

Monotone Additive Statistics

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Talk Overview

- Definition of **monotone additive statistics**.
- Characterization.
- Applications.
 - ▶ Posted prices for sacks of potatoes.
 - ▶ Fishburn-Rubinstein time preferences.
 - ▶ Rabin-Weizsäcker preferences over gambles.
- Monotone additive costs of **Blackwell experiments**
 - ▶ Different paper: “From Blackwell Dominance in Large Samples to Rényi Divergences and Back Again.”
 - ▶ Same authors.
 - ▶ Related ideas.
- Work in progress.

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Monotone Additive Statistics

- A **statistic** is a way of capturing distributions by a single number.
 - ▶ Expectation.
 - ▶ Median.
 - ▶ Value at risk.
 - ▶ Certainty equivalent.
- Let L^∞ be the set of all bounded random variables.
- A **statistic** is a map $\Phi: L^\infty \rightarrow \mathbb{R}$ such that
 - ① $\Phi(c) = c$.
 - ② If X and Y have the same distribution then $\Phi(X) = \Phi(Y)$.
- It is **monotone** if $X \geq_1 Y$ implies $\Phi(X) \geq \Phi(Y)$.
- Equivalently: it is monotone if $X \geq Y$ implies $\Phi(X) \geq \Phi(Y)$.
 - ▶ Because $X \geq_1 Y$ iff $\exists \tilde{X} \sim X, \tilde{Y} \sim Y$ s.t. $\tilde{X} \geq \tilde{Y}$ a.s.
- A statistic is **additive** if $\Phi(X + Y) = \Phi(X) + \Phi(Y)$ whenever X and Y are **independent**.

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Monotone Additive Statistics

- **Question:** What are the additive monotone statistics?

Examples of Monotone Additive Statistics

- $\mathbb{E}[X]$.
- $\max[X] = \sup\{c \in \mathbb{R} : \mathbb{P}[X \geq c] > 0\}$.
- $\min[X]$.
- For $a \neq 0$,

$$S_a(X) = \frac{1}{a} \log \mathbb{E}[e^{aX}].$$

- By continuity
 - ▶ $S_0(X) = \mathbb{E}[X]$,
 - ▶ $S_\infty(X) = \max[X]$,
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Characterization

- Is there anything beside the S_a 's?
- Main result: this is it.
- Well... we can also take weighted averages.

Theorem

Let Φ be a monotone additive statistic. Then there is a probability measure m on $\mathbb{R} \cup \{+\infty, -\infty\}$ such that

$$\Phi(X) = \int S_a(X) dm(a).$$

- $\{S_a\}$ are the extreme points of the set of additive monotone statistics.

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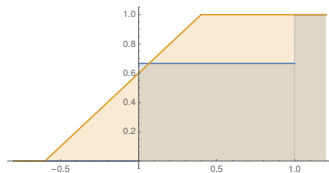
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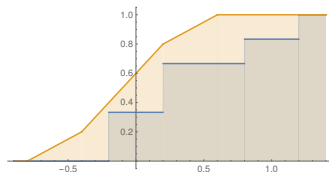
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Proof ideas

- Take X, Y that are not ranked under FOSD.
- Is it possible that there is a independent R such that $X + R \geq_1 Y + R$?
- Example: $X \sim B(1/3), Y \sim U([-3/5, 2/5])$.

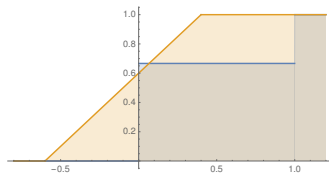


- Works for $\mathbb{P}[R = \pm 1/5] = 1/2$.

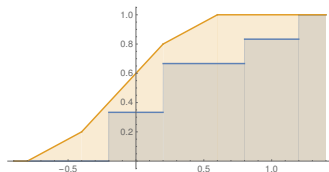


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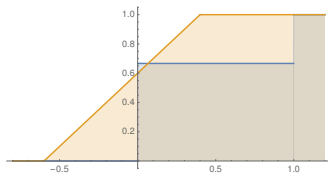


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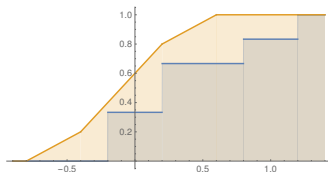


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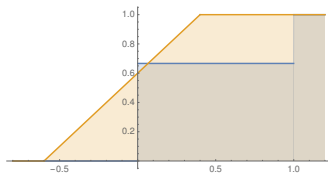


- Works for $\mathbb{P}[R = \pm 1/5] = 1/2$.

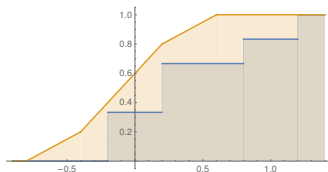


Proof ideas

- Take X, Y that are not ranked under FOSD.
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- Pomatto, Strack, Tamuz (2019): If $\mathbb{E}[X] > \mathbb{E}[Y]$ then $X + R \geq_1 Y + R$ for some independent R .
- Under what conditions on X, Y is there a **bounded** independent r.v. R such that $X + R \geq_1 Y + R$?
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For $X, Y \in L^\infty$, if $S_a(X) > S_a(Y)$ for all a , then there exists an $R \in L^\infty$ such that $X + R \geq_1 Y + R$.

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$$\begin{aligned} [(G - F) * h](y) &= \int_{-N}^N [G(x) - F(x)] \cdot h(y - x) dx \\ &= e^{-\frac{y^2}{2V}} \cdot \int_{-N}^N [G(x) - F(x)] \cdot e^{\frac{y}{V} \cdot x} \cdot e^{-\frac{x^2}{2V}} dx. \end{aligned}$$

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Application: Posted Prices for Sacks of Potatoes

- Consider a buyer who posts her prices for **potatoes**.
- Farmers come and sell her their crops.

Potatoes	Price
1	\$1
2	\$2
3	\$3.10
4	\$4
5	\$5
6	\$6
7	\$5

- Price $P: \mathbb{R}_+ \rightarrow \mathbb{R}_+$.
- **Free disposal:** $x \geq y$ implies $P(x) \geq P(y)$.
- **No mergers:** $P(x + y) \leq P(x) + P(y)$.
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Application: Fishburn-Rubinstein Time Preferences

- A pair (x, t) is a (positive) amount of **money** x at (non-negative) **time** t . The set of such pairs is $\Omega = \mathbb{R}_{++} \times \mathbb{R}_+$.
- Fishburn and Rubinstein consider preferences \succsim over Ω .

Axiom

- All such preferences come from exponential discounting.

Theorem (Fishburn and Rubinstein)

The axioms imply that \succsim is represented by $f(x, t) = u(x)e^{-rt}$ for some $r > 0$, and an increasing $u: \mathbb{R}_{++} \rightarrow \mathbb{R}_{++}$.

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- *If $x > y$ then $(x, t) \succ (y, t)$.*
 - *If $t < s$ then $(x, t) \succ (x, s)$.*
 - *If $(x, t) \succ (y, s)$ then $(x, t + \tau) \succ (y, s + \tau)$.*
 - *Upper and lower contour sets are closed.*
- All such preferences come from exponential discounting.

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The axioms imply that \succ is represented by $f(x, t) = u(x)e^{-rt}$ for some $r > 0$, and an increasing $u: \mathbb{R}_{++} \rightarrow \mathbb{R}_{++}$.

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- A pair (x, t) is a (positive) amount of **money** x at (non-negative) **time** t . The set of such pairs is $\Omega = \mathbb{R}_{++} \times \mathbb{R}_+$.
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- A pair (x, T) is a (positive) amount of money x at a **random** (non-negative) time T .

Axiom

- *Keep FR's axioms for deterministic times.*
 - *If $T <_i S$ then $(x, T) \succ (x, S)$.*
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- Example: $f(x, T) = u(x)\mathbb{E}[e^{-rT}]$.
 - ▶ Expectation of the Fishburn-Rubinstein utility.
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- Example: $f(x, T) = \frac{u(x)}{\mathbb{E}[e^{rT}]}$.
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- Let L^∞ be the set of **bounded gambles**.
- Consider an expected utility agent with an increasing utility function u for money.
- Write $X \succ Y$ if $\mathbb{E}[u(X)] > \mathbb{E}[u(Y)]$.

Axiom

Suppose X_1, X_2 are independent, Y_1, Y_2 are independent. If $X_1 \succ Y_1$ and $X_2 \succ Y_2$ then $Y_1 + Y_2$ does not stochastically dominate $X_1 + X_2$.

- What does this tell us about u ?

Theorem (Rabin-Weizsäcker)

The axiom implies that either $u(x) = ae^{ax}$ for some $a \neq 0$, or $u(x) = x$ (up to affine transformations).

- So **CARA** agents are the only ones that satisfy the axiom.

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Proposition

The axioms imply that \succ is represented by some monotone additive statistic.

- Φ is the average of CARA certainty equivalents $S_a(X) = \frac{1}{a} \log \mathbb{E} [e^{aX}]$.

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- What about general (non-expected utility) preferences?
- Write $X \succ Y$ if the agent strictly prefers X to Y .

Axiom

- 1 **Rabin-Weizsäcker.** *Suppose X_1, X_2 are independent, Y_1, Y_2 are independent. If $X_1 \succ Y_1$ and $X_2 \succ Y_2$ then $Y_1 + Y_2$ does not stochastically dominate $X_1 + X_2$.*
- 2 $X + \varepsilon \succ X$.
- 3 for all X there is a $c \in \mathbb{R}$ such that $X \sim c$.

- Such preferences can be represented by a monotone additive statistic.

Proposition

The axioms imply that \succ is represented by some monotone additive statistic.

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Monotone Additive Costs of Blackwell Experiments

- Binary state of the world $\theta \in \{0, 1\}$.
- A **Blackwell Experiment** is a pair $\mu = (\mu_0, \mu_1)$ of probability measures on some measurable space Ω .
- We say that it is **bounded** if $\log \frac{d\mu_0}{d\mu_1}$ is bounded.
- The collection of bounded experiments is \mathcal{B} .
- The Blackwell order captures a strong sense of when one experiment is more informative than another.
- The product experiment $\mu \otimes \nu$ is given by $(\mu_0 \times \nu_0, \mu_1 \times \nu_1)$.

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Theorem (Mu, Pomatto, Strack, Tamuz (2020))

Every monotone additive cost is a weighted sum of the KL-divergences and the Rényi divergences.

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