The Reversal Interest Rate*

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Abstract: The reversal interest rate is the rate at which accommodative monetary policy reverses and becomes contractionary for lending. Its determinants are 1) banks' fixed-income holdings, 2) the strictness of capital constraints, 3) the degree of pass-through to deposit rates, and 4) the initial capitalization of banks. Quantitative easing increases the reversal interest rate and should only be employed after interest rate cuts are exhausted. Over time the reversal interest rate creeps up since asset revaluation fades out as fixed-income holdings mature while net interest income stays low. We calibrate a New Keynesian model that embeds our banking frictions.

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1 Introduction

In most New Keynesian models, the economy enters a liquidity trap because of an exogenously assumed zero lower bound. This assumption has been called into question since a growing number of central banks – the Swedish Riksbank, Danmarks Nationalbank, the Swiss National Bank, the European Central Bank, and the Bank of Japan – have led money market rates into negative territory as a response to the Great Recession. In addition to going negative, these rates have been kept low for a long period. Although unusual by historical standards, this economic environment is likely to prevail amid the persistent decline in real and nominal interest rates over the last two decades.

This motivates the question: what is the effective lower bound on monetary policy? We suggest in this paper that it is given by the reversal interest rate, the rate at which accommodative monetary policy reverses its effect and becomes contractionary for lending. A monetary policy rate decrease below the reversal interest rate depresses rather than stimulates the economy.

Importantly, the reversal interest rate is not necessarily zero, as commonly assumed. In our model, when the reversal interest rate is positive, say 1%, a policy rate cut from 1% to 0.9% is already contractionary. On the other hand, if the reversal interest rate is -1%, policy rate cuts stay expansionary up to that point, even if their effectiveness might be impaired.

The reversal interest rate is endogenous, and its existence is guaranteed when banks' gains from maturity mismatch are insufficiently large. We formally demonstrate this result in a stylized model of monetary policy transmission through banks. Following an interest rate cut, two opposing forces affect banks' net worth. On the one hand, banks make capital gains on assets with long-term fixed-rate coupon payments (e.g., bonds). On the other hand, the rate cut shrinks banks' net interest income going forward. The yield they obtain per unit of liability employed decreases. Note that our result does not require the elasticity of deposit demand faced by banks to vary with the level of interest rates, even though properties of this elasticity matter in our quantitative application.

Our comparative statics reveal four key determinants of the reversal interest rate: 1) banks' holdings of long-term fixed-income assets, 2) banks' equity capitalization, 3) the tightness of capital constraints, and 4) the deposit supply elasticity faced by banks. Higher initial holdings of long-term fixed-income assets imply a larger maturity mismatch, resulting in a larger asset revaluation that decreases the reversal interest rate. Low initial bank capitalization and restrictive capital constraints both imply that the capital constraint tightens sooner following the drop in profitability caused by rate cuts, ceteris paribus. Finally, when the deposit supply elasticity increases as rates fall – due to consumer awareness of spreads or cash competition as rates approach zero – banks' profits shrink faster and the reversal interest rate is higher.

Quantitative easing (QE) increases the reversal interest rate, as it takes long-term fixed-income holdings out of bank balance sheets. Consequently, QE should only be employed after interest rate cuts are exhausted.

Our multiple-period extension shows that the effectiveness of a given *path* of policy rates depends on the duration of banks' initial fixed-income holdings. The negative effects on net interest income we described cumulate every period, while asset revaluation fades out as bank assets mature. In other words, a rate cut far in the future decreases banks' profitability but without revaluation gains if all initial fixed-income holdings have matured by then. In other words, the reversal interest rate "creeps up": said differently, the most stimulatory path of policy rates is increasing over time. "Low for long" rate environments can depress lending relative to policies that generate a relatively larger decrease in the slope of the yield curve while keeping the long end of that curve high.

The economics behind our results carry through in general equilibrium. After embedding our banking model in a New Keynesian macro model, we calibrate it in order to study quantitatively the importance of general equilibrium feedbacks. A new force emerges due to sticky prices. A policy rate cut generates a demand for credit that increases banks' intermediation and hence profits. This force, in loose terms, decreases the reversal interest rate. We find in our calibration that the monetary authority's ability to stimulate lending rates on impact declines with the size of the monetary shock and reverses at around -1% for the euro area. Given the persistence of our monetary shock, the negative effects are even more pronounced on lending rates one or two years ahead. This is due to our earlier "creeping up" effect: banks are shielded from rate cuts on impact, but not later. Once the reversal in bank lending is crossed, the economy's reliance on bank credit – the share of firms that are bank-dependent and the extent to which they are – dictates the aggregate implications for investment, output, and consumption. The reversal interest rate for these aggregate variables is lower, as other channels through which monetary policy operates – nonbank-dependent firms' funding costs and the inter-temporal substitution channel – remain active. Finally, we also show that a permanently lower steady-state real natural rate r^* leaves less leeway for monetary policy to be effective should the inflation target π^* stay unchanged.

The rest of this paper is organized as follows. First, we present a partial-equilibrium, two-period model where we provide explicit conditions for the existence of the reversal interest rate. In Section 2, we provide comparative statics and implications of our model for quantitative easing. In Section 3, we extend the model to multiple periods while keeping the analysis in partial equilibrium, and analyze how paths of policy rates transmit through banks. Finally, in Section 4 we introduce our simple banking model inside a New Keynesian model. We calibrate the model and illustrate its implications for impulse responses of aggregate variables to a monetary shock. The last section concludes.

Literature Review. A long-standing literature developed the concepts of the "balance sheet" and "bank lending" channels of monetary policy, emphasizing the importance of the balance sheet structure and the net worth of intermediaries for the transmission of monetary policy (Bernanke and Blinder, 1988; Bernanke and Gertler, 1995; Van den Heuvel, 2007). In our model, these objects are key determinants of the transmission of monetary policy.

From a theoretical standpoint, our microeconomic modeling of banks stands on the shoulders of a literature formally started by Klein (1971) and Monti (1972).[1] This literature emphasized the importance of market power when modeling banks.[2] Sharpe (1997) provides evidence of switching costs for depositors, and Kim et al. (2003) and Chodorow-Reich (2014) demonstrate the existence of relationship costs. Hainz et al. (2017) offer suggestive evidence that depositors' switching costs might change in low-rate environments, which we exploit in our quantitative exercise.

Banks' market power on their funding sources materializes in the impaired transmission of money market rates to bank deposit rates, affecting banks' interest rate risk exposure. Saunders and Schumacher (2000) and Maudos and Fernandez de Guevara (2004), among others, showed this fact empirically. Eggertsson et al. (2017) document a collapse of the pass-through of the monetary policy rate when the latter hits negative territory, owing to zero-interest-bearing cash becoming relatively more competitive. Work by Rognlie (2016) also suggests that the elasticity of demand changes at zero and sub-zero rates, affecting the pass-through. Drechsler et al. (2017) showed that quantities also respond to the spread that banks' charge. Importantly, this impaired transmission suggests a wedge between contractual and effective maturity of deposits, a fact long recognized by regulators when assessing banks' interest rate risk (Hoffmann et al., 2018).

[1] Santomero (1984) provides a good survey of this early theoretical literature.

[2] Petersen and Rajan (1995) in fact argue that banks need some monopoly power to sustain their businesses.

The interest rate exposure on banks' liabilities due to market power is balanced by banks' long-term fixed-income assets. Begenau et al. (2015) and Gomez et al. (2017) document that banks' assets are exposed to interest rate risk, which banks do not fully hedge in derivative markets (Abad et al., 2016; Hoffmann et al., 2018). Hoffmann et al. (2018) and Drechsler et al. (2018) show that this asset exposure is rationalized by banks' market power on deposits as it provides a natural hedge to the resulting interest rate risk. Drechsler et al. (2018) show effectively that due to this hedge, realized net interest income varies little with the level of policy rates: banks optimally choose a maturity mismatch in order to hedge the interest rate risk created by their market power on the liability side of their balance sheets. We view our study as complementary: we argue that the long-lasting low/negative interest rate period was largely unexpected and hence not hedged, and we study the implications of an unexpected shock. Di Tella and Kurlat (2017) offer an alternative rationalization with a similar result. In Brunnermeier and Sannikov (2016), banks also hold interest rate risk since appropriate monetary policy provides a "stealth recapitalization" in downturns. Importantly, these hedging strategies work in expectation. Hence, upon unusual interest rate realizations, the valuations of banks can fluctuate significantly, as in our model.

A recent empirical literature has shown real effects of low/negative rate environments on banks' profitability. English et al. (2012), Ampudia and Heuvel (2018), Claessens et al. (2017), Eisenschmidt and Smets (2018), and Wang et al. (2018) provide evidence that banks' net interest income and equity valuations vary with the level of interest rates, possibly in a nonlinear way. In particular, Claessens et al. (2017) find that a 1% policy rate drop implies, on average, a net interest margin 8 basis points, but that this magnitudes grows as rates move lower. This effect carries through bank profitability. Moreover, for each additional year of low rates, margins and profitability fall further. Altavilla et al. (2017) document that the ECB's introduction of negative interest rates was significantly detrimental to banks' net interest income, although increased intermediation activity as well as an improvement in the risk profile of banks' assets helped sustain returns on assets. Evidence provided by Ampudia and Heuvel (2018) suggest that banks' profitability response to interest rate cuts is non-monotonic: in normal times, interest rate cuts increase banks' valuations, although this does not hold in low-rate environments. Our DSGE model replicates this non-monotonicity.

Finally, we rely on a literature showing that the profitability of banks impacts their lending activities and hence the level of intermediation in the economy. Brunnermeier and Sannikov (2014) provide a theoretical foundation where intermediaries' profitability is key for the economy to function properly. Cavallino and Sandri (2018) obtain contractionary monetary easing in their theoretical model and explore the implications in an open economy context. Empirically, Chodorow-Reich (2014) shows real effects of bank lending frictions on firm employment. Heider et al. (2017) employ a difference-in-difference analysis using syndicated loans in the euro area to document that banks with a high deposit base decreased their lending relative to low-deposit, wholesale-funded banks following the ECB's decision to implement negative interest rates.[3] Importantly, Gropp et al. (2018) show that banks exposed to higher capital requirements decrease their risk-weighted assets instead of recapitalizing, as in our model.

2 Two-Period Partial Equilibrium Setup

In this section, we analyze a two-period banking model in partial equilibrium. That is, we hold fixed any aggregate prices and quantities that are not directly determined by banks' decisions.

Each of a continuum of (ex ante) identical banks is initially endowed with equity funding of E_0. In addition, banks can raise liabilities D in the form of bank deposits. On the asset side of the balance sheet, banks have two investment opportunities: loans L to firms and fixed-income assets S. Banks compete for loans to firms, while they take the return on fixed-income assets as well as their initial equity as given. Figure 1 displays a stylized balance sheet of a bank.

2.1 Timing of events

There are two periods, 0 and 1. We let i_0 denote the interest rate between time 0 and 1 that was expected before the beginning of period 0. At the beginning of period 0, the central bank sets the policy rate to i, which may differ from i_0. Observing i, banks then set new interest rates to maximize their period 1 net worth.

2.2 Bank assets

Bank loans. Each bank grants loans to a unit measure of its customer firms. We assume for simplicity that informational friction make firms locked in a relationship with their house

[3] Gomez et al. (2017) offer similar evidence, by studying two groups differentially exposed to interest rate risk. The group whose profitability is affected negatively (in relative terms) by a change in aggregate interest rates decreases its lending.

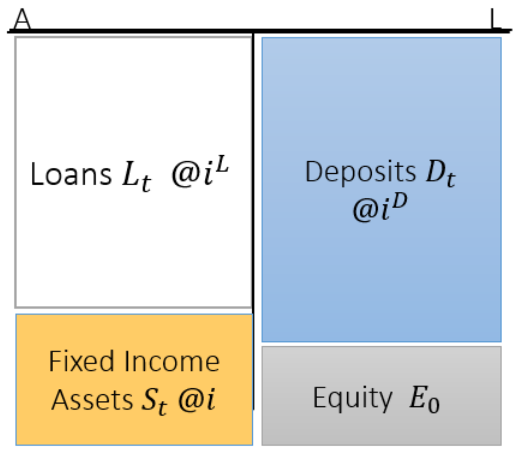

Figure 1: Bank's balance sheet: two-period model.

bank. The loan demand that bank j faces is denoted by $L(i_j^L)$, where i_j^L is the nominal rate on bank loans that bank j offers.[4]

Fixed-Income Assets. Each bank can also invest in fixed-income assets S. These assets are available in perfectly elastic supply, with a yield i that banks take as given.[5]

2.3 Bank Liabilities

Deposits. As with loans, each bank is naturally associated with depositors over which they have market power. We assume the demand schedule is given by $D(i^D)$, where $D(\cdot)$ is increasing. In the comparative statics section – as well as in our quantitative analysis – we'll consider ways in which $D(\cdot)$ might also depend on the current level of rates: we might expect, for example, competition for deposits to be tougher in low-rate environments.

Equity. Let $E_0(i_0)$ be the banks' book equity before the surprise policy rate change. We assume that banks' book equity after the surprise change, $E_0(i)$, is a function of the nominal policy interest rate i. This captures the fact that the value of banks' past assets and liabilities might change after monetary policy changes its stance. This revaluation can take the form of capital gains on mark-to-market assets, but include in spirit any asset revaluation, including,

[4] In our general equilibrium section, we micro-found the loan demand of firms and make them dependent on aggregate conditions beyond i_j^L.

[5] A broader interpretation of these assets would include equities, as banks take their returns essentially as given, and the risk-free component of these returns tracks the policy rate i.

for example, changes in loan-losses provisions. We decompose the equity after the monetary policy shock into $E_0(i) = \bar{e}_0 + e_0(i)$, where $\bar{e}_0$ is the interest-insensitive component of initial equity and $e_0(i)$ the interest-sensitive part. We assume that $\partial e_0(i)/\partial i < 0$, reflecting a maturity mismatch.[6]

2.4 Financial frictions

Banks face two forms of financial frictions. First, banks are subject to a capital constraint of the form

$$\psi^L L + \psi^S S \leq N_1,$$

where $\psi^L, \psi_S \geq 0$ are risk weights and N_1 denotes the bank's (nominal) net worth (defined below). That is, a weighted average of book assets must be covered by the value of the bank. We set $\psi^S = 0$ for concreteness.[7] The capital constraint captures economic and regulatory factors. Note that E_0 does enter the constraint indirectly through N_1: ceteris paribus, a larger E_0 leads to higher net worth.[8]

Second, banks face a liquidity constraint of the form

$$S \geq \psi^D D,$$

with $\psi^D > 0$. That is, each bank's fixed-income holdings must cover a certain fraction of deposits. Such a constraint captures the fact that banks need sufficient and easily accessible funds to avoid run risk.

2.5 Banks' problem

Finally, let $L + S = D + E_0(i)$ be the balance sheet identity of the bank. Then, we can write its problem as

$$\max_{i^L, i^D, L, D, S, N_1} \quad N_1 = (1 + i^L)L + (1 + i)S - (1 + i^D)D$$

$$L + S = D + E_0(i)$$

[6]For example, assume that banks enter the period with some equity $\bar{e}_0$ and an interest rate swap with notional value A_0. Setting $e_0(i) = \frac{i^0 - i}{1+i} A_0$ is isomorphic to including this derivative in the analysis.

[7]The analysis generalizes to the case $\psi_S < \psi_L$.

[8]Regulation often has an object closer to E_0 in the analysis. Such a constraint would not tighten in our two-period model, but would if time between these two periods is split into more sub-periods.

$$
\begin{aligned}
\psi^D D &\leq S \\
\psi^L L &\leq N_1 \\
L &= L_j(i_j^L) \\
D &= D_j(i_j^D; \boldsymbol{i}_{-j}^D, i).
\end{aligned}
$$

This problem offers no particular mathematical difficulties, and hence we omit conditions for existence and uniqueness of a symmetric equilibrium. In our micro-foundations of $D(\cdot), L(\cdot)$ in later sections, existence and uniqueness are straightforward to show.

3 Partial Equilibrium Reversal Interest Rate

3.1 Definition of the reversal interest rate

We now explicitly define the "reversal interest rate" as the rate at which a decrease in the nominal policy rate, i, stimulates lending if and only if the current level of the interest rate, i, is above the reversal interest rate i^{RR}.

Definition 1 (reversal interest rate)**.** *Let i^{RR} define the reversal interest rate such that*

1. *$i > i^{RR}$ implies $\frac{dL^*}{di} < 0$;*
2. *$i = i^{RR}$ implies $\frac{dL^*}{di} = 0$;*
3. *$i < i^{RR}$ implies $\frac{dL^*}{di} > 0$.*

In what follows, we first derive the bank's optimal setting rules. We then spell out sufficient conditions under which a reversal interest rate obtains.

3.2 Banks' rate-setting rules

Monetary policy affects the marginal investment opportunity of banks. Given that a bank can earn a return of i from holding a fixed-income asset, i encodes the opportunity cost of granting loans, and banks charge a mark up above it. Similarly, for deposits banks apply a mark down on the marginal investment yield i.

The constraints limit banks' portfolio choices. In particular, when the capital constraint binds, banks charge higher-than-desired lending rates in order to decrease their leverage.

Similarly, when the liquidity constraint binds, banks offer higher-than-desired lending and deposit rates in order to bring the liquidity ratio up.

Let ε^{f*} denote the semi-elasticity of the function f with respect to the relevant rate, evaluated at the optimal pricing rules.[9] The next lemma formally encodes these results.

Lemma 1 (rate-setting rules). *The optimal rate on loans is given by*

$$i^{L*} = \underbrace{i}_{\substack{\text{Marginal} \\ \text{opportunity cost}}} + \underbrace{\frac{1}{\varepsilon^{L*}}}_{\text{Mark up}} + \underbrace{\frac{\psi^L}{1+\psi^L}\lambda^{L*}}_{\text{Capital constraint}} . \tag{1}$$

The optimal rate on deposits is given by

$$i^{D*} = \underbrace{i}_{\substack{\text{Marginal} \\ \text{benefit}}} - \underbrace{\frac{1}{\varepsilon^{D*}}}_{\text{Mark down}} + \underbrace{\frac{\psi^D}{1+\psi^L}\lambda^{D*}}_{\text{Liquidity constraint}} \tag{2}$$

When constraints are slack, the Lagrangian multipliers are simply zero; when they do bind, the Lagrange multipliers are defined by the FOCs, and actual rates are given by the constraints themselves.[10]

3.3 Existence of i^{RR}

We now show how the constraints lead to a reversal of the bank lending channel. Remember that the capital constraint depends on how profitable the bank is. The next lemma shows that profits of banks have two components: net interest income (NII) and capital gains (CG). NII is defined as[11]

$$NII = \underbrace{i^{L*}L^* + i\,S^*}_{\text{Interest income}} - \underbrace{i^{D*}D^*}_{\text{Interest expenses}} .$$

[9]That is, $\varepsilon^{L*} = \frac{\partial \log L(i^L)}{\partial i^L}\Big|_{i^L = i^{L*}}$. Although mathematically these are semi-elasticities, economically they are elasticities since the units of i^L, i^D are percentage points.

[10]Smooth costs would enter in a similar way, increasing both lending and deposit rates.

[11]Note that our definition differs from measurements of *realized* net interest income as given, for example that in Drechsler et al. (2017). One can view our capital gains as the component of realized net interest income which is "shielded" from interest rate fluctuations.

Capital gains, on the other hand, are simply the change in initial equity (retained earnings) created by the surprise change in interest rates:

$$\text{CG} = E_0(i) - E_0(i_0).$$

The following lemma also shows that when there are no capital gains, that is, CG $= 0$, then the change in profits following an interest rate cut is strictly negative. We first state the result when the liquidity constraint binds and then show that it only works to increase the derivative.

Lemma 2 (profit response)**.** *Assume that $\psi^D = 0$. The change in profits following a change in i is then given by*

$$\frac{dN_1^*}{di} = \underbrace{(1+\lambda^{L*})}_{Amplification}\Big(\underbrace{\frac{dNII}{di}}_{NII>0} + \underbrace{(1+i)\frac{dE_0(i)}{di}}_{CG<0}\Big). \tag{3}$$

Moreover, if $\frac{dE_0(i)}{di} = 0$, then $\frac{dN_1^*}{di} > 0$.

Also, if $\psi^D > 0$, then the derivative is larger than $\frac{dN_1^*}{di}$ given in (3) above.

This result is intuitive. An interest rate cut depresses the return on new investments in fixed-income assets. Because fixed-income assets are always held by banks (whether the liquidity constraint binds or not), net interest income, and hence profits, decreases. That this is sufficient is a consequence of the envelope theorem – the fact that the first-order conditions described above apply. However, an interest rate cut also leads to an increase in $E_0(i)$, on the basis of the maturity mismatch. Without such gains, profits unambiguously decrease following an interest rate cut.

Moreover, an amplification occurs when the capital constraint binds (implying that $\lambda^{L*} > 0$). From that point onward, banks are forced to divert loan investment into fixed-income asset investment. Ceteris paribus, that makes further cuts more harmful to banks' profits.

Returning to our main result, when the capital gains are sufficiently small – that is, the change in $E_0(i)$ is small enough – then the NII channel dominates. Hence, profits decrease with a decline in i. Moreover, as long as the capital constraint does not bind, $di^{L*}/di > 0$, so that an interest rate cut lowers the loan interest rate, leading to more loans. Both forces tighten the constraint. Eventually, the constraint inevitably binds: at that point the policy rate hits the reversal interest rate, because any further decrease in i will decrease profits, which through the constraint must decrease L^*, so that dL^*/di flips sign.

Proposition 1. *(existence of i^{RR}) When capital gains $E_0(i)$ are sufficiently low (uniformly bounded by some number), there exists a finite reversal interest rate i^{RR}.*

Furthermore, nothing guarantees the reversal interest rate to be zero or any particular number. Instead, it has an endogenous number that depends on the state of the economy and in particular that of the banking sector, as highlighted by our comparative statics.

Corollary 1. *Generically, $i^{RR} \neq 0$.*

A numerical example. Figure 2 displays a numerical example of a reversal interest rate triggered by a binding capital constraint for a baseline interest rate of $i_0 = 1.5\%$. An interest rate cut lowers banks' net worth (lower-left panel), as the decline in net interest income exceeds the increase in capital gains (lower-right panel). The capital constraint tightens until it inevitably binds. At this stage, i falls below the reversal interest rate and a further decrease in the policy rate lowers loan volume (top-left panel). Interestingly, the loan interest rate then rises below that interest rate (top-right panel).

3.4 Comparative statics results

We now derive the comparative statics results. Unsurprisingly, the reversal interest rate is lower when capital constraints are looser or initial equity is low. This is consistent with Corbae and D'Erasmo (2014), who find in a structurally estimated banking model that an increase in capital requirements leads to a decline in aggregate loan supply and an increase in the loan interest rate.[12,13] Our third result in the next proposition states that, everything else equal, higher capital constraints on leverage make subsequent interest rate cuts below the reversal interest rate even more harmful for lending.

Proposition 2 (capital constraint and equity)**.** *The reversal interest rate i^{RR} has the following properties:*

1. *The reversal interest rate i^{RR} increases in the risk weight of the capital constraint ψ^L.*

[12]It is important to note that we neglect the potential risk-taking effects of decreasing interest rates, which might be the basis for a constraint – see, for example, Di Tella (2013) or Klimenko et al. (2015). In a theory encompassing both channels, a trade-off would emerge between the two; we are only modeling one side of a trade-off, and hence our results are unambiguous here.

[13]Note that for Propositions 2 and 3 below, the statements hold only weakly; however, for some reasonable constellations of parameters they hold strictly.

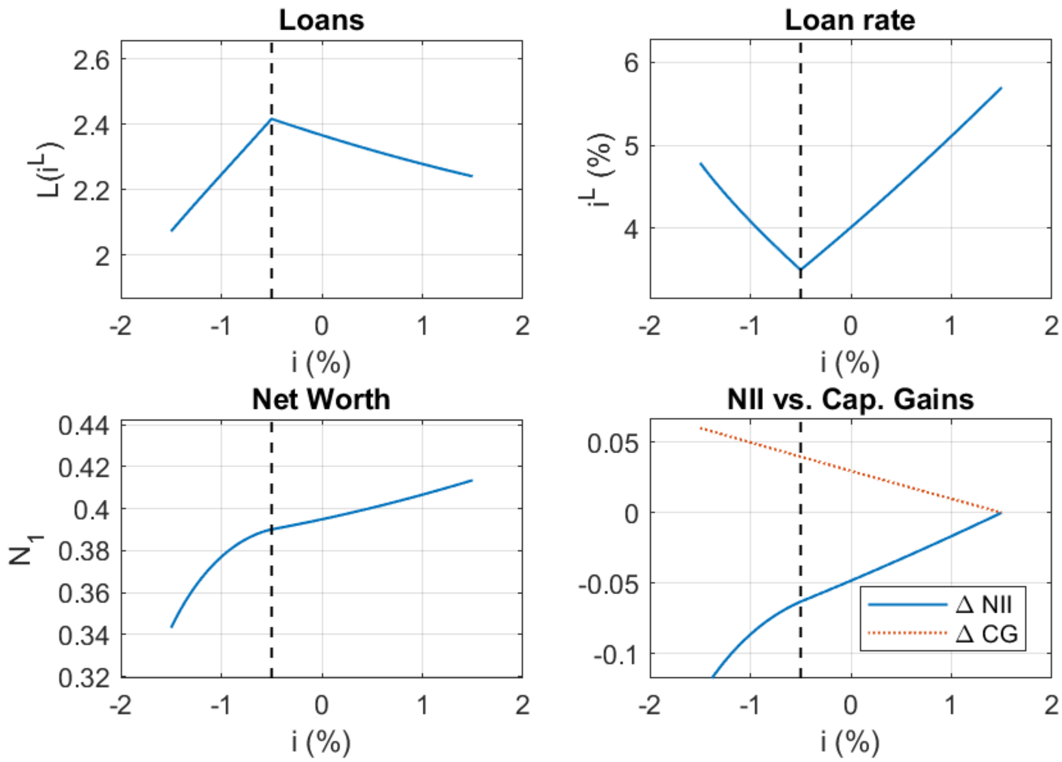

Figure 2: A numerical example of a reversal interest rate (dashed vertical line) due to a binding capital constraint. The constraint binds since the losses on net interest income (NII) are not sufficiently compensated by capital gains (CG). ΔCG and ΔNII refer to changes relative to their respective value at a baseline rate of $i_0 = 1.5\%$.

2. *The reversal interest rate i^{RR} decreases in the interest rate sensitivity of the initial equity $\partial e_0(i)/\partial i$ (keeping $E_0(i_0)$ constant).*
3. *An interest rate cut below the reversal rate is more detrimental for lending in an economy with a tighter capital constraint. Specifically, consider two economies, A and B, that have the same reversal interest rate $i_A^{RR} = i_B^{RR}$ and that are identical in all respects except that $\bar{e}_{0,A} < \bar{e}_{0,B}$ and $\psi_A^L < \psi_B^L$. Then, for any $i < i_A^{RR} = i_B^{RR}$, $L_A^*(i) > L_B^*(i)$.*

Arguably, one of the most striking features of our reversal result is that it does not rely on stickiness of the deposit rate. We highlight that, more generally, what matters is the fact that banks have more market power on the liability side of their balance sheet.

Hence, it follows that decreases in their market power coming from either the extensive or intensive margin of their supply of liabilities rise the reversal interest rate. Specifically, the next proposition shows the higher intensive margin deposit supply, coming, for example, from competition with other nominal saving vehicles such as cash also increases the reversal interest rate. We also show that if banks face a higher elasticity as rates fall due to heightened bank competition, the reversal interest rate increases as well. Hainz et al. (2017) document this phenomenon for Germany, where customers became more enclined to switch banks as the policy rate declined. In contrast, if banks are funded to a large extent with wholesale funding and the funding rate drops almost one-for-one with the policy rate, then the reversal interest rate is lower. Madaschi and Nuevo (2017) and Erikson and Vestin (2019) show that this was the case in Sweden. Finally, we note that larger liquidity constraints also work toward making the reversal interest rate higher.

Proposition 3 (liquidity constraint and deposit rate pass-through)**.** *The reversal interest rate i^{RR} has the following properties:*

1. *For $\underline{i}^D$, such that $d(i^D) = 0$ for all $i^D \leq \underline{i}^D$, where $d(\cdot)$ is the intensive margin of the deposit supply from depositors, i^{RR} is increasing in $\underline{i}^D$. In other words, the reversal interest rate is increasing with the lower bound on deposits.*

2. *Similarly, assume that the perceived elasticity ε_D^* is decreasing in i, leaving $D(\cdot)$ unchanged – e.g., individual banks face a higher perceived elasticity for extensive margin reasons. Then, the reversal interest rate is larger than with a constant ε_D^*.*

3. *The reversal interest rate i^{RR} increases with the tightness of the liquidity constraint ψ^D.*

We finally add an obvious but important result. Suppose that capital gains can be paid out as dividends: that is, equity is given by $E_0(i) = \bar{e}_0 + (1 - \upsilon_0)e_0(i)$, υ_0 is the dividend rate, and $e_0(i)$ represents capital gains (asset revaluation). Certainly, the larger υ_0 is, the less effective interest rate cuts are at recapitalizing gains for the purpose of making new loans, thus increasing the reversal interest rate. Our framework therefore rationalizes some of the recent policies implemented to restrict the dividend payments of banks.

Proposition 4 (dividends and the reversal interest rate)**.** *Suppose that equity is given by $E_0(i) = \bar{e}_0 + (1 - \upsilon_0)e_0(i)$, with υ_0 the dividend rate. Then, i^{RR} is increasing in υ_0.*

3.5 Optimal sequencing of QE

Our model also implies an optimal sequencing of interest rate policy and other monetary operations such as quantitative easing (QE). QE changes the bond holdings of the banking sector and hence its interest rate risk exposure. QE reduces the banks' holdings of long-term bonds; hence, after QE, the interest rate sensitivity of bank equity $\partial e(i)/\partial i$ is reduced, which increases the reversal interest rate.

Proposition 5 (QE and capital gains)**.** *Quantitative easing, which lowers the interest rate sensitivity of banks' initial equity $\partial e(i)/\partial i$ while leaving the overall level of $E_0(i_0)$ unchanged, lowers potential capital gains from a subsequent interest rate cut and hence increases the reversal interest rate i^{RR}.*

The optimal sequence of stimulating monetary policy is to cut the interest rate all the way toward the reversal interest rate before conducting QE measures.

We emphasize that this is a partial equilibrium result. In general equilibrium, other forces might pull toward an alternative sequencing.[14]

4 The "Creeping up" Effect

In this section, we extend the model to a three-period setting. This allows us to study how announcements about a *path* of policy rates impact the business of the bank, in particular net interest income in the future and the feedback on lending today. Our main result is that the optimal length of interest rate cuts should be related to the maturity of the banks' existing assets. The reason is as follows. As in the two-period model, a cut in an interest rate in the future has two effects: 1) fixed-income assets experience capital gains, while 2) net interest income will be depressed. Since the fixed-income assets mature over time, the first force slowly fades out whereas the loss in margins on future business does not. Hence, the interest rates that maximize lending "creep up" over time.

To make that intuition concrete, we consider banks in a setting similar to our two-period model, except that banks enter the period with two assets on their books: a one-period bond and a two-period bond. Moreover, their equity in the second-period is endogeneous and depends on profits that banks make in the first period. We then ask: what is the path of interest rates that maximizes banks' loan supply? Since fixed-income holdings of the first period are larger than those of the second period, the case for cutting the interest rate is stronger in the first period, where the capital gains are higher, while the effect on net interest income is similar across both periods. In that sense, the optimal path of (reversal) interest

[14] In particular, the sequencing described above stealthily recapitalizes banks, meaning that losses other sectors – the government, households, foreigners, or others – could distort other economic decisions.

rates is increasing, and an exceedingly long-lasting environment of low interest rates might hurt lending.

4.1 Three-period model extension

In our three-period model, the monetary authority controls a *path* of one-period interest rates $\{i_{0,1}, i_{1,2}\}$: the rate between the first two periods, $i_{0,1}$, and the rate between the last two periods, $i_{1,2}$. Banks enter the period with their past book, consisting of equity $\bar{e}_0$, one-period bonds $B_{0,1}$, and two-period zero-coupon bonds $B_{0,2}$.[15] Bonds are priced competitively at $p^B_{0,1} = \frac{1}{1+i_{0,1}}$ and $p^B_{0,2} = \frac{1}{(1+i_{0,1})(1+i_{1,2})}$. Hence the bank's equity entering the period is $E_0(i_{0,1}, i_{1,2}) = \bar{e}_0 + p^B_{0,1}B_{0,1} + p^B_{0,2}B_{0,2}$: when rates and therefore prices change, so too does equity entering the new period.

Each period, the bank is able to grant loans and deposits and to invest in fixed-income assets. The demand for loans $L(\cdot)$ and the supply of deposits $D(\cdot)$ is the same in both periods. We also assume that, before the policy experiment, the interest rates are equal in both periods, denoted by i^*. Hence, the two periods are identical in every aspect to repeating our two-period model twice, except that we have one- and two-period bonds and the equity level in the second period is now endogenously specified. As before, let N_1 be the net worth of the bank after optimization in the first period. We assume that part of the earnings is retained. Specifically, we assume a dividend (payout) rate $\upsilon \in (0,1)$ of the net worth, so that $E_1 = (1-\upsilon)N_1$. We further assume that υ is such that $E_0(i^*, i^*) = E_1$. In sum, the environment is totally stationary when there are no policy changes, and the bank makes similar decisions in both periods should rates stay at i^*. Finally, we keep the analysis in partial equilibrium, so there are no feedbacks from the policy changes and banks' endogenous responses.

4.2 Loan-maximizing policies

We define the loan-maximizing policies $i^P_{0,1}, i^P_{1,2}$ as those that maximize the discounted sum of loans:

$$(i^P_{0,1}, i^P_{1,2}) = \underset{i_{0,1}, i_{1,2}}{\text{argmax}} \; L^*_{0,1}(i_{0,1}, i_{1,2}) + \beta^P L^*_{1,2}(i_{0,1}, i_{1,2}),$$

[15] More generally, these bonds should represent the duration structure of banks' balance sheets as they enter the period, in the spirit of Begenau et al. (2015).

where $L^*_{0,1}, L^*_{1,2}$ are the optimal choices of banks' loan supply given the interest rates, and β^P is a "policy specific" discount factor (of the social planner). We assume that $\beta^P \leq 1$, that is, the policy cares more about present loans than future loans.

Our goal in this section is to characterize the choices $i^P_{0,1}, i^P_{1,2}$.

4.3 "Creeping up" result

Our main result is that, under mild conditions, $i^P_{0,1} < i^P_{1,2}$ – that is, the interest rate path "creeps up." The key reason for this result is that, although the loss in net interest income following an interest rate cut is similar in both periods, the capital gains from cutting the short-term rate are larger than from cutting the long-term rates since assets mature. In other words, a long-lasting low-interest-rate environment is going to hurt banks' flow profits in every period, while generating low capital gains in the later periods. As a consequence, it is optimal to cut the short-term rate more deeply than the long-term one.

One condition we need when capital gains on long-term assets are present, however, is that the policy makers must care about loans in the second period. To see this, suppose that $\beta^P = 0$, that is, the policy maker is myopic and cares only about current loan volume. Suppose, moreover, that $B_{0,2} > 0$, that is, there are capital gains to be made on long-term assets. The policy response will naturally be to decrease the long rate as much as possible, so as to maximally boost capital gains on long-term assets, which will drive down long-term loans to very low levels. Consequently, bank net worth would tank and, with it, loan volume in the second period. Hence, to avoid these myopic cases, we need β^P to be sufficiently close to one whenever $B_{0,2} > 0$.

Proposition 6. *Assume that $B_{0,1}$ and $B_{0,2}$ are small enough such that the loan-maximizing rates $i^P_{0,1}, i^P_{1,2}$ are well defined. Then $i^P_{0,1} < i^P_{1,2}$.*

Note that we have not assumed that a low net worth in the long term feeds back on the banks' ability to lend in the first period.[16] This would make the case for cutting the long rate even weaker, as there would be an additional motive to raise the long-term rate further in order to avoid the drop in long-term net worth that would feed back on the bank's risk-taking ability.

[16]That is, we could have assumed that $\psi^L L_{0,1} \leq N_1 + \beta N_2$, where β is some discount factor relevant to evaluating how much long-term net worth impacts a bank's ability to take on risk.

5 Reversal in a New Keynesian DSGE Model

We now ask whether a reversal interest rate still exists in a quantitatively realistic general equilibrium setting in which changes in the policy rate can stimulate aggregate demand due to nominal price rigidities. As in our partial equilibrium model, the capital gains are not enough to offset the fall in the net interest income of banks, decreasing their profitability and hence threatening their ability to provide productive lending to the economy. Moreover, the decreased pass-through to deposit rates weighs further on bank profitability. However, standard New Keynesian forces operating through inter-temporal substitution and price rigidity generate an increase in loan demand and hence lift bank profitability. If such lift is strong enough, it may overturn our partial equilibrium results.[17] The lack of deposit rate pass-through mitigates this channel, very much like an economy entering a liquidity trap.

Our calibration quantitatively pins down these forces. We find that a general equilibrium reversal interest rate still obtains. Moreover, even before the reversal interest rate is reached, the effectiveness of monetary policy decreases as it approaches the reversal interest rate: in that sense, our frictions smoothly affect monetary policy's effectiveness.

5.1 Environment

Time is discrete and the horizon is infinite. Households choose consumption, savings, and labor supply to maximize their lifetime utility over consumption and leisure. They own banks and all three types of firms in the economy: intermediate goods producers, retailers, and final goods producers. Some intermediate goods producers require bank loans to sustain their investment activities. Banks obtain deposits from households and invest these savings in government fixed-income assets and loans to the bank-dependent intermediate goods producers. Intermediate goods producers competitively sell goods to retailers. These retailers differentiate these goods at no cost and sell them to competitive final goods producers. Retailers are subject to price frictions, in a New Keynesian fashion. Final goods producers then bundle retail goods into final goods usable for consumption and capital. Finally, a monetary authority (government) supplies nominal fixed-income assets to banks elastically at a particular interest rate, taxing (or redistributing gains to) households lump-sum to finance such assets.

[17]With flexible prices, general equilibrium forces will work to alleviate the negative consequences of a reversal interest rate, but they cannot undermine its existence.

Households. A unit continuum of identical households with separable preferences over consumption and labor choose consumption C_t, labor supply l_t, and deposits D_t in order to maximize their lifetime utility:

$$\max_{\{C_t,l_t,D_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, C_{t-1}, l_t),$$

where the per-period utility is given by

$$U(C_t, C_{t-1}, l_t) = \frac{(C_t - hC_{t-1})^{1-\gamma}}{1-\gamma} - \chi \frac{l_t^{1+\varphi}}{1+\varphi}$$

subject to their budget constraint

$$p_t C_t + D_{t+1} = p_t w_t l_t + (1 + i^D_{t-1,t}) D_t + p_t \Pi_t - p_t T_t,$$

where Π_t denotes profits coming from retailers, intermediate goods producers, and banks, and T_t are government lump-sum transfers.

Final goods producers. Final goods producers purchase retail goods $j \in [0, 1]$ at price $p_t(j)$ and aggregates them into the final good, with production function

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

They then sell these final goods on competitive markets to households for consumption and to intermediate firms for capital investment.

Capital goods producers. Capital goods producers costlessly differentiate final output goods in an investment good that can be used by intermediate firms as capital. The market for such goods features perfect competition, with Q_t denoting the price of investment. Capital goods producers face adjustment costs on the rate of change of investment. They use the stochastic discount factor of households to discount profits. The capital accumulation equation reads:

$$K_{t+1} = (1-\delta)K_t + I_t \left(1 - \Xi(I_{t+1}/I_t)\right)$$

Where $\Xi(I_{t+1}/I_t)$ is an adjustment cost function. Given this setting we can write the problem of capital goods producers as:

$$\max \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[Q_t I_t \left(1 - \Xi(I_{t+1}/I_t)\right) - I_t \right]$$

We choose the following quadratic specification for the adjustment cost function:

$$\Xi(I_{t+1}/I_t) = \frac{\kappa_{\mathrm{AC}}}{2} \left(\frac{I_{t+1}}{I_t} - 1 \right)^2$$

Retailers. A unit continuum of retailers is indexed by $j \in [0,1]$. Each produces its own retail good variety j by costlessly transforming intermediate goods. They face the demand function for their retail variety derived from the problem of the final goods producers. Retailers are subject to Rotemberg price adjustment costs. The problem of a single retailer j is then summarized by

$$\max_{\{p_{t+s}(j)\}} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \Big[p_{t+s}(j)^{1-\epsilon} p_{t+s}^{\epsilon-1} Y_{t+s} - mc_{t+s} p_{t+s}(j)^{-\epsilon} p_{t+s}^{\epsilon} Y_{t+s} \right.$$
$$\left. - \frac{\theta}{2} \left(\frac{p_{t+s}(j)}{p_{t+s-1}(j)} - 1 \right)^2 p_{t+s}(j)^{1-\epsilon} p_{t+s}^{\epsilon-1} Y_{t+s} \Big] \right],$$

where real marginal costs mc_t are equal to p_t^I, the price of intermediate goods, and $\Lambda_{t+s} = U_C(C_{t+s})$, where C_{t+s} is the consumption of the representative household.

Intermediate firms. Labor and capital are combined to produce intermediate goods. These goods are then sold competitively to retailers. Labor is hired competitively at a wage rate w. Capital is purchased a period in advance from final goods producers, and depreciates slowly. There are two types of firms in the economy, which differ in two aspects: their access to financial markets and their productivity.

Intermediate firms are of two types. A share $1 - \xi$ of firms are neoclassical and obtain capital directly from households. Specifically, the problem of neoclassical firms is to maximize their profits, given that they hire labor at the real wage w_t and acquire capital a period in advance at price Q_t. In so doing, they value the opportunity cost of capital acquisition at the effective rental rate $r_{t-1,t}$ defined by

$$r_{t-1,t} = Q_t \frac{\Lambda_{t-1}}{\beta \Lambda_t} - Q_{t+1}(1-\delta) \tag{4}$$

where δ is the depreciation rate of capital and Q_{t+1} its resale price. They then produce output using a Cobb-Douglas production function with TFP $\bar{A}$ and sell it at the given price p_t^I. Hence their problem reads as

$$\max_{\ell_t, k_t} p_t^I \bar{A} \left(k_t^\alpha \ell_t^{1-\alpha}\right)^\nu - w_t \ell_t - r_{t-1,t} k_t.$$

The second type of intermediate firms, the remaining share ξ, are bank dependent. They produce by combining labor and capital in a Cobb-Douglas production function with productivity $\underline{A}$. These firms are born without financial resources, so their entire capital purchase must be financed with a bank loan; in subsequent periods, they can use retained earnings as a source of financing, which diminishes their need for bank loans. We introduce heterogeneous duration of firms' projects in order to obtain a maturity structure for bank loans. Specifically, when a bank-dependent firm is founded, it learns a specific duration $\tau \in \{1, ..., T\}$ of its project, drawn from a distribution Γ_τ. In their founding period, these firms can choose to enter long-term bank contracts that fix their loan and rate schedules; changing the schedule, once settled, requires paying a fixed cost F_b.[18] Specifically, firms can commit to an interest rate schedule $i^{L*}_{0,\dots,\tau}$ and associated loan schedule $L^*_{0,\dots,\tau}$. Similarly, we assume that these firms plan their entire investment path given prices as they draw a project. Deviating from this capital path in subsequent periods costs a fixed cost F_k.[19]

Finally, we assume that firms discount using the SDF of households, although frictions prevent them from being able to draw funds from them.[20,21]

Given this setting, a firm that drew a project of duration τ at time 0 and commits to a

[18]This helps us micro-found capital gains from loans on banks' balance sheets. Capital gains will also come from banks' holding of bonds.

[19]In a perfect foresight equilibrium, this is inconsequential. After an unexpected monetary shock, it acts as an adjustment cost, by allowing only firms with new projects (as well as non-bank-dependent ones) to adjust their capital stock in response to price changes. On top of generating hump-shaped responses in investment and loans, this assumption also greatly simplifies solving the banks' problem.

[20]Note that banks are special in our model because they have the ability to grant loans to these firms.

[21]Since $r_t^L > \frac{\Lambda_{t+1}}{\Lambda_t} - 1$ in equilibrium, the choice of the households' SDF is inconsequential.

plan solves

$$
\begin{aligned}
\max\ \mathbb{E}_0 \sum_{t=0}^{\tau} \beta^t \Lambda_t x_t \\
x_t + Q_t k_t - L_t &= p_t^I y_t + Q_t(1-\delta)k_{t-1} - w_t \ell_t - (1+r_t^L)L_{t-1} - F_b \mathbf{1}_{L_t \neq L_{t,0}^*} - F_k \mathbf{1}_{k_t \neq k_{t,0}^*} \\
y_t &= \underline{A} k_{t-1}^{\alpha} \ell_t^{1-\alpha} \\
1 + r_t^L &= \frac{1 + i_{t-1,0}^{L*}}{1+\pi_t} + \mathbf{1}_{L_t \neq L_{t,0}^*} \frac{i_{t-1}^L - i_{t-1,0}^{L*}}{1+\pi_t} \\
L_t &\geq 0,\ \ x_t \geq 0,
\end{aligned}
$$

with $k_{-1} = L_{-1} = 0$. In our calibration, it will be the case that the optimal payout rate is $x_t = 0$ for all $t < \tau$ – that is, firms will direct all retained earnings toward investment and minimize the reliance on bank loans. Also, note that in a perfect-foresight equilibrium, firms will be indifferent as to whether to commit to a plan or not.

Banks. A unit continuum of identical banks exists. We assume that banks pay a fixed dividend rate to their owners.[22] Solving the problem of each bank is hence identical to solving our two-period model repeatidely, using the dividend assumption to obtain the transition rule for equity. To keep the banks' problems differentiable, we replace our capital inequality constraint with smooth leverage costs. We also assume that there is perfect competition for loans, so that loans are priced at marginal costs that include costs from leverage. Specifically, marginal costs have three components: 1) the return on fixed-income assets i_t, which represent the marginal opportunity cost of lending; 2) a per-loan unit leverage cost $\varrho(\gamma_t^L)$, where γ_t^L is the aggregate leverage of the banking sector and $\gamma_t^L = \frac{L_t}{N_{1,t+1}}$ and $L_t, N_{1,t+1}$ are, respectively, the aggregate loans and next-period aggregate net worth of the banking system; and 3) a time-invariant per-loan unit cost c^L. The loan rate offered on new loans is then given by

$$
i_t^L = i_t + \varrho(\gamma_t^L) + c^L.
$$

[22]The fixed-dividend assumption makes certain that banks do not drive leverage costs $\varrho(\cdot)$ to zero by borrowing from households. This is consistent with the empirical evidence in Gropp et al. (2018). When undercapitalized, banks do not recapitalize but instead deleverage by decreasing the size of their balance sheets or rebalance their assets toward less risky ones. Debt overhang is one underpinning for this behavior (Admati et al., 2017).

However, as described in the previous sections, banks also offer firms long term loans with locked-in rates and quantities. They price these according to marginal costs.[23]

Next, we describe the capital accumulation process of banks between periods. We assume that banks accumulate real equity according to the following formula:

$$E_{t+1} = (1 - \upsilon) \frac{1}{1 + \pi_{t+1}} N_{t+1},$$

where υ is a fixed dividend rate, and N_{t+1} is the nominal (in period t terms) net worth of banks. Dividends are rebated directly to households.

Next, we need to parametrize deposit demand in the model and, in particular, how the margins on deposit vary with the level of interest rates. The data suggests that deposits rates are quite sticky, with a declining pass-through as money market rates decrease. We micro-found one explanation for this phenomenon at the extensive margin in Appendix B.[24] Simply put, we posit that households become more aware of spreads as rates approach zero or negative territory. This makes the extensive margin dominate bank mark downs on deposit rates so that we can simply set $i_t^D = i_t - \eta^D(i_t)$ and parametrize $\eta^D(\cdot)$ to fit the observed pass-through in the data. Importantly, once they post a rate, banks in our setting are willing to absorb as many deposits as are demanded by households.[25]

Finally, given the level of equity E_t, the loan demand given prices $L_t(i_t^L)$, and deposit demand $D_t(i_t^D)$, banks' balance sheet equation means that the total level of fixed-income assets invested by banks must be $S_t = E_t + D_t - L_t$. We furthermore allow banks to invest in fixed-income assets of different maturities τ. Let total holdings be $S_t = \sum_\tau S_{t,\tau}$. These assets are elastically supplied by the government at fair prices. In the perfect-foresight equilibrium that we study, banks are indifferent between maturity choices. We pick the maturity choices of banks to match their empirical maturity structure of assets.

Hence, the representative bank's net worth is

$$N_{t+1} = (1 + i_t^L) L_t(i_t^L) + \sum_\tau p_{t+1,\tau} S_{t,\tau} - (1 + i_t^D) D_t,$$

where $p_{t+1,\tau}$ is the price of the fixed-income asset with remaining maturity τ at time $t + 1$.

[23] After a surprise shock, though, banks still pay the leverage costs coming from these fixed-interest loans.

[24] The explanation at the intensive margin would be the presence of cash.

[25] This is important, as it can be that $i_t^D < i_t$. In our two-period model, the liquidity constraint played the role of preventing banks from shedding their fixed-income assets. Here, banks hold on to their deposits.

Government and Monetary Authority. The government taxes (or transfers) a lump-sum amount T_t to households, which allows it to pay the nominal interest rates i_t on fixed-income assets. Ricardian equivalence holds in our economy, making the timing of taxes irrelevant. The monetary authority follows a Taylor rule, which is assumed to take the following common form:

$$\frac{1+i_t}{1+i^*} = \left(\frac{1+i_{t-1}}{1+i^*}\right)^{\rho_i} \left(\frac{1+\pi_t}{1+\pi^*}\right)^{(1-\rho_i)\phi^\pi} e^{\varepsilon_t^m},$$

where i^* is the steady-state policy rate, π^* is the steady-state inflation rate, and ε_t^m is a monetary policy shock.

5.2 Solution concept

We calibrate our model to the euro area, where negative rates have been implemented since 2014.[26] We assume that every agent in the economy has perfect foresight over the future and solve the deterministic equilibrium after a one-time unexpected monetary policy shock ε_t^m. Our computational algorithm solves for the full nonlinear system of equations, and hence does not rely on perturbation techniques. This is important, since our economy inherently features large nonlinearities.

5.3 Calibration

We set the length of a period to one year. The calibration of the New Keynesian block of the model is standard, and the parameters are summarized in Table 1.

We calibrate the remaining parameters to match key moments about banks and the production sector's dependence on bank lending. Table 2 displays the resulting parameters. We choose the nominal fixed-income asset rate and households' deposit rates to be, respectively, 2.00% and 0.65%, corresponding to values prior to the low/negative rate environment. We use the EONIA for the fixed-income asset rate, and the deposit rate for euro-area households in the ECB MIR data. Next, we need to match the pass-through of fixed-income rates into deposit rates. We choose to do so using the flexible functional form $\eta^D(i) = i - \eta_1 - \eta_2 \exp(\eta_3 i)$, which allows for a decaying pass-through. Concretely, we fit the parameters η_1, η_2, and η_3 such that 1) the steady-state values satisfy the equation

[26] The euro area also has a higher share of small and medium-sized firms, and its firms are more bank dependent than firms in the U.S.

Table 1: Conventional DSGE Parameters.

Parameter	Description	Value
γ	IES parameter	2
h	Habit formation	0.6
ϕ	Disutility of labor	1
δ	Capital depreciation	0.1
α	Capital share	0.33
ν	Scale parameter	0.81
ϵ	Retail price elasticity	6
θ	Rotemberg cost	60
ϕ^π	Taylor rule coefficient	1.5
ρ^i	Taylor rule persistence	0.8

$i^D_{SS} = i_{SS} - \eta^D(i_{SS})$; 2) the average deposit rate during the negative rate environment, which was $i^D_{\text{neg}} = 0.1\%$, matches the average negative rate on fixed-income assets of i_{neg} =-0.3%, that is $i^D_{\text{neg}} = i_{\text{neg}} - \eta^D(i_{\text{neg}})$; and 3) the pass-through at steady-state value is perfect, that is, $\frac{\partial i^D}{\partial i} = 1$ in the steady state.[27] Figure 8 in the Appendix depicts the resulting pass-through. Next, we match banks' profits to assets to equal the equivalent ratio in the data, which is 1.80%.[28] Given steady-state lending costs, c_L adjusts to match this ratio. For leverage costs, we choose the specification $\varrho(\gamma) = \frac{\kappa_L}{\gamma - \bar{\gamma}}$ with parameter κ_L and $\bar{\gamma}$: it has an asymptote as $\gamma \to \bar{\gamma}$, mimicking our hard constraint in the two-period model. We pick a maximum leverage $\bar{\gamma}$ of 10%, in accordance with the guideline provided by the ECB for the minimum leverage ratio. Steady-state leverage costs depend on the capital ratio of banks in the steady state: we pin it down to 15.5%, the average value reported by Altavilla et al. (2017). As in Alpanda et al. (2014), we set the capital leverage cost parameter κ_L so that we match the elasticity of rates to changes in capital leverage in the MAG report of the Financial Stability Board (Macroeconomic Assessment Group, 2010). This report estimates that a 1% increase in capital leverage results in a 0.28% increase in capital funding costs. Next, we

[27]This is desirable, as IRFs in the steady state's vicinity are not affected by the lack of deposit pass-through, which facilitates the comparison with the vanilla New Keynesian model.

[28]We sum net interest income and net fee and commission income for euro-area domestic banks and divide that by the outstanding amounts of assets for the years 2007-2013, e.g., prior to the introduction of negative rates. The data is from the ECB's statistics on consolidated banking data.

obtain information on the maturity structure of banks from Hoffmann et al. (2018). Within fixed-income assets, we target a duration of 3.4 years, which we match using a geometrically decaying structure of banks' fixed-income asset purchases such that the share of fixed-income assets expiring in every period is $1 - \tau_S$. Within loans, we set the firm project duration (Γ_τ) such that a firm can draw either a short-term (one year) or long-term (three year) project upon its foundation, and set the fraction p for the former to 75% so that the average banks' loan maturity is 1.9 years. We set the fixed cost of renegotiating loans F_b and F_k large enough that no bank-dependent firms renegotiate their loans or investment plans in equilibrium.[29] We set loans to represent 60% of banks' assets, as in Hoffmann et al. (2018).[30] Loan demand then pins down the banks' balance sheet size. Next, we pick the investment adjustment cost parameter κ_{AC} to imply an elasticity of investment to a shock to the contemporaneous price of capital of about 3, generating a value close to Smets and Wouters (2003). Finally, we need to specify the relative characteristics of two intermediate producer types. We relate our bank-dependent firms to small and medium enterprises, which represent more than 99% of all enterprises in the euro area. We calibrate ξ to the value reported by Eurostat's statistics on SMEs. However, we adjust the relative productivity $\bar{A}/\underline{A}$ so that bank-dependent firms only end up producing 55.8% of output in steady state, consistent with the values reported in Eurostat. Our neoclassical firms end up being about three times more productive; moreover,by enjoying cheaper borrowing rates, they end up producing significantly more output than the average bank-dependent firm, which allows us to match the data.

5.4 Results

To study the impact of each marginal innovation on aggregate variables, we generate innovations ε_0^m to the Taylor rule of increasing magnitudes. In an economy log-linearized around its steady state, the resulting impulse responses scale up proportionality. Our economy, in contrast, features a a nonlinear and non-monotonic response to each marginal innovations.[31]

Specifically, we first study the effects of a marginal – 10 basis points – negative Taylor rule innovation in the vicinity of the steady state and report the resulting impulse response

[29] In the perfect-foresight equilibrium, firms are indifferent between locking in their loan interest rate and capital plans or not.

[30] A degree of freedom exists because total debt, and hence total assets, is not pinned down in equilibrium.

[31] Our economy naturally has strong sign dependence: monetary expansions are less effective than contractions, as our frictions (bank leverage costs, deposit rate pass-through) are one-sided frictions. We focus on expansions.

Table 2: Calibrated Parameters.

Parameter	**Description**	**Value**
i_{SS}	Steady-state policy rate	2.00%
c_L	Loan cost	0.73%
η_1	Deposit pass-through parameter	8.2e-4
η_2	Deposit pass-through parameter	2.6e-4
η_3	Deposit pass-through parameter	154
κ_{AC}	Investment AC parameter	0.15
κ_L	Leverage costs parameter	4.8e-3
$\bar{\gamma}$	Maximal equity-to-capital ratio	10%
L_SS/S_SS	Steady-state loans-to-safe-assets ratio	3/2
υ	Bank dividends	11.4%
τ_S	Fixed-income maturity parameter	0.7
p	Probability of three-year project draw	75%
ξ	Share of bank-dependent firms	99.8%
$\bar{A}/\underline{A}$	Relative firm productivity	2.83

function. Then, we generate innovations of larger sizes and study the effect of the last 10 basis point innovation. In other words, we compute three impulse responses: that of a small shock in the vicinity of the steady state, that of a large shock, and finally a small shock in addition to the large one; we then compare the first IRFs with the difference between the last two IRFs. We use a Newton algorithm with automatic differentiation as a solution procedure, iterating on 10 basis point innovations.

Before describing our results, we point out two subtleties of our analysis. First, a reversal within our experiment necessarily applies to a particular variable at a particular horizon. In our two-period model, we focused on the lending rate on impact. Due to the creeping up effect, the reversal rate for the lending rate one or two periods ahead may differ. Similarly, if one focuses on GDP, the reversal rate will also be different. Second, given that our economy's constraints are smooth – in contrast to the sharp constraints of our previous sections' stylized models – the economic mechanisms we highlight have consequences before aggregate variables display a full reversal, and work toward dampening the effectiveness of rate cuts before reversing it.

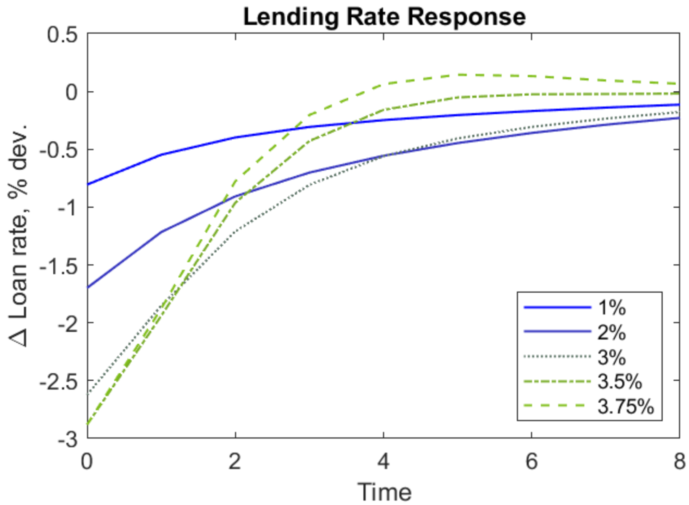

Figure 3: Impulse response of loan rates to Taylor rule innovations of increasing size, in deviations from the steady state. The legend contains the size of the respective innovations to the Taylor rule. The dashed line reflects a reversal of the response, including on impact. For the largest shock, the policy rate reaches about -1% on impact. The dotted line shows that a reversal occurs even for smaller innovations but at a later horizon, in line with our "creeping up" result.

Figure 3 depicts the response of bank lending rates to negative Taylor rule innovations of increasing size, and Figure 4 displays the response of physical capital from bank-dependent firms to the same innovations. As negative monetary innovations become larger, their ability to stimulate lending rates diminishes, eventually reverting (dashed line) when the level of nominal rates on impact reaches about -1%.[32] Hence, in this particular sense, we estimate the reversal interest rate to be about -1% in our calibration.

Figure 5 (left panel) reflects the same result, but instead computes the marginal response of the lending rates to a 0.1% shock after the steady state and after an innovation large enough to trigger a "reversal" of the impact response of lending rates.[33] Figure 5 also contains the response of loans, and Figures 6 and 7 depict the responses of additional variables.

Some variables experience reversals even at higher rates: in particular, the response of bank net worth to a marginal shock reverses when policy rates arrive in the vicinity of 0%; loans rates and loan quantities two periods ahead also reverse before -1% is reached. In contrast, some variables do not experience a reversal at -1%: output on impact, for example,

[32] Due to the inflation response in the Taylor rule, a one percent negative innovation to the Taylor rule decreases the actual nominal rate by less than one percent.

[33] Alternatively, taking the first curve, as well as the two differences in the last two curves of Figure 3, more or less replicates Figure 5 – except that the innovations have a size of 50 basis points.

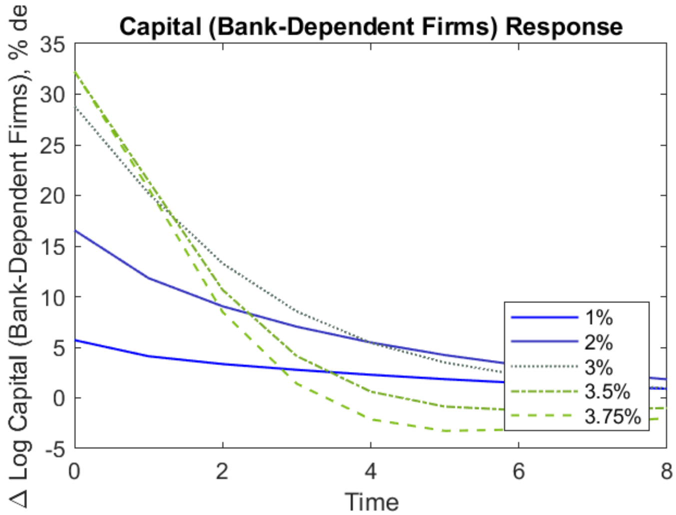

Figure 4: Impulse response of log capital of new bank-dependent firms to Taylor rule innovations of increasing size, in deviations from the steady state. The legend contains the size of the respective innovations to the Taylor rule. The dashed line reflects a reversal of the response, including on impact. For the largest shock, the policy rate reaches about -1% on impact. The dotted line shows that a reversal occurs even for smaller innovations but at a later horizon, in line with our "creeping up" result.

still rises following a marginal shock, although it does decline two periods ahead. The right panel of Figure 5 shows that the response of loans closely follows that of the loan rate, modulo changes in other prices.

Figure 6 confirms that the reversal in loan rates and loans has aggregate consequences for output and investment. On impact, the increase in aggregate demand generates an increase in output, as loans do not play a direct role in the production of period 0 output. Investment, moreover, still increases due to the response of neoclassical firms in the economy, which do not rely on loans.[34] However, in subsequent periods, the rise in loan rates results in depressed investment in output, which persists for one to four years. These results reinforce the idea that the contractionary aggregate effects of low or negative interest rate environments may take some time to materialize.

Finally, Figure 7 shows that the reversal is due to poor bank profitability and rising leverage costs. Note that bank net worth actually increases following a shock around the steady state, due to the intermediation boom that lifts banks' net interest income. In contrast, a cut into negative territory deeply depresses the net worth of banks. This is

[34]Costs of changing investments, as present in Christiano et al. (2005), for example, would likely weaken that mechanism.

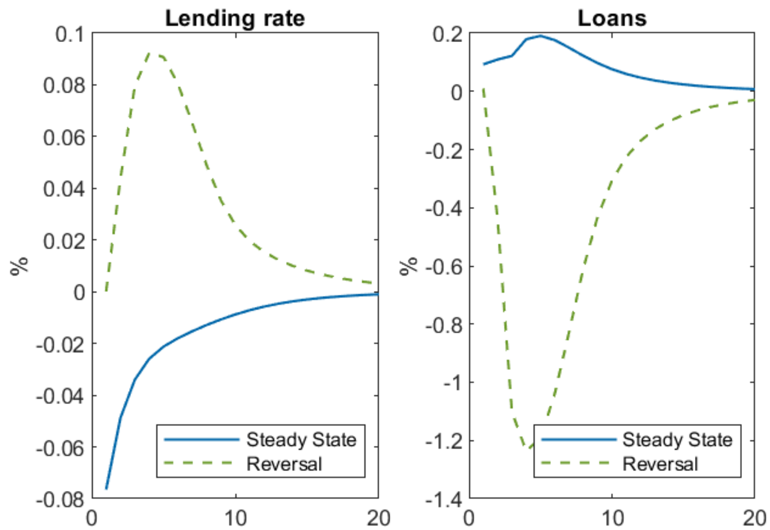

Figure 5: Marginal impulse responses of loan rates and loans to a 10 basis points innovation to the Taylor Rule in two economies. In the baseline economy (plain line), the shock occurs in the vicinity of the steady state. In the second economy, the marginal shock occurs on top of an innovation to the Taylor rule that, on its own, would depress the policy rate to about -1% on impact. The reversal in loan rates has been crossed at this stage.

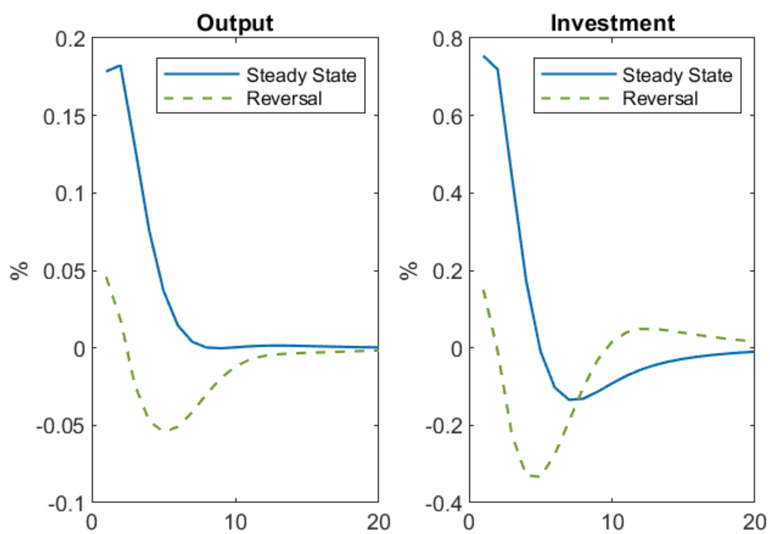

Figure 6: Marginal impulse responses of output and investment to a 10 basis points innovation to the Taylor rule in two economies. In the baseline economy (plain line), the shock occurs in the vicinity of the steady state. In the second economy, the marginal shock occurs on top of an innovation to the Taylor rule that, on its own, would depress the policy rate to about -1% on impact. The reversal in loan rates has been crossed at this stage.

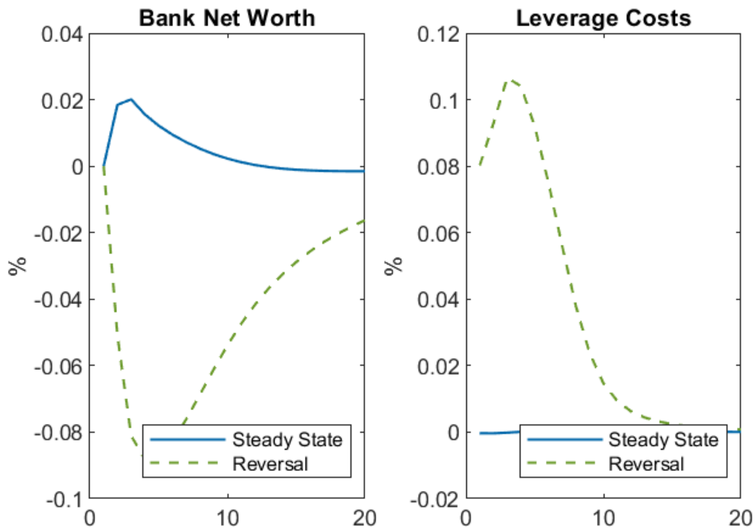

Figure 7: Marginal impulse responses of bank net worth and leverage costs to a 10 basis points innovation to the Taylor Rule in two economies. In the baseline economy (plain line), the shock occurs in the vicinity of the steady state. In the second economy, the marginal shock occurs on top of an innovation to the Taylor rule that, on its own, would depress the policy rate to about -1% on impact. The reversal in loan rates has been crossed at this stage.

consistent with the evidence documented in Ampudia and Heuvel (2018), who document that the response of banks' stock valuations to monetary policy shocks changes sign as the level of interest rates decreases. Following their decrease in net worth, banks are forced to increase their loan rates in order to lower their leverage costs. Quantitatively, three key forces shape the response of banks' net worth. First, banks' net worth on impact is well hedged from interest rate risk as they hold long-term assets. Without this hedge, the reversal rate would be substantially larger. However, these assets mature, making net worth more sensitive in subsequent periods. Second, the impaired deposit rate pass-through as policy rates decrease substantially lowers bank profitability, especially as rates enter negative territory. Finally, this impaired pass-through also weakens the inter-temporal substitution channel. This decreases the intermediation gains that generated the positive net worth response close to the steady state.

5.5 The effects of permanently lower rates

We now consider a permanent decrease in the natural rate r^* and suppose that the steady-state inflation target π^{SS} stays unchanged. This implies that the steady-state nominal rate i^{SS} decreases.[35] This permanent shift in the level of steady-state rates reduces the effectiveness of monetary policy for shocks of comparable magnitude. Concretely, for a shock of a similar magnitude to force our baseline economy into its reversal rate, the economy with a lower i^{SS} has a lower response of aggregates to that shock. This indicates that the reversal interest rate has not decreased one-for-one with the steady-state nominal rate change, leaving less leeway for monetary policy.[36]

6 Conclusion

We have shown the conditions for the existence of a reversal interest rate, the rate at which monetary policy stimulus reverses its intended effect and becomes contractionary. Its existence relies on the net interest income of banks decreasing faster than recapitalization gains from banks' initial holdings of fixed-income assets. We showed that its level depends on the magnitudes of these capital gains, the overall capitalization of banks, the strength of the leverage constraints faced by banks, and the deposit supply elasticity. The reversal interest rate creeps up over time, making steep but short rate cuts preferable to "low for long" interest rate environments. Finally, we included our banking frictions in a New Keynesian model and showed that the economics we described have meaningful effects for the transmission of monetary policy in general equilibrium.

For the sake of tractability, we have omitted other channels through which monetary policy can affect banks as well as the real economy. In particular, policies such as ECB's long term refinancing operations could have alleviated some of the low rates' effect on bank margins. Moreover, we have omitted the explicit modeling of risk; hence, we have remained agnostic on how low rates change nonperforming loans and the associated responses in provisions. We see these as important quantitative refinements for future research. Finally, we view our results as driven by unusual surprise movements in interest rates: low-for-long and negative rates were largely unforeseen events. It remains a question whether banks can and will adjust to a permanently lower interest rates environment – for example, by increasing their maturity mismatch. The competitive landscape faced by banks could also change, with depositors growing accustomed to the possibility of negative interest rates, hence supporting

[35] Effectively, β permanently shifts upward.

[36] The maintained assumptions here are that 1) banks do not change their maturity structure in response to that change, and 2) the pass-through of deposit rates stays as described by the data. Our analysis here is purely positive. We leave it to future research to determine whether such responses have happened empirically.

banks' profitability in negative-rate environments.

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A Proofs

Lemma 1

Proof. The Lagrangian of this problem is

$$\begin{aligned}\mathcal{L} =&(1+i^L)L + (1+i)S - (1+i^D)D - \mu(L+S-D-E_0(i)) + \lambda^D(\psi^D D - S)\\ &- \lambda^L(\psi^L L - (1+i^L)L - (1+i)S + (1+i^D)D).\end{aligned}$$

The first-order conditions with respect to S, i^L and i^D are

$$\mu = (1+\lambda^L)(1+i)$$

$$1+i^L = \frac{1}{1+\lambda^L}\left(\mu - (1+\lambda^L)\frac{L}{L'} + \lambda^L\psi^L\right)$$

$$1+i^D = \frac{1}{1+\lambda^L}\left(\mu - (1+\lambda^L)\frac{D}{D'} + \lambda^D\psi^D\right).$$

Define $\frac{D'}{D} = \epsilon^D$, $\frac{L'}{L} = -\epsilon^L$. Rearrangement of these first-order conditions then yields the equations in Lemma 1.

□

Lemma 2

Proof. We can write a bank's problem as

$$\begin{aligned}N(i) = &\max_{i^L, i^D, S} (1+i^L)L + (1+i)S - (1+i^D)D\\ &s.t.\;\; L+S = D+E_0(i),\;\; \psi^L L \leq (1+i^L)L + (1+i)S - (1+i^D)D,\;\; \psi^D D \leq S.\end{aligned}$$

The envelope condition of this problem implies

$$\frac{dN}{di} = \mu E_0'(i) + (1+\lambda^L)S,$$

where the Lagrange multipliers μ and λ^L are as defined in Lemma 1. This yields

$$\frac{dN}{di} = (1+\lambda^L)(S + (1+i)E_0'(i)).$$

Write

$$CG = E_0'(i), \quad NII = i^L L + iS - i^D D.$$

Combining these definitions with the expression for $\frac{dN}{di}$, we obtain

$$\frac{dN}{di} = (1+\lambda^L)\left(\frac{dNII}{di} + (1+i)CG\right)$$

as desired.

Finally, note that when $\psi^D > 0$, banks' holding of fixed-income assets (weakly) increases, making the derivative higher. □

Proposition 1

Proof. Define $N(i)$ as in the proof of Lemma 2. First, we show that $N(i)$ is an increasing function. From the previous lemma above, we know that $\frac{dN}{di} \geq \mu E_0'(i) + (1+\lambda^L)S$. Given that we can bound the first term by assumption, it suffices to show that $S > 0$. But that is immediate given our assumption of a liquidity constraint.

Thus the derivative of $N(i)$ is bounded away from 0 for $i < i_0$. That is, there exists some $G(i_0)$ such that:

$$N(i) \leq N(i_0) - G(i_0)(i_0 - i).$$

When the capital constraint does not bind, the quantity of loans made by the bank is given by $L(i^{L*})$, where i^{L*} satisfies the equation

$$i^{L*} = i + \frac{1}{\varepsilon^L(i^{L*})}.$$

Note that $L(i^{L*})$ is decreasing in i. For sufficiently low i, then,

$$\psi^L L(i^{L*}) \geq N(i_0) - G(i_0)(i_0 - i) \geq N(i),$$

meaning there exists a largest interest rate $\hat{i}$ such that the capital constraint binds for all $i < \hat{i}$. In this region, $L(i^L) = \frac{1}{\psi^L}N(i)$, so $\frac{dL}{di} = \frac{1}{\psi^L}\frac{dN}{di} < 0$. Therefore $i^{RR} = \hat{i}$, since $\frac{dL}{di} < 0$ for all $i < \hat{i}$ and $\frac{dL}{di} > 0$ for $i > \hat{i}$. □

Main Lemma

Suppose $i_0 > i^{RR}$ when the parameters of the bank's problem are $\theta = (\psi^L, \psi^D, \bar{e}_0, e_0(i))$. Under an alternative set of parameters $\hat{\theta} = (\hat{\psi}^L, \hat{\psi}^D, \hat{\bar{e}}_0, \hat{e}_0(i))$ such that $N(i,\theta) > N(i,\hat{\theta})$ for $i \leq i_0$, the reversal interest rate is lower under parameters θ than under θ' (so long as it is unique under both sets of parameters).

Proof. Define $i^{L*}(i)$ implicitly as the solution to the equation

$$i^L - \frac{L(i^L)}{L'(i^L)} = i.$$

Note that $i^{L*}(i)$ is increasing in i, so $L(i^{L*}(i))$ is decreasing in i. Furthermore, i^{L*} does not depend on parameters. The reversal interest rate i^{RR} is the solution to the equation

$$\psi^L L(i^{L*}(i)) = N(i, \theta).$$

Let $i^{RR}(\theta)$ be the reversal interest rate under parameters θ. With parameters θ', for any $i \leq i^{RR}(\theta)$ we have

$$\psi^L L(i^{L*}(i)) \geq N(i, \theta) > N(i, \theta').$$

This is because by the definition of the reversal interest rate, the function $N(i, \theta)$ must be increasing in i in the region $i \leq i^{RR}(\theta)$. Thus it cannot be that $i^{RR}(\theta') \leq i^{RR}(\theta)$. □

Proposition 2

Proof.

1. Clearly, an increase in the capital constraint ψ^L weakly decreases $N(i)$ for all i. Then, by the main lemma, the reversal interest rate must increase.

2. Consider a shift in the interest rate sensitivity of equity such that $E_0(i_0)$ remains constant but $\frac{de_0}{d_i}$ is uniformly increased for all i. Then $N(i)$ is uniformly increased for all $i \leq i_0$, since capital gains following an interest rate cut are larger. By the main lemma, the reversal interest rate must decrease.

3. Note that the first-order condition for i^D implies that i^D is the same in both economies for a given level of $i < i^{RR}$, so $D_A(i) = D_B(i)$. Then the equation

   $$L_j(i) + S_j(i) = D_j(i) + E_0^j(i)$$

   for $j \in \{A, B\}$ implies $L_A(i) + S_A(i) < L_B(i) + S_B(i)$, since $E_0^A(i) < E_0^B(i)$ for all i. Furthermore, note that when $i = i^{RR}$, i^L is the same in both economies, so it must be that $S_A(i) < S_B(i)$. Thus

   $$\frac{dN}{di} = (1 + \lambda^L)(S(i) + (1 + i)E_0'(i))$$

   must be larger in economy B when evaluated at i^{RR} because $S(i^{RR})$ is larger. Thus

   $$\frac{dL_A(i)}{di} = \psi_A^L \frac{dN_A(i)}{di} < \psi_B^L \frac{dN_B(i)}{di} = \frac{dL_B(i)}{di}$$

   at i^{RR}, so the balance sheet constraints $L_j(i) + S_j(i) = D(i) + E_0^j(i)$ yield $\frac{dS_A(i)}{di} > \frac{dS_B(i)}{di}$ at $i = i^{RR}$. But clearly, then, we can keep iterating this argument to obtain $S_A(i) < S_B(i)$ for all $i < i^{RR}$, which then implies $L_A(i) > L_B(i)$ for all $i < i^{RR}$ through the equation for $\frac{dN}{di}$ above (using the fact that the constraint is tighter in economy B).

□

Proposition 3

Proof.

1. A higher lower bound on the deposit rate lowers $N(i)$, so the main lemma implies that i^{RR} increases.

2. Lower markdowns as rates decrease must lower $N(i)$ uniformly, so again the main lemma applies.

3. An increase in the liquidity coefficient ψ_D can only lower $N(i)$. Therefore, i^{RR} must increase by the main lemma.

□

Proposition 4

Proof. A larger v_0 must make $N(i)$ (weakly) increase, so i^{RR} must weakly decrease by the main lemma. □

Proposition 5

Proof. Here we consider a perturbation $E_0(i) \to \tilde{E}_0(i)$ such that $\tilde{E}_0(i) \geq E_0(i)$ if and only if $i \geq i_0$. In particular, for $i < i_0$, $\tilde{E}_0(i) < E_0(i)$, so $N(i)$ is shifted uniformly downward to below i_0. Hence, by the main lemma, i^{RR} must increase. □

Proposition 6

Proof. There are four possible cases: 1) the capital constraint does not bind in either period at an optimum, 2) the capital constraint binds in both periods, 3) the capital constraint binds only in the first period, and 4) the capital constraint binds only in the second period. We consider these cases in turn.

1. If the bank is unconstrained in both periods, $i_t^L = i_{t-1,t} + \frac{1}{\varepsilon_t^L}$ for $t = 1, 2$. Thus it is possible to increase the quantity of loans made in both periods by decreasing both $i_{0,1}$ and $i_{1,2}$ by a small constant $\epsilon > 0$. Therefore, case 1) is never optimal for the central bank.

2. If the capital constraint binds in both periods, it is possible to increase the bank's net worth in both periods by increasing $i_{0,1}$ as long as $B_{0,1}$ and $B_{0,2}$ are sufficiently low that $\frac{dNII_1}{di_{0,1}} > \frac{d}{di_{0,1}}(\frac{B_{0,1}}{1+i_{0,1}} + \frac{B_{0,2}}{(1+i_{0,1})(1+i_{1,2})})$. This increase in interest rates at $t = 1$ then increases the quantity of loans made at $t = 1$ and $t = 2$, so it is never optimal for the constraint to bind in both periods.

3. When the capital constraint binds only in the first period, the central bank can increase the net worth of the bank (and thus the quantity of loans made) by cutting $i_{1,2}$, since this increases the value of equity $E_0(i_{0,1}, i_{1,2}) = \frac{B_{0,1}}{1+i_{0,1}} + \frac{B_{0,2}}{(1+i_{0,1})(1+i_{1,2})}$ and N_1 is increasing in equity. This cut in $i_{1,2}$ does not change the quantity of loans made at $t = 2$ because the bank's choice of i_2^L is unconstrained by assumption. Hence it cannot be that the capital constraint binds only in the first period.

4. Given the analysis of the three cases above, it must be that the capital constraint binds only in the second period. We now argue that it must be *exactly* binding (in the sense that $\lambda_2^L = 0$) when $B_{0,2}$ is sufficiently low. Suppose $\lambda_2^L > 0$. Then the quantity of loans made in the second period is increasing in the bank's net worth N_2 in period 2, as $\psi^L L_2 = N_2$. A change in the interest rate at $t = 2$ has two effects: it changes the value of bank equity at $t = 1$, which feeds into $t = 2$ net worth, and it directly impacts net interest income at $t = 2$. Formally,

$$N_2(i_{0,1}, i_{1,2}) = \max_{i^L, i^D, S} (1 + i^L)L + (1 + i_{1,2})S - (1 + i^D)D$$
$$s.t.\ L + S = D + E_1(i_{0,1}, i_{1,2}),\ \psi^L L \leq N_2,\ \psi^D D \leq S.$$

Then the envelope theorem implies

$$\frac{dN_2}{di_{1,2}} = (1 + \lambda^L)\left(S^* + (1 + i_{1,2})\frac{dE_1}{di_{1,2}}\right).$$

Note that using the formula $E_1 = (1 - \nu)N_1$, we can write

$$\begin{aligned}\frac{dE_1}{di_{1,2}} &= (1 - \nu)\frac{dN_1}{dE_0}\frac{dE_0}{di_{1,2}} \\ &= -(1 - \nu)\frac{dN_1}{dE_0}\frac{B_{0,2}}{(1 + i_{0,1})(1 + i_{1,2})^2}.\end{aligned}$$

Recall from the proof of Lemma 1 that $\frac{dN_1}{dE_0} = 1 + i_{0,1}$ when the capital constraint does not bind. Thus

$$\frac{dN_2}{di_{1,2}} = (1 + \lambda^L)\left(S^* - (1 - \nu)\frac{B_{0,2}}{1 + i_{1,2}}\right),$$

so net worth is increasing in $i_{1,2}$ when $B_{0,2}$ is sufficiently small. Therefore, when $B_{0,2}$ is small and the constraint in the second period binds, loans are increasing in $i_{1,2}$. Loans in the first period do not depend on $i_{1,2}$ because the capital constraint is slack at $t = 1$. The capital constraint must then bind *exactly* at $t = 2$.

The proof in part 1) to raise interest rates in the first period as long as the constraint in the first period is slack as well. Consider setting $i_{0,1} = i_{0,2} = \hat{i}$ such that $\hat{i}$ is the highest interest rate for which the constraint binds in the second period. We now show that under these interest rates, the constraint in the first period will be slack. By assumption, $B_{0,1}$ and $B_{0,2}$ are small enough that $N_1(\hat{i}, \hat{i}) < N_1(i^*, i^*)$. Then

$$E_1(\hat{i}, \hat{i}) = (1 - \nu)N_1(\hat{i}, \hat{i}) < (1 - \nu)N_1(i^*, i^*) = E_1(i^*, i^*) = E_0(i^*, i^*),$$

so equity in the second period is lower than in the first. By the main lemma, the reversal interest rate must be higher in the second period because net worth is increasing in equity. Hence at $\hat{i}$, the capital constraint in the first period must be slack, so by the argument above it is never optimal to increase $i_{0,1}$ from $\hat{i}$, meaning that at an optimum $i_{0,1} \leq i_{1,2}$. □

B A Micro-Foundation for Deposit Stickiness

We describe in this section a micro-foundation for the lack of pass-through on deposits. Each bank is naturally associated with a continuum of depositor households. Their deposit supply is sticky in the sense that the depositors shop around for better deposit rates if the spread between i and the deposit rate i_j^D from their associated bank j is larger than some threshold, the "activation level," $\eta^D(i)$. Hence, banks compete on prices, but only if the spread they charge relative to some baseline rate i is large enough.[37]

Importantly, we assume that the search "activation level" $\eta^D(i)$ is decreasing in the interest rate i. In other words, depositors become more sensitive to spreads when the policy rate is low. This generates pressure on the extensive margin of banks' deposit margins as rates decrease. For example, depositors are more prone to switch banks if the interest rate is negative, as empirically documented in Hainz et al. (2017). In addition, bank deposit rate choices are also driven by intensive margin considerations. That is, conditional on keeping a customer, the bank might decide to offer an attractive interest rate to ensure that the customer supplies a sufficient amount of deposits instead of simply consuming his income or substituting to alternative savings vehicles like cash.

Concretely, each depositor household $h \in [0,1]$ in the continuum associated with bank j has an activation level $\eta^D(i)$. He only considers looking at the rates offered by competing bank j' if the rate offered by his bank, i_j^D, is below $i - \eta^D(i)$. We assume that $\partial \eta^D(i)/\partial i \geq 0$, that is, the activation level is increasing with the interest rate level, so that spreads are less tolerated at low levels of the policy rate.

Let us denote $\boldsymbol{i}_{-j}^D \equiv \{i_{j'}^D\}_{j' \neq j}$ as the vector of competitors' deposit rates. Hence, the share of costumers ϑ_j^D that actually stay with bank j is

$$\vartheta_j^D(i_j^D; \boldsymbol{i}_{-j}^D, i) \equiv \mathbf{1}_{\{i - i_j^D \leq \eta^D(i) \,\vee\, i_j^D > \max_{j' \neq j} i_{j'}^D\}}.$$

We can then decompose the residual deposit supply faced by bank j as consisting of an extensive and an intensive margin:

$$D_j(i_j^D; \boldsymbol{i}_{-j}^D, i) = \underbrace{\vartheta_j^D(i_j^D; \boldsymbol{i}_{-j}^D, i)}_{\text{Extensive margin}} \times \underbrace{d(i_j^D)}_{\text{Intensive margin}}$$

Provided the intensive demand for deposits is sufficiently inelastic – which is true in our calibration – the extensive margin dominates and the rates are given by

$$i_t^D = i_t - \eta^D(i_t)$$

Figure 8 depicts the pass-through of i_t in i_t^D for our calibration of $\eta^D(i_t)$.

[37] Varying markdowns at the extensive margin can be modeled in numerous ways. A large literature focuses on switching costs (Klemperer, 1995), which is sometimes applied to banking (as in Sharpe (1997)). Our goal here is to have a realistic yet parsimoniously parametrized model that easily fits in a New Keynesian model.

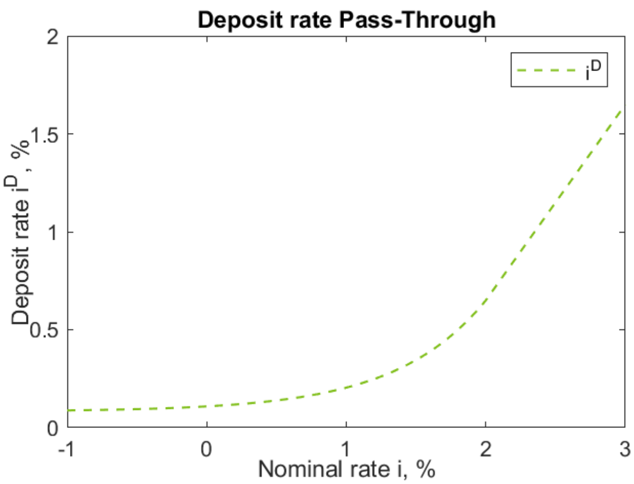

Figure 8: Pass-through of nominal rates into deposit rates given the calibrated values of η_1, η_2, η_3. The pass-through is assumed to be one-for-one for policy rates above the steady state ($i > i^{SS}$).