete with bank payments as an efficient means of payment, all transaction payments using bank deposits and digital currency occur costlessly and simultaneously within a period. This precludes the channel for digital currency to have a positive value purely from a direct means-of-payment liquidity premium. A liquidity premium value for an outside money would be equal to the present value of future payment liquidity services for non-instantaneous transactions, as in Bias et al. (2018b). More efficient payments imply a lower liquidity premium value for outside money. With the simplification of assuming instant bank and digital currency payments, there is no liquidity premium value.

Deposits and loans At $t = 0$, consumers sell their goods for fiat money or private digital currency to deposit at banks, directly lend to firms, or directly hold. A firm can finance buying goods to invest and store by borrowing either L_0^c from a consumer, or $L_0^{f^\iota}$ from a bank taking consumer deposits D^ι , in either currency $\iota \in \{s, v\}$. At period $t = 1$, the firm rolls over $L_1^c(\lambda, r_2)$ or $L_1^{f^\iota}(\lambda, r_2)$ of its borrowing. A firm can only borrow from either a single bank or consumer at $t = 0$ and can only borrow from that lender at $t = 1$ within the date.

$R_t^{k\iota}(\lambda, r_2)$ is the return paid at period $t \in \{1, 2\}$, in the currency $\iota \in \{s, v\}$ borrowed, on a loan or deposit indexed by $k \in \{c, f, d\}$, which corresponds to consumer loans, bank loans, and deposits, respectively. The macro state (λ, r_2) is publicly ob-

servable at period $t = 1$, when it is realized, but the state is not verifiable. Since depositor types and the macro state (λ, r_2) are not verifiable, there are incomplete contracts for deposits and loans made at $t = 0$. $\hat{R}_t^{k\iota}$ denotes a contracted return, which is not type or state-contingent but may be subject to a non-strategic default by the borrowing bank or firm. The fraction $\delta_t^k(\lambda, r_2) \leq 1$ is paid of the contracted return $\hat{R}_t^{k\iota}$, where $R_t^{k\iota}(\cdot) \equiv \delta_t^k(\cdot)\hat{R}_t^{k\iota}$.

Hence, $\delta_t^k(\cdot) < 1$ signifies a default, which requires the borrower to pay all revenues possible to maximize δ_t^k . Hence, strategic defaults are not permitted, and borrowing must be repaid in the currency $\iota \in \{s, v\}$ borrowed.³¹ Returns $\hat{R}_2^{f\iota}(\lambda, r_2) = R_2^{f\iota}(\lambda, r_2)$ and $\hat{R}_2^{c\iota}(\lambda, r_2) = R_2^{c\iota}(\lambda, r_2)$ for loans made at $t = 1$, as well as period $t = 1$ and $t = 2$ quantities and prices, are state contingent.

Monetary policy The central bank pays the monetary policy rate $R_t^s(\lambda, r_2)$ on fiat money to optimize the expected utility from investment and economic output, with a policy discount factor β^{cb} . For convenience of language, the term ‘rate’ is used interchangeably with ‘return’ throughout the paper to refer to a gross rate of return, except when a net rate of return is clearly intended.

Dates In order to permit money to have a continuation value after $t = 2$, the main model with three periods $t = 0, 1, 2$ occurs within individual, non-overlapping dates τ that repeat indefinitely: $\tau = 0, 1, \dots, \infty$. The initial supplies of money M^s and M^v are created only once at date $\tau = 0$. Fiat money and digital currency can be stored across periods and dates, while goods are perishable after period $t = 2$ of each date. Within each date, a new generation of consumers endowed with goods is born at $t = 0$ and live only within that date. All other agents are infinitely-lived.

³¹For example, a deposit of D^c has a contracted return $\hat{R}_t^{d\iota}$. If the bank defaults on withdrawals at $t = 1$, with $R_t^{d\iota} < \hat{R}_t^{d\iota}$ hence $\delta_1^d < 1$, the bank has to pay out all of its revenues at $t = 1$ for withdrawals, which implies that the bank cannot rollover any lending to its firms, $L_1^{f\iota} = 0$. Hence, the bank will not have any revenues and has a complete default at $t = 2$, $\delta_2^d = 0$. Such a bank is referred to as liquidated at $t = 1$, since it has no loan assets after $t = 1$. The bank’s firms cannot rollover any of their loans at $t = 1$. If these firms default at $t = 1$, $\delta_1^f < 1$, the firms must fully liquidate their assets, $a_1 = a_0$, to sell goods and repay as much of their loans at $t = 1$ as possible. For simplicity, I assume that in case of a bank default $\delta_t^d < 1$ at period $t \in \{1, 2\}$, there is a pro rata sharing rule among deposits withdrawn at that period. Results do not change if there were instead any type of priority rule, such as is there were a sequential service constraint at $t = 1$ in which some deposit withdrawals incurred no default, $\delta_1^d = 1$, and the remaining withdrawals incurred a complete default, $\delta_1^d = 0$.

Notation Throughout the paper, uppercase letters denote nominal variables, and lowercase letters denote real variables. For simplicity of notation, the state (λ, r_2) is typically suppressed in the writing of state-contingent variables after they are first introduced, except where the state dependence is included for particular clarity or emphasis. In addition, the subscript for the generic date τ is typically omitted from all variables, except where the date is included to refer to a particular non-generic date or to provide extra clarity when examining variables across different dates.

2.3 Optimizations

Consumers At period $t = 0$, a consumer sells e^ι of her unit of endowment goods, $e \equiv 1$, for currency $\iota \in \{s, v\}$ to deposit (D^ι), directly lend to firms ($L_0^{c\iota}$), and directly hold as digital ($M_0^{c\iota}$):

$$D^\iota + L_0^{c\iota} + M_0^{c\iota} \leq e^\iota P_0^\iota \text{ for } \iota \in \{s, v\} \quad (1)$$

At $t = 1$, an early consumer buys goods using proceeds in currency $\iota \in \{s, v\}$:

$$c_1 = \sum_{\iota \in \{s, v\}} \frac{1}{P_1^\iota} (D^\iota R_1^{d\iota} + L_0^{c\iota} \delta_1^{e\iota} \hat{R}_1^{c\iota} + M_0^{c\iota}). \quad (2)$$

A late consumer withdraws a fraction $w^\iota \in [0, 1]$ of her deposit early, rolls over $L_1^{c\iota}$ of her loan, and holds $M_1^{c\iota}$ digital until $t = 2$. She buys goods and consumes c_1^ℓ from remaining proceeds at $t = 1$ and c_2 from proceeds at $t = 2$:

$$c_1^\ell = \sum_{\iota \in \{s, v\}} \frac{1}{P_1^\iota} \left[\overbrace{w^\iota D^\iota R_1^{d\iota}}^{\text{withdrawal at } t=1} + \underbrace{(L_0^{c\iota} \delta_1^{e\iota} \hat{R}_1^{c\iota} - L_1^{c\iota})}_{\text{amount of loan not rolled over at } t=1} + M_0^{c\iota} - M_1^{c\iota} \right] \quad (3)$$

$$c_2 = \sum_{\iota \in \{s, v\}} \frac{1}{P_2^\iota} [(1 - w^\iota) D^\iota R_2^{d\iota} + L_1^{c\iota} R_2^{c\iota} + M_1^{c\iota} R_2^\iota], \quad (4)$$

where R_2^ι is the policy return R_2^s on fiat money and is $R_2^v \equiv 1$ otherwise.

Consumers have a standard utility function $u(\cdot)$ that is twice continuously differentiable, strictly concave, and satisfies Inada conditions $u'(0) = \infty$ and $u'(\infty) = 0$. Late consumers have a discount factor β^ℓ on $t = 2$ consumption, which for simplicity

is set to one, $\beta^\ell = 1$. The consumer optimization is:

$$\left. \begin{aligned} \max_{Q^c} EU^c &= E[\lambda u(c_1) + (1 - \lambda)u(c_1^\ell + \beta^\ell c_2)] \\ \text{s.t.: } \quad t=0: & \text{ Eq (1), budget constraint} \\ & \sum_{\iota \in \{s,v\}} e^\iota \leq e \equiv 1, \text{ feasibility constraint} \end{aligned} \right\} \quad (5)$$

where choice variables are $Q^c \equiv \{e^\iota, D^\iota, L_0^{c\iota}, L_1^{c\iota}(\lambda, r_2), M_0^{c\iota}, M_1^{c\iota}(\lambda, r_2), w^\iota(\lambda, r_2)\}_\iota$. Note that strategic behavior among late consumers is not considered here but is analyzed in Appendix 8.

Banks For currency $\iota \in \{s, v\}$, a bank lends to firms ($L_0^{f\iota}$) and holds reserves ($M_0^{b\iota}$) at $t = 0$ out of its new deposits (D^ι) and existing reserves ($M_{2,\tau-1}^{b\iota}$) held over from $t = 2$ of the previous date $\tau - 1$. At the initial date $\tau = 0$, the bank's existing reserves are $M_{2,-1}^{bs} \equiv M^s$ and $M_{2,-1}^{bv} \equiv 0$. The bank rolls over loans ($L_1^{f\iota}$) and holds reserves ($M_1^{b\iota}$) after paying deposit withdrawals at $t = 1$, and pays remaining withdrawals at $t = 2$.

Because of free entry, the bank maximizes its depositors' expected utility from consumption bought using returns on their deposits:

$$\left. \begin{aligned} \max_{Q^b} EU^b &= E \left[\lambda u \left(\frac{D^\iota R_1^{d\iota}}{P_1^\iota} \right) + (1 - \lambda) u \left(\frac{w^\iota D^\iota R_1^{d\iota}}{P_1^\iota} + \beta^\ell \frac{(1-w^\iota)D^\iota R_2^{d\iota}}{P_2^\iota} \right) \right] \\ \text{s.t.: } \quad t=0: & \quad L_0^{f\iota} + M_0^{b\iota} \leq D^\iota + M_{2,\tau-1}^{b\iota} \quad \forall \iota \\ t=1: & \quad [\lambda + w^\iota(1 - \lambda)]D^\iota R_1^{d\iota} \leq (L_0^{f\iota} R_1^{f\iota} - L_1^{f\iota}) + M_0^{b\iota} - M_1^{b\iota} \quad \forall \iota \\ t=2: & \quad (1 - w^\iota)(1 - \lambda)D^\iota R_2^{d\iota} \leq L_1^{f\iota} R_2^{f\iota} + M_1^{b\iota} R_2^\iota - M_2^{b\iota} \quad \forall \iota, \end{aligned} \right\} \quad (6)$$

where $Q^b \equiv \{D^\iota, L_0^{f\iota}, L_1^{f\iota}(\cdot), M_0^{b\iota}, M_t^{b\iota}(\cdot), \delta_t^d(\cdot)\}_{t \in \{1,2\}, \iota}$, and the three inequalities are the bank's budget constraints.

Firms At each date τ , a firm borrows in a currency $\iota \in \{s, v\}$ at $t = 0$ to buy q_0^ι goods, of which it invests a_0 and stores g_0 . At $t = 1$, the firm repays its borrowing that is not rolled over ($L_0^{k\iota} R_1^{k\iota} - L_1^{k\iota}$) by selling q_1^ι goods. The firm liquidates $a_1(\lambda, r_2)$ of its investment if required, or else stores $g_1(\lambda, r_2)$ any remaining goods. At $t = 2$, the firm sells q_2^ι goods to repay its rolled over borrowing, $L_1^{k\iota} R_2^{k\iota}$, and consumes any remaining goods c_2^f as profit.

The investment program is specified by $a_0, g_0, a_1(\cdot), g_1(\cdot)$, where the assets a_0 and storage g_0 at $t = 0$ is referred to as the initial investment, and the ongoing assets

$a_0 - a_1(\lambda, r_2)$ not liquidated and ongoing storage $g_1(\lambda, r_2)$ at $t = 1$ is referred to as the continuing investment.

The firm maximizes its expected profit over all dates as follows:

$$\left. \begin{aligned} & \max_{Q^f} E[\sum_{\tau=0}^{\infty} c_{2,\tau}^f] \\ \text{s.t.:} \quad & t=0: \quad q_0^l P_0^l \leq L_0^{kl} \quad a_0 + g_0 \leq q_0^l \\ & t=1: \quad L_0^{kl} R_1^{kl} - L_1^{kl} \leq q_1^l P_1^l \quad q_1^l \leq g_0 + a_1 r_1 - g_1, \quad a_1 \leq a_0 \\ & t=2: \quad L_1^{kl} R_2^{kl} \leq q_2^l P_2^l \quad q_2^l \leq (a_0 - a_1) r_2 + g_1 - c_2^f, \end{aligned} \right\} \quad (7)$$

which includes the firm's budget and feasibility constraint for each period $t = 0, 1, 2$ within date τ , and where $Q^f \equiv \{g_0, a_0, q_0^l, g_1(\cdot), a_1(\cdot), q_1^l(\cdot), L_0^{kl}, L_1^{kl}(\cdot), \delta_1^k(\cdot)\}_{t \in \{1,2\}, k \in \{c,b\}, l, \tau}$. While in principle firms could hold digital currency, in equilibrium they would not and so is not considered.

Private issuer At each date τ , the issuer uses its private digital $M_{2,\tau-1}^{yv}$ held from the previous date to buy q_0^y goods at $t = 0$ in order to consume c_2^f goods as profit at $t = 2$, where $M_{2,-1}^{yv} \equiv M^v$ at date $\tau = 0$. The issuer has the technology to store goods, g_t^y at $t = 1, 2$, but not to invest goods. The issuer can also sell q_t^y goods at $t = 1, 2$ and hold M_t^{yv} private digital at $t = 0, 1, 2$. The issuer's maximization of expected profit over all dates is:

$$\left. \begin{aligned} & \max_{Q^y} E[\sum_{\tau=0}^{\infty} c_{2,\tau}^y] \\ \text{s.t.:} \quad & t=0: \quad g_0^y P_0^v + M_0^{yv} \leq M_{2,\tau-1}^{yv} \quad q \\ & t=1: \quad M_1^{yv} \leq q_1^y P_1^v + M_0^{yv} \quad q_1^y \leq g_0^y - g_1^y \\ & t=2: \quad M_2^{yv} \leq q_2^y P_2^v + M_1^{yv} \quad q_2^y \leq g_1^y - c_2^y, \end{aligned} \right\} \quad (8)$$

which includes the issuer's budget and feasibility constraint for each period $t = 0, 1, 2$ within date τ , and where $Q^y \equiv \{g_0^y, g_1^y(\cdot), q_0^y, q_t^y(\cdot), M_0^{yv}, M_t^{yv}(\cdot)\}_{t \in \{1,2\}, \tau}$.

Central bank The central bank sets the monetary policy rate $R_2^s(\lambda, r_2)$ to affect investment based on the expected utility from short-term output at $t = 1$, which is consumed as c_1 and c_1^ℓ , relative to longer-term output at $t = 2$, which is discounted according to the policy factor β^{cb} and consumed as c_2 :

$$\max_{R_2^s(\cdot)} EU^{cb} = E[\lambda u(c_1) + (1 - \lambda)u(c_1^\ell + \beta^{cb} c_2)]. \quad (9)$$

The policy rate is set at $t = 1$, based on the realized macro state (λ, r_2) , and determines nominal lending rates at $t = 1$, which in turn can influence fiat inflation and real lending rates because of the nominal rigidity of deposit rates and the partial irreversibility of firm investment set at $t = 0$. For simplicity, and without loss of generality, the policy rate is paid only at $t = 2$ on fiat money agents hold over from $t = 1$.

The central bank has more of a focus on short-term output than late consumers do if $\beta^{cb} < \beta^\ell$, and a more long-term focus if $\beta^{cb} > \beta^\ell$.

Equilibrium

Definition 1 *An equilibrium consists of prices and returns*

$$\{P_{0,\tau}^\iota, \{P_{t,\tau}^\iota(\lambda_\tau, r_{2,\tau}), \hat{R}_{t,\tau}^{d\iota}\}_{t \in \{1,2\}}, \{\hat{R}_{1,\tau}^{k\iota}, \hat{R}_{2,\tau}^{k\iota}(\lambda_\tau, r_{2,\tau})\}_{k \in \{c,f\}}, R_{2,\tau}^s(\lambda_\tau, r_{2,\tau})\}_{\iota, \tau},$$

and quantities $\{Q_\tau^c\}_\tau, \{Q_\tau^b\}_\tau, Q^f, Q^y$, such that the following conditions are satisfied.

The quantities for consumers $\{Q_\tau^c\}_\tau$, banks $\{Q_\tau^b\}_\tau$, firms Q^f and the private issuer Q^y satisfy their optimizations given by (5), (6), (7) and (8) respectively.

The returns $\{R_{2,\tau}^s(\cdot)\}_\tau$ satisfy the central bank optimization given by (9).

For both currencies $\iota \in \{s, v\}$ and all dates τ , markets clear for:

- (a) deposits: D_τ^ι at $t = 0$;
- (b) loans to firms: $\{L_{0,\tau}^{k\iota}\}_{k \in \{c,f\}}, L_{0,\tau}^c$ at $t = 0$ and $\{L_{1,\tau}^{k\iota}(\lambda_\tau, r_{2,\tau})\}_{k \in \{c,f\}}$ at $t = 1$;
- (d) private digital currency: $\sum_{\kappa \in \{c,b,y\}} M_{t,\tau}^{k\iota v}(\lambda_\tau, r_{2,\tau}) = M^v$ at $t \in \{0, 1, 2\}$;
- (e) fiat money: $\sum_{\kappa \in \{c,b\}} M_{t,\tau}^{k\iota s}(\lambda_\tau, r_{2,\tau}) = \prod_{\hat{\tau}=0}^{\tau-1} M^s R_{2,\hat{\tau}}^s(\cdot) R_{t,\tau}^s(\cdot)$ at $t \in \{0, 1, 2\}$;
- (f) goods at $t \in \{0, 1, 2\}$:
 - $t = 0$: $\sum_{j \in \{s,v,y\}} q_{0,\tau}^j = \sum_{\iota \in \{s,v\}} e^\iota$,
 - $t = 1$: $\lambda_\tau c_{1,\tau}(\lambda_\tau, r_{2,\tau}) + (1 - \lambda_\tau) c_{1,\tau}^\ell(\lambda_\tau, r_{2,\tau}) = \sum_{j \in \{s,v,y\}} q_{1,\tau}^j(\lambda_\tau, r_{2,\tau})$,
 - $t = 2$: $(1 - \lambda_\tau) c_{2,\tau}(\lambda_\tau, r_{2,\tau}) = \sum_{j \in \{s,v,y\}} q_{2,\tau}^j(\lambda_\tau, r_{2,\tau})$;

where $\{M_{0,\tau}^{k\iota v}\}_{\kappa \in \{c,b,y\}}, \{M_{0,\tau}^{k\iota s}\}_{\kappa \in \{c,b\}}$, and $\{R_{t,\tau}^s\}_{t \in \{0,1\}} \equiv 1$ are not state-contingent on $(\lambda_\tau, r_{2,\tau})$.

3 Fiat money

The proceeding analysis first examines the impact on investment of digital relative to fiat, without the potential threat of bank runs, under the restriction of no early

withdrawals by late consumers: $w^t \equiv 0$. This restriction is then relaxed in section 5 to analyze the threat of digital currency runs on investment.

3.1 Optimal allocation

The full-information optimal allocation in the real economy at date τ consists of initial investment at $t = 0$, a_0^*, g_0^* , and continuing investment at $t = 1$, $a_0^* - a_1^*(\lambda, r_2)$ and $g_1^*(\lambda, r_2)$, that determines output consumed at $t = 1$, $c_1^*(\cdot) + c_1^{\ell*}(\cdot)$, and output discounted for any factor $\beta \leq 1$ consumed at $t = 2$, $c_2^*(\cdot)$, for all states (λ, r_2) , according to the optimization:

$$\left. \begin{aligned} \max_{g_0, a_0, g_1(\cdot), a_1(\cdot)} EU(\beta) &= E[\lambda u(c_1) + (1 - \lambda) u(c_1^\ell + \beta c_2)] \\ \text{s.t.: } t=0: & a_0 + g_0 \leq 1 \\ t=1: & \lambda c_1 \leq g_0 + a_1 r_1 - g_1 \quad \forall(\lambda, r_2) \\ t=2: & (1 - \lambda)(c_1^\ell + c_2) \leq (a_0 - a_1)r_2 + g_1 \quad \forall(\lambda, r_2), \end{aligned} \right\} \quad (10)$$

where the three constraints are feasibility conditions at periods $t = 0, 1, 2$, respectively.

The first-order conditions and binding constraints for the optimization problem result in:

$$E[u'(c_1^*)] = E[r_2 u'(\beta c_2^*)] \quad (11)$$

$$c_1^* = \frac{g_0^* - g_1^* + a_1^* r_1}{\lambda} \quad (12)$$

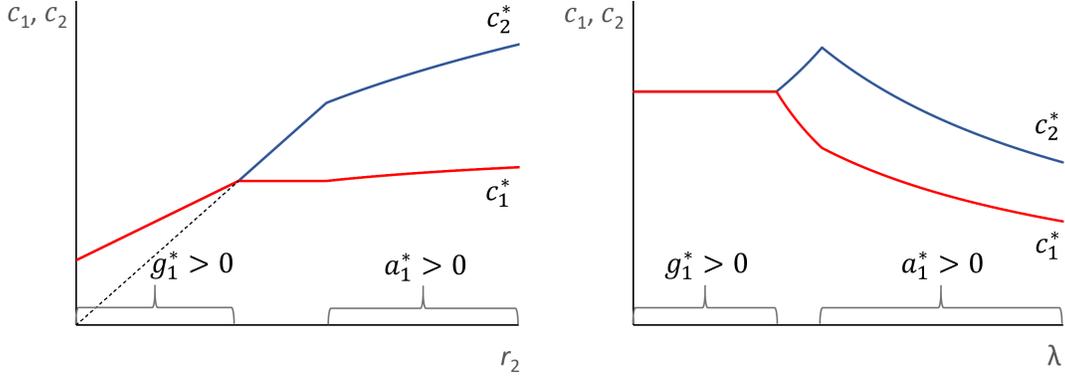
$$c_2^* = \frac{(a_0^* - a_1^*)r_2 + g_1^*}{1 - \lambda} \quad (13)$$

$$c_1^{\ell*} = 0. \quad (14)$$

Equation 11 is an Euler equation showing that initial investment, a_0^*, g_0^* , is chosen at $t = 0$ such that in expectation, the marginal utility of output at $t = 1$ consumed by early types is equal to that for discounted output at $t = 2$ consumed by late types at the marginal rate of transformation r_2 , which gives optimal risk-sharing among consumers over the aggregate liquidity and asset risk of the macro state (λ, r_2) , and for consumers' idiosyncratic liquidity risk of being an early consumer.

At $t = 1$, continuing investment has positive storage $g_1^* = (1 - \lambda)g_0^* - \lambda a_0^* r_2 > 0$ and no liquidation ($a_1^* = 0$) when λ and r_2 are relatively low, written as $\lambda < \check{\lambda}(r_2) \equiv \frac{g_0^*}{g_0^* + a_0^* r_2}$, which gives equal consumption $c_2^* = c_1^*$. There is no storage ($g_1^* = 0$) and

Figure 1: Optimal Consumption
 For a constant realization of λ For a constant realization of r_2



positive liquidation $a_1^* > 0$ when λ, r_2 are relatively high, written as $\lambda > \hat{\lambda}(r_2)$, with $\hat{\lambda}(r_2)$ and a_1^* implicitly defined by $\frac{u'(\check{c}_1)}{u'(\check{c}_2)} = \frac{r_2}{r_1}$ for $\check{c}_1 \equiv \frac{g_0^*}{\lambda}$, $\check{c}_2 \equiv \frac{a_0^* r_2}{1-\lambda}$. There is no storage or liquidation, $g_1^* = a_1^* = 0$, for relatively moderate λ, r_2 , written as $\lambda \in [\check{\lambda}(r_2), \hat{\lambda}(r_2)]$.

Lemma 1 *The optimal amount of storage at $t = 1$ is positive, $g_1^* > 0$, when there are relatively few early consumers and low asset returns. Conversely, the optimal amount of asset liquidation at $t = 1$ is positive, $a_1^* > 0$, when there are relatively many early consumers and high asset returns.*

A comparison of optimal consumption for early and late consumers, and the ex-interim investment policy represented by a_1^* and g_1^* , is illustrated in the two diagrams in Figure 1. For a low realization of r_2 or λ , there is an initial amount of storage $g_1^* > 0$ at $t = 1$ to provide equal consumption $c_2^* = c_1^*$. As r_2 increases for a constant realization of λ (shown in the left diagram) and as λ increases for a constant realization of r_2 (shown in the right diagram): the storage amount decreases to zero ($g_1^* = 0$) and there is also no liquidation ($a_1^* = 0$), and then eventually there is liquidation ($a_1^* > 0$) and in increasing amounts to provide a partial transfer of late consumers' increasing consumption to early consumers.

3.2 Fiat reserves equilibrium

I initially analyze the baseline economy without digital currency, where only banks hold fiat money as fiat reserves. In equilibrium, the central bank policy rate determines state-contingent fiat inflation to buffer the economy against aggregate liquidity

and asset risk, which with bank lending implements the optimal real allocation of investment and output consumed for any discount factor β^{cb} on $t = 2$ output. If the central bank has a shorter-term focus than late consumers, $\beta^{cb} < \beta^\ell$, there is higher fiat inflation, and lower long-term investment and output, than preferred by late consumers, which provides the rationale for private digital currency analyzed in the next section.

Banks lend without consumers holding digital currency, $M_t^{cu} \equiv 0$ for $t = 0, 1$. Consumers deposit proceeds from selling their unit of goods at $t = 0$, $D^s = P_0^s$, and withdraw to buy goods at $t = 1, 2$ at equilibrium prices

$$P_1^s(\lambda, r_2) = \frac{\lambda + w^\ell(1-\lambda)D_0^s R_1^{ds}(\lambda, r_2)}{q_1^s(\lambda, r_2)} \quad (15)$$

$$P_2^s(\lambda, r_2) = \frac{(1-w^\ell)(1-\lambda)D_0^s R_2^{ds}(\lambda, r_2)}{q_2^s(\lambda, r_2)}. \quad (16)$$

Prices for each period reflect the amount of money withdrawn and spent by consumers in the numerator divided by the amount of goods sold by firms in the denominator.

Investment Banks lend $L_0^{f\ell}$ and rollover $L_1^{f\ell}$ loans at $t = 0, 1$ for returns $R_1^{f\ell}$ and $R_2^{f\ell}$ at $t = 1, 2$, respectively. Investment by firms is analyzed first at $t = 1$ and then at $t = 0$.

At $t = 1$, all uncertainty is resolved, and the real rate on loans to firms is defined according to a simple application of the Fisher equation as $r_2^{fs}(\lambda, r_2) \equiv \frac{R_2^{fs}(\lambda, r_2)}{\Pi_2^s(\lambda, r_2)}$.

First order conditions for the firm's optimization require that if there is positive storage at $t = 1$, $g_1 > 0$, then the equilibrium real rate must equal one and there is no liquidation, $r_2^{fs} = 1$ and $a_1 = 0$. Conversely, if there is liquidation, $a_1 > 0$, then the real rate r_2^{fs} must equal $\frac{r_2}{r_1}$, which is the marginal rate of transformation for the illiquid asset not being liquidated (r_2) relative to being liquidated (r_1), and hence there is no storage at $t = 1$, $g_1 = 0$. Hence, if there is neither storage nor liquidation at $t = 1$, $g_1 = a_1 = 0$, the real rate must be relatively moderate $r_2^{fs} \in (1, \frac{r_2}{r_1})$.

As a result, continuing investment in assets ($a_0 - a_1$) and goods g_1 follows from the equilibrium real rate:

$$r_2^{fs} = \begin{cases} 1 & \Rightarrow g_1 \geq 0, a_1 = 0 \\ \in (1, \frac{r_2}{r_1}) & \Rightarrow g_1 = a_1 = 0 \\ \frac{r_2}{r_1} & \Rightarrow g_1 = 0, a_1 \geq 0. \end{cases}$$

In particular, for a moderate realization of (λ, r_2) given by $\lambda \in [\check{\lambda}(r_2), \hat{\lambda}(r_2)]$, the equilibrium price levels at $t = 1$ and $t = 2$ are moderate, with $P_1^s = \frac{\lambda D_0^s R_1^{ds}}{g_0^*}$ and $P_2^s = \frac{(1-\lambda)D_0^s R_2^{ds}}{a_0^* r_2}$, and the real rate is the optimal $r_2^{fs*} \in [1, \frac{r_2}{r_1}]$. Firms sell at $t = 1$ all of their goods stored from $t = 0$ and sell at $t = 2$ the returns on all their assets.

For a low realization of (λ, r_2) given by $\lambda < \check{\lambda}(r_2)$, there is downward pressure on P_1^s and upward pressure on P_2^s , with optimal real rate $r_2^{fs*} = 1$. With fewer early consumers, the amount of money spent for goods is reduced at $t = 1$ and increased at $t = 2$. With lower returns, fewer goods produced by assets are available to sell at $t = 2$. Firms respond to these market prices by storing the optimal amount g_1^* of their goods at $t = 1$ to sell at $t = 2$, which provides for equal consumption among early consumers withdrawing at $t = 1$ and late depositors withdrawing at $t = 2$.

Conversely, for a high realization of (λ, r_2) given by $\lambda > \hat{\lambda}(r_2)$, there is relative upward pressure on P_1^s and downward pressure on P_2^s , with optimal real rate $r_2^{fs*} = \frac{r_2}{r_1}$. Firms respond by liquidating the optimal amount a_1^* of their assets to sell additional goods at $t = 1$, which results in the marginal rate of substitution between early and late consumers equal to the marginal rate of transformation between asset returns and liquidation returns: $\frac{u'(c_1^*)}{u'(c_2^*)} = \frac{r_2}{r_1}$.

At $t = 0$, firms choose their initial investment a_0, g_0 , according to the Euler equation that results from the firm's optimization, $E[r_2^{fs*}(\cdot)] = \bar{r}_2$.

Fiat bank lending The central bank chooses its state-contingent policy rate $R_2^s(\beta^{cb}, \lambda, r_2)$ based on its discount factor $\beta^{cb} \leq 1$, which, through bank lending, firm investment, and inflation, implements in decentralized equilibrium the optimal allocation in the real economy for $\beta = \beta^{cb}$.

The first order conditions for the bank's optimization require that the return on bank loans to firms is equal to the policy rate paid on holding fiat reserves, with $R_1^{fs} = R_1^s = 1$ and $R_2^{fs}(\lambda, r_2) = R_2^s(\lambda, r_2)$. Bank lending and firm investment determine the optimal state-contingent inflation $\Pi_1^{s*}(\lambda, r_2)$ and $\Pi_2^{s*}(\lambda, r_2)$. The market provides the optimal rationing of goods between early and late consumers through the optimal quantity of goods sold by firms at each period, $q_1^{s*} = \frac{g_0^* + a_1^* r_1 - g_1^*}{\lambda} = \frac{c_1^*}{\lambda}$ and $q_2^{s*} = \frac{(a_0^* - a_1^*)r_2 + g_1^*}{1-\lambda} = \frac{c_2^*}{1-\lambda}$. Firms have zero consumption $c_2^f = 0$ for all states (λ, r_2) .

Optimal allocation for $\beta = \beta^{cb}$ First consider the case of $\beta^{cb} = \beta^\ell$, in which the central bank does not have a short-term bias.

Proposition 1 *If the central bank does not have short-term bias, $\beta^{cb} = 1$, the central bank sets the optimal policy rate R_2^{s*} such that with optimal inflation Π_t^{s*} , banks provide sufficient credit for optimal initial and continuing investment by firms, a_t^* and g_t^* , which results in the optimal consumption c_t^* , with no defaults by banks or firms, $\delta_t^d = \delta_t^f = 1$, for periods $t = 1$ and $t = 2$ and for all realizations of the macro state (λ, r_2) .*

Since deposits pay out nominal amounts, the bank can pay fixed promises in terms of money as numeraire with no bank defaults, yet depositors' real consumption can adjust through optimal inflation. The real return per unit on deposits provides consumption contingent on the aggregate state (λ, r_2) for early and late types, $c_1 = \frac{D_0^s R_1^{ds}}{P_1^s} = \frac{q_1^s}{\lambda}$ and $c_2 = \frac{D_0^s R_2^{ds}}{P_2^s} = \frac{q_2^s}{1-\lambda}$, respectively.

Corollary 1 *Expected inflation equals the expected optimal policy rate: $E[\Pi_1^{s*}(\lambda, r_2)] = 1$ at period 1 and $E[\Pi_2^{s*}(\lambda, r_2)] = E[R_2^{s*}(\lambda, r_2)]$ at period 2*

The following corollary shows comparative statistics for how equilibrium variables respond to changes in the central bank's discount factor, where the comparative statics for equilibrium values of the real allocation are equal to those for the optimal allocation with $\beta = \beta^{cb}$.

Corollary 2 *A lower discount factor β^{cb} increases the nominal policy and lending rates $R_2^{fs} = R_2^s$, real lending rate r_2^{fs} , and inflation Π_2^s , which lowers initial a_0 and continuing $(a_0 - a_1)$ investment, resulting in higher output q_1^s and consumption c_1 at $t = 1$ at the expense of lower output q_2^s and consumption c_2 at $t = 2$.*

Allocation relative to $\beta^\ell < \beta^{cb}$ If the central bank has a bias for higher short-term consumption and output that arises from a lower discount factor $\beta^{cb} < 1$ than consumers' discount factor at period $t = 2$, which has been implicitly set equal to one. The central bank short-term bias can take two different forms, which are analyzed in turn. One form is that the central bank's bias comes as a surprise to the public at period $t = 1$, after initial investment decisions are made at period $t = 0$. The second form is that the central bank's bias is known by the public at period $t = 0$.

For the first form of bias, suppose the public expects at $t = 0$ that the central bank has a discount factor $\beta^{cb} = 1$ and will set the optimal policy rate at R^{s*} , and firms choose a_0^* as their asset investment. At $t = 1$, the central bank will unexpectedly set a higher nominal rate $\acute{R}^s > R_2^{s*}$ to maximize consumers' expected utility with the lower discount factor $\beta^{cb} < 1$, with a real central bank rate of $r_2^s \equiv \frac{R_2^s}{\Pi_2^s}$. For a state (λ, r_2) with $g_1^*(\lambda, r_2) = 0$, increasing \acute{R}_2^s above R_2^{s*} implies that firms excessively liquidate assets at $t = 1$, where $\acute{a}_1(\acute{R}_2^s) > a_1^*(R_2^{s*})$ and $\acute{r}_2^{fs} = \acute{r}_2^s = \frac{r_2}{r_1}$.

For the second form of central bank bias, the public knows the central bank's discount factor $\beta^{cb} < 1$. Rather than a greater amount of output at $t = 1$ through excessive asset liquidation at $t = 1$, firms in anticipation instead store excessive goods and hold lower investment at $t = 0$ than the first best: $\acute{a}_0 < a_0^*$.

Corollary 3 *For either an unexpected or expected central bank short-term bias of $\beta^{cb} < 1$, there is excessive inflation at $t = 2$ of $\acute{\Pi}_2^s > \Pi_2^{s*}$ through the central bank setting $\acute{R}_2^s > R_2^{s*}$ at $t = 1$, which increases output at $t = 1$ to $\acute{q}_1^s > q_1^{s*}$ and decreases output at $t = 2$ to $\acute{q}_2^s < q_2^{s*}$. There are no bank or firm defaults, with $\delta_1^d = \delta_2^d = \delta_1^f = 1$. However, early consumers receive higher consumption than optimal, $\acute{c}_1 \geq c_1^*$ with $E[\acute{c}_1] > E[c_1^*]$, and late consumers receive lower consumption than optimal, $\acute{c}_2 \leq c_2^*$ with $E[\acute{c}_2] < E[c_2^*]$.*

I proceed by assuming that the central bank's discount factor $\beta^{cb} \leq 1$ is known by the public at $t = 0$, such that distortionary fiat inflation is fully anticipated when the central bank has a short-term bias with $\beta^{cb} < 1$.

4 Digital currency

In this section, I first consider the potential use of private digital currency and then public digital currency.

4.1 Private digital currency

Private digital currency is not inflationary or affected by fiat inflation, and it has appeal to consumers when the central bank has a more short-term focus than late consumers. I first demonstrate how private digital currency can be utilized in the

economy, and then analyze its impact on investment and consumption based on consumers directly holding it, lending it to firms, or depositing it at banks.

At period $t = 0$ of any date τ , the private issuer uses its private digital currency $M_{2,\tau-1}^{yv}$ brought into that date to buy goods g_0^y and hold M_0^{yv} until period $t = 1$, where at the initial date $\tau = 0$, $M_{2,\tau-1}^{yv} \equiv M^v$ is the private digital currency the issuer originally creates. The private issuer's budget constraint at period $t = 0$ of date τ is $g_0^y P_0^v + M_0^{yv} \leq M_{2,\tau-1}^{yv}$.

Consumers can sell e^v of their endowment for $e^v P_0^v$ private digital currency and store the amount M_0^{cv} , lend the amount L_0^{cv} directly to a firm, and deposit D^v , where $M_0^{cv} + L_0^{cv} + D^v \leq e^v P_0^v$. Banks can take deposits D^v to store M_0^{bv} as private reserves and make loans L_0^{fv} to firms using the private digital currency. Firms use loans from consumers or banks to buy q_0^v goods. Market clearing for the goods market using private digital currency at $t = 0$ is represented by $e^v P_0^v = (q_0^v + q_0^y) P_0^v$.

At periods $t = 1$ and $t = 2$, firms sell q_t^v of their output goods for private digital currency to repay loans, and the private issuer can sell q_t^y of its goods for private digital currency. Market clearing at $t = 1$ and $t = 2$ for the goods market using private digital currency is represented by $c_t^v P_t^v = (q_t^v + q_t^y) P_t^v$, where c_t^v denotes consumption from goods that consumers buy using private digital currency.

There is no inflation in the private digital currency price level of goods within a date or across dates, with $\Pi_{1,\tau}^v \Pi_{2,\tau}^v \leq 1$. This result arises as a requirement for agents to be willing to hold private digital currency and obtains because the supply of private digital currency has a fixed quantity. Hence, the price level has an effective cap at the end of each date, with $P_{2,\tau}^v \leq P_{0,\tau}^v$.

Lemma 2 *There is no inflation for private digital currency within or across dates: $\Pi_{1,\tau}^v \Pi_{2,\tau}^v \leq 1$ and $\frac{P_{0,\tau+1}^v}{P_{0,\tau}^v} = 1$ for all τ .*

The exchange rate value of private digital currency in terms of fiat money at period t of date τ is defined as the fiat price of goods relative to the private currency price of goods, $X_{t,\tau} \equiv \frac{P_{t,\tau}^s}{P_{t,\tau}^v}$. Hence, $X_{t,\tau}$ is the quantity of fiat money per unit of private digital currency that can be obtained by using a unit of private currency to buy goods that are sold for fiat money, or equivalently could be obtained in a direct exchange rate trade of private currency for fiat, and which in a vice-versa manner would provide for exchanging fiat for private money. Since private digital currency

maintains a constant real value across dates, the exchange rate for the fiat value of private digital currency, $X_{t,\tau}(\lambda, r_2)$, increases by $R_{2,\tau}^s(\lambda, r_2)$ in expectation across dates: $E[\frac{X_{0,\tau+1}}{X_{0,\tau}}] = E[R_{2,\tau}^s]$ for all dates τ .

Private digital currency deposits In the case of banks with fiat deposits, optimal investment and risk-sharing for consumption when $\beta^{cb} = 1$ is implemented through a high optimal level of inflation to decrease the real cost of firm and bank liabilities when investment returns r_2 are low or consumption demands by the fraction $1 - \lambda$ of late consumers are high at $t = 2$.

While private digital currency does not permit such inflation, private digital currency that is deposited at banks implements optimal investment and consumption risk-sharing instead through effectively negative nominal interest rates on loans to firms and on bank deposits for such macro states in which high optimal inflation occurs in the fiat case. Negative rates on deposits take the form of a partial bank default on withdrawals at $t = 2$. Despite the default, the informal incentive constraint for late consumers holds, $c_2(\cdot) \geq c_1(\cdot)$. However, the current initial assumption that late consumers cannot withdraw at $t = 1$ is critical, as the ability for late consumers to withdraw and store digital currency at $t = 1$ will be shown as cause for digital currency runs in section 5.

The return to a consumer from using private digital currency can be measured by her consumption per unit of private digital currency that she receives, for endowment sold, and that she deposits, stores, or lends.

For a deposit of $D^v = 1$, the consumer receives a withdrawal of R_t^{dv} private digital currency to buy goods for consumption of $c_t = \frac{R_t^{dv}}{P_t^v}$ at either $t = 1$ or $t = 2$. The bank lends L_t^{fv} and holds M_t^{bv} as private digital currency reserves at periods $t = 0$ and $t = 1$.

At periods $t = 1$ and $t = 2$, consumers withdraw deposits and buy goods from firms at equilibrium prices equivalent to the expressions for the fiat case in equations (15) and (16) but with the superscripts s for fiat replaced by v for private digital currency.

The return on loans must equal one for the bank to make loans while also holding private digital currency at period $t \in \{0, 1\}$. At $t = 0$, the bank holds private digital currency as reserves that, depending on the state (λ, r_2) , are paid out for withdrawals

at $t = 1$ and/or at $t = 2$.

Investment is determined by the first order conditions for the bank and firm optimizations that give the Euler equation, $E[u'(c_1^*)] = E[r_2 u'(c_2^*)]$, and real rate on loans at periods 1 and 2, $r_1^{fv}(\cdot) = \frac{1}{\Pi_1^v(\cdot)}$ and $r_2^{fv}(\cdot) \equiv \frac{R_2^{fv}(\cdot)}{\Pi_2^v(\cdot)}$, which are equivalent to the real state-contingent loan rates in the optimal fiat case with $\beta^{cb} = 1$, and thus results in optimal firm initial investment a_0^* at $t = 0$ and continued investment represented by $g_1^*(\cdot)$ and $a_1^*(\cdot)$ according to $r_2^{fv}(\cdot) = r_2^{fs}(\cdot) \in [1, \frac{r_2}{r_1}]$.

Deflation When there are low investment returns r_2 and high consumption demands with a large fraction $1 - \lambda$ of late consumers at $t = 2$, which are jointly expressed by $\lambda < \check{\lambda}(r_2)$, the optimum requires the storage of goods $g_1^* > 0$ from $t = 1$ to $t = 2$ to provide late consumers with consumption equal to that of early consumers, $c_2(\cdot) = c_1(\cdot)$, which represents the optimal macro risk sharing between early and late consumers when r_2 and λ are low, and which requires that the real lending return equals one ($r_2^{fv} = 1$) for firms to store goods ($g_1 > 0$) at $t = 1$ in the decentralized implementation.

However, with the cap on inflation under private digital currency, there is deflation $\Pi_2^v = \frac{P_2^v}{P_1^v} < 1$ in these states with $\lambda < \check{\lambda}(r_2)$. Yet, the optimal real return of $r_2^{fv} = 1$ is achieved through a negative nominal net interest rate on loans rolled over at $t = 1$, where the nominal return on loans of less than one can be seen in terms of the Fisher equation as $R_2^{fv} = r_2^{fv} \Pi_2^v = \Pi_2^v < 1$.

In addition, a partial default is required on deposit withdrawals at $t = 2$, with $\delta_2^d(\lambda, r_2) < 1$, in these states with low r_2 and λ . However, as long as $r_2 \geq 1$, such that there is no actual loss on investment, the absolute return on deposit withdrawals is greater at $t = 2$ than at $t = 1$: $R_2^{dv} = \delta_2^d \hat{R}_2^{dv} \geq R_1^{dv} = 1$.

But when there is an investment loss, $r_2 < 1$, as well as a very high fraction $1 - \lambda$ of late consumers with consumption demand at $t = 2$, expressed as $\lambda \leq \check{\lambda}^v(r_2 < 1)$ for a cutoff $\check{\lambda}^v(r_2) \in (0, \check{\lambda}(r_2))$, the cap on inflation at $t = 2$ requires not only a partial bank default of $\delta_2^d(\lambda, r_2) < 1$, but an absolute default of $R_2^{dv} = \delta_2^d(\lambda, r_2) \hat{R}_2^{dv} < 1$ on late consumer deposit withdrawals. This is termed an absolute default to reflect that relative to deposit withdrawals of $R_1^{dv} = 1$ at $t = 1$, the implicit net interest rate on deposits between $t = 1$ and $t = 2$ is negative, since $\frac{\delta_2^d(\lambda, r_2) \hat{R}_2^{dv}}{R_1^{dv}} < 1$, and this negative rate is known by late consumers at $t = 1$ since there is no uncertainty once the macro

state (λ, r_2) is realized and fully observable at $t = 1$.

Regardless, the informal incentive constraint for late consumers, $c_2(\cdot) \geq c_1(\cdot)$, does continue to hold. It holds as a strict inequality when $\lambda(r_2) > \check{\lambda}(r_2)$ and as an equality when $\lambda(r_2) \leq \check{\lambda}(r_2)$, as is equivalent for the first best allocation.

In particular, for the states with $r_2 < 1$ and $\lambda \leq \check{\lambda}^v(r_2 < 1)$ such that the absolute default on $t = 2$ deposits occurs, firms optimally store goods from $t = 1$ to $t = 2$. Early and late consumers have equal consumption, $c_2(\cdot) = c_1(\cdot) < 1$, where $c_2^* = \frac{R_2^{dv}}{P_2^v}$ with $R_2^{dv} = \delta_2^d \hat{R}_2^{dv} < 1$ and $c_1^* = \frac{R_1^{dv}}{P_1^v}$ with $R_1^{dv} = \delta_1^d \hat{R}_1^{dv} = 1$. Consumption has to be less than one, as is the case for the optimal allocation, since with $r_2 < 1$, the total output per unit of endowment is $\frac{g_0 + a_0 r_2}{e} < \frac{g_0 + a_0}{e} = 1$.

In equilibrium, late consumers are indifferent between withdrawing and buying goods at $t = 1$ relative to withdrawing and buying goods at $t = 2$.

As with all states (λ, r_2) in which $c_1(\cdot) = c_2(\cdot)$, late consumers are indifferent between withdrawing and buying goods at $t = 1$ relative to withdrawing and buying goods at $t = 2$, since $c_1^\ell = c_1^* = c_2$. Early withdrawals by late consumers buying goods at $t = 1$ would have no real effect. The bank would reduce its rollover of loans to firms by the amount of the early withdrawals, firms would store fewer goods from $t = 1$ to $t = 2$ by the amount of consumption by late consumers at $t = 1$, and there would continue to be no defaults at $t = 1$ or any change to consumption $c_1(\cdot) = c_2(\cdot)$.

As a comparison, when $r_2 < 1$ and $\lambda(r_2) < \check{\lambda}(r_2)$ in the case of fiat deposits, there is sufficiently high fiat inflation through a high enough price level P_2^s to lower real consumption for late consumers to equal that of early consumers without any bank default at $t = 2$.

Proposition 2 *For banks with private digital currency deposits, there is optimal investment and output at each period for all macro states (λ, r_2) . Consumption for early and late consumers has the optimal risk sharing, and the informal incentive constraint $c_2(\cdot) \geq c_1(\cdot)$ holds for late consumers, for all states (λ, r_2) .*

Despite the optimal investment and output, and the optimal risk sharing among consumers for the aggregate state risk (λ, r_2) and idiosyncratic liquidity risk for early consumers, in some of the states (λ, r_2) consumers do not receive all of the output for their own consumption, but rather the private digital currency issuer shares in a part of the output with a positive amount of consumption. Consumption by the

private issuer does not entail any inefficiency, but making a welfare evaluation between consumption by consumers and the private issuer is not considered here since this case does not lend itself to a straightforward social welfare function to evaluate, whereas welfare in consideration of just consumers can be based on the ex-ante expected utility of an individual representative consumer.

Holding private digital currency Rather than the consumer depositing private digital currency, holding private digital currency at period $t = 0$ gives a nominal return of one for both an early consumer and late consumer for all states (λ, r_2) . This return is equal to the deposit return for an early consumer, $\hat{R}_1^{dv} = 1$, but is lower than the expected deposit return in both risk-neutral and risk-adjusted term for a late consumer. Likewise, expected consumption from storing rather than depositing private digital currency is the same for an early consumer but is lower in both risk-neutral and risk-adjusted terms for a late consumer, which reflects that storing private digital currency does not provide credit in the form of any type of lending to firms to enable investment.

Direct lending private digital currency If a consumer directly lends private digital currency to a firm, the long term amount of credit to firms is still lower than as provided by banks. Direct lending does not provide the maturity and risk transformation of banks. Early consumers do not rollover loans at $t = 1$, which requires firms to liquidate more at $t=1$. This results in inefficient underinvestment by firms in terms of both continued investement at $t = 1$ and the amount of initial investment at $t = 0$. Both early and late consumers have lower consumption through direct lending than depositing private digital currency.

A consumer loan to a firm at $t = 0$ has a return of $R_1^{cv} \geq 1$ at $t = 1$ and can be rolled over at $t = 1$ for a return of R_2^{cv} at $t = 2$. The firm uses the direct loan to buy $q_0^c = e^v = L_0^{cv} = 1$ goods to store and invest at $t = 0$, $g_0^c + a_0^c = q_0^c$, and sells its output of goods to repay its loan at $t = 1$ and $t = 2$.

A late consumer rolls over the full amount of the loan, and the firm does not default. The firm repays $L_0^{cv} R_1^{cv} R_2^{cv} = q_2^\ell P_2^v$ at $t = 2$, and the late consumer has consumption of $c_2 = \frac{R_1^{cv} R_2^{cv}}{P_2^v} = q_2^\ell = g_0^c + a_0^c r$. However, an early consumer does not rollover any amount of the loan to the firm at $t = 1$. The firm sells $q_1^e = g_0^c +$

$a_1^e r_1 \leq q_0^c = 1$, with a strict inequality if there is any initial investment $a_0^c > 0$. The firm repays $\delta_1^e L_0^{cv} R_1^{cv} = q_1^e P_1^v$, which gives a real return in terms of consumption of $c_1 = \frac{\delta_1^e R_1^{cv}}{P_1^v} = \frac{q_1^e}{q_0^c} < 1$, which implies $c_1 < \frac{\delta_1^d R_1^{dv}}{P_1^v}$.

Lower consumption for early consumers reflects that credit to firms is only for short term. Lower consumption for late consumers reflects that even though credit is extended for long term, the firm ex-ante invests less than the optimal first best, $a_0^c < a_0^*$, because the firm has the ex-ante risk of having to fully liquidate its investment if its lender is realized as an early consumer ex-interim at $t = 1$. The underinvestment at $t = 0$ is ex-interim inefficient for the late consumer and the firm. However, the underinvestment is ex-ante constrained efficient for consumers and the firm because it decreases the ex-interim inefficiency of the amount liquidated at $t = 1$ for the early consumer and firm.

Direct lending does not provide as sufficient credit to firms as bank lending does because direct lending does not have the leverage for increased lending that banks can attain through maturity transformation of longer term loans that can support more investment in long term illiquid assets.

Consumers have lower expected consumption from storing or direct lending of private digital currency to firms than from depositing.

Lemma 3 *Consumers prefer holding private digital currency in the form of bank deposits, which provides for greater investment through bank lending, compared to consumers holding or lending private digital currency directly.*

If the central bank has a short-term bias, with $\beta^{cb} < 1$, bank deposits in private digital currency provide for greater investment and risk sharing than , deposits in fiat money.

4.2 Public digital currency

Now consider the potential for a central bank with a short-term bias $\beta^{cb} < 1$ to use public digital currency to compete back against private digital currency. As with private digital currency, public digital currency provides the opportunity for consumers to hold it directly or lend directly to firms as well as hold it in the form of

bank deposits. However, this section shows that a central bank with a short-term bias is not able to use public digital currency to compete with private digital currency.

As with private digital currency, at period $t = 0$ of any date τ , consumers can sell e^s of their endowment for $e^s P_0^s$ public digital currency and store the amount M_0^{cs} , lend the amount L_0^{cs} directly to a firm, and deposit the amount D^s , where $M_0^{cs} + L_0^{cs} + D^s \leq e^s P_0^s$. As with private digital currency, the return to a consumer from using public digital currency can be measured by her consumption from selling endowment for a unit of public digital currency that she deposits, stores, or lends.

If the central bank has a short-term bias, the impact of excessive fiat inflation above the optimum between periods $t = 1$ and $t = 2$, $\dot{\Pi}_2^s > \Pi_2^{s*}$, impacts the period $t = 1$ and $t = 2$ real value of public digital currency in an equivalent manner regardless of whether it is stored, directly lent to a firm, or deposited by a consumer at $t = 0$.

Deposits of public digital currency are equivalent to fiat deposits. Bank lending, firm investment, and withdrawal returns to consumers are the same. Holding public digital currency rather than depositing it gives a nominal return and a real return in terms of consumption that is equivalent for early consumers and is lower for late consumers, which reflects that bank lending to firms provides for investment. If a consumer directly lends public digital currency to firms, the long term amount of credit to firms is lower than as provided by banks, resulting in lower initial and continuing investment and lower consumption, as in the case of private digital currency.

Hence, regardless of whether or not the central bank has a short-term bias, consumers prefer to deposit public digital currency rather than directly hold or lend it.

Lemma 4 *Consumers hold bank deposits rather than hold or lend public digital currency directly, regardless of fiat inflation.*

If the central bank has a short-term bias $\beta^{cb} < 1$, consumers hold private digital currency, in the form of bank deposits, and do not hold fiat money deposits or public digital currency in any form.

Proposition 3 *If $\beta^{cb} < 1$, banking with private digital currency provides for greater investment and risk sharing than banking with fiat or public digital currency.*

Private digital currency does not act as a threat to merely discipline a central bank that has a short-term bias to lower excessive fiat inflation, because the central

bank faces an inherent time-inconsistency problem. The central bank would not be credible if it tried to commit to lower inflation, and the central bank cannot constrain itself through public digital currency.

5 Digital currency runs

The potential for bank runs is now analyzed.

Financial stability without digital currency In order to distinguish the threat of bank runs caused by digital currency, the contrasting result of financial stability for banks without digital currency is first briefly reviewed.

The model in sections 3 and 4, which does not consider early withdrawals by late consumers, can be considered as a special case of the general model in this section in which the additional constraint $w^\iota = 0$ for $\iota \in \{s, v\}$ is added to the consumer's optimization. For convenience, this additional constraint and hence the special case of the model is referred to by $w^\iota \equiv 0$.

With fiat deposits in the special case of the model with $w^s = w^0$, the banking system is fully hedged from any defaults for all macro states (λ, r_2) . This holds regardless of whether the central bank has a short-term bias, as shown by proposition 1 and corollary 3 in section 3.

In the general model with public digital currency, for any potential fraction $w^s \in [0, 1]$ of withdrawals by late consumers buying goods at $t = 1$ for any realized state (λ, r_2) . The total amount of withdrawals by early and late consumers at $t = 1$ can be defined as $\lambda^w(w^s) \equiv \lambda + (1 - \lambda)w^s$, with $\lambda^w(w^s) \in [\lambda, 1]$ for $w^s \in [0, 1]$ and $\lambda^w(1) = 1$ for $w^s = 1$. For $w^s \leq 1$, total withdrawals at $t = 1$ is $\lambda^w(w^s \leq 1) \leq 1$, which is equivalent to the special case of the model without early withdrawals, $w^s = w^0$ in which the realized state (λ', r_2) has a fraction of early consumers $\lambda' = \lambda^w(w^s \leq 1) \leq 1$, there are no bank or firm defaults, and the optimal allocation obtains with weakly greater consumption for late consumers than early consumers, $c_2^*(\lambda', r_2) \geq c_1^*(\lambda', r_2)$.

Consumption per unit withdrawn at $t = 1$ for late consumers equals that for early consumers: $c_1^\ell = c_1^* \leq c_2^*$, with total consumption for late consumers of $c_2(w^s) = w^s c_1^* + (1 - w^s) c_2^*$. For $\lambda > \hat{\lambda}$, $c_2 > c_1$, so $w^s = 0$. For $\lambda < \hat{\lambda}$, $c_2 = c_1$, so $w^s \in [0, 1]$ is not determined and has no impact and without loss of generality can be set to

$w^s = 0$. Hence, late consumers never withdraw early: $w^s = 0$.

Lemma 5 *Without digital currency, there are no bank runs for all macro states (λ, r_2) at any date τ .*

When there is a real loss on investments, $r_2 < 1$, the per capita consumption that is available to depositors is less than one. To avoid bank runs and complete asset liquidation when there is a real loss on assets, $r_2 < 1$, there is sufficient inflation for banks to remain solvent on their nominal deposit contracts.

Firms optimally respond to what would otherwise be even higher inflation caused by the low return $r_2 < 1$, by storing goods at $t = 1$ to sell at $t = 2$, when there is also a sufficiently low enough amount of early consumers $\lambda < \check{\lambda}(r_2) = \frac{g_0^*}{g_0^* + a_0^* r_2}$, that allows late consumers to have consumption c_2^* that is optimally equal to that of early consumers, c_1^* , by withdrawing at $t = 2$ and not running on the bank.

Inflation on fiat money that enables the risk sharing of macro risks (λ, r_2) between early and late consumers also enables financial stability against the two primary risks inherent in the banking system. One risk is solvency-based bank runs from the potential insolvency of the banking system in the case of low real returns on assets, r_2 . The second risk is liquidity-based bank runs from the potential illiquidity of the banking system in the case of a large fraction of early consumers, λ , or early withdrawals by late consumers, $w^s > 0$.

First, consider the risk of insolvency in the case of low realizations of r_2 . P_2^i increases due to the reduction in goods available to sell at $t = 2$. This leads firms to hold over goods from $t = 1$ to sell at $t = 2$, such that late consumers do not receive any greater consumption by running the bank to buy goods at $t = 1$. Moreover, banks are effectively hedged on their nominal deposit liabilities at $t = 2$. The equilibrium price level at $t = 2$ remains elevated even with the counterbalancing effect of firms selling more goods at $t = 2$. The elevated price level implies that the real cost of bank deposit liabilities at $t = 2$ falls enough that banks do not default.

Second, consider the risk of the bank defaulting when there is a large realization of early consumers, λ , and/or early withdrawals by late consumers, $w \in (0, 1]$. P_1^i increases from the larger amounts of money spent for goods at $t = 1$. This leads firms to liquidate a greater amount of assets to sell additional goods at $t = 1$. While additional goods sold provides a partial counterbalancing effect on the price level, P_1^i

is still sufficiently elevated such that firms do not default on their loans to banks, and banks do not default on paying withdrawals. Banks continue to rollover loans to firms, which enables firms to only liquidate assets to the extent that it is profit-maximizing according to selling goods at $t = 1$ relative to at $t = 2$, and which provides the optimal allocation between consumption from withdrawals at $t = 1$ relative to at $t = 2$. Consumption from withdrawing and buying goods at $t = 2$ relative to at $t = 1$, $\frac{c_2}{c_1}$, actually increases in the fraction of early consumers, λ , and early withdrawals by late consumers, w^s , because of the relatively higher nominal deposit return, $\frac{R_2^d}{R_1^d}$, and lower price level $\frac{P_2^s}{P_1^s}$, which reflects the relatively higher investment return, r_2 when there is no asset liquidation ($a_1 = 0$) and $\frac{r_2}{r_1}$ when there is asset liquidation ($a_1 > 0$), for $t = 2$ relative to $t = 1$. Hence, late consumers do not withdraw at $t = 1$, $w^s = 0$.

Private digital currency runs As shown by proposition 2 in section 4, banks with private digital currency deposits, for which early withdrawals by late consumers is not considered, a real asset loss $r_2 < 1$ and few early consumers $\lambda \leq \check{\lambda}^v(r_2 < 1)$ cause an absolute bank default at $t = 2$, with $\delta_2^d \hat{R}_2^{dv} < 1$. With the cap on private digital currency inflation at $t = 2$, the absolute default instead efficiently lowers the nominal return paid on late consumer withdrawals, while firms optimally store $g_1^*(\lambda, r_2) > 0$ goods at $t = 1$ until $t = 2$ to provide late consumers with consumption that is equal to that of early consumers regardless of whether late consumers withdraw and buy goods at $t = 1$ or at $t = 2$.

However, the ability for late consumers to withdraw and store private digital currency at $t = 1$ triggers a bank run in the states with $r_2 < 1$ and $\lambda \leq \check{\lambda}^v(r_2 < 1)$, despite that consumption from withdrawals to buy goods at $t = 1$ and $t = 2$ would otherwise be equal.

In these states, there is deflation $\Pi_2^v(\lambda, r_2) < 1$ of private digital currency prices between periods $t = 1$ and $t = 2$, which can be seen by the outcome of equal consumption for withdrawals that are used to buy goods at each period $t = 1$ and $t = 2$, while nominal withdrawal returns are higher at $t = 1$ than $t = 2$.

Since $R_1^{dv} > R_2^{dv}$, rather than accept the nominal return on deposit withdrawals at $t = 2$ that is less than one, $\delta_2^d \hat{R}_2^d < 1$, late consumers will attempt to withdraw early the nominal return of one that the bank has to pay in order to not default at $t = 1$, $\delta_1^d \hat{R}_1^{dv} = 1$. Late consumers attempt to withdraw for the higher nominal payout at

$t = 1$ only to hold the private digital currency, outside of the banking system, to buy goods at the $t = 2$ price that is lower than at $t = 1$ with deflation $\Pi_2^v < 1$.

Consumption from withdrawing and holding private digital currency to buy goods at $t = 2$ price $P_2^{dv} = 1$ would equal to one, $\frac{\hat{R}_1^{dv}}{P_2^v} = 1$. Consumption from buying goods at $t = 1$ price $P_1^{dv} > 1$ would be $\frac{\hat{R}_1^{dv}}{P_1^v} < 1$, and from withdrawing at $t = 2$ would be $\frac{\delta_2^d \hat{R}_2^{dv}}{P_2^v} < 1$.

However, with all late consumers withdrawing $w^v = 1$ and holding private digital currency, the bank would default at $t = 1$, $\delta_1^d < 1$, since the bank's $t = 0$ loans to firms are too illiquid to receive their par value repaid back at $t = 1$ to pay out all withdrawals at $t = 1$, and in particular does not have enough private reserves held from $t = 0$ to pay all of the late consumer demands for withdrawals in the form of digital currency.

In equilibrium, at $t = 1$ the bank defaults, $\delta_1^d < 1$, has to pay out as much as possible to withdrawals, and cannot rollover any loans to firms. Without loan rollovers, firms cannot fully repay the initial $t = 0$, default at $t = 1$ with $\delta_1^f < 1$, and have to completely liquidate their investments to repay as much as possible, $a_1 = a_0$. The bank has no revenues for a complete default at $t = 2$, with $\delta_2^d = 0$.

Hence, in equilibrium, late consumers fully withdraw $w^v = 1$ and receive pro rata with early consumers the bank default fraction $R_1^{dv} = \delta_1^d = \delta_1^f \hat{R}_1^{fv} + M_0^{bv} < 1$ at $t = 1$. Consumption is lower than in the case of no run, with $c_1^l = c_1 = \delta_1^d = g_0 + a_1 r_1 < g_0 + a_1 r_2$.

Proposition 4 *With private digital currency deposits, when there is a loss on the asset $r_2 < 1$ and a large fraction of late consumers $1 - \check{\lambda}^v(r_2)$, there is a digital currency run in the form of digital currency withdrawals by late consumers that create a complete liquidation of the banking system.*

When there is a real investment loss, $r_2 < 1$, and the fraction of late consumers is not too high, $1 - \lambda < 1 - \check{\lambda}^v(r_2)$, the ex-post efficient consumption allocation obtains and the late consumer incentive constraint holds, $c_2(\cdot) \geq c_1(\cdot)$. While there may be a partial bank default $\delta_2^d < 1$, there is no absolute bank default, $\delta_2^d \hat{R}_2^{dv} > 1$, at $t = 2$. Hence, late consumers withdraw only at $t = 2$.

When there is not a real investment loss, $r_2 \geq 1$, there are no early withdrawals by late consumers, $w^v = 0$, and the ex-post efficient consumption allocation obtains,

with $c_2(\cdot) \geq c_1(\cdot)$, regardless of the fraction of late consumers $1 - \lambda \leq 1$.

For example, for $r_2 = 1 - \varepsilon$, without digital currency, banks are only slightly insolvent and efficiently survive without runs, but private digital currency allows for a liquidity run that magnifies the liquidity run even as the insolvency and $\varepsilon \rightarrow 0$.

Public digital currency runs The bank runs that occur for $r_2 < 1$ and $\lambda \geq \check{\lambda}(r_2)$ with private digital currency deposits is caused by private digital currency not allowing for inflationary prices, with its nominal price level at $t = 2$ capped $P_2^v \leq 1$, and which gives it a lower bound on its real value of one, $\frac{1}{P_2^v} \geq 1$.

This is the contrast from the case of fiat deposits shown above, as well as the case of public digital currency deposits, for which there are no bank runs for any state (λ, r_2) because of fiat money inflation in states of low returns r_2 and many late consumers $1 - \lambda$.

Fiat money has sufficient inflation $P_2^s(\cdot) > 1$ such that the the bank never has even a partial default at $t = 2$, nor at $t = 1$, $\delta_2^d = \delta_1^d = 1$, for all states (λ, r_2) . Since the optimal consumption obtains, $c_2(\cdot) = \frac{R_2^{ds}}{P_2^s} = \frac{\bar{r}_2}{P_2^s} = c_2^*(\cdot)$ and $c_1(\cdot) = \frac{R_1^{ds}}{P_1^s} = c_1^*(\cdot)$, with $c_2^*(\cdot) \geq c_1^*(\cdot)$, late consumers do not withdraw early to buy goods at $t = 1$. Since there are no bank defaults, late consumers do not withdraw early to directly hold public digital currency. Deposits pay a nominal amount at $t = 2$, $R_2^{ds} = \bar{r}$, that is equal to what late consumers would receive by withdrawing at $t = 1$, $R_1^{ds} = 1$, and holding it until $t = 2$ for a total return, with interest on public digital currency, of $R_1^{ds} R_2^s = R_2^s = \bar{r}$.

Corollary 4 *With public digital currency deposits, there are no bank runs for all states (λ, r_2) at any date τ .*

Consumers do not hold public digital currency directly but rather only in the form of deposits.

5.1 Equilibrium for the economy

Private digital currency avoids the distortion of high fiat inflation. However, if it is held directly, it loses the value creation from bank maturity and risk transformation. If held in the form of deposits, it creates the risk of digital currency runs. In equilibrium,

whether private digital currency is used, and whether it is lent to firms through the banking system or directly by consumers, is determined by consumers who choose among these options. Their choice depends on the relative benefits and costs from high fiat inflation, bank maturity and risk transformation, and the probability and losses of digital currency runs.

Since consumers are ex-ante identical at $t = 0$ when they sell their endowment and choose their portfolio among the options of depositing, direct lending, and holding fiat money and private digital currency, the equilibrium for the economy can be analyzed based on the type of money and lending that maximizes consumers' expected utility. Holding digital currency is always dominated by either banking or direct lending, and direct lending is dominated by banking for the case of fiat money. Hence, the type of equilibrium for an economy is either fiat money with bank lending, or private digital currency with either bank lending or consumer lending.

The efficiency of equilibrium investment and expected consumption in the economy can be compared relative to the optimal allocation for the real economy based on the late consumers' discount factor β^ℓ , the investment liquidation return r_1 , the joint distribution of the random macro state (λ, r_2) , and consumers' risk aversion and intertemporal substitution preference over consumption expressed by their utility function $u(c_t)$.

This efficiency for consumers weakly decreases with higher fiat inflation by a central bank with discount factor $\beta^{cb} < \beta^\ell$, higher insolvency risk for banks, and investment liquidation cost, and increases with the value of bank maturity transformation. For $\beta^{cb} < \beta^\ell$, fiat inflation decreases in β^{cb} . The insolvency risk for banks decreases in the probability $\rho \equiv \Pr(r_2 \geq 1)$. Investment liquidation cost decreases in r_1 . The value of maturity transformation which increases in $\bar{\lambda}$ and \bar{r}_2 .

The determination for an economy of the equilibrium type can be characterized by these parameters of the economy, β^{cb} , ρ , r_1 , $\bar{\lambda}$, and \bar{r} , each relative to corresponding endogenous cutoffs $\hat{\beta}^{cb}(\cdot)$, $\hat{\rho}(\cdot)$, $\hat{r}_1(\cdot)$, $\dot{\lambda}(\cdot)$, and $\dot{r}_2(\cdot)$, respectively, where each cutoff is a function of the other non-corresponding parameters of the economy.

Proposition 5 *The equilibrium for an economy is:*

- i) Fiat money with bank lending if $\beta^{cb} \geq \hat{\beta}^{cb}(\cdot)$ or $\bar{\lambda} \geq \dot{\lambda}(\cdot)$;*
- ii) Private digital currency with bank lending if $\rho \geq \hat{\rho}(\cdot)$ or $\bar{r}_2 > \dot{r}_2(\cdot)$;*
- iii) Private digital currency with consumer lending if $r_1 \geq \hat{r}_1(\cdot)$.*

If the central bank has a significant enough short-term bias, reflected by a low enough β^{cb} , fiat money and public digital currency is not held and is driven out by private digital currency.

6 Regional altcoin

While private digital currency does not allow for inflation to prevent digital currency runs on banks with losses on lending investments, liquidity-based runs do not occur for fundamentally solvent banks. For $r_2 \geq 1$, there are no runs for any level of liquidity demand, since the price level in the economy is partially elastic relative to the macroeconomy's liquidity demand reflected by λ .

Applying the model to an international setting, a private digital currency that is used throughout the global economy, as is intended for prominent cryptocurrencies such as bitcoin, would have a price level and value that is partially flexible relative to the global macro state. Such a globally-used private digital currency does not have any price level flexibility relative to the macro shocks that are idiosyncratic to a small developing county.

A global currency has a fixed value relative to the country's regional macro shocks and would act as a traditional hard currency, used by many developing countries, through current forms such as a dollar-pegged currency and dollarization, and historically through a gold standard. A hard currency avoids fiat inflation but exacerbates recessions and creates a fragile banking system that is more susceptible to bank runs.

Instead, a regionally-used altcoin form of private digital currency is required to have a flexible price relative to regional macro shocks and to limit bank runs in the region. A regional altcoin can prevent fiat inflation, as with a traditional hard currency, but has a more flexible value that lessens the impact of recessions, supports greater investment, gives better risk sharing among consumers for macro asset and liquidity risk, and reduces bank runs relative to using a hard currency.

6.1 Extended model

In order to analyze a regional altcoin, assume that there is a small regional economy and a global economy that are each represented by the model of the economy and private digital currency developed above, with a separate set of agents for each economy.

There are two distinctions between the regional and global economies. First, the regional economy has the random macro state (λ, r_2) as in the model heretofore. The global economy is assumed for simplicity to have a constant macro state $(\bar{\lambda}, \bar{r}_2)$. This represents that the regional economy is small and has macro shocks that do not impact the global economy. Allowing for global macro risk, with a random global macro state (λ^z, r_2^z) that may or may not impact the regional economy, would not qualitatively change the results as long as the regional and global macro states are not perfectly correlated.

Second, the private digital currency issued in the regional economy, and referred to as an altcoin, is a separate digital currency than what is referred to as a global private digital currency issued in the global economy. The analysis and results would be similar if a fiat money, either in the form as reserves held only by banks or as a public digital currency, and issued by a central bank with no short term bias in a large country with a constant macro state $(\bar{\lambda}, \bar{r}_2)$ relative to the regional economy, were to be considered instead of the global private digital currency and global economy.

If there were no frictions between the economies, the regional economy could be fully integrated within the global economy and use the global currency to achieve the optimal allocation of investment and consumption.

In order to consider the regional economy having exposure to its regional macro shocks, a partial segmentation between the regional and global economies is required. I assume that both are open economies and a single market without segmentation for trading goods, the regional altcoin, and the global currency between the economies at each period. However, there is segmentation for credit and investment, which could be interpreted as based on asymmetric information between the economies, and which prevents risk-sharing between the economies. Specifically, consumers, banks, and firms can only make deposits, loans and investments within their own economy.

6.2 Equilibrium results

Consider the regional economy using the global currency rather than the regional altcoin or regional fiat money. Across both economies, the global currency has a fixed value and constant price level of goods, which is determined based on the global economy with its constant macro state $(\bar{\lambda}, \bar{r}_2)$ and the regional economy being small. The price level can be normalized to one, $P_t^z = 1$ $t = 0, 1, 2$, and the superscript $\iota = z$ denotes variables in the regional economy when the global currency is used. With a fixed nominal price level, the model of each economy is equivalent to that of a real model with goods as numeraire for deposit and loan contracts.

In the regional economy, banks have excessive liquidation and liquidity-based runs even when they are solvent. For a moderately high realization of early consumers $\lambda > \hat{\lambda}^z \equiv g_0 + g_0^z$, regardless of solvent investment returns $r_2 \geq 1$, banks can only roll over a limited amount of lending to firms. In order for the bank not to default at $t = 1$, $R_1^{dz} = \delta_1^d \hat{R}_1^{dz} = 1$, a marginal increase in λ requires the bank to increase its marginal withdrawal payout at a constant real amount of one, $c_1 = \frac{R_1^{dz}}{P_1^g} = 1$, since the global currency price level is not elastic to the regional economy and P_1^z does not increase with an increase in λ . To repay the greater amount of loans not rolled over at $t = 1$, firms are forced to liquidate $a_1 = \frac{\lambda - \hat{\lambda}^{dz}}{r_1} > 0$ of investments. The liquidation is in excess of the optimal amount (if any) of liquidation, and consumption for late consumers is below the optimal allocation.

When the liquidity demand is very high, for $\lambda > \hat{\lambda}^z(r_2) \equiv 1 - a_0(\frac{1-r_1}{r_2-r_1})r_2$, where $\hat{\lambda}^z(r_2) \in [\hat{\lambda}^z, 1)$ for $r_2 \geq 1$, there is a complete bank run and liquidation for any arbitrarily high investment return.

To analyze “before the run occurs” with $w^z = 0$, the required investment liquidation at $t = 1$ is large enough that firms and banks will have a substantial default at $t = 2$, reflecting a substantial shortage of remaining investments. Consumption for late consumers withdrawing at $t = 2$ is less than for early consumers: $c_2(w^z = 0) = \frac{\delta_2^d \hat{R}_2^{dz}}{P_2^z} < c_1(w^z = 0) = \frac{\delta_1^d R_1^{dz}}{P_1^z} = 1$.

A marginal withdrawal by late consumers at $t = 1$ further decreases bank lending and firm liquidation. A marginal increase of investment liquidation a_1 gives a marginal return of less than one, $r_1 < 1$, that firms can pay to the bank for paying out the marginal withdrawal of one, $c_1^\ell = 1$, required to not default at $t = 1$. And the

liquidation reduces the return on continuing investment that firms can pay the bank for withdrawals at $t = 2$. The marginal increase in liquidation decreases investment returns that can be paid out at $t = 2$ by $r_2 \geq 1$, which is more than the decrease in the marginal withdrawal of c_2 at $t = 2$.

Liquidating investments to pay out late consumer withdrawals at $t = 1$ creates a downward spiral in the amount that firms and hence the bank can pay at $t = 1$. Late consumers run on the bank, with $w^z = 1$. The bank and firms have a default and complete liquidation at $t = 1$, with $\delta_1^d < 1$ and $a_1 = a_0$.

Because of the excessive liquidation caused by a moderately high λ and complete liquidation and runs caused by a very high λ , there is a lower amount of initial investment a_0 than otherwise optimal. However, the lower initial investment does not eliminate excessive liquidation and bank runs, unless expected returns are low enough and liquidation costs are high enough such that initial investment amount is zero.

There is also an increase in insolvency-based bank runs, which occur for all realizations of $\lambda \in [0, 1]$ whenever there is a loss on investment, $r_2 < 1$.

In contrast, if the regional altcoin is used in the regional economy instead of the global currency, the results for the regional economy are equivalent to those of the paper for a single economy, and the international context with a global economy does not play a role.

Liquidity-based bank runs do not occur. When there is a high amount of early consumers, $\lambda > \hat{\lambda}^v$, the regional currency has a partially flexible value, with $P_1^v > 1$ and $\frac{P_2^v}{P_1^v} < 1$, such that the bank and firm do not default on early consumer withdrawals at $t = 1$. There is sufficient rollover lending to firms that late consumers receive a greater amount by withdrawing at $t = 2$ than $t = 1$. Insolvency-based bank runs occur only when there is a high level of late consumers, $1 - \lambda \geq 1 - \check{\lambda}^v(r_2)$, and investment loss, $r_2 < 1$.

Lemma 6 *Using a global private digital currency in a regional economy, there is lower initial and continuing investment, less risk sharing of regional macro risk (λ, r_2) between early and late consumers, liquidity-based bank runs, and more insolvency-based banks runs, relative to using a regional altcoin.*

The equilibrium for the regional and global economies are that each economy exclusively uses its own private digital currency or fiat currency. For each economy,

the equilibrium is determined as a single economy according to proposition 5 rather than the global digital currency in the regional economy.

Proposition 6 *Either the regional altcoin or regional fiat currency is used exclusively in the regional economy, which provides the constrained efficient allocation for investment and consumption withno liquidity-based bank runs. The global digital currency or global fiat currency is used exclusively in the global economy.*

The distinction between the global and regional private currencies is that the global version acts as a traditional hard currency while the regional version acts similar to a traditional fiat domestic currency, but without central bank discretion to create excessive fiat inflation.

Historically, gold has acted as the traditional hard currency. Under the gold standard, countries only issued fiat money fully backed and redeemable at a fixed conversion rate to gold. In more recent decades, countries such as many emerging market economies with high fiat inflation have at times adopted a hard currency by using or pegging their currency to a stable foreign fiat currency. One example is dollarizing the economy, in which only U.S. dollars are used as money and the domestic currency is no longer used. Dollarization can occur through government mandate, such as the case of Ecuador, or through the economy shifting to only dollar use if the government does not try or have the ability to ban dollar use. Countries have also pegged their domestic currency to the U.S. dollar using a currency board that only issues domestic currency one-for-one in exchange with the dollar, such as in the case of Argentina.

In the absense of a regional altcoin, a global private digital currency effectively provides the benefits but also the costs of a traditional hard currency to a country that otherwise has large fiat inflationary problems. A hard currency avoids the distortions of high fiat inflation but has a fixed real value that exacerbate recessions and enables more bank runs.

A regional altcoin provides better outcomes to avoid fiat inflation than a traditional hard currency or a global private digital currency. A regional altcoin has a partially elastic value that somewhat buffers the economy against recessions and the banking system against liquidity-based runs and partially against insolvency-based runs.

7 Concluding remarks

A major theme in the academic literature since the financial crisis is investigating causes of fragility in the leveraged financial system. Now, with the heightened interest and concern about the potential impact on the financial system that may come from fintech, understanding the financial fragility that major financial technologies may bring is crucial.

This paper provides a first examination within the burgeoning literature on fintech of the potential impact of digital currency on economic investment and the stability of the banking system. Digital currency permits but does not necessarily lead to the ex-ante disintermediation of the banking system. Consumers may deposit digital currency at banks because of the benefit of maturity and risk transformation that increases investment and consumption. Banks are partially buffered from macro liquidity and return risk using a private digital currency. The disintermediation threat takes the form of digital currency runs that create fragility of the banking system.

There is an important trade-off between the features of privately issued digital currency, such as bitcoin, and fiat money whether in traditional form or as central bank digital currency. Central bank discretion permits fiat inflation that buffers the economy and banking system from macro risks but also can lead to excessive distortionary inflation. Private digital currency precludes fiat inflation but also creates more rigidity in the banking system. A regionally used private digital currency altcoin can limit excessive inflation in developing countries, while limiting the bank fragility that traditional hard currencies or a globally used digital currency create.

8 Appendix: Strategic digital currency runs

The potential for bank runs caused by late consumers who act strategically can be considered with a slight generalization of the model. Digital currency creates a new form of the classic threat of liquidity-based runs, on banks that are otherwise fundamentally solvent, based on strategic complementarities among late consumers.

The model is generalized to allow for late consumers to act strategically over their early withdrawal fraction of deposits at $t = 1$ and the amount held as digital currency until $t = 2$. The central bank can act as lender of last resort by lending fiat money to banks at $t = 1$.

Specifically, at period $t = 1$ of each date τ , the fraction $1 - \lambda$ consumers who have the realization of being late types are indexed by $i \in I \equiv [\lambda, 1]$. Each late consumer $i \in I$ has a withdrawal strategy, for her deposit $D^\iota \geq 0$ of currency $\iota \in \{s, v\}$, which is defined as

$$\sigma^{i\iota}(\lambda, r_2) \equiv (w^{i\iota}(\lambda, r_2), M_1^{cwi}(\lambda, r_2)) \text{ for } \iota \in \{s, v\}.$$

The strategy is comprised of her early withdrawal fraction $w^{i\iota} \in [0, 1]$ for an amount $w^{i\iota} R_1^{d\iota} D^\iota$ at $t = 1$, of which $M_1^{cwi} \in [0, w^{i\iota} R_1^{d\iota} D^\iota]$ is stored as digital currency to buy goods at $t = 2$, and the remainder is spent to buy goods at $t = 1$. Her withdrawal strategy across currencies is defined as $\sigma^i(\lambda, r_2) \equiv \{\sigma^{i\iota}(\cdot)\}_{\iota \in \{s, v\}}$. The set of withdrawal strategies for all late consumers is defined as $\sigma(\lambda, r_2) \equiv \{\sigma^i\}_{i \in I}$ or equivalently as $\sigma(\lambda, r_2) \equiv \{\sigma^\iota\}_{\iota \in \{s, v\}}$ where $\sigma^\iota \equiv \{\sigma^{i\iota}\}_{i \in I}$.

A Nash equilibrium for early withdrawals by late consumers is the set σ of early withdrawals in which the withdrawal strategy σ^i for each late consumer $i \in I$ is a best response to the withdrawal strategies $\{\sigma^{i'}(\cdot)\}_{i' \in I}$ for all other late consumers $i' \in I$, which is expressed as follows.

Definition 2 *A Nash equilibrium of early withdrawals at $t = 1$ is defined as:*

$$\sigma(\lambda, r_2) \equiv \{\sigma^i(\lambda, r_2) | \{\sigma^{i'}(\lambda, r_2)\}_{i' \in I}\}_{i \in I, \iota \in \{s, v\}}.$$

Banks can borrow $L_1^s(\lambda, r_2)$ in fiat reserves or public digital currency from the central bank at $t = 1$ and repay at the return on fiat money $R_2^s(\lambda, r_2)$ at $t = 2$. The

bank budget constraints are updated as:

$$t=1: [\lambda + w^\iota(1 - \lambda)]D^\iota R_1^{d\iota} \leq L_0^{f\iota} R_1^{f\iota} - L_1^{f\iota} + M_0^{b\iota} - M_1^{b\iota} + L_1^s$$

$$t=2: (1 - w^\iota)(1 - \lambda)D^\iota R_2^{d\iota} \leq L_1^{f\iota} R_2^{f\iota} + M_1^{b\iota} R_2^\iota - M_2^{b\iota} - L_1^s R_2^s$$

$$Q_\tau^b(\cdot) \equiv \{D^\iota, L_0^{f\iota}, M_0^{b\iota}, \{L_1^{f\iota}, L_1^s, \{M_t^{b\iota}, \delta_t^d\}_{t \in \{1,2\}}\}_{\lambda, r_2}\}_\iota,$$

where w^ι and $M_1^{c\iota w}$ are now defined as the average of late consumers' early withdrawal fractions and average amount of which they store as digital currency, respectively: $w^\iota(\lambda, r_2) \equiv \int_{i \in I} w^{i\iota}(\lambda, r_2)$ and $M_1^{c\iota w}(\lambda, r_2) \equiv \int_{i \in I} M^{c\iota w i}(\lambda, r_2)$ for $\iota \in \{s, v\}$.

The amount of bank borrowing $L_{t,\tau}^s$ is also added to the aggregate supply of fiat money, as shown on the RHS of the updated market clearing equation for fiat money:

$$\sum_{\kappa \in \{c,b\}} M_{t,\tau}^{\kappa s}(\cdot) = \prod_{\hat{\tau}=0}^{\tau-1} R_{2,\hat{\tau}}^s(\cdot) R_{t,\tau}^s(\cdot) M^s + \mathbf{1}_{t=1} L_{1,\tau}^s \text{ at } t \in \{0, 1, 2\},$$

where $\mathbf{1}_{t=1}$ is the indicator function. The borrowed amount $L_{1,\tau}^s$ gives only a temporary increase in fiat money, at period $t = 1$ of date τ . On the LHS of the equation, there is an increase at this period equal to $L_{1,\tau}^s$ in aggregate fiat money held by consumers and banks that receives the return $R_{2,\tau}^s$ at $t = 2$, and hence equals the amount $L_{1,\tau}^s R_{2,\tau}^s$ repaid by the borrowing bank to the central bank at $t = 2$.

The withdrawal run threat is first analyzed in the absence of the central bank as lender of last resort, in which case $L_1^s \equiv 0$.

No digital currency In a regime without private or public digital currency, the banking system is fully hedged from bank runs for all states (λ, r_2) , and late consumers never withdraw early.

Lemma 7 *Without digital currency, there are no bank runs for all realizations of (λ, r_2) at each date τ :*

$$w^{i\iota} = M_1^{c\iota w i} = 0 \quad \text{for all } i \in I, \iota \in \{s, v\} \text{ and } (\lambda, r_2).$$

Under the first threat, with $w^\iota = 1$ and $M_1^{c\iota w} = 0$, all late consumers run on the banking system in order to buy goods at $t = 1$. Similar to above, the impact would be an increase in P_1^ι , which would lead firms to liquidate a greater amount of assets than otherwise in order to sell additional goods at $t = 1$. While additional goods sold

would provide a partial counterbalancing effect on the price level, P_1^t would still be sufficiently elevated such that firms would not default on their loans to banks, and banks would not default on paying withdrawals. Banks could continue to rollover loans to firms, which enables firms to only liquidate assets to the extent that it is profit-maximizing for selling goods at $t = 1$ relative to at $t = 2$. A marginal late consumer would prefer to deviate from the strategy of withdrawing to buy goods at $t = 1$ in order to withdraw instead at $t = 2$ for the higher nominal deposit return as well as relatively lower price level P_2^t . Thus, a marginal late consumer who deviates and withdraws instead at $t = 2$ has greater consumption. Hence, with $M_1^{cww} = 0$, all late consumers would prefer to withdraw at $t = 2$, and such liquidity-based runs do not occur in equilibrium. The outcome of no bank runs, $w^t = M_1^{cww} = 0$ is a Nash equilibrium, and there are no defaults: $\delta_t^k = 1$ for all $k \in \{d, f, c, cb\}$, $t \in \{1, 2\}$.

Under the second threat, with $w^t = 1$ and $M_1^{cww} \in (0, D^t R_1^{dt}]$, the withdrawal run equilibrium may exist. The bank defaults at $t = 1$ if $(1 - \lambda)M_1^{cww} > M_0^{bt}$. In particular, for $M_1^{cww} = D^t \delta_1^d \hat{R}_1^{dt}$, this bank default condition is

$$(1 - \lambda)D^t \hat{R}_1^{dt} > M_0^{bt},$$

which can be simplified as

$$\lambda < \frac{1}{1 + m_{0,\tau}^t} \in (\frac{1}{2}, 1)$$

where $m_{0,\tau}^t \equiv \frac{M^t}{P_{0,\tau}^t} \in (0, 1)$ is defined as the real value at date τ of the digital currency M^t . Counterintuitively, a withdrawal run equilibrium can only occur at dates when there is a sufficiently low realization λ of early consumers. This is because with a greater amount of late consumers, there is a larger amount of digital currency withdrawals under a withdrawal run threat at $t = 1$ that has greater ability to deplete the bank, cause a bank default, and enable the withdrawal run threat to sustain as an equilibrium run.

8.1 Strategic runs

With public digital currency, the ability for late consumers to withdraw and store digital currency at $t = 1$ to buy goods at $t = 2$ allows for the potential of bank runs in which the bank defaults at $t = 1$ if there is a greater demand for digital currency withdrawn and stored by late consumers beyond what the bank holds as reserves.

Corollary 5 *With public digital currency deposits, there exists strategic digital currency runs in the form of digital currency withdrawals by late consumers if there is a sufficient amount of late consumers $1 - \lambda$.*

8.2 Central bank as lender of last resort

The central bank can act as lolr to prevent runs for banks with public but not private digital currency. The central bank even with $\beta^{cb} = 1$ allows for optimal sufficient inflation created by its lolr such that banks are not insolvent for $r_2 < 1$.

The central bank has the ability and discretion to create an additional quantity of the supply of fiat money, which gives the central bank a natural monopoly over the outside supply of liquidity available to banks. Because of this, the central bank has the unique ability to act as lender of last resort to banks with public digital currency deposits by issuing an additional quantity of public digital currency that is lent to banks facing runs at $t = 1$.

Regardless of the seniority of the central bank's loans to banks, the central bank can create and lend large enough amounts to such illiquid banks to ensure they do not default at $t = 1$ and $t = 2$. Hence, the the central bank does not face any risk of banks defaulting on the loans. Borrowing banks can repay the loans, comprised of outside digital currency at $t = 1$, in kind at $t = 2$ with public digital currency received from their returns on loans to firms.

The withdrawal run threat on a bank is that late consumers withdraw $w^t > 0$ and store an amount of the withdrawal as digital currency at $t = 1$: $M_1^{csw} = (0, w^t D^t R_1^{dt}]$. Banks with public digital currency deposits can borrow this amount $L_1^{cb} = (1 - \lambda)M_1^{csw}$ in public digital currency from the central bank, and the bank does not default. Late consumers prefer to not withdraw at $t = 1$, $w^s = M_1^{csw} = 0$. Withdrawing at $t = 2$ provides the late consumer a greater withdrawal return and hence a greater amount of goods bought at $t = 2$ for consumption. Hence, the digital currency run does not occur.

In equilibrium, banks do not borrow from the central bank. The potential case of a digital currency run is an out-of-equilibrium threat that is prevented from occurring as an equilibrium because of the ability and willingness of the central bank to elastically supply its digital currency as lender of last resort.

Corollary 6 *With a central bank as lender of last resort, there are no digital currency runs for banks with public digital currency deposits for all realizations of (λ, r_2) .*

Public versus private digital currency The central bank cannot lend private digital currency to banks with private digital currency deposits that face digital currency runs. Hence, digital currency runs occur for low asset returns $r_2 < 1$. The central bank is not able to act as lender of last resort because it cannot create the private digital currency required to lend. While a private digital currency does not cause a digital currency run equilibrium to occur, the private digital currency enables it to happen.

Corollary 7 *For banks with public digital currency deposits facing a digital currency run threat, the central bank acts as lender of last resort by providing an elastic outside money supply. The digital currency run does not occur. Whereas, for banks with private digital currency deposits, the central bank cannot act as lender of last resort, and the digital currency run equilibrium exists at dates with $r_2 < 1$.*

The central bank lending to such banks with fiat money or private digital currency bought with fiat would create spiraling inflation and depreciation of fiat money. The result reflects the contrast of the elastic supply of public digital currency but inelastic supply of private digital currency. For a public digital currency, the central bank can elastically supply its own digital currency to banks. For a private digital currency, the central cannot create the private digital currency required for lender of last resort.

This result also highlights a distinction between an elastic value yet inelastic supply of a private digital currency. Even with an inelastic supply of the digital currency, prices are elastic and permits the optimal equilibrium, even with the realization of low asset returns and high early consumer liquidity needs. However, an inelastic supply of the digital currency also permits the digital currency run equilibrium, which elastic prices do not prevent. There is a trade-off for private digital currency deposits, which avoid the costs of distortionary central bank fiat inflation but are subject to digital currency runs.

Digital currency can be quickly and easily withdrawn in very large quantities to hold and transact with outside of the financial system, which allows liquidity runs to be an even greater threat than such types of runs based on withdrawing deposits for paper currency or gold historically or other financial instruments in modern times.

9 Appendix: Proofs

Section 3: Fiat money

Proof for Lemma 1. The planner's optimization (10) gives binding budget constraints, which imply consumption equations (12)-(14); and first order conditions for $EU(\beta)$ with respect to i) a_0 , which gives the Euler equation (11), ii) $g_1(\lambda, r_2)$, which gives $g_1^* = (1 - \lambda)g_0^* - \lambda a_0^* r_2 > 0$ for $\lambda < \check{\lambda}(\cdot)$ and $g_1^* = 0$ for $\lambda \geq \check{\lambda}(\cdot)$, and iii) $a_1(\lambda, r_2)$, which gives $a_1^* > 0$ for $\lambda > \check{\lambda}(\cdot)$ and $a_1^* = 0$ for $\lambda < \check{\lambda}(\cdot)$.

Specifically, define consumption as if there were no storage or liquidation for any realization of (λ, r_2) , $g_1(\lambda, r_2) = a_1(\lambda, r_2) = 0$, as $\check{c}_1 \equiv \frac{g_0^*}{\lambda}$, $\check{c}_2 \equiv \frac{a_0^* r_2}{1 - \lambda}$. For $u'(\check{c}_1) < u'(\check{c}_2)$, there is positive storage $g_1^* = (1 - \lambda)g_0^* - \lambda a_0^* r_2 > 0$ to equalize marginal utilities between early and late consumers such that $u'(c_1^*) = u'(c_2^*)$. As a result, $c_1^* = c_2^* = g_0^* + a_0^* r_2$. This outcome occurs for a low enough joint realization of (λ, r_2) , which can be expressed as $r_2 < \check{r}_2(\lambda) \equiv \frac{(1 - \lambda)g_0^*}{\lambda a_0^*}$ and $\lambda < \check{\lambda}(r_2) \equiv \frac{g_0^*}{g_0^* + a_0^* r_2}$ that implies a threshold $(\check{\lambda}, \check{r}_2)$. When the illiquid asset return or the aggregate liquidity need for early consumers is small enough, positive storage of goods from $t = 1$ to $t = 2$ enables late consumers to share equally with early consumers in the total goods available at $t = 1$ and $t = 2$. The marginal rate of substitution between late and early consumers equals the marginal rate of transformation of one on storage between $t = 2$ and $t = 1$.

For $u'(\check{c}_1) > \frac{r_2}{r_1} u'(\check{c}_2)$, which holds with an implicit $(\hat{\lambda}, \hat{r}_2)$ for $r_2 > \hat{r}_2(\lambda)$ and $\lambda > \hat{\lambda}(r_2)$, such a high enough joint realization of (λ, r_2) implies there is instead positive liquidation $a_1^* > 0$ implicitly defined by $u'(c_1^*) = \frac{r_2}{r_1} u'(c_2^*)$. When the illiquid asset return or the aggregate liquidity for early consumers is large enough, asset liquidations allows for early consumers to share in part of the abundance of goods that are available at $t = 2$. The marginal rate of substitution between late and early consumers equals the marginal rate of transformation between assets' return at $t = 2$ and liquidation return at $t = 1$.

Otherwise, for $u'(\check{c}_1) \in \left[u'(\check{c}_2), \frac{r_2}{r_1} u'(\check{c}_2) \right]$, for moderate realizations of (λ, r_2) , there is no storage or liquidation, $g_1^* = a_1^* = 0$, hence $u'(c_1^*) \in \left[u'(c_2^*), \frac{r_2}{r_1} u'(c_2^*) \right]$. These

results for optimal consumption, storage, and liquidation are summarized as

$$u'(c_1^*) = \begin{cases} u'(c_2^*), & \text{with } g_1^* > 0, a_1^* = 0, \text{ for } \lambda \in [0, \check{\lambda}(r_2)), \\ \in [u'(c_2^*), \frac{r_2}{r_1}u'(c_2^*)], & \text{with } g_1^* = a_1^* = 0, \text{ for } \lambda \in [\check{\lambda}(r_2), \hat{\lambda}(r_2)], \\ \frac{r_2}{r_1}u'(c_2^*), & \text{with } g_1^* = 0, a_1^* > 0, \text{ for } \lambda \in (\hat{\lambda}(r_2), 1]. \end{cases} \quad (17)$$

Proof for Proposition 1. The requirement that the bank and firm maximize repayment on deposits and loans, respectively, in case of a default at a period $t \in \{1, 2\}$, can be written in the form of complementary slackness conditions. Specifically, for every state $(\lambda_\tau, r_{2,\tau})$ at each date τ , complementary slackness conditions for the bank are $(1 - \delta_1^d)\phi_1^d = 0$ for $\phi_1^d \in \{L_1^{f\iota}, M_1^{b\iota}\}$ and $(1 - \delta_2^d)M_2^{b\iota} = 0$, and for the firm are $(1 - \delta_1^f)\phi_1^f = 0$ for $\phi_1^f \in \{L_1^{f\iota}, g_1, a_1 - a_0\}$, where $\iota = s$.

Necessary first order conditions and sufficient second order conditions hold for the consumer, bank, and firm optimization. Market clearing for goods at $t \in \{0, 1, 2\}$ requires that all constraints bind for the optimizations of the consumer, bank, and firm given by optimization equations (5)-(7), with the exception of the firm's constraint $a_1 \leq a_0$.

Thus, the market equilibrium exists and is unique up to an indeterminate price level at $t = 0$, $P_{0,0}^s$, with equilibrium prices $P_t^s(\lambda, r_2)$ at $t \in \{1, 2\}$ given by equations (15) and (16), and where first order conditions for the bank's optimization determine optimal deposit and loan rates as $\hat{R}_1^{ds*} = \hat{R}_1^{fs*} = 1$ and $\hat{R}_2^{ds*} = \hat{R}_2^{fs*} = \bar{r}_2$. Note that \hat{R}_1^{ds*} , \hat{R}_2^{ds*} and \hat{R}_1^{fs*} are required to be constants not dependent on the state (λ, r_2) , since the rates are contracted at $t = 0$ and cannot be made contingent on the realization of (λ, r_2) , which is not contractible. The return \hat{R}_2^{fs*} on loans made at $t = 1$ can be contracted on (λ, r_2) realized at $t = 1$, but since it is determined in equilibrium as $\hat{R}_2^{fs*} = \hat{R}_2^{ds*}$, it also is constant and not contingent on (λ, r_2) .

Substituting with equilibrium prices from equations (15) and (16) into the budget constraints for the consumer, bank, and firm; applying market clearing conditions; and simplifying; there is no bank borrowing from the central bank, $L_1^{cb}(\lambda, r_2) = 0$, and the firm and bank default fractions equal one, showing no bank defaults, $\delta_t^d(\lambda, r_2) = 1$ for $t \in \{1, 2\}$, for any $\beta^{cb} \leq 1$.

With $\beta^{cb} = 1$, since the central bank's objective function is equivalent to that for banks, the expected utility of consumers EU , the central bank optimally sets its rates on reserves and loans, R_2^s and R_2^s equal to the market equilibrium rate R_2^{fs*} on

loans to firms made at $t = 1$ that exists without consideration of the central bank optimization (9): $R_2^{s*} = R_2^{cb*} = R_2^{fs*} = \bar{r}_2$, not contingent on (λ, r_2) .

Loans to firms made at $t = 1$ have a real return $r_2^{fs*}(\lambda, r_2) \equiv \frac{R_2^{fs*}(\lambda, r_2)}{\Pi_2^s(\lambda, r_2)}$. The firm's first order conditions with respect to $\{g_t, a_t\}_{t \in \{1, 2\}}$ determine $a_t(\lambda, r_2) = a_t^*(\lambda, r_2)$ and $g_t(\lambda, r_2) = g_t^*(\lambda, r_2)$ for $t \in \{0, 1\}$, where for $\lambda < \check{\lambda}(r_2)$, $r_2^{fs} = 1$; for $\lambda \in (\check{\lambda}(r_2), \hat{\lambda}(r_2))$, $r_2^{fs} \in (1, \frac{r_2}{r_1})$; and for $\lambda \geq \hat{\lambda}(r_2)$, $r_2^{fs} = \frac{r_2}{r_1}$. Thus, $q_1^s = q_1^{s*} = g_0^* + a_1^* r_1 - g_1^*$, and $q_2^s = q_2^{s*} = (a_0^* - a_1^*) r_2 + g_1^*$. From the consumption equations (2) - (4) and prices in equations (15) and (16), consumption for early and late consumers can be solved as

$$c_1 = \frac{\delta_1^d D_0^s R_1^{ds}}{P_1^s} = \frac{q_1^s}{\lambda} \quad (18)$$

$$c_2 = \frac{\delta_2^d D_0^s R_2^{ds}}{P_2^s} = \frac{q_2^s}{1-\lambda}, \quad (19)$$

which since $\delta_t^d = 1$ and $q_t^s = q_t^{s*}$ gives $c_t = c_t^*$ for $t \in \{1, 2\}$.

Proof for Corollary 1. Since consumers have nominal revenues at $t = 0$ of P_0^s from selling their one unit of goods endowment, their deposits are $D_0^s = P_0^s$, and expected prices are

$$\begin{aligned} E[P_{1,\tau}^s(\lambda, r_2)] &= E\left[\frac{\lambda g_0^*}{\lambda c_1^*} P_{0,\tau}^s\right] = E[P_{0,\tau}^s] \\ E[P_{2,\tau}^s(\lambda, r_2)] &= E\left[\frac{(1-\lambda)a_0^* \bar{r}_2}{(1-\lambda)c_2^*} P_{0,\tau}^s\right] = E[P_{0,\tau}^s]. \end{aligned}$$

Since the period $t = 0$ price level at date $\tau = 0$ is normalized to one, $P_{0,0}^s = 1$, $E[P_{t,0}^s] = 1$. Since $P_{0,\tau}^s = P_{2,\tau-1}^s$, we have $E[P_{0,1}^s] = 1$, and hence $E[P_{t,\tau}^s] = 1$, which implies $E[\Pi_{t,\tau}^s] = 1$ and $E[\frac{P_{t,\tau+1}^s}{P_{t,\tau}^s}]$ for $t \in \{0, 1, 2\}$.

Proof for Corollary 2. This proof is to be completed, which will show the following:

$$\begin{aligned} \partial R_2^s / \partial \beta &< 0, \quad \partial R_2^{fs} / \partial \beta < 0, \quad \partial r_2^{fs} / \partial \beta < 0, \quad \partial \Pi_2^s / \partial \beta < 0, \quad \partial a_1 / \partial \beta \leq 0, \\ \partial a_0 / \partial \beta &> 0, \quad \partial (a_0 - a_1) / \partial \beta > 0, \\ \partial q_1 / \partial \beta &< 0, \quad \partial c_1 / \partial \beta < 0, \quad \partial q_2 / \partial \beta > 0, \quad \partial c_2 / \partial \beta > 0 \end{aligned}$$

Proof for Corollary 3. From the central bank's optimization (9), the first order condition with respect to $R_2^{cb}(\lambda, r_2)$ implies that $\hat{R}_2^s(\lambda, r_2) > R_2^{fs*}(\lambda, r_2)$. The bank's first order conditions with respect to L_1^{fs} and L_1^{cb} require $R_2^{fs} = R_2^s$, hence $R_2^{fs}(\lambda, r_2) > R_2^{fs*}(\lambda, r_2)$ and $r_2^{fs}(\lambda, r_2) > r_2^{fs*}(\lambda, r_2)$.

If $\beta^{cb} < 1$ is unexpected, then $a_0 = a_0^*$ and $g_0 = g_0^*$ are unchanged. The firm's first order conditions imply that $g_1 \leq g_1^*$ and $a_1 \geq a_1^*$, with $\dot{q}_1^s > q_1^{s*}$ and $\dot{q}_2^s < q_2^{s*}$. If $\beta^{cb} < 1$ is expected, the firm's first order conditions imply that $a_0 < a_0^*$ and $g_0 > g_0^*$, which implies that $\dot{q}_1^s > q_1^{s*}$ and $\dot{q}_2^s < q_2^{s*}$. Hence, in either case, $\dot{c}_1 > c_1^*$, $\dot{c}_2 < c_2^*$, and $\dot{\Pi}_2^s > \Pi_2^{s*}$.

Section 4: Digital currency

Proof for Lemma 2. The private issuer's three budget constraints for $t = 0, 1, 2$ can be combined as the single budget constraint $M_{2,\tau}^y \leq q_1^y P_1^v + q_2^y P_2^v - q_0^y P_0^v + M_{2,\tau-1}^y$. The private issuer's first order conditions with respect to $M_{2,\tau}^y$, q_0^y and q_1^y are $P_{2,\tau}^v = E[P_{2,\tau}^v]$, $E[P_2^v] = P_0^v$, and $E[P_1^v] = P_0^v$ with complementary slackness conditions. The proof is to be completed and to show $E_{0,\tau}[\frac{P_{0,\tau+1}^v}{P_{2,\tau}^v}] = 1$, $E_{0,\tau}[\frac{P_{1,\tau+1}^v}{P_{2,\tau}^v}] = 1$, and $E_{0,\tau}[\frac{P_{2,\tau+1}^v}{P_{2,\tau}^v}] = 1$.

Proof for Proposition 2. The proof follows similar to the proof for fiat deposits in proposition 1. The complementary slackness conditions in relationship to a default by the bank or firm are as given in that proof but with $\iota = v$. Necessary first order conditions and sufficient second order conditions hold for the consumer, bank, firm, and issuer optimizations. Market clearing for goods at $t \in \{0, 1, 2\}$ requires that all constraints bind for the optimizations of the consumer, bank, firm, and issuer given by optimization equations (5)-(8), with the exception of the firm's constraint $a_1 \leq a_0$. Thus, the market equilibrium exists and is unique. The market clearing conditions for $t = 1$ and $t = 2$ are represented by $\lambda D^v R_1^{dv} = (q_1^v + q_1^y) P_1^v$ and $(1 - \lambda) D^v R_2^{dv} = (q_2^v + q_2^y) P_2^v$, with equilibrium prices $P_t^v(\lambda, r_2)$ at $t \in \{1, 2\}$ given by equations (15) and (16) and consumption given by equations (18) and (19), in which the superscript s is replaced by v .

The bank's first order conditions give the bank's Euler equation, $E[u'(c_1) \frac{1}{P_1}] = E[u'(c_2) \frac{1}{P_2} R_2^{fv}]$ with complementary slackness condition $(R_2^{fv} - 1) M_1^{bv} = 0$. The first order conditions for the bank's optimization and determine deposit and loan rates $R_1^{dv*} = R_1^{fv*} = 1$, $R_2^{dv*} = \bar{r}_2$ that are equivalent and $R_2^{fv*}(\cdot)$ that is different to those for the fiat case.

The firm's first order condition with respect to a_0 gives the firm's Euler equation, $E[\frac{R_2^{fv}(\cdot)}{\Pi_2^v(\cdot)}] = \bar{r}$, which for the real return $r_t^{fv}(\lambda, r_2) = \frac{R_t^{fv^*}(\lambda, r_2)}{\Pi_t^v(\lambda, r_2)}$ with the nominal rate functions determined by the bank and the equilibrium price functions, gives $r_2^{fv^*}(\lambda, r_2) = r_2^{fs^*}(\lambda, r_2)$.

At $t = 1$, for $\lambda < \check{\lambda}(r_2)$, $r_2^{fv^*} = 1$ and firms optimally store $g_1 > 0$. For $\lambda \in [\check{\lambda}(r_2), \hat{\lambda}(r_2)]$, $r_2^{fv^*} \in [1, \frac{r_2}{r_1}]$ and firms do not store or liquidate with $g_1 = a_1^* = 0$. For $\lambda > \hat{\lambda}(r_2)$, $r_2^{fv^*} = \frac{r_2}{r_1}$ and firms partially liquidate $a_1^* > 0$. For all states (λ, r_2) , firms have zero consumption: $c_2^f = 0$.

Define $\alpha \equiv \frac{a_0}{g_0 + a_0}$ and $x \equiv g_0 + a_0$. Substituting with equilibrium prices into the budget constraints for the consumer, bank, firm, and issuer; applying market clearing conditions; and simplifying to solve for defaults; the equilibrium at $t = 1$ has no defaults with $\delta_1^d(\lambda, r_2) = \delta_1^f(\lambda, r_2) = 1$ for all λ, r_2 , and at $t = 2$ has no absolute default with $\delta_2^d(\lambda, r_2) \hat{R}_2^{dv}(\lambda, r_2) \geq 1$ if either $r_2 \geq 1$ or $\lambda(r_2) > \check{\lambda}^v(r_2) \equiv \frac{1 - \alpha + (1 - x)\alpha r_2}{1 - \alpha + \alpha r_2}$ and has an absolute default with $\delta_2^d(\lambda, r_2) \hat{R}_2^{dv}(\lambda, r_2) = 1 - \alpha + \alpha r_2 < 1$ if $r_2 < 1$ and $\lambda(r_2) \leq \check{\lambda}^v(r_2)$, where $\delta_2^d(\lambda, r_2) \hat{R}_2^{dv}(\lambda, r_2) < 1$ since $g_0 + g_0^y + a_0 = 1$.

Deposits give a real return in terms of consumption of $c_1^{dv}(\cdot) = \frac{R_1^{dv}}{P_1^v} = \frac{1}{P_1^v} = \frac{q_1^v + q_1^y}{\lambda}$ for early consumers and $c_2^{dv}(\cdot) = \frac{R_2^{dv}}{P_2^v} = \frac{q_2^v + q_2^y}{1 - \lambda}$ for late consumers per unit deposited.

These consumption functions $c_1^{dv}(\cdot)$ and $c_2^{dv}(\cdot)$, with $r_2^{fv^*}(\lambda, r_2) = r_2^{fs^*}(\lambda, r_2)$ and the Euler equations for the bank and firm, give the optimal Euler equation $E[u'(c_1^*(\cdot))] = E[r_2 u'(c_2^*(\cdot))]$ of the first best case and the fiat case for $\beta^{cb} = 1$, which implies the optimal investment a_0^* at $t = 0$ and optimal investment functions $a_1^*(\lambda, r_2)$ and $g_1^*(\lambda, r_2) = g_1(\lambda, r_2) + g_1^y(\lambda, r_2)$ at $t = 1$ for all states (λ, r_2) .

Combined with the goods sold and consumed by the private issuer at $t = 1$ and $t = 2$, total output equals the first best for all states (λ, r_2) , which at $t = 1$ is $q_1^v + q_1^y + c_1^y = q_1^*(\cdot) = g_0^* + a_1^* r_1 - g_1^* = \lambda c_1^*$, and at $t = 2$ is $q_2^v + q_2^y + c_2^y = q_2^*(\cdot) = (a_0^* - a_1^*) r_2 + g_1^* = (1 - \lambda) c_2^*$, corresponding to the state (λ, r_2) according to $\lambda < \check{\lambda}(r_2)$, $\lambda \in [\check{\lambda}(r_2), \hat{\lambda}(r_2)]$, and $\lambda > \hat{\lambda}(r_2)$.

With consumption of $c_1^{dv}(\cdot) = \frac{q_1^v(\cdot) + q_1^y(\cdot)}{\lambda}$ and $c_2^{dv}(\cdot) = \frac{q_2^v(\cdot) + q_2^y(\cdot)}{1 - \lambda}$, the incentive constraint for late consumers holds, $c_2^{dv}(\cdot) \geq c_1^{dv}(\cdot)$ for all (λ, r_2) .

Proof for Lemma 3. If a consumer lends private digital currency at $t = 0$, the initial return is $\hat{R}_1^{cv} = \hat{R}_1^{fv} \geq 1$ since the loan market is competitive at $t = 0$.

The firm's budget constraint at $t = 1$ is $\delta_1^e \hat{R}_1^{cv} = q_1^e P_1^v$. The amount of goods the firm sells to repay its loan is $q_1^e = g_0^c + a_1^e r_1 - g_1^e - c^f$. Even by fully liquidating its assets, with $a_1^e = a_0^c$, the maximum the firm can sell is $q_1^e = g_0^c + a_0^c r_1 \leq q_0^c \leq 1$. Since the price level of goods in terms of private digital currency as established above is $P_t^v \leq 1$ for $t = 1, 2$, the return that the firm repays is $\delta_1^e \hat{R}_1^{cv} = q_1^e P_1^v \leq 1$. Since $\hat{R}_1^{cv} \geq 1$, $\delta_1^e < 1$ if either $q_1^e P_1^v < 1$ or $\hat{R}_1^{cv} > 1$. In particular, if the firm invests any amount $a_0^c > 0$, $q_1^e < 1$ and the firm defaults, $\delta_1^e < 1$, requiring full asset liquidation $a_1^e = a_0^c$.

The early consumer has a real return in terms of consumption of $c_1 = \frac{\delta_1^e \hat{R}_1^{cv}}{P_1^v} \leq \frac{\hat{R}_1^{dv}}{P_1^v}$, with $c_1 < c_1^* = \frac{\hat{R}_1^{dv}}{P_1^v}$ if $a_0^c > 0$. The consumption is $c_1 = \frac{\delta_1^e \hat{R}_1^{cv}}{P_1^v} = q_1^e \leq 1$, with $c_1 < 1$ if $a_0^c > 0$.

Proof for Lemma 4. With public digital currency, inflation at $t = 2$ of any date τ , $\Pi_{2,\tau}^s(\lambda, r_2) = \frac{P_2^s}{P_1^s}$, is independent of $D_\tau^s(M^s)$. Hence, the firm's real return $r_2^{fs}(\lambda, r_2) \equiv \frac{R_2^{fs}(\lambda, r_2)}{\Pi_2^s(\lambda, r_2)}$ is independent of $D_\tau^s(M^s)$, which implies that for $t \in \{1, 2\}$, q_t^s , and thus c_t given by equations (18) and (19), are independent of $D_\tau^s(M^s)$ and M^s .

With public digital currency, equilibrium prices at $t \in \{1, 2\}$ are

$$\begin{aligned} P_1^s(\lambda, r_2) &= \frac{\lambda(D_0^s R_1^{ds} + M_0^{cs}) + (1-\lambda)(M_0^{cs} - M_1^{cs})}{q_1^s} \\ P_2^s(\lambda, r_2) &= \frac{(1-\lambda)(D_0^s R_2^{ds} + M_1^{cs})}{q_2^s}. \end{aligned}$$

Hence, $\frac{R_t^{ds}}{P_t^s} > \frac{1}{P_t^s}$ for $t \in \{1, 2\}$, which implies from the consumer's first order conditions that $M_0^{cs} = M_1^{cs} = 0$.

Proof for Proposition 3. From corollary 3, for $\beta^{cb} < 1$, the expected utility of fiat money deposits and public digital currency is less than that of the optimal consumption allocation $\{c_t^*\}_{t \in \{1,2\}}$. From the proof of proposition 2, regardless of β^{cb} , the expected utility of private digital currency deposits is equal to that of the optimal consumption allocation $\{c_t^*\}_{t \in \{1,2\}}$. Hence, consumers do not hold fiat money deposits or public digital currency, $D_0^s = M_0^{cs} = 0$, and only hold private digital currency deposits D_0^v .

Section 5: Digital currency runs

Proof for Lemma 5. Consumption for late consumers withdrawing i) early at $t = 1$ equals that of early consumers, $c_1^l = c_1$, and ii) at $t = 2$ equals c_2 , for c_1 and c_2 determined in the proofs for proposition 1 if $\beta^{cb} = 1$ and corollary 3 if $\beta^{cb} < 1$.

Proof for Proposition 4. Binding budget constraints consolidated for the bank and firm are:

$$\begin{aligned} t=1: \quad & \lambda\delta_1 = (g_0 + a_1r_1 - g_1)P_1^l + M_0^{bl} - M_1^{bl} \\ t=2: \quad & (1 - \lambda)\delta_2R_2^{dl} = [g_1 + (a_0 - a_1)r_2]P_2^l + M_1^{bl} \\ t=1,2: \quad & \lambda\delta_1 + (1 - \lambda)R_2^{dl}(g_0 + a_1r_1 - g_1)P_1^l + [g_1 + (a_0 - a_1)r_2]P_2^l + M_0^{bl} \end{aligned}$$

With all late consumers withdrawing $w^v = 1$ and holding private digital currency, the bank would default at $t = 1$, $\delta_1^d < 1$, since the bank's $t = 0$ loans to firms are too illiquid to receive their par value repaid back at $t = 1$ to pay out all withdrawals at $t = 1$, and in particular does not have enough private reserves held from $t = 0$ to pay all of the late consumer demands for withdrawals in the form of digital currency, since $1 - \check{\lambda}^v(r_2) \geq \frac{\alpha r_2}{1 - \alpha + \alpha r_2}$.

First, we show using a proof by contradiction that for $r_2 < 1$, the firm defaults at $t = 1$ with $\delta_1^f \hat{R}_1^{fv} < 1$ or at $t = 2$ with $R_2^{fv} < 1$. Suppose instead that for $r_2 < 1$, $\delta_1^f \hat{R}_1^{fv} \geq 1$ and $\hat{R}_2^{fv} \geq 1$. The firm's budget constraints for $t = 0, 1, 2$ in equation set (7) can be combined into the single budget constraint $(\delta_1^f q_0^v P_0^v \hat{R}_1^{fv} - q_1^v P_1^v) \hat{R}_2^{fv} \leq q_2^v P_2^v$. With $\delta_1^f \hat{R}_1^{fv} \geq 1$ and $\hat{R}_2^{fv} \geq 1$; and with $P_0^v = 1$, $P_1^v \not\leq 1$ and $P_2^v \leq 1$ from lemma (2), the single budget constraint can be written as $q_0^v \leq q_1^v + q_2^v$. Substituting with the firm's feasibility constraints in equation set (7) and simplifying, the budget constraint is $a_0(r_2 - 1) - a_1(r_2 - r_1) > 0$, which is a contradiction for $r_2 < 1$ since $a_0 > 0$.

Proof for Corollary 4. To be completed.

Proof for Proposition 5. To be completed.

Section 6: Regional altcoin

Proof for Lemma 6. For the global economy using the global private digital currency, the equilibrium is determined for prices following the proof for lemma 2 with

the macro state $(\bar{\lambda}, \bar{r}_2)$, as $P_{t,\tau}^\psi = 1$ for all $t \in \{0, 1, 2\}$ and all τ , where the superscript $\iota = \psi$ is used to denote the global currency. within the global economy.

The equilibrium is determined for bank, firm, global currency issuer, and consumer quantities and returns following the proof for proposition 2 with macro state $(\bar{\lambda}, \bar{r}_2)$ and $P_{t,\tau}^\psi = 1$. The bank's first order conditions give the bank's Euler equation, $E[u'(c_1)] = E[u'(c_2)r_2^{f\psi}]$, and loan and deposit returns $r_1^{f\psi^*} = 1$, $r_2^{f\psi^*} = \bar{r}_2$, $r_1^{d\psi^*} = c_1^*$, and $r_2^{d\psi^*} = c_2^*$. The firm's and issuer's first order conditions gives the optimal investment $a_0 = a_0^*$, $g_0 + g_0^\psi = g_0^*$, $a_1 = a_1^* = 0$, $g_1 = g_1^\psi = g_1^* = 0$. The equilibrium has no defaults, $\delta_t^d = \delta_t^f = 1$ since $r_2 > 1$. Hence, $w^\psi = 0$, $c_t = r_t^{d\psi} = c_t^*$ for $t = 1, 2$, and $c_2^f = c_2^\psi = 0$. The first best allocation obtains since there is no loss on assets, $r_2 > 1$.

For the regional economy using the global private digital currency with banking, the equilibrium is determined for bank, firm, and consumer quantities and returns following the proof for proposition 2 with macro state (λ, r_2) but where prices $P_{t,\tau}^z \equiv 1$ for all $t \in \{0, 1, 2\}$ and all τ .

The bank's first order conditions give the bank's Euler equation, $E[u'(c_1)] = E[u'(c_2)r_2^{fz}(\cdot)]$ with complementary slackness condition $(r_2^{fz} - 1)m_1^{bz} = 0$ and loan and deposit returns $r_1^{fz} = 1$, $r_2^{fz}(\cdot)$, r_1^{dz} , and r_2^{dz} .

The firm's first order conditions give $a_1(\cdot)$, and $g_1(\cdot)$ and the firm's Euler equation, $E[r_2^{fz}(\cdot)] = \bar{r}$, which with $r_2^{fz}(\cdot)$ from the bank's first order condition, give a_0 and g_0 . Consumers deposit $d^z = 1$ and have consumption $c_1 = \delta_1^d r_1^{dz}$, $c_1^\ell = w^z \delta_1^d r_1^{dz}$, and $c_2 = (1 - w^z) \delta_2^d r_2^{dz}$, where as in the case of the global economy, digital currency that is withdrawn by late consumers at $t = 1$ and stored to buy goods at $t = 2$ does not need to be considered without loss of generality.

The budget constraints for the bank and firm can be consolidated as:

$$\begin{aligned}
t=0: & \quad g_0 + a_0 + m_0^{bz} = 1 \\
t=1: & \quad [\lambda + (1 - \lambda)w^z] \delta_1 r_1^{dz} = g_0 + a_1 r_1 - g_1 + m_0^{bz} - m_1^{bz} \\
t=2: & \quad (1 - \lambda)(1 - w^z) \delta_2 r_2^{dz} = g_1 + (a_0 - a_1) r_2 + m_1^{bz} \\
t=1,2: & \quad \lambda \delta_1^{dz} r_1^{dz} + (1 - \lambda) \delta_2 r_2^{dz} = g_0 + a_0 r_2 - a_1 (r_2 - r_1) + m_0^{bz}
\end{aligned}$$

First consider if $w^z = 0$. The consolidated budget constraint for $t = 1$ requires that for the bank not to default, $\delta_1^d = 1$, then for $\lambda \leq \hat{\lambda}^z \equiv \frac{g_0 + M_0^{bz}}{R_1^{dz}}$, there is an amount of combined goods and digital currency that is available to store until $t = 2$,

$g_1 + m_1^{bz} = \lambda r_1^{dz} - (1 - \alpha^z) \geq 0$, whereas if $\lambda > \hat{\lambda}^z \equiv \frac{g_0 + M_0^{bz}}{R_1^{dz}}$, there is a required amount of investment liquidation, $a_1 = \frac{\lambda r_1^{dz} - (1 - \alpha^z)}{r_1} = \frac{\lambda r_1^{dz} - g_0 - m_0^{bz}}{r_1} > 0$.

Substituting for these into the consolidated budget constraint at $t = 2$ and simplifying, the incentive constraint for the late consumer to not withdraw at $t = 1$, $\delta_2^d r_2^{dz} < r_1^{dz}$, is violated if either $r_2 < \frac{r_1^{dz} - g_0 - m_0^{bz}}{a_0}$ for any λ or if $r_2 < \frac{(1 - \lambda)r_1^{dz} r_1}{g_0 + m_0^{bz} + a_0 r_1 - \lambda r_1^{dz}}$ for the following.

For $\lambda > \hat{\lambda}^z(r_2)$, the amount of liquidation a_1 is so large that the bank has a large enough default at $t = 2$ such that the late consumer's incentive constraint is violated, $\delta_2^d c_2 < c_1$. Late consumers run the bank and fully withdraw at $t = 1$, $w^z = 1$. The bank and firm default and fully liquidate assets at $t = 1$, $\delta_1^d < 1$, $\delta_1^f < 1$, and $a_1 = a_0$.

Based on $c_1 = 1$, for $\lambda \in (\hat{\lambda}^{dv}, \hat{\lambda}(r_2))$, where $\hat{\lambda}^{dv} \equiv g_0 + g_0^y$ and $\hat{\lambda}^{dv} \in (\check{\lambda}(r_2), \hat{\lambda}(r_2))$, there is excessive liquidation required for the firm and bank not to default at $t = 1$, which gives suboptimal consumption for late consumers: $a_1(\lambda, r_2) = \frac{\lambda - \hat{\lambda}^{dv}}{r_1} > a_1^*(\lambda, r_2) = 0$ and $c_2(\lambda, r_2) < c_2^*(\lambda, r_2)$.

For $\lambda \in [\check{\lambda}(r_2), \hat{\lambda}(r_2)]$, $r_2^{fz^*} \in [1, \frac{r_2}{r_1}]$ and firms do not store or liquidate with $g_1 = a_1^* = 0$. For $\lambda > \hat{\lambda}(r_2)$, $r_2^{fz^*} = \frac{r_2}{r_1}$ and firms partially liquidate $\alpha_1^* > 0$. For all states (λ, r_2) , firms have zero consumption: $c_2^f = 0$.

Substituting with equilibrium prices into the budget constraints for the consumer, bank, firm, and issuer; applying market clearing conditions; and simplifying to solve for defaults; the equilibrium at $t = 1$ has no defaults with $\delta_1^d(\lambda, r_2) = \delta_1^f(\lambda, r_2) = 1$ for all λ, r_2 , and at $t = 2$ has no absolute default with $\delta_2^d(\lambda, r_2) R_2^{dz}(\lambda, r_2) \geq 1$ for either $r_2 \geq 1$ or $\lambda(r_2) > \check{\lambda}^z(r_2) \equiv \frac{1 - \alpha + (1 - x)\alpha r_2}{1 - \alpha + \alpha r_2}$ and has an absolute default with $\delta_2^d(\lambda, r_2) R_2^{dz}(\lambda, r_2) = 1 - \alpha + \alpha r_2 < 1$ if $r_2 < 1$ and $\lambda(r_2) \leq \check{\lambda}^z(r_2)$, where $\delta_2^d(\lambda, r_2) R_2^{dz}(\lambda, r_2) < 1$ since $g_0 + g_0^y + a_0 = 1$.

Deposits give a real return in terms of consumption of $c_1^{dz}(\cdot) = \frac{\delta_1^d R_1^{dz}}{P_1^z} = \frac{1}{P_1^z} = \frac{q_1^z + q_1^y}{\lambda}$ for early consumers and $c_2^{dz}(\cdot) = \frac{\delta_2^d R_2^{dz}}{P_2^z} = \frac{q_2^z + q_2^y}{1 - \lambda}$ for late consumers per unit deposited.

These consumption functions $c_1^{dz}(\cdot)$ and $c_2^{dz}(\cdot)$, with $r_2^{fz^*}(\lambda, r_2) = r_2^{fs^*}(\lambda, r_2)$ and the Euler equations for the bank and firm, give the optimal Euler equation $E[u'(c_1^*(\cdot))] = E[r_2 u'(c_2^*(\cdot))]$ of the first best case and the fiat case for $\beta^{cb} = 1$, which implies the optimal investment a_0^* at $t = 0$ and optimal investment functions $a_1^*(\lambda, r_2)$ and $g_1^*(\lambda, r_2) = g_1(\lambda, r_2) + g_1^y(\lambda, r_2)$ at $t = 1$ for all states (λ, r_2) .

Combined with the goods sold and consumed by the private issuer at $t = 1$ and $t = 2$, total output equals the first best for all states (λ, r_2) , which at $t = 1$ is $q_1^z + q_1^y + c_1^y = q_1^*(\cdot) = g_0^* + a_1^* r_1 - g_1^* = \lambda c_1^*$, and at $t = 2$ is $q_2^z + q_2^y + c_2^y = q_2^*(\cdot) = (a_0^* - a_1^*) r_2 + g_1^* = (1 - \lambda) c_2^*$, corresponding to the state (λ, r_2) according to $\lambda < \check{\lambda}(r_2)$, $\lambda \in [\check{\lambda}(r_2), \hat{\lambda}(r_2)]$, and $\lambda > \hat{\lambda}(r_2)$.

There is the optimal rationing of goods between early and late consumers through the optimal quantity of goods sold by firms and the private issuer at $t = 1$ and $t = 2$ for all macro states (λ, r_2) .

With consumption of $c_1^{dz}(\cdot) = \frac{q_1^z(\cdot) + q_1^y(\cdot)}{\lambda}$ and $c_2^{dz}(\cdot) = \frac{q_2^z(\cdot) + q_2^y(\cdot)}{1 - \lambda}$, the incentive constraint for late consumers holds, $c_2^{dz}(\cdot) \geq c_1^{dz}(\cdot)$ for all (λ, r_2) .

Proof for Proposition 6. To be completed.

Section 8: Strategic digital currency runs

Proof for Lemma 7. Consider a withdrawal strategy set σ without early withdrawals, $w^t(\lambda, r_2) = 0$, which for feasibility requires $M_1^{cuw}(\lambda, r_2) = 0$, for all $\lambda \in (0, 1)$, $r_2 \in (0, r_2^{\max})$, and $i \in I$. Consumption for depositors is equivalent to that from proposition 1, corollary 3 and lemma 2, with optimal consumption for fiat money deposits with $\beta^{cb} = 1$ and for private digital currency deposits, and with suboptimal consumption for fiat money deposits with $\beta^{cb} < 1$.

In particular, a late consumer's consumption at $t = 2$ is $c_2 = \frac{D^\iota R_2^{d\iota}}{P_2^\iota}$ for $\iota \in \{s, v\}$. Suppose there is a deviation withdrawal strategy $\sigma^{i''}$ by a late consumer i'' , such that $w^{i''}(\lambda, r_2) > 0$ and $M_1^{i''}(\lambda, r_2) \leq w^{i''}(\lambda, r_2)$ for any $\lambda \in (0, 1)$ and $r_2 \in (0, r_2^{\max})$. This late consumer's consumption is $c_1^{i''} + c_2^{i''}$, where $c_1^{i''} = \sum_\iota \frac{w^{i''} D^\iota R_1^{d\iota} + M_0^{c\iota} - M_1^{c\iota i} - M_1^{c\iota w i}}{P_1^\iota}$, $c_2^{i''} = \frac{(1 - w^{i''}) D^\iota R_2^{d\iota} + M_1^{c\iota i} - M_1^{c\iota w i}}{P_2^\iota}$, and hence $c_1^{i''} + c_2^{i''} < c_2$. Thus, given the withdrawal strategy set σ , including the withdrawal strategies for late consumers $i' \neq i$, $\{\sigma^{i'}\}_{i' \in I}$, where $\sigma^{i'} = \{0, 0\}$; $\sigma^i = \{0, 0\}$ is a weakly best response for all (λ, r_2) and a strictly best response for $\lambda(r_2) > \check{\lambda}(r_2)$. Hence, σ is a Nash equilibrium of the withdrawal game.

Proof for Corollary 5. Consider a withdrawal strategy set σ with complete early withdrawals, $w^{i\iota} = 1$ in the form of demands for digital currency, $M^{cuw i} = D^\iota \hat{R}_1^{d\iota}$,

for all late consumers $i \in I$. For the case of $M_0^{bc} < D^t \hat{R}_2^{dt}$, it is not feasible to pay these early withdrawal demands in digital currency, which implies the bank defaults at $t = 1$. For the case of $(1 - \lambda)D^t \hat{R}_1^{dt} > M_0^{bc}$, the bank's budget constraint at $t = 1$ implies that the bank defaults at $t = 1$, $\delta_1^d < 1$, does not roll over any lending to firms, $L_1^f = 0$, and hence has no revenues for withdrawals at $t = 2$, for a complete default at $t = 2$, $\delta_2^d = 0$.

Suppose there is any deviation in the withdrawal strategy $\sigma^{i''}$ by any late consumer i'' . For $w^{i''} < 1$, the late consumer receives no amount for the withdrawal of $(1 - w^{i''})$ at $t = 2$. For an early withdrawal demand not in digital currency, consumption $c_1^{i''} + c_2^{i''}$ is unchanged. Hence, σ is a Nash equilibrium.

Proof for Corollary 6. Consider any withdrawal strategy set σ with positive early withdrawals for any set $I' \in I$ of late consumers. The bank can borrow from the central bank the amount of the public digital currency withdrawals at $t = 1$: $L_1^{cb} = (1 - \lambda) \int_{i' \in I'} M_1^{cswi'}$. There is no default for the bank, which implies that the withdrawal strategy with a positive amount of early withdrawals for each late consumer $i' \in I'$ is not a best response. Hence, the Nash equilibrium without early withdrawals is unique.

Proof for Corollary 7. Following from the proof of corollary 6, for banks with public digital currency deposits, there is a unique Nash equilibrium without early withdrawals by late consumers. For banks with private digital currency deposits, for $\lambda < \frac{1}{1+m_{0,\tau}^b}$, consider complete withdrawals in the form of digital currency by all late consumers. For any positive amount of bank borrowing in the form of fiat money from the central bank, $L_1^{cb} > 0$, the bank would default on repaying the central bank at $t = 2$, $\delta_2^{cb} < 1$, which rules out such borrowing in equilibrium: $L_1^{cb} = 0$. Hence, following the proof of corollary 5, the withdrawal run is a Nash equilibrium.

References

- Abadi and Brunnermeier (2018). “Blockchain Economics,” working paper.
- Acharya, Viral and David Skeie (2011). “A Model of Liquidity Hoarding and Term Premia in Inter-Bank Markets” *Journal of Monetary Economics* 58(5).
- Aghion, Philippe, Mathias Dewatripont and Patrick Bolton (2000). “Contagious Bank Failures in a Free Banking System,” *European Economic Review* 44.
- Allen, Franklin, Elena Carletti and Douglas Gale (2014). “Money, Financial Stability and Efficiency,” *Journal of Economic Theory* 149.
- Allen, Franklin and Douglas Gale (1998). “Optimal Financial Crises,” *Journal of Finance* 53(4).
- Allen, Franklin and Douglas Gale (2000a). “Financial Contagion,” *Journal of Political Economy* 108(1).
- Allen, Franklin and Douglas Gale (2000b). “Optimal Currency Crises,” *Carnegie-Rochester Series on Public Policy* 53.
- Allen, Franklin and Douglas Gale (2004). “Financial Intermediaries and Markets,” *Econometrica* 72.
- Allen, Franklin and David Skeie (2018). “Financial Stability with Commercial and Central Banking and Money,” work in progress.
- Andolfatto (2018). “Assessing the Impact of Central Bank Digital Currency on Private Banks,” working paper.
- Ashcraft, Adam, James McAndrews and David Skeie (2011). “Precautionary Reserves and the Interbank Market,” *Journal of Money, Credit and Banking* 43(s2).
- Bagehot, Walter (1873). *Lombard Street*, (H.S. King, London).
- Balyuk and Davydenko (2018). “Reintermediation in FinTech: Evidence from Online Lending,” working paper.
- Bech, Morten and Garratt, Rodney (2017). “Central Bank Cryptocurrencies,” *BIS Quarterly Review*.
- Bhattacharya, Sudipto and Douglas Gale (1987). “Preference Shocks, Liquidity and Central Bank Policy,” in William Barnett and Kenneth Singleton, eds: *New Approaches to Monetary Economics* (Cambridge University Press, New York, N.Y.).

- Bhattacharya, Sudipto and Paolo Fulghieri (1994). “Uncertain Liquidity and Inter-bank Contracting,” *Economics Letters* 44.
- Biais, Bisiere, Bouvard and Casamatta (2018). “The Blockchain Folk Theorem,” working paper.
- Biais, Bisiere, Bouvard, Casamatta and Menkveld (2018). “Equilibrium Bitcoin Pricing,” working paper.
- Bohannon (2016). “Why Criminals Can’t Hide behind Bitcoin,” *Science Magazine* March 9.
- Bolt and van Oordt (2016). “On the Value of Virtual Currencies,” working paper.
- Bordo, Michael D. and Levin, Andrew T. (2017). “Central Bank Digital Currency and the Future of Monetary Policy,” Hoover Institution Economics Working Paper 17104
- Broadbent, Ben (2016). “Central Banks and Digital Currencies,” available at <https://www.bankofengland.co.uk/speech/2016/central-banks-and-digital-currencies>.
- Brunnermeier, Markus and Martin Oehmke (2013). “The Maturity Rat Race,” *Journal of Finance* 68(3).
- Brunnermeier, Markus K, and Yuliy Sannikov (2016). “The I Theory of Money,”. NBER Working Paper No. 22533.
- Cao, Cong and Yang (2018). “Auditing and Blockchains: Pricing, Misstatements and Regulation,” working paper.
- Cawrey, Daniel (2014). “Icelandic Parliament Committee Holds Closed Session to Discuss Auroracoin,” *Coindesk* March 14.
- Champ, Bruce, Bruce D. Smith and Stephen D. Williamson (1996). “Currency Elasticity and Banking Panics: Theory and Evidence,” *Canadian Journal of Economics* 29(4).
- Chang, Roberto and Andres Velasco (2000). “Financial Fragility and the Exchange Rate Regime,” *Journal of Economic Theory* 92.
- Cong, Lin William and Zhiguo He (2018). “Blockchain Disruption and Smart Contracts,” working paper.
- Cong, He and Li (2018). “Decentralized Mining in Centralized Pools,” working paper.
- Cong, Li and Wang (2018). “Tokenomics: Dynamic Adoption and Valuation,” working paper.

- Cruz, Kevin (2015). “BTC.sx Rebrands as Magnr, Introduces Bitcoin Savings Accounts,” *Bitcoin Magazine* June 26.
- Dang, Tri Vi, Gary Gorton and Bengt Holmström (2013). “Haircuts and Repo Chains,” working paper.
- DeAngelo, Harry and René M Stulz (2015). “Liquid-Claim Production, Risk Management, and Bank Capital Structure: Why High Leverage is Optimal for Banks,” *Journal of Financial Economics* 116(2).
- Diamond, Douglas W. and Philip H. Dybvig (1983). “Bank Runs, Deposit Insurance, and Liquidity,” *Journal of Political Economy* 91(3).
- Diamond, Douglas W. and Raghuram G. Rajan (2001). “Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking,” *Journal of Political Economy* 109(2).
- Diamond, Douglas W. and Raghuram G. Rajan (2005). “Liquidity Shortages and Banking Crises,” *Journal of Finance* 60.
- Diamond, Douglas W. and Raghuram G. Rajan, (2006), “Money in a Theory of Banking,” *American Economic Review* 96.
- Easley, O’Hara and Basu (2017). “From Mining to Markets: The Evolution of Bitcoin Transaction Fees,” working paper.
- Fernandez-Villaverde and Sanches (2017a). “Cryptocurrencies: Some Lessons from Monetary Economics,” working paper.
- Fernandez-Villaverde and Sanches (2017b). “Can Currency Competition Work?” working paper.
- Flannery, Mark J. (1996). “Financial Crises, Payment System Problems, and Discount Window Lending,” *Journal of Money, Credit, and Banking* 28(4).
- Freixas, Xavier and Cornelia Holthausen (2001). “Interbank Market Integration Under Asymmetric Information,” ECB Working Paper Series, Working Paper #74.
- Freixas, Xavier, Antoine Martin and David Skeie (2011). “Bank Liquidity, Interbank Markets and Monetary Policy,” *Review of Financial Studies* 24(8).
- Freixas, Xavier, Bruno M. Parigi, and Jean-Charles Rochet (2000). “Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank,” *Journal of Money, Credit, and Banking* 32(3).

- Freixas, Xavier, Bruno M. Parigi, and Jean-Charles Rochet (2003). “Interbank Market and Lender of Last Resort Policy,” working paper.
- Furfine, Craig H. (1999). “The Microstructure of the Federal Funds Market,” *Financial Markets, Institutions & Instruments* 8(5).
- He, Zhiguo and Wei Xiong (2012). “Dynamic Debt Runs,” *Review of Financial Studies* 25(6).
- Gale, Douglas and Xavier Vives (2002). “Dollarization, Bailouts, and the Stability of the Banking System,” *Quarterly Journal of Economics* 117(2).
- Gertler, Mark and Nobu Kiyotaki (2015). “Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy,” *American Economic Review* 105(7).
- Goldstein, Itay and Ady Pauzner (2005). “Demand–Deposit Contracts and the Probability of Bank Runs,” *Journal of Finance* 60(3).
- Goodfriend, Marvin and Robert G. King (1988). “Financial Deregulation, Monetary Policy, and Central Banking,” in William S. Haraf and Rose Marie Kushmeider, eds: *Restructuring Banking and Financial Services in America* (American Enterprise Institute for Public Policy Research, Washington, D.C.).
- Gorton, Gary (1988). “Banking Panics and Business Cycles,” *Oxford Economic Papers* 40.
- Gorton, Gary and George Pennachi (1990). “Financial Intermediaries and Liquidity Creation,” *Journal of Finance* 45(1).
- Hancock, Diana and James A. Wilcox (1996). “Intraday Management of Bank Reserves: The Effects of Caps and Fees on Daylight Overdrafts,” *Journal of Money, Credit, and Banking* 28(4).
- Hayek, Friedrich A. (1976). *Denationalisation of Money: The Argument Refined*, Institute of Economic Affairs.
- Henckel, Timo, Alain Ize, and Arto Kovanen (1999). “Central Banking Without Central Bank Money,” *Working Paper of the International Monetary Fund*, WP/99/92.
- Holmström, Bengt and Jean Tirole (1998). “Private and Public Supply of Liquidity,” *Journal of Political Economy* 106(1).
- Hosain, Md Sajjad (2018). “Bitcoin: Future transaction currency?” Munich Personal RePEc Archive Paper No. 87588.

Jack, William, and Tavneet Suri (2011). “Mobile Money: The Economics of M-Pesa,” NBER Working Paper No. 16721.

Jacklin, Charles J. (1987). “Demand Deposits, Trading Restrictions, and Risk Sharing,” in Edward C. Prescott and Neil Wallace, eds: *Contractual Arrangements for Intertemporal Trade* (University of Minnesota Press, Minneapolis).

Jawaheri, Sabah, Boshmaf and Erbad (2018). “When A Small Leak Sinks A Great Ship: Deanonimizing Tor Hidden Service Users Through Bitcoin Transactions Analysis,” arXiv:1801.07501v2.

Johnson, Amanda B. (2015). “How Can a Bitcoin Bank Pay Interest?” *Cointelegraph* June 19.

Keister and Sanches (2018). “Managing Aggregate Liquidity: The Case of a Central Bank Digital Currency,” working paper.

Kharif, Olga (2019). “Lenders Are Thriving on Bitcoin’s Bust by Aiding Both Fanatics and Shorts,” *Bloomberg* January 2.

Kim, Thomas (2015). “The Predecessors of Bitcoin and Their Implications for the Prospect of Virtual Currencies,” *PLoS ONE* 10(4).

Konig, J.P. (2014). “Fedcoin,” available at <http://jpkoning.blogspot.com/2014/10/fedcoin.html>.

Kroll, Davey and Felten (2013). “The Economics of Bitcoin Mining,” working paper.

Li and Mann (2018). “Initial Coin Offering and Platform Building,” working paper.

Lielacher, Alex (2017). “Japan to Receive Its First Interest-Paying Bitcoin Deposit Accounts,” *Bitcoin Magazine* May 10.

Madore, P. H. (2018). “Mastercard Wants to Patent a Fractional Reserve Cryptocurrency Bank,” *CCN* October 26.

Malinova, Katya and Andreas Park (2017). “Market Design with Blockchain Technology,” working paper.

Martin, Antoine, James McAndrews, Ali Palida and David Skeie (2018). “Federal Reserve Tools for Managing Rates and Reserves,” working paper.

Martin, Antoine, James McAndrews and David Skeie (2016). “Bank Lending in Times of Large Bank Reserves” *International Journal of Central Banking* 12(4).

Martin, Antoine, David Skeie and Ernst-Ludwig von Thadden (2014a). “Repo Runs,” *Review of Financial Studies* 27(4).

Martin, Antoine, David Skeie and Ernst-Ludwig von Thadden (2014b). “The Fragility of Short-Term Secured Funding Markets,” *Journal of Economic Theory* 149(1).

Martin, Antoine, Michael Orlando and David Skeie (2008). “Payment Networks in a Search Model of Money,” *Review of Economic Dynamics* 11(4).

Meiklejohn, Pomarole, Jordan, Levchenko, McCoy, Voelker and Savage (2016). “A Fistful of Bitcoins: Characterizing Payments Among Men with No Names,” *Communications of the ACM* 59(4).

Mundell, Robert (1998). “Uses and Abuses of Gresham’s Law in the History of Money,” *Zagreb Journal of Economics* 2(2).

Nakamoto, Satoshi (2008). “Bitcoin: A Peer to Peer Electronic Cash System,” available at <https://bitcoin.org/bitcoin.pdf>.

Nelson, Bill (2017). “Financial Stability and Monetary Policy Issues Associated with Digital Currencies,” Remarks at Fintech – the Impact on Consumers, Banking, and Regulatory Policy.

Pagnotta and Buraschi (2018). “An Equilibrium Valuation of Bitcoin and Decentralized Network Assets,” working paper.

Parham (2017). “The Predictable Cost of Bitcoin,” working paper.

Parlour, Rajan and Walden (2017). “Making Money: Commercial Banks, Liquidity Transformation and the Payment System,” working paper.

Payments Canada, Bank of Canada and R3 (2018). “Project Jasper: A Canadian Experiment with Distributed Ledger Technology for Domestic Interbank Payments Settlement,” white paper.

Poon, Joseph and Thaddeus Dryja (2016). “The Bitcoin Lightning Network: Scalable Off-Chain Instant Payments,” working paper.

Popper, Nathaniel (2015). “Can Bitcoin Conquer Argentina?” *The New York Times* April 29.

Posner, Eric (2015a). “Vigna & Casey: Bitcoin and Economic Decentralization,” blog post February 4, <http://ericposner.com/vigna-casey-bitcoin-and-economic-decentralization>.

Posner, Eric (2015b). “Vigna & Casey: Bitcoin and Regulation,” blog post February 5, <http://ericposner.com/vigna-casey-bitcoin-and-regulation>.

Rajan, Raghuram G. (1992). “Insiders and Outsiders: The Choice between Informed and Arm’s-Length Debt,” *Journal of Finance* 47(4).

- Raskin, Max (2012). “Dollar-Less Iranians Discover Virtual Currency,” *Bloomberg News* November 29.
- Raskin, Max and David Yermack (2018). “Digital Currencies, Decentralized Ledgers, and the Future of Central Banking,” *Research Handbook on Central Banking*. Edward Elgar Publishing.
- Rochet, Jean-Charles and Jean Tirole (1996). “Interbank Lending and Systemic Risk,” *Journal of Money, Credit, and Banking* 28(4).
- Rochet, Jean-Charles and Xavier Vives (2004). “Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?” *Journal of the European Economic Association* 2(6).
- Rosenfeld, Everett (2015). “Ecuador Becomes the First Country to Roll Out Its Own Digital Cash,” *CNBC* February 9.
- Saleh, Fahad (2018a). “Blockchain Without Waste: Proof-of-Stake,” working paper.
- Saleh, Fahad (2018b). “Volatility and Welfare in a Crypto Economy,” working paper.
- Schilling and Uhlig (2018). “Some Simple Bitcoin Economics,” working paper.
- Shapiro, Joel and David Skeie (2015). “Information Management in Banking Crises,” *Review of Financial Studies* 28(8).
- Skeie, David.R. (2004). “Money and Modern Bank Runs,” Society for Economic Dynamics Meeting Paper Series No. 785.
- Skeie, David R. (2008). “Banking with Nominal Deposits and Inside Money,” *Journal of Financial Intermediation* 17.
- Sockin and Xiong (2018). “A Model of Cryptocurrencies,” working paper.
- Suberg, William (2018). “Mastercard Patent Claims Cryptocurrency Can Benefit From Fractional Reserve Banking,” *Cointelegraph* October 26.
- Tasca, Hayes, and Liu (2018). “The Evolution of the Bitcoin Economy: Extracting and Analyzing the Network of Payment Relationships,” *The Journal of Risk Finance* 19(2).
- Tomasicchio, Amelia (2016). “Bitcoin Holders Can Now Receive Daily Interest Payments On Coinbase,” *Cointelegraph* January 23.
- Urban, Rob (2017). “Bitcoin Is the New Crisis Currency,” *Bloomberg* November 16.

U.S. Patent and Trademark Office (2018). Patent Application No. 20180308092, October 25.

Vigna, Paul and Michael J. Casey (2015). *The Age of Cryptocurrency: How Bitcoin and Digital Money Are Challenging the Global Economic Order* (St. Martin's Press, New York).

von Mises, Ludwig (1912). *The Theory of Money and Credit*.

Winkler, Robin (2015). "Fedcoin: How Banks Can Survive Blockchains," *Konzept* 6.

Wolfson, Martin H. (1994). *Financial Crises: Understanding the Postwar U.S. Experience*, Second Edition (M.E. Sharpe, Armonk, N.Y.).

Woodford, Michael (2000). "Monetary Policy in a World Without Money," *International Finance* 3(2).