

Financial Fragility with Collateral Circulation*

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Abstract

We present a model of secured credit chains in which the circulation of risky collateral generates fragility. An intermediary stands between an investor and a financier. The intermediary borrows to finance a profitable investment opportunity of his own, subject to a moral hazard problem, and can also intermediate funds. He will only do so if he can re-use the collateral pledged by the investor when borrowing from the financier. We show that when the re-pledged collateral is sufficiently risky and the loans that it secures are recourse, the circulation of collateral generates fragility in the chain, by undermining the intermediary's incentives. The arrival of news about the value of the re-used collateral further increases fragility. This fragility channel of collateral re-use generates a premium for safe or opaque collateral. The environment considered in our model applies to various situations, such as trade credit chains, securitization and repo markets.

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1 Introduction

Lenders often require borrowers to pledge assets to secure their exposures. When lenders themselves need to borrow, they may be able to repledge the collateral or the cash flows of the loan backed by this collateral. Through this process, collateral circulates along credit chains. There are various examples of such collateral circulation. Firms that extended trade credit often use their account receivables or invoices as collateral when they borrow from banks (see [Berger and Udell \(1990\)](#) or [Omiccioli \(2005\)](#)). Through securitization, banks can secure fundings for new loans by pledging the cash flows from other loans. This form of collateral circulation is sometimes referred to as pyramiding (see [Geanakoplos and Zame \(2010\)](#)). In markets for repurchase agreements (repo), intermediaries protect lenders by re-pledging financial securities such as T-bills they themselves received as collateral, a practice known as collateral rehypothecation.¹ In segmented markets, collateral circulation allows funds to flow from ultimate lenders to ultimate borrowers. [Gottardi and Kubler \(2015\)](#) and [Gottardi et al. \(2019\)](#) show that when funding constraints are tight, pyramiding and collateral re-use can generate additional borrowing and increase welfare.

Financial regulators have raised concerns that secured lending based on pyramiding or collateral re-use may generate risks and weaken credit chains.² Such concerns about the fragility of a *secured* credit chain are intriguing, because all lenders that form the chain are supposed to be protected against the default of their creditors, which should mute the contagion of adverse shocks along the chain. However financial authorities provide little discussion about the potential contagion channels along a secured credit chain. The objective of this work is to provide an answer to the following questions. Can secured credit chains be inefficient? Can pyramiding or collateral re-use trigger a higher default probability of a borrower? How can this happen? How is collateral quality related to risks and fragility in a secured credit chain?

To address these questions, we examine a simple model with three classes of risk-neutral agents: borrowers, lenders and intermediaries. Both borrowers and intermediaries have profitable investment opportunities but no funds of their own. Lenders have deep pockets but no investment opportunity. Hence, there are gains from trade but credit is subject to two frictions. First, intermediaries' investment are subject to moral hazard, as in [Holmstrom and Tirole \(1997\)](#). Intermediaries can choose the probability of success of their investment but this choice is costly and

¹At the macro level, [Singh \(2019\)](#) ?? [Singh \(2011\)](#) shows that collateral velocity measured as the ratio between the total collateral pledged to the total collateral available is about 3, so the same piece of collateral is used to secure three loans on average.

²For example, the Financial Stability Board ([FSB, 2017](#)) remarks that “Collateral re-use can increase the interconnectedness among market participants and potentially contributes to the formation of contagion channels and risks.”

effort is unobservable. Second, the market is segmented as borrowers can trade with intermediaries but not, directly, with lenders. To channel funding between lenders and borrowers through a credit chain and finance their own investment, intermediaries must take a larger debt position.

Each loan must be backed by collateral. The investment returns are pledgeable and can thus be used as collateral. We say the intermediary can re-use collateral when the loan to the borrower can be pledged as collateral with the lender. The ability to re-use collateral allows the intermediary to pledge more assets to secure the larger debt position required to finance his own investment and his intermediation activity. The larger debt can be raised as two separate loans, backed by the intermediary's own investment and the re-used collateral, respectively. We assume, however, that the borrower cannot ring-fence the assets used as collateral, that is, each loan provides recourse to other assets on the borrower's balance sheet. We argue in Section 7 that this feature is relevant to the various situations to which our model applies: factoring, repo, and securitization.

The first insight from our analysis is that, in the environment considered, collateral re-use is essential for intermediation. Without re-use, the intermediary's entire debt is only backed by his own investment, which undermines his incentives to exert effort. The possibility to re-use collateral increases the intermediaries' ability and willingness to take a larger loan to intermediate funds between the lender and the borrower.³ The intermediation profits earned by channeling the lenders' funding to the borrower reduce the (net) value of the debt incurred by the intermediary's to finance his own investment. Compared to the no re-use case, the intermediary has so more skin-in-the game in his own project and, therefore, exerts more effort. When the re-used collateral is safe enough, we show that re-pledging the loan to the lender is equivalent to selling it. In this case, the only effect of re-use is the greater skin-in-the-game effect described above. Re-use is always profitable and the intermediary's incentives improve, thus decreasing his probability of default, and making the chain less fragile.

The second insight from our analysis is that, when the re-used collateral is sufficiently risky, re-use can still be profitable but lead instead to an increase in fragility, as FSB (2017) suggests. In this case, we identify a negative hedging effect which may overcome the positive skin-in-the-game effect described above. To understand this other effect, note that, when the yield of the re-used collateral is sufficiently risky, its payoff in case of success may allow the intermediary to repay his whole debt to lenders and leave him some positive payoff even when his own investment fails. This provides some hedging to the intermediary against the default of his own project, which, in turn, weakens his incentives to exert effort and increases the default probability of his own investment.

³For instance, a dealer bank who is financing a reverse repo with a borrower may only do so if it has the right to re-use the collateral pledged by this borrower in a repo with a lender. Singh (2011) argues that dealer banks' ability to re-use collateral is essential to their role as repo intermediaries.

We show this negative hedging effect is exacerbated by the recourse feature of the loans.

We then show that, when collateral risk exceeds a given threshold, the hedging effect dominates the skin-in-the game effect: incentives are weaker and hence the probability of default is higher when the intermediary chooses a larger debt exposure in order to intermediate funds than when he cannot re-use collateral. Hence, collateral circulation along the credit chain generates fragility. Provided collateral risk is not too high, however, the intermediary still prefers the large loan because the gains from intermediation exceed the losses from the negative effect on his incentives. Our fragility result depends on the following features: market segmentation generates a role for the circulation of collateral and intermediation, which then interacts, when collateral is risky, with the moral hazard problem faced by the intermediary in funding his own investment.

In Section 7, we describe three environments to which our model and hence our results about fragility apply: trade credit, securitization or repos. In these markets, some intermediary, either a bank or a firm, re-use collateral by pledging assets or loans. As argued above, a key feature to generate our fragility results is that the intermediary's debt backed by re-used collateral is recourse. This recourse feature is present in all three markets considered. In securitization, for instance, sponsors provided guarantees to the creditors of their Special Purpose Vehicle (SPV), beyond the value of the loans held by the SPV. Hence, while securitization allows banks to capture additional intermediation gains, their balance sheet becomes more fragile when the SPV loans are risky. Fragility is the price to pay for the development of secured credit chains with risky collateral.

We then show that collateral re-use can further increase fragility through an additional *news channel*. We extend the model to allow the intermediary to receive news about the payoff of the re-used collateral before he chooses the level of effort for his own investment. The intermediary can then optimally adjust his effort to this information. Intuitively, he will exert less effort when he learns the collateral value is low because he understands lenders will claim most of the cash flow generated by his own investment. This induces a positive correlation between the cash flow of the borrower's project and the one of the intermediary's own project. This endogenous correlation effect is akin to contagion: the negative shock to the yield of the re-pledged collateral increases the default risk of the intermediary.

But there is more than just correlation: The collateral value is low exactly at times when the lender needs it, when the intermediary's project is more likely to fail. As a result, the re-used collateral has a lower value for the lender. He then charges a higher interest rate and the intermediary chooses a lower expected level of effort. Therefore, in line with the claims of FSB (2017), our analysis shows that collateral re-use can amplify a negative shock about the value of the collateral: the expected level of effort is unconditionally lower than without the news channel.

We also endogenize the arrival of news by assuming that intermediaries can choose to pay some

cost to acquire information about the value of the re-pledged collateral and the decision to acquire information is not observable by lenders. In this situation an exogenous increase in collateral risk may induce the intermediary to acquire information (since its benefit, for a given lending contract, increase in such risk). Anticipating this, lenders will charge a higher interest rate, as we explained above. Ex-ante intermediaries' expected utility is lower with news, which implies they prefer an opaque environment where information about collateral is hard to obtain, or to be able to commit not to acquire information.⁴

Finally, we endogenize the quality of the collateral that the intermediary re-uses. To do so, we assume the borrower also faces a moral hazard problem. In this situation, the riskiness of the cash-flow of the loan granted varies with the face value of the loan. The higher the face value, the lower the incentive of the borrower to exert effort. We show that collateral re-use can give some incentive to the intermediary to sacrifice intermediation profits by lowering the face value of the loan granted in order to reduce collateral riskiness. Hence, intermediaries are willing to pay a re-use premium for safer collateral. Through an example, we show that despite these incentives fragility is still an equilibrium phenomenon.

Literature review

Our paper contributes both to the literature on collateral re-use and securitization. The role of collateral re-use in shadow banking is discussed by [Singh and Aitken \(2010\)](#) and subsequent papers by these authors. Several theoretical works studied collateral re-use (see [Andolfatto et al., 2017](#), [Infante, 2019](#)) and its role in expanding borrowing (see for instance [Bottazzi et al., 2012](#)). In [Gottardi et al., 2019](#), we showed that collateral re-use can also explain the formation of intermediation chains, a feature we take as exogenous in this paper, as we focus on the role of collateral re-use for the fragility of credit chains.

Pyramiding resembles collateral re-use, except that what is re-used as collateral by the lender is the cash flow of the loan granted rather than the (financial or tangible) asset pledged by the borrower to secure the loan. An analogous outcome is also achieved with securitization, which entails the outright sale of the entire cashflow of the loans or just a fraction of it.⁵ As for collateral re-use, [Gottardi and Kubler \(2015\)](#) show that pyramiding can relax collateral constraints by making an efficient use of the collateral (see also [Geanakoplos and Zame, 2010](#)). The 2007 financial crisis triggered a debate about the effect of securitization on lenders' incentives to monitor loans in the mortgage market. [Keys et al. \(2010\)](#) report evidence that securitization led issuers to apply lax standards for subprime loans, a finding questioned by [Bubb and Kaufman \(2014\)](#). [Plantin](#)

⁴This echoes the result by [Dang et al. \(2015\)](#) and [Monnet and Quintin \(2017\)](#) that ignorance is bliss.

⁵[Maurin \(2017\)](#) shows that pyramiding is a more efficient way to reuse collateral if loans are non-recourse. For a joint analysis of pyramiding and rehypothecation, see [Muley \(2016\)](#).

(2011) shows theoretically that more securitization and less screening by lenders need not be inefficient. Chemla and Hennessy (2014) and Vanasco (2017) highlight the trade-off faced by intermediaries when screening between higher borrowers' quality and the resulting information asymmetry between the intermediaries and their creditors. These works show that securitization can reduce the quality of the marginal loan extended by an intermediary. Our model shows that securitization can make intermediaries' balance sheet more fragile by increasing the default risk of other investments.

More broadly, our paper relates to a large literature on fragility and contagion in credit chains and networks. Kiyotaki and Moore (1997) studied the propagation of default along credit chains and Allen and Gale (2000) showed that the structure of the network affects the propagation of risk. Following works extended these results by considering either simple interactions in complex networks or richer relationships in simplified networks. Eisenberg and Noe (2001), Acemoglu et al. (2015) and Cabrales et al. (2017) belong to the first category with the latter works analyzing the topology of resilient networks. Our work with endogenous lending contracts belongs to the second category, together with Farboodi (2017) and Di Maggio and Tahbaz-Salehi (2015). Like in our model, the intermediary in Di Maggio and Tahbaz-Salehi (2015) is subject to a moral hazard problem. Their intermediaries have no investment opportunity, however, while we are interested in the contagion between the intermediary's own investment and his intermediation business. Like in Farboodi (2017), the intermediary in our model voluntarily chooses to expose himself to fragility to reap intermediation profits. Unlike in her paper, we model explicitly the role of collateral and study contagion between different activities.

Finally, we identify a news channel for fragility whereby the access to a technology to produce more accurate information about the value of collateral increases default risk when intermediaries re-use collateral. Our results suggest that opacity about assets' payoffs can be optimal when they are used as collateral, a theme shared with Dang et al. (2015) and Gorton and Ordoñez (2014). Our news channel, however, is different from the Hirshleifer (1971) effect at play in these papers. In our model, the intermediary acquires information to correlate the effort choice on his own investment with the value of the collateral and we show this correlation generates fragility.

The rest of the paper is structured as follows. Section 2 present the model. The benchmark case without collateral re-use is studied in Section 3. Our main results about collateral re-use and fragility are gathered in Section 4. Section 5 shows that fragility worsens in the presence of news about the collateral value. In Section 6, we endogenize the quality of the re-used collateral. Finally, Section 7 discusses applications of our model and Section 8 concludes. All proofs are in the Appendix.

2 Model

2.1 Technology and Preferences

The model has three periods $t = 0, \frac{1}{2}, 1$. Period $\frac{1}{2}$ plays no role until Section 5. There is one consumption good called cash for simplicity. The economy is populated by three types of risk-neutral agents, borrowers, intermediaries and lenders called A , B and C respectively. Agent C has a large initial endowment of cash. Agents A and B have no initial wealth but they are both endowed with a risky project that requires an initial investment of size 1.⁶ So the overall financing need of this economy is 2.

The project of agent A matures at date 1 and pays off X_A with probability p_A and 0 otherwise. We assume this investment has positive net present value but the expected return $R_A \equiv p_A X_A$ is not too high.

Assumption 1. *Expected return on A 's investment*

$$1 < R_A < 2$$

The upper bound on the expected return R_A ensures that the expected value of A 's investment cannot cover by itself the total financing need of the economy. In our analysis, we will often take the expected value R_A as given and let the probability of success p_A vary adjusting X_A accordingly. Then, an increase in p_A corresponds to a decrease in the variance of the payoff of A 's investment.

The project of agent B also matures at date 1 and pays off X_B in case of success and 0 otherwise. But agent B 's investment is subject to moral hazard: at the interim date, she chooses the probability of success p_B and incurs a utility cost $\frac{1}{2}X_B p_B^2$. This effort choice is not contractible. In the following, we refer to p_B as the probability of success or as the effort choice of agent B . In Section 6, we analyze the case in which A 's investment is also subject to moral hazard.⁷

The specification of the effort cost implies that the socially optimal choice of effort, obtained as solution of

$$\max_{p_B} U_B = p_B X_B - \frac{1}{2} X_B p_B^2$$

is $p_B^* = 1$, that is, such that B 's project always succeeds. Because the effort choice of agent B

⁶Although we consider for simplicity the case where the intermediary has no funds of his own, the moral hazard friction we introduce below induces more generally a cost of deploying own capital to fund intermediation activity. Whenever the intermediary's own funds are limited, the intermediary would rather use his capital for his own investment. If intermediation gains are small, expanding borrowing may prove too costly.

⁷The quadratic cost function is used for tractability. All that matters is that the cost is increasing and convex in the probability of success.

is not contractible as in [Holmstrom and Tirole \(1997\)](#), agent B chooses a strictly lower level of effort when he borrows to finance the project, as the analysis will make clear. To streamline the analysis, the yield X_B of B 's project in case of success is then assumed to be sufficiently high that the expected net present value of investing is positive even when the investment is financed entirely with debt. Given the functional form for the effort cost, this assumption takes the following form.

Assumption 2. *High return on B 's investment:*

$$X_B \geq 4$$

Intuitively if X_B is high enough, agent B can pledge enough of this cash flow to repay a loan without undermining incentives too much.

2.2 Frictions and Contracting

There are obvious gains from trade in our model. Agents A and B have investment opportunities but no funds while agent C has funds but no investment opportunity. Absent other frictions, agent C would lend to agent A and B against a piece of their investments and the only inefficiency would stem from agent B 's inability to commit to the ex-ante optimal effort choice. We impose, however, some restrictions on admissible trades to capture frictions in actual financial markets mentioned in the Introduction.

Segmented market. Agent B can trade with agents A and C but agents A and C cannot trade directly. As a result, agent B has two potential roles in the model: he borrows from agent C to finance his own project but he can also intermediate funds from C in order to lend to A .

Pledgeability. When borrowing from agent B , agent A pledges his own investment as collateral. When it comes to the loan from agent C to agent B , we will compare two different cases. In the first case, agent B can only pledge his own investment as collateral. In this case, any payoff agent B receives from lending to agent A cannot be seized by agent C . In the second case, agent B can in addition re-pledge the collateral pledged by A when borrowing from agent C .⁸

Non-contingent loan terms. Loan contracts between C and B and between B and A specify the amount borrowed, the collateral pledged and the repayment required. The latter is assumed not to be contingent on the realized yield of the collateral. In the repo market for example, the repurchase price is fixed in advance.

⁸Technically, agent B can re-pledge the loan he made to A rather than the whole investment of A implicitly backing the loan as collateral. Because we consider the case where B has all the bargaining power, this distinction becomes moot since B will appropriate all the cash flows from A 's investment.

Given the assumed segmentation of the market, we can also refer in what follows to agent A as the borrower, to B as the intermediary and to C as the lender. In what follows, we focus on the case in which the intermediary has all the bargaining power in both his lending relationships.⁹ This assumption simplifies the analysis because it implies that B will ultimately own A 's investment. However, most of our results are robust to different specifications of the bargaining power. In Appendix [A](#) we show that our results extend, qualitatively, to the case where in each transaction all the bargaining power rests with the borrower side, that is, with A when borrowing from B and with B when borrowing from C .

3 No Re-use

In this section, we first show that agent B may not be willing to intermediate funds between C and A when he cannot re-use A 's collateral. Agent B must decide how much to borrow from C . If he borrows 1 unit, he can invest in his own project only. If he borrows 2 units, he can also lend 1 unit to agent A . In the second case, without collateral re-use, his own investment alone must back the bigger loan. Under moral hazard, we will show that the negative effect on incentives can be so strong as to induce B to forgo a profitable intermediation opportunity.

To show that such a trade-off only arises in the presence of moral hazard, consider the benchmark where agent B can commit ex-ante to the optimal choice of effort $p_B^* = 1$. If he lends to A , agent B can capture entirely the surplus from the investment by charging a face value equal to X_A for the 1 unit loan required for the investment. His utility when getting a small 1 unit loan or a large 2 units loan is given respectively by

$$U_{B,1}^* = X_B - 1 \tag{1}$$

$$U_{B,2}^* = p_A X_A + X_B - 2 > U_{B,1}^* \tag{2}$$

where the inequality follows from Assumption [1](#), stating that the net present value of A 's investment is positive. It follows that agent B would always choose to raise 2 units from agent C to invest and intermediate. In the analysis below, we show that the moral hazard problem of agent B generates a trade-off between these two options.

Let $R_{B,l}$ be the face value of agent B 's debt when he gets a loan of size $l \in \{1, 2\}$ from agent C . Because the investment of agent B succeeds with probability $p_{B,l}$, the participation constraint

⁹For example, in the repo market, dealer banks earn a spread on their matched repo book and they can benefit by charging higher haircuts as lenders. See for example [Infante \(2019\)](#).

of agent C is

$$p_{B,l}R_{B,l} \geq l. \quad (3)$$

Recall that agent B has all the bargaining power when he lends to A . So he can capture the entire expected payoff $R_A = p_A X_A$ from this investment by setting the face value of the loan equal to X_A . Agent B then chooses his effort level solving the following problem,

$$U_{B,l} = (p_A X_A) \mathbb{I}_{l=2} + \max_{p_{B,l}} p_{B,l} \max \{X_B - R_{B,l}, 0\} - X_B p_{B,l}^2 \quad (4)$$

In this case the loan to A cannot be (re)-used as collateral when borrowing from C . So any loan is backed entirely by B 's project. For any feasible face value $R_{B,l} \leq X_B$, solving for $p_{B,l}$ in equation (5), we find the optimal choice of effort:

$$p_{B,l} = \frac{X_B - R_{B,l}}{X_B}. \quad (5)$$

It is intuitive that the effort choice is decreasing in the debt face value $R_{B,l}$. The higher the repayment due, the lower the share of the total payoff left for agent B and the lower his incentives to exert effort. When agent B borrows 2 units from agent C , the face value of the loan $R_{B,2}$ is larger than when agent B only borrows 1 unit from agent C , with face value $R_{B,1}$. As a consequence agent B 's will exert less effort as he takes a larger loan from C . Substituting for the optimal choice of effort in equation (4) shows that agent B 's expected utility is decreasing in $R_{B,l}$. Therefore, the optimal value of $R_{B,l}$ for agent B is the lowest one that satisfies the participation constraint of agent C ,

$$(X_B - R_{B,l}) R_{B,l} \geq l X_B \quad (6)$$

Solving for this value gives the equilibrium face value with a loan of size l .

Proposition 1. *Agent B can only borrow 2 units from agent C if $X_B \geq 8$. When agent B borrows $l \in \{1, 2\}$ units from agent C , the face value of the loan is*

$$R_{B,l} = \frac{2l}{1 + \sqrt{1 - \frac{4l}{X_B}}} \quad (7)$$

Agent B 's effort choice and his utility are then given respectively by

$$p_{B,l} = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{l}{X_B}} \quad (8)$$

$$U_{B,l} = p_A X_A \mathbb{I}_{l=2} + \frac{1}{2} p_{B,l}^2 X_B \quad (9)$$

We see that $R_{B,l} > l$, that is the repayment due always exceeds the loan amount l . Agent C charges a positive interest rate because agent B can default as he chooses an effort level lower than $p_B^* = 1$. The interest rate $R_{B,l}/l$ charged by the lender is increasing with the loan size l to compensate for the larger probability of default with a 2 unit loan. The values of the repayment, effort and agent B 's utility for any admissible loans size are as stated in in Proposition 1.

The negative effect of the loan size on the effort choice explains why agent B may prefer to forgo a profitable intermediation opportunity between A and C . When agent B can commit to the optimal effort level, he always prefers to take a large loan from C because the expected value of the (intermediated) loan to A , equal to R_A , more than exceeds the extra borrowing cost, equal to 1. Without commitment, however, the extra borrowing cost for a large loan is higher than 1 because agent B then reduces the effort on his own investment. Hence, even if the loan to A is profitable, agent B now faces a trade-off between capturing intermediation profits and decreasing effort on his own investment. The comparison between agent B 's utility with a small loan and a big loan, respectively $U_{B,1}$ and $U_{B,2}$, leads to the following result:

Corollary 1. *Without re-use, agent B chooses to get a small loan from agent C if $X_B \leq 8$ or if $X_B \geq 8$ and*

$$p_A X_A - 1 \leq \frac{1}{2} X_B [p_{B,1} - p_{B,2}] - \frac{1}{2} \quad (10)$$

with $p_{B,l}$ the effort choice with a loan of size l , given in Proposition 1.

When the yield of his project is too low, that is $X_B \leq 8$, agent B chooses a small loan because it is the only available option. When $X_B \geq 8$, both options are feasible but agent B still prefers a small loan if condition (10) holds. The left-hand side of (10) is a measure of B 's intermediation profit and the right-hand side captures the incentive cost of a bigger loan. It is immediate to verify that this term is decreasing in X_B . When X_B is large, equation (5) shows that the effort choice is less sensitive to the debt face value, that is, the negative effect on incentives from borrowing more is then smaller. Overall, agent B chooses to borrow 1 unit if the foregone intermediation profit is low compared to the negative incentive effect of getting a bigger loan.

In the following, we assume that parameters are such that agent B prefers a small loan without collateral re-use. When referring to the contracts chosen by agent B without re-use, we now drop

the subscript referring to the size of the loan since it is always equal to 1.

Assumption 3. *Either $X_B \leq 8$ holds or $X_B \geq 8$ and (10) hold.*

4 Collateral re-use

In this section, we examine the case in which agent B can re-use the collateral pledged by A. Agent B's borrowing from C can now be backed both by his own investment and the re-used collateral from A. To borrow 2 units, B can either get a single 2 unit loan collateralized by both assets or two distinct loans of one unit backed by each piece of collateral. It is immediate to verify that these two approaches are equivalent if lending is recourse. A recourse loan grants a creditor a claim to the borrower's balance sheet if the asset he receives as collateral falls short of the promised repayment.¹⁰ To simplify the exposition, we adopt the first approach with a single 2 unit loan.

Suppose then that agent B gets a loan of size 2 from agent C with a repayment of R_B^r where the superscript r is for re-use. As in the previous section, agent B has all the bargaining power when he lends to agent A, so agent B sets the repayment of agent A to X_A . With re-use, agent B can now pledge as collateral, in addition to his own investment, an asset worth X_A with probability p_A and 0 with probability $1 - p_A$.¹¹ The payoffs of agents B and C then depend on the joint outcome of the loan to A and B's own investment. We guess and verify that the repayment of the loan from C to B is lower than B's investment payoff when it succeeds, that is $R_B^r \leq X_B$. Then, agent B chooses effort p_B^r to solve the following problem,

$$\max_{p_B^r} p_B^r (X_B + p_A X_A - R_B^r) + (1 - p_B^r) p_A \max \{X_A - R_B^r, 0\} - \frac{1}{2} X_B (p_B^r)^2 \quad (11)$$

In case of success of his own project, the payoff of agent B includes the expected value of A's collateral. If his project fails, agent B may still obtain a positive payoff if the collateral pays off, and X_A is large enough to fully cover the total repayment to agent C. In all other cases, agent B is protected by limited liability and gets zero payoff. The payoff of agent C is thus given by

$$U_C^r = p_B^r R_B^r + (1 - p_B^r) p_A \min \{R_B^r, X_A\} \quad (12)$$

Agent C gets the face value of the loan whenever agent B's project is successful. When B project

¹⁰See Appendix B for details. In all the applications considered in Section 7, loans have this recourse feature. For completeness, we also study the case in which no loan is recourse in this Appendix.

¹¹Since he has all the bargaining power, agent B by lending to A effectively acquires possession of his investment. As we show in Appendix A, our qualitative results however do not depend on this specific allocation of the bargaining power..

fails, agent C can seize the collateral and obtains $\min\{R_B^r, X_A\}$ when the collateral payoff is positive. We can now define the contract choice with collateral re-use.

Definition 1. The optimal (large) loan contract with re-use is given by a repayment value R_B^r such that agent C breaks even, that is $U_C^r \geq 2$, where the effort choice p_B^r solves (11).

Before stating our results, we provide some intuition about the effect of collateral re-use on agent B 's incentives. When B takes a large loan and re-uses collateral, the optimal choice of effort is obtained from (11):

$$p_B^r = \frac{X_B - R_B^r + p_A X_A - p_A \max\{X_A - R_B^r, 0\}}{X_B} \quad (13)$$

We first describe a skin-in-the-game effect, whereby agent B may exert more effort with a large loan and re-use than with a small loan. To isolate this first effect, consider the case in which $R_B^r \geq X_A$. Equation (13) shows in this case that the relevant variable for the effort decision of agent B is the repayment due, net of the expected value of the collateral, given by $R_B^r - p_A X_A$. Using equation (12), the participation constraint of agent C , given by $U_C^r \geq 2$, can be written as

$$p_B^r (R_B^r - p_A X_A) \geq 2 - p_A X_A \quad (14)$$

The right hand side of (14) is the total financing need of B net of the expected value of the collateral. By analogy with the no re-use case in Section 3, the left-hand-side of the (14) is the expected repayment to C for a loan with net face value $R_B^r - p_A X_A$ backed only by B 's own investment. Hence, a 2 unit loan with collateral re-use is equivalent to the outright sale of B 's loan to A and a smaller loan, of $2 - p_A X_A$ units with a repayment of $R_B^r - p_A X_A$, backed entirely by agent B 's investment. As lending to A is profitable, that is, $p_A X_A > 1$, effective borrowing by agent B is in fact lower with a 2 unit loan when he re-uses collateral than with a 1 unit loan.¹² Because agent B has more skin in the game compared to the no re-use case, he exerts more effort and obtains then a higher utility.

The argument above may suggest that agent B always prefers a large loan with re-use. While this is always true if $R_B^r \geq X_A$, we will show this condition is not necessarily satisfied. When $R_B^r < X_A$, an additional hedging effect arises, which is detrimental to incentives. In this situation, equation (13) shows that agent B is in fact partly hedged against the default of his own project because he obtains a positive payoff when agent A 's project succeeds. This hedge against a failure of his own investment weakens agent B 's incentives compared to the case analyzed above (in which

¹²By effectively selling to agent C his loan to A , agent B acquires a positive amount $p_A X_A - 1$ of cash which he can use as equity in his own investment, thus reducing the amount of borrowing needed.

$R_B^r \geq X_A$). Holding fixed the expected value of the yield $p_A X_A$ of the re-used collateral, this effect is stronger when p_A is smaller, that is, when collateral is riskier. We will show that if the re-used collateral is very risky, this negative hedging effect of collateral re-use on incentives can be so strong as to trump the positive skin-in-the-game effect described above.

In the proposition below, we derive the contract chosen by B with collateral re-use, solving for the value of the loan repayment R_B^r . By keeping fixed the expected value of the collateral $R_A = p_A X_A$, we can then study the effect of an increase in the riskiness of the collateral by decreasing the probability of success p_A . We then derive the conditions under which B prefers to re-use collateral.

Proposition 2. *Fix the expected value of A 's investment R_A . When agent B can re-use collateral, under Assumption 3 there exists thresholds $\underline{p}_A, \bar{p}_A$ with $0 < \underline{p}_A < \bar{p}_A \leq 1$ such that*

1. *When $p_A \leq \underline{p}_A$, agent B borrows 1 unit to finance only his own investment.*
2. *When $p_A \in (\underline{p}_A, \bar{p}_A)$, B borrows 2 units. The face value and the effort choice are given by*

$$R_B^r = \frac{4}{1 + \sqrt{1 - \frac{8(1-p_A)^2}{X_B}}} \leq X_A \quad (15)$$

$$p_B^r = \frac{1}{2} - \frac{p_A}{2(1-p_A)} + \sqrt{\frac{1}{4(1-p_A)^2} - \frac{2}{X_B}} \quad (16)$$

3. *When $p_A \geq \bar{p}_A$, B borrows 2 units. The face value and the effort choice are given by*

$$R_B^r = p_A X_A + \frac{2(2 - R_A)}{1 + \sqrt{1 - \frac{4(2-R_A)}{X_B}}} \geq X_A \quad (17)$$

$$p_B^r = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2 - R_A}{X_B}} \quad (18)$$

Proposition 2 shows that total borrowing weakly increases with collateral re-use. Recall that under Assumption 3 agent B only takes a small 1 unit loan from agent C if he cannot re-use collateral. Maintaining Assumption 3, Proposition 3 shows that when $p_A \geq \underline{p}_A$, that is when re-pledged collateral is not too risky, agent B chooses to take a 2 unit loan - and hence to intermediate funds between A and C - when he can re-use collateral.

Proposition 2 further partitions the region where B chooses the larger loan into two sub-regions, characterized by different levels of risk of the collateral. When $p_A \geq \bar{p}_A$, that is, when collateral is relatively safe, the face value of B 's loan satisfies the condition $R_B^r \geq X_A$. In this case, only the

skin-in-the-game effect due to the lower effective loan size is present. As we explained, both agent B 's utility and his effort level are higher with re-use.¹³ The reduction in net borrowing from 1 to $2 - R_A$ maps into the effort choice, as can be seen by comparing Proposition 1 and 2:

$$p_B^r = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2 - R_A}{X_B}} > \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{X_B}} = p_B$$

When $p_A \geq \bar{p}_A$, neither the face value R_B^r of agent B 's loan from agent C nor his effort level p_B^r depend on p_A , that is on the riskiness of the payoff of the collateral pledged. Hence, collateral payoff risk is irrelevant when collateral is safe enough.

When instead $p_A \in [\underline{p}_A, \bar{p}_A]$, that is when collateral risk is intermediate, the value of the re-used collateral when A 's project succeeds exceeds the face value of B 's loan, that is $R_B^r < X_A$. When $R_B^r < X_A$, as argued above, the hedging effect is also present, with detrimental consequences on incentives. Proposition 2 shows that in this region the effort choice p_B^r increases with p_A . This result arises because the hedging effect is stronger the riskier is collateral since a high payoff in case of success compensates for the high collateral default risk. Since agent B is more likely to default, agent C must raise the loan repayment R_B^r to break even. Hence, R_B^r is decreasing with p_A . When collateral is very risky, that is p_A is very low, namely $p_A \leq \underline{p}_A$, the negative hedging effect is so strong that agent B prefers not to re-use collateral.

The above result shows that when collateral is sufficiently risky, that is when $p_A \leq \bar{p}_A$, the expected level of effort with collateral re-use decreases with respect to the level of risk of the collateral. One may think that agent B would no longer choose to re-use collateral when his level of effort falls below the level without re-use. However, we show below that agent B may still prefer a 2 unit loan with re-use even if his expected level of effort decreases. In other words, agent B may voluntarily expose himself - and the lender - to fragility.

Proposition 3. *Fix the expected value R_A of A 's investment. There exists $p_A^* \in (\underline{p}_A, \bar{p}_A)$ such that when $p_A \in (\underline{p}_A, p_A^*)$, agent B strictly prefers a 2 unit loan with re-use and exerts less effort than when re-use is not allowed, that is, $p_B^r < p_B$.*

To get some intuition for the result, it is useful to rewrite agent B 's utility using equations (11) and (13). We obtain:

$$U_B^r = \frac{1}{2}(p_B^r)^2 X_B + p_A \max\{X_A - R_B^r, 0\}, \quad (19)$$

where the second term is positive when $p_A \leq \bar{p}_A$ since in that case $X_A > R_B^r$, as shown by

¹³Effort strictly increases with re-use because B has all the bargaining power. When A has all the bargaining power, Appendix A shows that for high values of p_A , agent B breaks even on the loan to A , which implies that his effort choice with re-use is the same as without re-use.

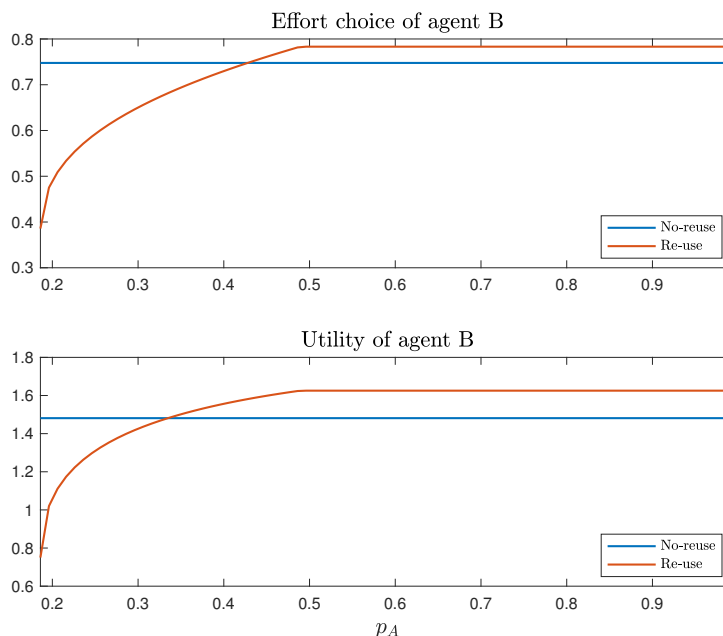


Figure 1: Fragility with collateral re-use. *The payoff in case of success X_A is given by $X_A = 1/p_A$. Parameter values: $X_B = 5.3$, $R_A = 1.1$. Thresholds: $\underline{p}_A = 0.34$, $p_A^* = 0.43$, $\bar{p}_A = 0.48$.*

Proposition 2. Compared to the expression for the utility in the no re-use case, given by equation (9), equation (19) shows that B can accept an increase in default risk with re-use (lower p_B) because of the residual intermediation profit when his investment fails - the second term of (19). This explains why the circulation of risky collateral can create fragility.

These results are illustrated in Figure 1. The top (bottom) panel of Figure 1 shows the level of effort (utility) of agent B as a function of the probability of success of agent A 's investment p_A . Below the threshold \bar{p}_A , both the level of effort and the utility are increasing with p_A when agent B re-uses collateral, and thus decreasing with the collateral payoff risk. In contrast, without re-use, collateral risk plays no role. In the top (bottom) panel, the intersection between the blue line and the red line defines the threshold p_A^* (\underline{p}_A). The Figure illustrates the fragility region $[\underline{p}_A, p_A^*]$ in which agent B prefers to re-use collateral despite his exerting less effort.

5 Information and Fragility

In this section, we show that collateral re-use can generate additional fragility through a news channel. We extend the set-up to allow the intermediary to acquire costly information about the collateral value before it pays off. At the intermediate date $1/2$ agent B , before choosing the level of effort on his own investment, can pay a cost γ to learn perfectly the outcome of A 's investment. In

Sections 5.1 and 5.2 we consider first the case where such information is acquired and characterize the news-driven fragility channel. In Section 5.3 we then show that, despite the increased fragility, agent B still chooses to acquire information at the interim stage, provided the cost γ is not too high.

5.1 No Re-use Benchmark

The benchmark situation without collateral re-use is unchanged in the presence of news. When agent B takes a small loan, agent A cannot invest and information is then irrelevant. When agent B takes a large loan, his incentives to exert effort are independent of the news received because the loan from C is backed only by B 's own investment. As a result, the trade-off between a small and a large loan is the same as in Section 3 and hence agent B prefers to take a small loan of 1 unit from agent C under Assumption 3.

5.2 Collateral re-use

We now analyze the case with re-use when B receives news about the payoff of A 's investment at the interim date. Let us denote $R_B^{r,n}$ the repayment due for the 2 unit loan agent C makes to agent B , with the new superscript n for news. The key difference with our previous analysis is that, at date $1/2$, the effort decision of agent B is contingent on the news. The subscript $s \in \{b, g\}$ denotes the state of the world, with b standing for the bad state where A 's investment fails and g standing for the good state where the investment succeeds. With a slight abuse of notation, we let X_{As} denote the payoff in each of the two states, with $X_{Ab} = 0$ and $X_{Ag} = X_A$.

Learning about the realized value of the collateral pledged by A affects agent B 's incentives. In each state $s \in \{b, g\}$, agent B now chooses the probability of success p_{Bs} as a function of the value X_{As} of this collateral. The payoff of agent B in each state can be written as follows:

$$U_{Bs}^{r,n} := p_{Bs}^{r,n} (X_B + X_{As} - R_B^{r,n}) + (1 - p_{Bs}^{r,n}) \max \{X_{As} - R_B^{r,n}, 0\} - \frac{1}{2} X_B (p_{Bs}^{r,n})^2 \quad (20)$$

As in the previous section, we conjecture and then verify that the value of the repayment due to C is such that B can repay the loan if his investment succeeds, that is, $R_B^{r,n} \leq X_B$. Under this conjecture, the effort choice of agent B in state s is given by

$$p_{Bs}^{r,n} = \frac{X_B + X_{As} - R_B^{r,n} - \max \{X_{As} - R_B^{r,n}, 0\}}{X_B} \quad (21)$$

Equation (21) shows that the effort choice of agent B is positively correlated with the value of the

collateral. When the value of the collateral is low, a high share of the payoff of B 's investment will be used to service the debt. Since agent B captures a lower share of the investment returns, he exerts less effort. Because an exogenous drop in the collateral value lowers B 's effort and hence increases the probability that B defaults, we say that news generate contagion.

One could expect that, despite contagion, the expected level of effort of B is the same with or without news. The correlation between the effort choice and the collateral payoff is not neutral, however, as it reduces the lender's expected payoff, everything else equal. Intuitively, agent C seizes the collateral when agent B fails but agent B is more likely to fail when the collateral value is low, when he receives news. To see that C 's expected payoff is lower with news, suppose that the repayment faced by B is the same with or without news, that is, $R_B^{r,n} = R_B^r$. Equations (13) (21) show that the expected level of effort with news would be equal to the effort level without news. The negative effect of news shows up instead in the expected payoff to agent C given by:

$$U_C^{r,n} = E[p_{B_s}^{r,n}]R_B^r + p_A(1 - p_{B_g}^{r,n}) \max \{X_A, R_B^r\} < U_C^r, \quad (22)$$

where U_C^r is agent C 's payoff in the model without news and a face value $R_B^r = R_B^{r,n}$. The last inequality follows from the fact that $p_{B_g}^{r,n} > E[p_{B_g}^{r,n}]$. Hence, although B 's own investment succeeds with the same average probability, the contagion effect makes C worse off. To break even, agent C should thus charge a higher rate $R_B^{r,n} > R_B^r$ when B receives news, which, in turn, explains why the expected probability of success of B should be lower. The following Proposition formalizes this argument.

Proposition 4. *If agent B re-uses collateral, his utility and the expected level of effort are strictly lower with news than without.*

Proposition 4 shows that agent B is unambiguously worse-off when he receives interim news about the value of the collateral. His expected level of effort is lower compared to the case without news as agent C charges a higher repayment rate. In our terminology, fragility is higher with news when agent B re-uses collateral,. Figure 2 illustrates these results with the same parameters value chosen for Figure 1.¹⁴ The top (bottom) panel shows that the probability of success (the utility) of agent B is lower with news than without. Proposition 4 has direct implications for agent B 's optimal choice between a large loan with re-use and a small loan.

¹⁴Note that the lower bound for p_A is higher in Figure 2 because there is no equilibrium with collateral re-use in the presence of news for low values of p_A .

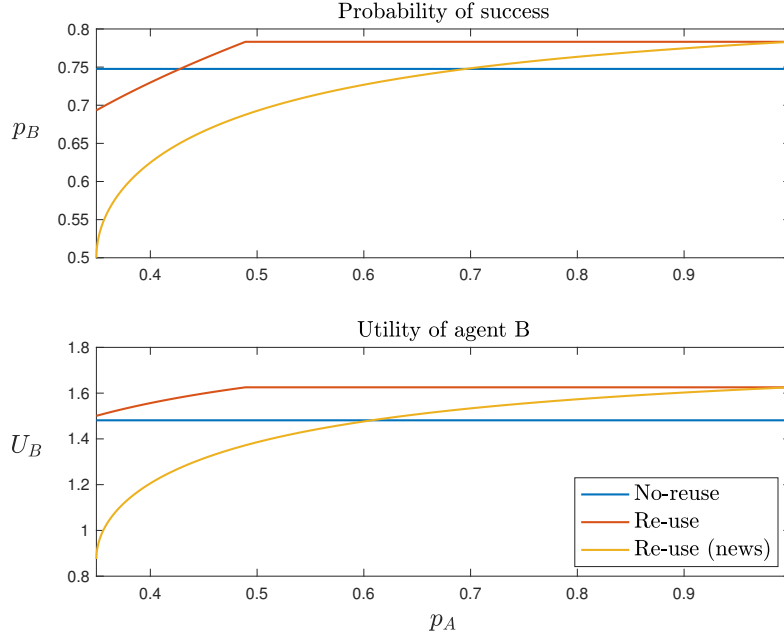


Figure 2: News-driven fragility. The payoff in case of success X_A is given by $X_A = 1/p_A$. Parameter values: $X_B = 5.3$, $R_A = 1.1$. Thresholds: $\underline{p}_A = 0.34$, $p_A^* = 0.43$, $\bar{p}_A = 0.48$. News: $\underline{p}_A^n = 0.61$, $p_A^{n,*} = 0.70$.

Corollary 2. *There exists threshold \underline{p}_A^n and $p_A^{*,n}$ such that with news, agent B re-uses collateral if and only if $p_A \geq \underline{p}_A^n$ and there is fragility if $p_A \in [\underline{p}_A^n, p_A^{*,n}]$. The thresholds satisfy $\underline{p}_A^n > \underline{p}_A$ and $p_A^{*,n} > p_A^*$. In particular, with news, fragility arises for lower levels of collateral risk.*

The results above follow directly from Proposition 4. Fragility materializes for lower level of collateral risks with news because agent B 's probability of success is lower. Similarly, agent B switches to a small loan for lower levels of collateral risk because re-using collateral is less attractive with news. These results are illustrated on Figure 2. With the parameter values chosen, agent B does not re-use collateral when $p_A \leq 0.61$ if he receives news. Without news, collateral re-use is optimal for all $p_A \geq 0.34$. In addition, the fragility threshold increases from 0.43 to 0.70.

We showed that if agent B re-uses collateral, fragility is higher with news. Fragility, however, can be lower in the model with news if agent B chooses not to re-use collateral. Corollary 2 implies that for all values of p_A in $[\underline{p}_A, \min\{p_A^*, \underline{p}_A^n\}]$, re-use is only optimal without news and agent B exerts less effort when he re-uses collateral. Then, agent B exerts more effort in the model with news because he cuts borrowing compared to the model without news. In Figure 2, this case arises for all $p_A \in [0.34, 0.43]$.

We conclude this section by discussing agent B 's optimal decision to increase fragility with collateral re-use. In Proposition 3, we already showed that B tolerates and increase in his own investment default risk to capture intermediation profits. With news, this effect is not the only

reason B likes fragility. To highlight the channel specific to the case with news, suppose collateral risk is low such that $p_A \geq \bar{p}_A$. In this parameter range, collateral re-use leads to a higher effort choice without news. To see why this result does not necessarily hold in the model with news, let us write the utility of agent B in this case:

$$U_B^{r,n} = \frac{1}{2}E \left[(p_{Bs}^{r,n})^2 \right] X_B = \frac{1}{2} \left\{ (E[p_{Bs}^{r,n}])^2 + Var[p_{Bs}^{r,n}] \right\} X_B \quad (23)$$

where we used equation (20) and the fact that $R_B^r \geq X_A$ when $p_A \geq \bar{p}_A$. Because the utility of agent B without re-use is given by $U_B = \frac{1}{2}p_B^2 X_B$, it can be that $U_B^{r,n} > U_B$ while the average effort choice is lower with re-use, that is $E[p_{Bs}^{news}] < p_B$. The fact that B enjoys the variability in his effort choice is the corollary to the fact that C suffers from the contagion effect. As shown by Proposition (3), however, B is unambiguously worse-off ex-ante in the presence of news. The variability in effort choice only yields an ex-post gain, once the face value of the debt is set. This intuition will be useful for the analysis in the next Section.

5.3 Information Acquisition

Interim news is a new source of fragility when agent B re-uses collateral. So far, we took news arrival as exogenous. We now consider the endogenous information acquisition choice of agent B . We showed in Proposition 4 that agent B is unambiguously worse-off in the presence of news. Hence, it would be optimal to commit ex-ante not to acquire news at the interim stage. Our discussion above, however, suggests that such commitment is not credible if information is cheap. We formalize this observation below, focusing on the case $p_A \geq \bar{p}_A$ for simplicity.

Proposition 5. *Under limited commitment, agent B acquires information at the interim stage if the cost γ is low enough. When $p_A \geq \bar{p}_A$, this condition writes*

$$\gamma \leq \frac{V_A}{2X_B} \quad (24)$$

where V_A is the variance of the re-used collateral.

Proposition 5 shows that agent B cannot credibly commit not to acquire information when the cost γ is low. Ex-ante, agent B anticipates that if he receives news about the collateral value, the lender will charge a higher interest rate. Once the face value of the loan R_B is determined, however, information is beneficial to agent B since he can then tailor his effort choice to the value of the collateral. Intuitively, this benefit is higher when the variability of the collateral payoff increases, which explains why the value of information is proportional to the variance of the collateral payoff.

We thus show that information production about collateral payoff is harmful but that it cannot be avoided when information costs are low. This optimal opacity result is reminiscent of [Dang et al. \(2015\)](#) and others who show that information acquisition about collateral may be detrimental to lending and welfare. The cost of information these papers highlight, however, is different. In these works, if lenders acquire information ex-ante, they do not lend to positive NPV borrowers with bad collateral while, under opacity, all borrowers would have received financing, a variant of the [Hirshleifer \(1971\)](#)'s effect. Instead, in our model, information is costly because borrowers use it to correlate the effort on their investment with the collateral at the expense of lenders. Lenders who anticipate this behavior charge a higher interest rate.

6 Endogenous Collateral Quality

We showed that intermediaries become fragile when they re-use risky collateral. In this section, we endogenize the choice of collateral risk by intermediaries. In our model, the collateral re-used by B is the loan extended to A . We thus endogenize collateral risk by assuming that A chooses the probability of success of its investment p_A at a cost $\frac{1}{2}c_A p_A^2 X_A$ with $c_A \geq 1$ and $X_A \in [4c_A, 8c_A]$. The assumption about X_A is the counterpart of Assumption 1 with endogenous risk. The bounds on X_A ensures that lending to A is profitable but not so profitable that B would only need to pledge the loan as collateral to obtain a 2 unit loan from C . The assumption $c_A \geq 1$ implies that the first-best level of effort is lower than one. We explain below the role of this assumption for the analysis. The setup is otherwise identical to Section 4, that is, we focus on the version of the model without news for simplicity

We first determine the effort choice of agent A . Agent B lends one unit to A and sets the face value F_A of the loan. Building directly on our results in Section 3, A 's optimal choice of effort is

$$p_A = \frac{X_A - F_A}{c_A X_A} \quad (25)$$

Hence, the higher F_A , the lower the effort exerted by A who has less skin in the game. The key insight is that the lower effort choice by A makes the collateral riskier for B .

Before stating our results, it is useful to present the outcome in which B would simply seek to maximize the expected revenue from the loan to A , equal to $p_A F_A$. Given the effort choice of A , given by equation (25), the face value F_A^* to maximize this expected value is

$$F_A^* = \frac{X_A}{2}, \quad p_A^* = \frac{1}{2c_A}, \quad R_A^* = \frac{X_A}{4c_A}$$

Setting the face value at F_A^* maximizes the revenue from lending to A . As we will show, revenue maximization is not always optimal for agent B , because he re-uses the collateral to raise financing from agent C . Our analysis in Section 4 shows that B prefers to repledge safer collateral, for a given expected value. When A 's investment risk is endogenous, B can reduce the collateral riskiness by lowering the face value F_A of the loan below F_A^* , as shown by equation (25). Hence, collateral re-use can incentivize agent B to sacrifice intermediation profits in order to reduce collateral riskiness.

Proposition 6. *There exists a threshold $\underline{X}_A < 8c_A$ such that for all $X_A \geq \underline{X}_A$, B re-uses collateral and $p_A^{**} > p_A^*$, that is, collateral is safer than in the benchmark.*

Proposition 6 shows that intermediaries who re-use collateral are willing to sacrifice intermediation profits in exchange for collateral safety. Increasing collateral safety is costly in our model because the intermediary must leave more rents for the borrower to exert more effort. Collateral safety materializes is beneficial, however, because the intermediary repledges the collateral, as shown in Section 4. This effect materializes when the payoff X_A of A 's investment in case success is high because the investment return is then also more volatile with our two state payoff specification. Although the trade-off between collateral return and risk is due to moral hazard in our model, we believe the insight is more general. Collateral re-use may incentivize intermediaries to source safer collateral, despite the lower returns, because there is an endogenous premium for safe collateral when it circulates along collateral chains. This finding resonates with the evidence that re-used collateral in the swaps and derivative markets is mostly in the form of high liquid and safe Treasuries (see e.g. ISDA (2019)).

Our results also imply that A 's profit is higher when he borrows from an intermediary who re-uses collateral. Suppose instead A can borrow from C directly, leaving the bargaining power to the lender for symmetry. C would simply maximize the expected value of the loan and thus choose face value F_A^* . Agent B , however, sets a lower face value $F_A^{**} < F_A^*$ which increases A 's profit from the transaction. This result is in line with the observation that counterparties who agree to the re-use of their collateral typically enjoy a discount on their borrowing terms (Monnet (2011)).

Despite B 's incentives to source safe collateral, fragility may still arise in equilibrium. We provide a numerical illustration in Figure 3. Observe that the baseline probability of success $p_A^* = \frac{1}{2c_A}$ lies below the fragility threshold p_A characterized in Section 4. This implies that the collateral risk channel is active and that B has incentives to increase the probability of success of A above this baseline. For very low values of X_A , B does not re-use collateral because of the risk effect. For low values of X_A , the optimal choice of collateral riskiness by B is such that p_A^{**} exceeds the fragility threshold. When X_A is large however, fragility arises in equilibrium, despite B 's incentive to reduce collateral risk. Hence, fragility may still arise even if B can mitigate the

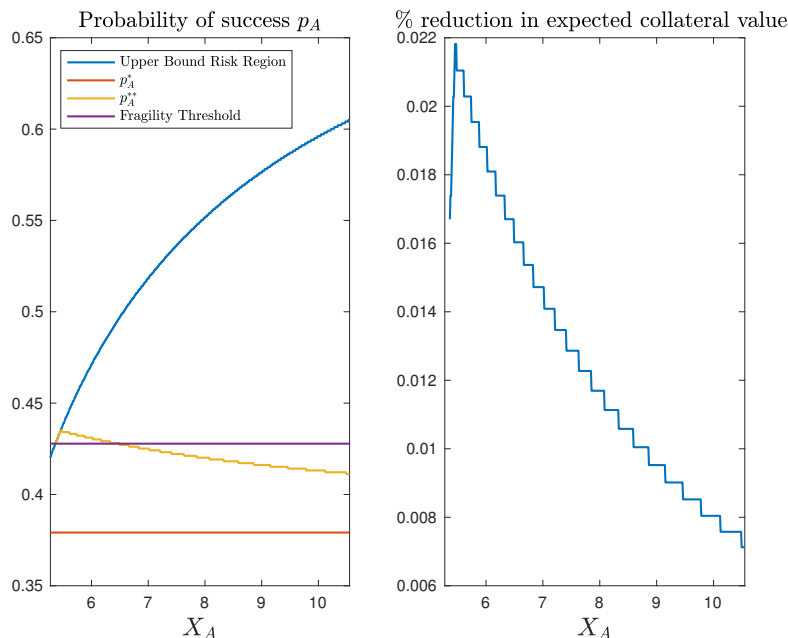


Figure 3: Optimal Collateral Risk. *Parameter Values:* $X_B = 5.3$, $c_A = 1.32$

collateral risk channel by sourcing safer collateral.

7 Applications

Our model is stylized and arguably does not fit perfectly one specific economic application. However, we believe that our analysis highlights a mechanism that applies to several situations. In this section, we discuss three such applications in details: securitization, trade credit, and the market for repurchase agreements (repos). We rely on the interpretation of the model with two distinct loans, secured respectively by the re-used collateral and B 's own investment. As explained in Section 4, this interpretation is theoretically equivalent to our single loan benchmark when loans provide recourse to B 's balance sheet. The necessary ingredient for fragility in our model is that B is “on the hook” to repay the loan he contracted in order to lend to A when the re-used collateral fails. In all three applications, we argue that the creditor whose loan is backed by the re-used collateral indeed has recourse to the balance sheet of B . Hence, the fragility channel we identified is likely to be active.¹⁵ We discuss these applications in more details below.

¹⁵Whether other loans taken by the intermediaries also provide recourse, as in our model, is not instrumental to the fragility result. As we show in Appendix B, it is only necessary that loans secured with re-used collateral provide recourse. In fact, we show that, if only these loans have recourse features, fragility is even stronger than in our benchmark analysis.

Trade Credit

Our first application regards chains of trade credit. Trade credit is one of the major sources of funds for corporate firms. Instead of borrowing money from a bank, a firm can obtain the inputs it requires by using trade credit. In this process, the firm (the borrower) obtains inputs from a supplier by promising to pay for those inputs at a later date. The supplier records these loans as “account receivables” on its balance sheet. This has some analogy with the relationship between agents A and B in our model. In turn, the supplier may obtain funding from a financial lender (agent C), by pledging or selling the trade receivable. This practice is sometimes known as factoring when the supplier uses invoices in order to borrow. Factoring can be recourse or non-recourse. With non-recourse factoring, the factoring firm is left empty-handed if the borrower (A) fails to pay. With recourse factoring, the factoring firm will sell back the trade receivable claim to the supplier (B) who is on the hook if the borrower fails. In Europe, while non-recourse factoring is increasing, recourse factoring has so far been the preferred factoring method.¹⁶

The findings by [Petersen and Rajan \(2015\)](#) also suggest that such credit chains are a common arrangement. They show that firms with better access to credit from financial institutions offer more trade credit, that is, they may play a role as intermediaries. In our model, only agent B has access to credit from *C* who can fund the trade credit position extended by *B* to *A*. In addition, [Berger and Udell \(1990\)](#) and, using data from Italian firms, [Omiccioli \(2005\)](#) show that firms use account receivables to secure borrowing from banks. This is akin to agent B pledging the cash flow of his loan to A to secure his loan from C and is a key aspect of our analysis. Interestingly, [Omiccioli \(2005\)](#) shows that this behavior is concentrated among small and risky firms. Our model makes the case that the use of account receivables as collateral contributes to making these firms riskier. Hence, causality could run both ways.

Securitization

To some extent, the trade credit narrative is related to the second application of our model, securitization, because trade credit is a form of asset-based financing. With securitization, an intermediary (the originator) can park loans off-balance sheet in a Special Purpose Vehicle (SPV). The originator can thus free balance sheet space. The SPV funds these loans by selling bonds.¹⁷ The intermediary who sets up the SPV is called the sponsor and, in our framework, is the same

¹⁶Data from Factor Chain International, the industry representative body, show that more than 60% of factoring happens in Europe (see here: <https://fci.nl/en/industry-statistics>)

¹⁷This process often involves the pooling of different loans and their tranching into different debt claims to cater to an heterogeneous investor clientele. These features of securitization are important but they are not relevant to our argument.

agent as the loan originator. In this interpretation of our model, B is the intermediary and the loan to A is held by the SPV while B only keeps his own project on his balance sheet.

As we explained, this arrangement can generate fragility if the creditors of the SPV have recourse to the balance sheet of the sponsor. In practice, sponsors often extend implicit or explicit guarantees to their SPVs in order to improve the rating of the SPV's debt, (see [Acharya et al. \(2013\)](#)). The simplest forms of credit enhancement is an explicit recourse arrangement, whereby the creditors of the SPV would receive a payment directly from the enhancer should the SPV fail to pay. A more common form of credit enhancement is an irrevocable letter of credit. With such credit enhancement, the creditors of the SPV effectively have recourse to the balance sheet of the sponsor. Hence, as in our model, the intermediary is on the hook to repay the creditors of the SPV

Viewing credit enhancement as a direct recourse arrangement, our model sheds light on how the securitization process can benefit the originator bank by allowing it to expand its lending activity while making its balance sheet more risky. We show that cross-subsidization between banking activities, through SPV credit enhancements, can generate contagion and fragility, formalizing the argument in [Acharya et al. \(2013\)](#). Our mechanism is different from the narrative in [Keys et al. \(2010\)](#) and others who argue that securitization leads to fragility because banks have no incentive to exert due diligence for loans they know they will sell. In our model, the enhancement guarantee puts the balance sheet of the intermediary at stake and can increase the probability of default for on-balance sheet assets. Since cross-subsidization sometimes both increases lending and reduces fragility in our model, pure ring-fencing between banks' own trading activities and their intermediation business may not always be efficient.

Repurchase Agreements

The third application of our model is the bilateral market for repos (repurchase agreements). In this market, financial institutions borrow funds usually short term by selling assets (collateral) with the agreement to buy them back at a later date at an agreed price. Essentially, the sale of these assets amounts to collateralizing the loan. Risky assets such as MBS or equity can be used as collateral in the repo market.¹⁸ Dealer banks often act as intermediaries in repo markets. For example [Aldasoro and Ehlers \(2018\)](#) provide evidence that French banks (among others) are intermediating the US dollar funding needs of Japanese banks from onshore US money market funds. Key to this intermediation process is the ability for financial institutions to re-use the asset they obtained in a previous repo. [Infante et al. \(2018\)](#) document high re-use rate of collateral even

¹⁸See [Julliard et al. \(2019\)](#) for the UK and [Baklanova et al. \(2015\)](#), for the US.

for non-Treasuries among US dealers and the [FSB \(2017\)](#) has identified re-use as a key source of risk. Finally, repos are recourse loans, as we point out in [Gottardi et al. \(2019\)](#). So a lender in the repo market has an (unsecured) claim to the borrower entire balance sheet in case the collateral value is not high enough to cover the borrower's debt. Hence, our model can explain why secured credit chains in repo markets are caused for concerns.

8 Conclusion

Our paper studies the trade-off between total borrowing and fragility when intermediaries can re-use collateral. We first show that collateral re-use can both increase lending and reduce fragility when the collateral is safe. When the re-used collateral is risky, however, intermediaries choose to expose themselves to fragility in order to reap intermediation profits. We show that such fragility is exacerbated in the presence of news about the value of the collateral. Hence, our model justifies claims that collateral re-use can lead to contagion and fragility along credit chains. Due to this fragility effect, intermediaries are willing to pay a premium for safe collateral since it reduces the fragility of the secured credit chain. Our findings apply both to the explicit re-use of collateral through rehypothecation, as in the repo market, and to the implicit re-use of collateral through securitization and other forms of asset-based financing.

While we model a short intermediation chain for simplicity, it would be interesting to see how our results change when considering a longer chain of trades or a richer network of credit relationships. While our results imply that credit chains built on safe assets are both efficient and robust, it is an open empirical question whether the supply of safe assets such as Treasuries can meet the demand for collateral from the broad financial system.

Appendix

A Bargaining Power to Borrowers

In this section, we consider the case where borrowers have the bargaining power, that is, A sets the terms when borrowing from B and B sets the terms when borrowing from C . We now denote R_A the face value of the loan from B to A with $R_A \leq X_A$ with X_A the payoff of A 's investment in case of success. Collateral re-use now means that B can pledge to C the loan extended to A . Effectively, the loan is an asset with payoff R_A (resp. 0) with probability p_A (resp. $1-p_A$). We focus on the case without news for simplicity.

A.1 No re-use

The analysis of the no-reuse case is identical to Section 3 despite the different allocation of bargaining power. Intuitively, agent A can promise to repay up to X_A to agent B . If agent B is not willing to lend in this case, as per Assumption 3, he would not be willing to lend for any $R_A \leq X_A$. Hence, under Assumption 3, the equilibrium without re-use is unaffected by the allocation of bargaining power.

A.2 Collateral Re-use

We now turn to the case where B can re-use the as collateral the loan to A . Let $\underline{R}_A := 1/p_A$ be the break-even rate. The break-even rate is the minimum face value a lender would accept to lend 1 unit to A given that the probability of success is p_A . We show below that our main results from Section 4 survive and in particular, that collateral re-use can generate fragility.

Proposition A.1. *There exists a threshold $p_A^{**} > \underline{p}_A$ such that*

1. *Agent B takes a 2 unit loan with re-use if and only if $p_A \geq \underline{p}_A$*
2. *Agent B exerts less effort with re-use than without if $p_A \in [\underline{p}_A, p_A^{**}]$.*

*The face value of the loan to A satisfies $R_A > \underline{R}_A$ in the fragility region $[\underline{p}_A, p_A^{**}]$, while $R_A = \underline{R}_A$ for $p_A \geq p_A^{**}$.*

Proof. With re-use, the objective of A is to minimize the face value of the loan extended by B while ensuring B 's participation constraint, that is

$$\min R_A \quad \text{subject to} \quad U_B^r(R_A) \geq U_B \tag{A.1}$$

with $U_B^r(R_A)$ the utility of B for a given face value R_A . The first step of the analysis is to determine $U_B^r(R_A)$. In the second step, we solve for R_A .

For the first step, observe that the only difference when A has the bargaining power is the payoff of B 's collateral in case of success, which is R_A instead of X_A . Thanks to this analogy, we can use the results in Proposition 2 to characterize $U_B(R_A)$. In particular, for any $R_A \geq \underline{R}_A$, extending the notation of Proposition 2, there exists thresholds $\underline{p}_A(R_A)$ and $\bar{p}_A(R_A)$ such that the statements of Proposition 2 hold, substituting X_A with R_A .

In the second step of the analysis, we determine R_A using (A.1). Conjecture first that R_A is such that $p_A \geq \bar{p}_A(R_A)$ which implies that $R_A \leq R_B^r$. Since $U_B(R_A) = \frac{1}{2} [p_B^r(R_A)]^2 X_B$, the participation constraint of agent B binds if and only if

$$p_B = p_B^r(R_A) = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2 - p_A R_A}{X_B}}$$

where we used equation (18) to derive the right hand side of the equation above. It follows immediately that $R_A = \underline{R}_A$ is the solution to problem (A.1). Let thus $p_A^{**} := \bar{p}_A(\underline{R}_A)$ be the threshold below which $R_B^r \geq \underline{R}_A$ does not hold.

We now turn to the case $p_A \leq p_A^{**}$. By construction, in this case, the face value of the loans satisfy $R_A \geq R_B^r$. Using the results from Proposition 2 and the characterization of $U_B^r(R_A)$ in the proof, we obtain

$$U_B^r(R_A) = \frac{1}{2} [p_B^r(R_A)]^2 X_B + p_A(R_A - R_B^r(R_A)) = \frac{X_B}{2} + R_A - 1 - \frac{2}{1 + \sqrt{1 - \frac{8(1-p_A)^2}{X_B}}} \quad (\text{A.2})$$

where $p_B(R_A)$ and $R_B(R_A)$ are given by equations (16) and (15) respectively. The threshold below which A cannot borrow from B is the minimum value of p_A such that agent B prefers not re-use even if he gets all the profit from A 's investment. By definition, this threshold is $\underline{p}_A = \underline{p}_A(X_A)$. By the participation constraint of agent B , we have $U_B(R_A) = U_B$, which given equation (A.2), implies that $p_B^r < p_B$ when $p_A \in [\underline{p}_A, p_A^{**})$. Finally, we show that $R_A > \underline{R}_A$ when $p_A \leq p_A^{**}$. To see this suppose by contradiction that $R_A = \underline{R}_A$. Equation (A.2) then shows that U_B^r is increasing with p_A . By continuity, we have

$$U_B^r(\underline{R}_A)_{p_A=p_A^{**}} = U_B,$$

it follows that the participation constraint cannot be satisfied when $p_A < p_A^{**}$. Hence, it must be that $R_A > \underline{R}_A$ when $p_A \in [\underline{p}_A, p_A^{**})$.

□

B Recourse Loans and Fragility

In this Section, we analyze the version of the model in which B obtains two distinct loans of one unit each. These two loans could be financed by the same investor, but, for ease of exposition, we call C_B the creditor secured by B 's own investment and C_A the creditor secured by B 's loan to A . With two distinct loans, an important feature of the lending relationships between B and his creditors is whether loans provides recourse. We say that a creditor has recourse if he has an (unsecured) claim to B 's assets when the payoff of the asset he receives as collateral falls short of the promised repayment of the loan.

We first show in Section B.1 that the model with two loans is equivalent to our benchmark model if both creditors have recourse. Motivated by the empirical applications discussed in Section 7, we then show in Section B.2 that fragility is even stronger if recourse is only given to creditor C_A whose claim is secured by the re-used collateral.

B.1 Symmetric Recourse

In this case, for $i \in \{A, B\}$, creditor C_i has an unsecured claim on the asset pledged by B to creditor C_j with $j \neq i$. We let F_{BA} and F_{BB} denote the face value of the loan secured by the re-used collateral and B 's own investment respectively. We guess and verify that the face value of the loans are such that $X_B > F_{BA} + F_{BB}$, that is, B can repay both loans only out of the cash flow of his own investment when it succeeds. Hence, we are left to determine the agents' payoff when B 's own investment fails but the loan to A succeeds. Creditor C_A receives $F_{BA} < X_A$. Creditor C_B receives $\min\{X_A - F_{BA}, F_{BB}\}$ and B receives $\max\{0, X_A - F_{BB} - F_{BA}\}$. Creditors C_A and C_B 's participation constraint are respectively

$$\begin{aligned} p_B F_{BA} + (1 - p_B) p_A F_{BA} &\geq 1 \\ p_B F_{BB} + (1 - p_B) p_A \min\{X_A - F_{BA}, F_{BB}\} &\geq 1 \end{aligned}$$

while agent B 's effort decision is the solution to the following problem:

$$\max_{p_B} p_B (X_B - F_{BB} - F_{BA} + p_A X_A) + (1 - p_B) p_A \max\{0, X_A - F_{BB} - F_{BA}\} - \frac{1}{2} p_B^2 X_B \quad (\text{B.1})$$

Denoting $F_B = F_{BA} + F_{BB}$, and adding the binding participation constraints, we obtain

$$p_B F_B + (1 - p_B) p_A \min\{X_A, F_B\} = 2$$

while the B 's effort choice is given by

$$p_B = \frac{X_B + p_A X_A - F_B - p_A \max\{0, X_A - F_B\}}{X_B} \quad (\text{B.2})$$

Observe that F_B and p_B are determined by the same equations as in Section 4 in which we assumed a single creditor. Hence, the equilibrium face value of the total debt incurred by B and the equilibrium effort choice are again given by Proposition 2. This observation also implies that our conjecture $X_B > F_B$ is satisfied under the assumptions of the model.

B.2 Asymmetric Recourse

In this case, only the loan extended by C_A is recourse. We guess and verify again that $X_B > F_{BA} + F_{BB}$. In this case, only the participation constraint of creditor C_B is different with respect to Section B.1. Since creditor C_B receives a payoff of zero when B 's investment fails, the participation constraint is given by:

$$p_B F_{BB} \geq 1.$$

The effort decision of agent B is the solution to the following problem

$$\max_{p_B} p_B (X_B - F_{BB} - F_{BA} + p_A X_A) + (1 - p_B) p_A (X_A - F_{BA}) - \frac{1}{2} p_B^2 X_B \quad (\text{B.3})$$

The second term of (B.3) is different from the second term of (B.1) since when B 's own investment fails, creditor C_B does not have recourse to the payoff of the loan to A . We can then prove the following result.

Proposition B.1. *With asymmetric recourse, fragility with re-use is higher than with symmetric recourse and than without reuse for all values of p_A . Despite the additional fragility due to asymmetric recourse, B prefers to re-use collateral when it is sufficiently safe, that is, when p_A is high enough.*

Proof. To prove the first result, let us derive the effort choice of B . Solving for p_B in (B.3) gives

$$p_B = \frac{X_B - F_{BB} - (1 - p_A) F_{BA}}{X_B} \quad (\text{B.4})$$

Comparing equations (B.4) and (B.2), it follows immediately, that for given values F_{BA} and F_{BB} , the effort choice is strictly lower with asymmetric recourse. Comparing now the participation constraints of creditor C_B , for a given effort choice p_B , the face value F_{BB} must be strictly higher with asymmetric recourse. From these two observations, we can conclude that the effort choice of agent B is weakly lower with recourse. A similar argument shows that the effort choice is also lower with re-use and asymmetric recourse than without re-use.

To prove the second result, consider the limit case when $p_A \rightarrow 1$. Then, comparing equations (B.4) and (8), the effort choice is the same with reuse and asymmetric recourse than without re-use. The utility

derived by agent B with re-use is given by

$$U_B = \frac{1}{2}p_B^2 X_B + X_A - 1,$$

which is strictly higher than the utility level without re-use. Hence, by continuity, for p_A close enough to 1, agent B prefers to re-use collateral in the asymmetric recourse model.

□

C Proofs

C.1 Proof of Proposition 1

Letting the participation constraint of C , equation (6) bind we obtain

$$R_{B,l}^2 - X_B R_{B,l} + lX_B = 0$$

The value of $R_{B,l}$ is the lowest of the roots of this second-order equation. The discriminant is $\Delta_l = X_B^2 - 4lX_B$. We have

$$R_{B,l} = \frac{X_B - \sqrt{\Delta_l}}{2}$$

Replacing Δ_l by its value, we obtain equation (7). Expression (8) follows by plugging equation (7) in equation (5). Finally, observe that

$$U_{B,l} = p_{B,l}(X_B - R_{B,l}) - \frac{1}{2}X_B p_{B,l}^2 = X_B p_{B,l}^2 - \frac{1}{2}X_B p_{B,l}^2$$

where we used (8) to substitute for $R_{B,l}$. Equation (9) immediately follows.

C.2 Proof of Corollary 1

The result is obvious when $X_B \leq 8$ since then a large loan of 2 units is not feasible. When $X_B \geq 8$, agent B prefers a small loan if

$$\begin{aligned} \frac{1}{2}p_{B,1}^2 X_B &\geq p_A X_A + \frac{1}{2}p_{B,2}^2 X_B \\ \frac{X_B}{2} \left(\frac{1}{2} - \frac{1}{X_B} + \sqrt{\frac{1}{4} - \frac{1}{X_B}} - \frac{1}{2} + \frac{2}{X_B} - \sqrt{\frac{1}{4} - \frac{2}{X_B}} \right) &\geq p_A X_A \\ \frac{1}{2}X_B \left(\sqrt{\frac{1}{4} - \frac{1}{X_B}} - \sqrt{\frac{1}{4} - \frac{2}{X_B}} \right) + \frac{1}{2} &\geq p_A X_A \end{aligned}$$

The last inequality is equivalent to condition (10).

We now prove the claim that the right-hand side of (10) is decreasing in X_B and equal to 0 when $X_B \rightarrow \infty$. Setting $x := X_B$, let us call $f(x)$ the right hand side of equation (10). We have

$$\begin{aligned} f(x) &= \frac{x}{2} \left(\sqrt{\frac{1}{4} - \frac{1}{x}} - \sqrt{\frac{1}{4} - \frac{2}{x}} \right) - \frac{1}{2} \\ &= \frac{x}{2} \frac{1}{x} \left(\sqrt{\frac{1}{4} - \frac{1}{x}} + \sqrt{\frac{1}{4} - \frac{2}{x}} \right)^{-1} \end{aligned}$$

which is strictly increasing in x and positive.

C.3 Proof of Proposition 2

The first step is to characterize the outcome with a 2 units loan and collateral re-use. We then analyze whether this choice dominates the 1 unit loan outcome characterized in Proposition 1.

Step 1. *Equilibrium with re-use.*

Case i) Conjecture first that $R_B^r \in [X_A, X_B]$. Solving for the optimal value of p_B in equation (11), we find

$$p_B = \frac{X_B - R_B^r + p_A X_A}{X_B} \quad (\text{C.5})$$

The participation constraint of agent C is satisfied if

$$\begin{aligned} p_B^r R_B + (1 - p_B^r) p_A X_A &\geq 2 \\ p_B^r (R_B^r - p_A X_A) &\geq 2 - p_A X_A \\ (X_B - R_B^r + p_A X_A)(R_B^r - p_A X_A) &\geq X_B(2 - R_A) \end{aligned}$$

Defining $\tilde{R}_B^r = R_B^r - p_A X_A$, we obtain that the face value of the loan is the smallest root of the following equation

$$\left(\tilde{R}_B^r\right)^2 - X_B \tilde{R}_B^r + X_B(2 - R_A) = 0$$

Solving for the smallest root of the equation above, we obtain

$$R_B^r = R_A + \frac{1}{2} \left(X_B - \sqrt{X_B^2 - 4X_B(2 - R_A)} \right)$$

which is equivalent to expression (17). The value of p_B^r is obtained by plugging equation (17) in (C.5). Finally, the expression for U_B^r obtains by plugging (17) in equation (11). We are left to derive conditions such that the conjecture $R_B^r \in [X_A, X_B]$ is satisfied. The condition $R_B^r \leq X_B$ is equivalent to

$$2R_A \leq X_B + \sqrt{X_B^2 - 4X_B(2 - R_A)}$$

which holds under Assumptions 1 and 2. The condition that $R_B^r \geq X_A$ writes

$$\begin{aligned} X_B - \sqrt{X_B^2 - 4X_B(2 - p_A X_A)} &\geq 2(1 - p_A)X_A \\ p_A &\geq \bar{p}_A := \frac{2R_A}{X_B + 2R_A - \sqrt{X_B^2 - 4X_B(2 - R_A)}} \end{aligned} \quad (\text{C.6})$$

where $\bar{p}_A \leq 1$.

Case ii). Conjecture now that $R_B^r \leq \min\{X_A, X_B\}$. Using equation (11), the optimal choice of effort

by agent B is given by

$$p_B^r = \frac{X_B - (1 - p_A)R_B^r}{X_B} \quad (\text{C.7})$$

The participation constraint of agent C now writes

$$\begin{aligned} p_B^r R_B^r + (1 - p_B^r) p_A R_B^r &\geq 2 \\ X_B p_A R_B^r + (X_B - (1 - p_A)R_B^r)(1 - p_A)R_B^r &\geq 2X_B \\ -(1 - p_A)^2 (R_B^r)^2 + X_B R_B^r - 2X_B &\geq 0 \end{aligned}$$

A solution to this equation exists if and only if $X_B \geq 8(1 - p_A)^2$, that is $p_A \geq \hat{p}_A := 1 - \sqrt{\frac{X_B}{8}}$. In this case, R_B^r is given by the smallest root of the second order polynomial above, that is

$$R_B^r = \frac{X_B - \sqrt{X_B^2 - 8X_B(1 - p_A)^2}}{2(1 - p_A)^2} \quad (\text{C.8})$$

which is equivalent to expression (15). The values for p_B^r and U_B^r are obtained by plugging equation (15) in equations (C.7) and (15) respectively. We are left to verify the conjecture that that $R_B^r \leq \min\{X_A, X_B\}$. The condition $R_B^r \leq X_A$ is equivalent to $p_A \leq \bar{p}_A$. Expression (15) shows that $R_B^r \leq 4$ which implies that $R_B^r \leq X_B$ under Assumption 2.

Step 2. *Optimality of re-use*

We now derive the conditions such that re-use is preferred by agent B compared to a 1 unit loan without re-use. We show first that it is the case when $p_A \geq \bar{p}_A$. Then, we have $U_B^r = \frac{1}{2} (p_B^r)^2 X_B$. Hence, it is enough to show that $p_B^r \geq p_B$. From equation (18), we have

$$p_B^r = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2 - R_A}{X_B}} \geq \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{X_B}} = p_B$$

where the inequality follows from Assumption 1. This proves that $U_B^r \geq U_B$ for $p_A \geq \bar{p}_A$.

Consider now the case $p_A \leq \bar{p}_A$. We first show that U_B^r is decreasing in p_A . Using equation (11), we

have

$$\begin{aligned}
U_B^r &= \frac{1}{2} (p_B^r)^2 X_B + R_A - p_A R_B^r \\
&= \frac{1}{2} \left(1 - \frac{(1-p_A)}{X_B} R_B^r \right)^2 X_B + R_A - p_A R_B^r \\
&= \frac{X_B}{2} + R_A - R_B^r \left(1 - \frac{(1-p_A)^2}{2X_B} R_B^r \right) \\
&= \frac{X_B}{2} + R_A - \frac{R_B^r}{4} \left(4 - 1 + \sqrt{1 - \frac{8(1-p_A)^2}{X_B}} \right) \\
&= \frac{X_B}{2} + R_A - \frac{R_B^r}{2} - \frac{X_B}{8(1-p_A)^2} \left(1 - \sqrt{1 - \frac{8(1-p_A)^2}{X_B}} \right) \left(1 + \sqrt{1 - \frac{8(1-p_A)^2}{X_B}} \right) \\
&= \frac{X_B}{2} + R_A - 1 - \frac{2}{1 + \sqrt{1 - \frac{8(1-p_A)^2}{X_B}}}
\end{aligned} \tag{C.9}$$

where to derive the second, third and final line, we used respectively equation (C.7), (C.8) and (16). It follows from the last expression that U_B^r is strictly increasing in p_A when $p_A \leq \bar{p}_A$.

Two cases are then possible. If $U_B^r(\hat{p}_A) \geq U_B$, then define $\underline{p}_A := \hat{p}_A$. If instead $U_B^r(\hat{p}_A) < U_B$, then \underline{p}_A is the value of p_A implicitly defined by $U_B^r(\underline{p}_A) = U_B$. Since we showed that $U_B^r(\bar{p}_A) > U_B$ and that U_B^r is strictly increasing in p_A , it follows that $\underline{p}_A \in (\hat{p}_A, \bar{p}_A)$ in this second case. This concludes the proof.

C.4 Proof of Proposition 3

The first part of the result follows directly from Proposition 2 since we showed that a 2 unit loan with re-use is preferred to a 1 unit loan without re-use when $p_A \geq \underline{p}_A$.

For the second part of the result, we first study the monotonicity of p_B^{reuse} as a function of p_A . Since $p_A X_A$ is assumed to be constant, it follows that p_B^r is constant for $p_A \geq \bar{p}_A$. We showed in the proof of Proposition 2 that in this case, $p_B^r > p_{B,1}$. Let us now consider the case $p_A \leq \bar{p}_A$. Then p_B^r is given by equation (16). Hence,

$$\frac{\partial p_B^r}{\partial p_A} = -\frac{1}{2(1-p_A)^2} + \frac{1}{4(1-p_A)^3} \frac{1}{\sqrt{\frac{1}{4(1-p_A)^2} - \frac{2}{X_B}}}$$

Hence, p_B^r is increasing with p_A since

$$0 \leq 1 - 2(1-p_A) \sqrt{\frac{1}{4(1-p_A)^2} - \frac{2}{X_B}} = 1 - \sqrt{1 - \frac{8(1-p_A)^2}{X_B}}$$

Given the monotonicity of p_B^r and our first result that $p_B^r(\bar{p}_A) > p_B$, we are left to show that $p_B^r(\underline{p}_A) < p_B$. Consider the two cases analyzed in the proof of Proposition 2. Suppose first that $\underline{p}_A = \hat{p}_A$, which is

the case when $U_B^r(\hat{p}_A) > U_B$. Then, we have

$$p_B^r(\hat{p}_A) = \frac{1}{2} - \frac{\hat{p}_A}{1 - \hat{p}_A} = 1 - \sqrt{\frac{2}{X_B}}$$

Then, using equation (??), we have $p_B^r(\hat{p}_A) < p_B$ iff

$$\begin{aligned} \frac{1}{2} - \sqrt{\frac{2}{X_B}} &\leq \sqrt{\frac{1}{4} - \frac{1}{X_B}} \\ \Leftrightarrow \frac{1}{4} - \sqrt{\frac{2}{X_B}} + \frac{2}{X_B} &\leq \frac{1}{4} - \frac{1}{X_B} \end{aligned}$$

The last equation holds since $X_B \geq 4$. Consider now the case $\underline{p}_A > \hat{p}_A$. Then, by definition of \underline{p}_A , we have

$$U_B = \frac{1}{2} p_B^2 X_B = U_B^r(\underline{p}_A) = \frac{1}{2} \left(p_B^r(\underline{p}_A) \right)^2 X_B + \underline{p}_A \left[X_A - R_B^r(\underline{p}_A) \right]$$

Since $X_A > R_B$ when $p_A < \bar{p}_A$, this implies that $p_B^r(\underline{p}_A) < p_B$. Hence, in both cases, there exists $p_A^* \in (\underline{p}_A, \bar{p}_A)$ such that $p_B^r < p_B$ for all $p_A \in (\underline{p}_A, p_A^*)$. We can derive an analytical solution for p_A^* by solving for the equation $p_B = p_B^r(p_A)$. We obtain

$$\begin{aligned} \sqrt{\frac{1}{4} - \frac{1}{X_B}} &= -\frac{p_A^*}{2(1-p_A^*)} + \sqrt{\frac{1}{4(1-p_A^*)^2} - \frac{2}{X_B}} \\ \frac{1}{4} - \frac{1}{X_B} + \frac{(p_A^*)^2}{4(1-p_A^*)^2} + \frac{p_A^*}{1-p_A^*} \sqrt{\frac{1}{4} - \frac{1}{X_B}} &= \frac{1}{4(1-p_A^*)^2} - \frac{2}{X_B} \\ \frac{1}{4} + \frac{1}{X_B} + \frac{p_A^*}{1-p_A^*} \sqrt{\frac{1}{4} - \frac{1}{X_B}} &= \frac{1+p_A^*}{4(1-p_A^*)} \\ \Rightarrow p_A^* &= \frac{2}{X_B + 2 - \sqrt{X_B^2 - 4X_B}} \end{aligned}$$

C.5 Proof of Proposition 4

The proof is in several steps. We first derive the values of $R_B^{r,n}$ and $p_{Bs}^{r,n}$ under the two different cases $R_B^{r,n} \geq X_A$ and $R_B^{r,n} \leq X_A$. We then compare the expected level of effort and the utility of agent B in the two regions $p_A \geq \bar{p}_A$ and $p_A < \bar{p}_A$, characterized in Proposition 2.

Step 1. *Values of $R_B^{r,n}$ and $p_{Bs}^{r,n}$ when $R_B^{r,n} \geq X_A$*

In this case, using equations (21) and (22), the participation constraint of agent C writes

$$(X_B - R_B^{r,n} + R_A)R_B^{r,n} + (R_B^{r,n} - X_A)R_A \geq 2X_B$$

Denoting $\tilde{R}_B^{r,n} = R_B^{r,n} - R_A$, we have

$$\left(\tilde{R}_B^{r,n}\right)^2 - X_B\tilde{R}_B^{r,n} + X_B(2 - R_A) + p_A(1 - p_A)X_A^2 = 0$$

This second order equation has real solutions if and only if

$$0 \leq X_B^2 - 4X_B(2 - R_A) - 4R_A(X_A - R_A)$$

which is equivalent to

$$p_A \geq \hat{p}_A := \frac{4R_A^2}{4R_A^2 + X_B^2 - 4X_B(2 - R_A)} \quad (\text{C.10})$$

The loan face value is then the lowest root of the second order polynomial, given by

$$\begin{aligned} R_B^{r,n} &= R_A + \frac{1}{2} \left(X_B - \sqrt{X_B^2 - 4X_B(2 - R_A) - 4R_A(X_A - R_A)} \right) \\ &= R_A + 2 \frac{2 - R_A + \frac{R_A(X_A - R_A)}{X_B}}{1 + \sqrt{1 - 4\frac{2 - R_A}{X_B} - \frac{4R_A(X_A - R_A)}{X_B^2}}} \end{aligned} \quad (\text{C.11})$$

The expression for $E[p_{B_s}^{r,n}]$ is obtained by plugging the expression for $R_B^{r,n}$ above in equation (21) and taking the average over the states $s \in \{b, g\}$. We obtain

$$E[p_{B_s}^{r,n}] = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{(2 - R_A)}{X_B} - \frac{R_A(X_A - R_A)}{X_B^2}} \quad (\text{C.12})$$

Since $E[p_{B_s}^{r,n}]$ is decreasing with X_A and that $X_A = R_A/p_A$ where R_A is fixed, it follows that $E[p_{B_s}^{r,n}]$ is decreasing in p_A .

Finally, the ex-ante utility of agent B is given by

$$\begin{aligned}
U_B^{r,n} &= \frac{1}{2} E \left[(p_{Bs}^{r,n})^2 \right] X_B \\
&= \frac{1}{2X_B} E \left[(X_B - R_B^{r,n} + A_s)^2 \right] \\
&= \frac{1}{2X_B} \left[(X_B - R_B^{r,n} + R_A)^2 + \text{Var}[A_s] \right] \\
&= \frac{1}{8X_B} \left[\left(X_B + \sqrt{X_B^2 - 4X_B(2 - R_A) - 4R_A(X_A - R_A)} \right)^2 + 4R_A(X_A - R_A) \right] \\
&= \frac{1}{8X_B} \left[2X_B^2 - 4X_B(2 - 2p_A X_A) + 2X_B \sqrt{X_B^2 - 4X_B(2 - R_A) - 4R_A(X_A - R_A)} \right] \\
&= \frac{1}{4} \left[X_B + \sqrt{X_B^2 - 4X_B(2 - R_A) - 4R_A(X_A - R_A) - 2(2 - R_A)} \right] \tag{C.13}
\end{aligned}$$

Since $U_B^{r,n}$ is decreasing with X_A and that $X_A = R_A/p_A$ where R_A is fixed, it follows that $U_B^{r,n}$ is decreasing in p_A .

Step 2. *Values of $R_B^{r,n}$ and $p_{Bs}^{r,n}$ when $R_B^{r,n} < X_A$*

The conjecture $R_B^{r,n} < X_A$ together with equation (21) imply that

$$p_{Bg}^{r,n} = 1, \quad p_{Bb}^{r,n} = \frac{X_B - R_B}{X_B}$$

Using again equations (21) and (22) with the effort choices derived above, the participation constraint of agent C now writes

$$\begin{aligned}
p_A R_B^{r,n} + (1 - p_A) p_{Bb}^{r,n} R_B^{r,n} &\geq 2 \\
-(1 - p_A) (R_B^{r,n})^2 + R_B^{r,n} X_B - 2X_B &\geq 0
\end{aligned}$$

This second-order equation has a solution if $X_B \geq 8(1 - p_A)$. Then, the solution is the smallest root of the second-order polynomial above, given by

$$R_B^{r,n} = \frac{X_B - \sqrt{X_B^2 - 8X_B(1 - p_A)}}{2(1 - p_A)} = \frac{4}{1 + \sqrt{1 - \frac{8(1 - p_A)}{X_B}}} \tag{C.14}$$

The expected level of effort in this case is obtained thanks to equation (21)The

$$\begin{aligned}
E[p_{Bs}^{rn}] &= p_A + (1 - p_A) \left[1 - \frac{R_B^{r,n}}{X_B} \right] \\
&= 1 - \frac{X_B - \sqrt{X_B^2 - 8X_B(1 - p_A)}}{2X_B} \\
&= \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{8(1 - p_A)}{X_B}}
\end{aligned} \tag{C.15}$$

The expression above shows that $E[p_{Bs}^{rn}]$ is also increasing with p_A in this case. Finally, the ex-ante utility of agent B is given by

$$\begin{aligned}
U_B^{r,n} &= p_A \left(\frac{X_B}{2} - R_B^{r,n} + X_A \right) + \frac{1}{2}(1 - p_A) \left(p_{B,b}^{r,n} \right)^2 X_B \\
&= p_A \frac{X_B}{2} - p_A R_B^{r,n} + R_A + \frac{1}{2}(1 - p_A) \left[X_B - 2R_B^{r,n} + \frac{(R_B^{r,n})^2}{X_B} \right] \\
&= \frac{X_B}{2} + R_A - R_B^{r,n} \left(1 - \frac{(1 - p_A)R_B^{r,n}}{2X_B} \right) \\
&= \frac{X_B}{2} + R_A - \frac{R_B^{r,n}}{4} \left(4 - 1 + \sqrt{1 - \frac{8(1 - p_A)}{X_B}} \right) \\
&= \frac{X_B}{2} + R_A - \frac{R_B^{r,n}}{2} - 1
\end{aligned} \tag{C.16}$$

The utility $U_B^{r,n}$ of agent B is increasing with p_A since $R_B^{r,n}$ is decreasing with p_A , as can be seen from equation (C.14).

Step 3. *Proof that $U_B^{r,n} \leq U_B^r$ and $E[p_{B,s}^{r,n}] < p_B^r$ for $p_A \geq \bar{p}_A$.*

By definition of \bar{p}_A , the face value of the loan in the absence of news satisfies $R_B^r \geq X_A$. Comparing equation (17) for R_B^r and equation (C.11) for $R_B^{r,n}$ shows that $R_B^{r,n} \geq R_B^r$. Hence, the conjecture $R_B^{r,n} \geq X_A$ is satisfied for $p_A \geq \bar{p}_A$. The result that the expected level of effort goes down follows from the comparison between equations (18) and (C.12). For the comparison between utility levels, observe that

$$\begin{aligned}
U_B^r &= \frac{1}{2}(p_B^r)^2 X_B = \frac{1}{2} \left(\frac{1}{2} - \frac{2 - R_A}{X_B} + \sqrt{\frac{1}{4} - \frac{2 - R_A}{X_B}} \right) X_B \\
&= \frac{1}{4} \left(X_B - 2(2 - R_A) + \sqrt{1 - 4(2 - R_A)X_B} \right) \\
&= U_{B|p_A=1}^{r,n}
\end{aligned}$$

Since $U_B^{r,n}$ is increasing with p_A , $U_B^r \geq U_B^{r,n}$ for all $p_A \geq \bar{p}_A$, with a strict inequality for $p_A < 1$. This concludes the proof for the case $p_A \geq \bar{p}_A$.

Step 4. *Proof that $U_B^{r,n} \leq U_B^r$ and $E[p_{B,s}^{r,n}] < p_B^r$ for $p_A < \bar{p}_A$*

When $p_A < \bar{p}_A$, the face value of the loan in the absence of news satisfies $R_B^r \leq X_A$ by definition of \bar{p}_A . In the model with news, two cases are possible with either $R_B^{r,n} \leq X_A$ or $R_B^{r,n} > X_A$. Consider first the case $R_B^{r,n} \leq X_A$. Then the comparison between equations (16) and (C.15) shows that $E[p_{B,s}^{r,n}] < p_B^r$ since $(1 - p_A)^2 < (1 - p_A)$. The comparison between equations (C.9) and (C.16) shows that $U_B^{r,n} < U_B^r$ for the same reason.

Suppose now that the equilibrium with news is such that $R_B^{r,n} > X_A$. We first show that the expected level of effort is lower than in the model without interim news. Using equation (21), we obtain

$$E[p_{B,s}^{r,n}] = \frac{X_B + p_A X_A - R_B^{r,n}}{X_B} \leq \frac{X_B - (1 - p_A)R_B^{r,n}}{X_B} \leq \frac{X_B - (1 - p_A)R_B^r}{X_B} = p_B^r$$

where the first inequality follows from $X_A \leq R_B^{r,n}$ and the second inequality is implied by $R_B^{r,n} \geq R_B^r$.

We are then left to show that $U_B^{r,n} \leq U_B^r$ in this case. For this, consider the fictitious case where agent B can commit to the maximum effort level $\tilde{p}_{Bg} = 1$ in state g . Given the face value $\tilde{R}_B^{r,n}$ that agent C would require, B 's effort choice in state b would be given $\tilde{p}_{Bb}^n = \frac{X_B - \tilde{R}_B^{r,n}}{X_B}$. Fixing the face value of the loan $\tilde{R}_B^{r,n}$, these effort levels are the same than in the case analyzed in Step 2. Hence, the fictitious face value $\tilde{R}_B^{r,n}$ and agent B 's utility $\tilde{U}_B^{r,n}$ would be given by equation (C.14) and (C.16), respectively. We have shown above that $\tilde{U}_B^{r,n} \leq U_B^r$ for all values of p_A . Since the ability to commit in state g is valuable, we have $U_B^{r,n} \leq \tilde{U}_B^{r,n}$ which implies that $U_B^{r,n} \leq U_B^r$ also in the case when $R_B^{r,n} > X_A$. This concludes the proof for the case $p_A < \bar{p}_A$.

C.6 Proof of Corollary 2

To prove the results, we consider the two cases $R_B^{r,n} \geq X_A$ and $R_B^{r,n} < X_A$ in turn.

Step 1. Case $R_B^{r,n} \geq X_A$

We showed in the proof of Proposition 4 that an equilibrium with this feature can exist if $p_A \geq \hat{p}_A$ where \hat{p}_A is defined in equation (C.10). Then, two cases are possible.

Suppose first that $U_B^{r,n}(\hat{p}_A) > U_B$ which implies that the lowest value to consider is $\underline{p}_A^n = \hat{p}_A$. Using equation (C.12), the expected level of effort with re-use for $p_A = \underline{p}_A^n$ is given by

$$E[p_{B,s}^{r,n}]_{p_A = \underline{p}_A^n} = \frac{1}{2} < \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{X_B}} = p_B$$

where we used equation (??) for the last equality. Since $E[p_{B,s}^{r,n}]$ is increasing in p_A , there is a region $[\underline{p}_A^n, p_A^{n,*}]$ where agent B prefers to re-use and the expected level of effort is lower than without re-use.

Suppose now that $U_B^{r,n}(\hat{p}_A) > U_B$ and let \underline{p}_A^n be the value of p_A such that $U_B^{r,n}(\underline{p}_A^n) = U_B$. By equation

(C.13), \underline{p}_A^n exists and it is strictly lower than 1 since $U_B^{r,n}$ is increasing in p_A and

$$U_B^{r,n}(1) = U_B^r(1) > U_B$$

where the last inequality follows from Proposition (2). Then, we are left to show that the expected level of effort with re-use when $p_A = \underline{p}_A^n$ is strictly lower than in the benchmark case without re-use. We have

$$U_B^{r,n} = \frac{1}{2}E \left[(p_{Bs}^{r,n})^2 \right] X_B = \frac{1}{2} (E [p_{Bs}^{r,n}])^2 X_B + \frac{1}{2} Var [p_{Bs}^{r,n}] X_B$$

Hence, since $U_B = \frac{1}{2}p_B^2 X_B$, if $U_B^{r,n}(\underline{p}_A^n) = U_B$, then it must that

$$E [p_{Bs}^{r,n}]_{|p_A=\underline{p}_A^n} < p_B$$

This proves the claim in this case.

Step 2. Case $R_B^{r,n} < X_A$.

We showed in Proposition (4) that such an equilibrium can exist if $p_A \geq \hat{p}_{A,2}^n = 1 - \frac{X_B}{8}$. Again, two cases are possible.

Suppose first that $U_B^{r,n}(\hat{p}_{A,2}^n) > U_B$ which implies that the lowest value to consider is $\underline{p}_A^n = \hat{p}_{A,2}^n$. Using equation (C.12), the expected level of effort with re-use for $p_A = \underline{p}_A^n$ is given by

$$E[p_{B,s}^{r,n}]_{|p_A=\underline{p}_A^n} = \frac{1}{2} < \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{X_B}} = p_B$$

where we used equation (??) for the last equality. Since $E[p_{B,s}^{r,n}]$ is increasing in p_A , there is a region $[\underline{p}_A^n, p_A^{n,*}]$ where agent B prefers to re-use and the expected level of effort is lower than without re-use.

Suppose now that $U_B^{r,n}(\hat{p}_{A,2}^n) > U_B$ and let \underline{p}_A^n be the value of p_A such that $U_B^{r,n}(\underline{p}_A^n) = U_B$. By equation (C.13), \underline{p}_A^n exists and it is strictly lower than 1 since $U_B^{r,n}$ is increasing in p_A and

$$U_B^{r,n}(1) = U_B^r(1) > U_B$$

where the last inequality follows from Proposition (2). Then, we are left to show that the expected level of effort with re-use when $p_A = \underline{p}_A^n$ is strictly lower than in the benchmark case without re-use. We have

$$\begin{aligned} U_B^{r,n} &= \frac{1}{2}E \left[(p_{Bs}^{r,n})^2 \right] X_B + p_A(X_A - R_B^{r,n}) \\ &= \frac{1}{2} (E [p_{Bs}^{r,n}])^2 X_B + \frac{1}{2} Var [p_{Bs}^{r,n}] X_B + p_A(X_A - R_B^{r,n}) \end{aligned}$$

Hence, since $U_B = \frac{1}{2}p_B^2 X_B$, if $U_B^{r,n}(\underline{p}_A^n) = U_B$, then it must that

$$E [p_{Bs}^{r,n}]_{|p_A=\underline{p}_A^n} < p_B$$

This result proves the claim in this case and concludes the proof.

C.7 Proof of Proposition (5)

Suppose that agent B takes a 2 unit loan and re-uses the collateral of agent A . Let then R_B^r be the face value of the loan. Since $p_A \geq \bar{p}_A$, this face value satisfies $R_B^r \geq X_A$ by Proposition 2. The effort choice of agent B when he does not acquire information is then given by

$$p_B^r = \frac{X_B - R_B^r + R_A}{X_B} \quad (\text{C.17})$$

where this expression is derived from equation (11). Agent B 's utility is then given by $U_B^r = \frac{1}{2} (p_B^r)^2 X_B$. Consider now agent B 's incentives to acquire information at the interim stage. If he does acquire information, his effort choice in state s is given by equation (21) and his utility is now $\tilde{U}_B^{r,n} = \frac{1}{2} E \left[(p_{Bs}^{r,n})^2 \right] X_B$. Equations (C.17) and (21) show that $E[p_{Bs}^{r,n}] = p_B^r$. Hence, the value of acquiring information is given by

$$\tilde{U}_B^{r,n} - U_B^r = \frac{1}{2} \text{Var}[p_{Bs}^{r,n}] X_B = \frac{1}{2} \frac{V_A}{X_B}$$

The comparison of the benefit of information above with the cost γ leads to Condition (24).

C.8 Proof of Proposition 6

We will show that the threshold \underline{X}_A exists. We need to find conditions such that B reuses collateral and sets $F_A^{**} < F_A^*$.

We first derive the condition such that $F_A^{**} < F_A^*$ if agent B re-uses collateral. Denote $\bar{p}_A(R_A)$ the threshold \bar{p}_A introduced in Proposition 2 where the dependence of \bar{p}_A on the expected value of collateral R_A is emphasized. Suppose first that $p_A > \bar{p}_A(R_A)$ holds in equilibrium, with $R_A = p_A X_A (1 - c_A p_A)$ the expected value of collateral. If $p_A > \bar{p}_A(R_A)$, Case 2 of Proposition 2 applies. Agent B 's utility is increasing with the expected value of the collateral and it does not depend on other moments of the distribution of the collateral payoff. This implies that the profit-maximizing face value is $F_A^{**} = F_A^* = \frac{X_A}{2}$ and $p_A^{**} = p_A^*$. We are left to verify that the initial conjecture $p_A^{**} > \bar{p}_A(R_A^{**})$ holds. Using equation (C.6), which defines \bar{p}_A , this condition writes

$$1 > \frac{X_A}{X_B + \frac{X_A}{2c_A} - \sqrt{X_B^2 - 4X_B \left(2 - \frac{X_A}{4c_A}\right)}} \quad (\text{C.18})$$

The right hand side of this inequality is increasing with and it is equal to $2c_A \geq 2$ for $X_A = 8c_A$. Hence, there exists $\underline{X}_{A,1} < 8c_A$ such that $p_A^{**} > \bar{p}_A(R_A^{**})$ holds if and only if $X_A < \underline{X}_{A,1}$.

If instead $X_A \geq \underline{X}_{A,1}$, it must be that $p_A^{**} \leq \bar{p}_A(R_A^{**})$. This implies that Case 1 of Proposition 2

applies. We showed in the proof of this Proposition that agent B 's utility is given by equation C.9. Hence, B 's optimization problem is given by

$$\begin{aligned} \max_{p_A} U_B^r(p_A) &= \frac{X_B}{2} + p_A(1 - c_A p_A)X_A - 1 - \frac{2}{1 + \sqrt{1 - \frac{8(1-p_A)^2}{X_B}}} \\ \text{subject to } p_A &\leq \bar{p}_A (p_A(1 - c_A p_A)X_A) \end{aligned} \quad (\text{C.19})$$

The constraint ensures that the optimal choice of p_A lies below the threshold \bar{p}_A so that agent B 's utility is indeed given by $U_B^r(p_A)$ for any feasible choice p_A . As will be clear shortly, this constraint is redundant. The second term of the objective function is increasing in p_A , which implies agent B chooses $p_A^{**} > \frac{1}{2c_A}$. Observe that there is no benefit in increasing p_A beyond \bar{p}_A . Indeed, the expected value of the collateral would further decrease without any risk reduction benefit since collateral risk is irrelevant for $p_A \geq \bar{p}_A$. This observation confirms that the constraint is redundant.

Finally, we are left to verify that agent B re-uses collateral in equilibrium. Re-use is preferred if $U_B(p_A^{**}) > U_B$ with $U_B(p_A)$ defined in equation (C.19) and p_A^{**} the profit maximizing choice. Since p_A^{**} is preferred to $p_A^* = \frac{1}{2c_A}$ when $X_A \geq \bar{X}_A(X_B)$, a sufficient condition for the result is that $U_B(p_A^*) \geq U_B$. This condition writes

$$U_B \leq \frac{X_B}{2} + \frac{X_A}{4c_A} - 1 - \frac{2}{1 + \sqrt{1 - \frac{8(1-\frac{1}{2c_A})^2}{X_B}}}$$

Since the left-hand side of the inequality is independent of X_A and the right-hand side is increasing with X_A , this condition defines a lower bound $\underline{X}_{A,2}$ on X_A . It is easy to verify that the condition holds strictly for $X_A = 8c_A$ and thus that $\underline{X}_{A,2} < 8c_A$. Hence, defining $\underline{X}_A := \max\{\underline{X}_{A,1}, \underline{X}_{A,2}\}$, we obtain the desired result.

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