E-commerce Platforms and Credit*

Matthieu Bouvard, Catherine Casamatta and Rui Xiong†

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Abstract

We develop a model where an e-commerce platform can benefit from offering credit to merchants in addition to access to its commercial services. While the volume of credit offered by the platform increases with its ability to monitor merchants, the platform may provide credit even when its monitoring technology is inferior to that of a competitive banking sector. Credit provided by the platform endogenously selects more financially constrained merchants and co-exists with bank credit. Relative to a benchmark where only banks provide credit, overall trading on the platform, as well as access fees paid by merchants and consumers, are higher when the platform provides credit. Welfare improves for the more financially constrained merchants but worsens for the less financially constrained ones. Overall, our model suggest that the expansion of e-commerce platforms into the credit market is related to the market power they hold vis-à-vis merchants.

Keywords: Two-sided markets, credit, banks

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†Matthieu Bouvard and Catherine Casamatta are at Toulouse School of Economics, Université Toulouse Capitole (TSM-Research), Rui Xiong is at Toulouse School of Management, Université Toulouse Capitole (TSM-Research).

1 Introduction

The expansion of big tech platforms into credit has been a major shift in an activity traditionally dominated by banks. Croxson, Frost, Gambacorta, Valletti, et al. (2022) report that from 2018 onwards credit issuance from big tech platforms has also overcome other types of fintech lenders. While some of this expansion may have substituted for bank credit, evidence suggests that credit provided by commercial platforms has also worked at the extensive margin by providing loans to smaller and more financially constrained firms (Hau, Huang, Shan, and Sheng (2019)). In that sense, big tech credit has been a vector of financial inclusion. One explanation for this spectacular growth can be rooted in higher efficiency in providing credit. In particular, platforms may have an advantage over banks when screening borrowers ex ante based on a wider access to data. For instance, Berg, Burg, Gombović, and Puri (2020) show how a German e-commerce platform leverages non-financial data to assess the credit-worthiness of customers. Ant Financial services uses data collected from Alibaba e-commerce activity to generate an automated credit scoring system. Platforms may also have an advantage ex post in monitoring borrowers and enforcing repayments. For instance, the platform may directly seize some of the revenue the borrower is generating on the platform to service the debt (Huang (2021)). However, beyond efficiency, e-commerce platforms may also have different *incentives* to provide credit than the banking system. Where a bank evaluates the profitability of granting a loan solely based on the cash-flows this loan will generate, a platform may internalize that access to credit allows merchants to develop, and generate traffic or trading on the platform. This additional benefit can be amplified by the network effects inherent to multi-sided businesses. In this paper, we study how the nature of platforms' commercial activities affects their decisions to enter the credit market, and conversely how providing credit affects management of platforms' commercial activities.

Our approach combines two streams of literature. The first one studies platform or two-sided markets. A central question in this literature is how a platform should optimally price on both sides (e.g., for merchants and for buyers) given the existence of an externality between users on those two sides (Rochet and Tirole (2006)). We show that allowing a monopoly commercial platform to be active in the credit market affects the balance of this optimal pricing exercise and the composition of the participants active on the platform. To make creditors' decisions meaningful, we draw on a second stream of literature in corporate finance that grounds financial frictions in incentive problems. Specifically, we build on Holmstrom and Tirole (1997) where firms limited pledgeable income stems from a moral hazard problem. Importantly, this model incorporates a costly monitoring technology that can mitigate financial frictions and allow us to capture the efficiency of the platform at providing credit relative to the banking system. We show that in their credit activities, platforms differ from banks in three important ways. First, they target marginal merchants, which, due to financial constraints are unable to access funding from banks. Second, platforms are willing to tolerate losses on their lending activity because they can recover them on the commercial side. Third, platforms may be willing to engage in lending even when their monitoring technology is inferior to that of the banking system. That is, while technological superiority makes the platform's entry into the credit market more likely, it is not a requirement. Our approach suggests that a platform's entry in the credit market and pricing on the platform should be analyzed as joint decisions.

2 Model

Our model of a two-sided market borrows from Rochet and Tirole (2003). Consider a platform serving two groups of agents, buyers and merchants. There is a continuum of buyers indexed by $i \in (0,1)$ who derive value V_i^b from transacting with each merchant. V_i^b is distributed according to the cumulative distribution function $F^b(.)$ over $[0, \overline{V^b}]$. There is a continuum of merchants indexed by $j \in (0,1)$ who, for simplicity, generate the same profit V^m from each transaction with a buyer. When providing a transaction service, the platform charges a per-transaction price P^b to buyers and P^m to merchants, and incurs a per-transaction cost $c > 0$. Denote N^b the number of buyers on the platform.

To transact on the platform, a merchant needs to invest in a risky project, which outcome can be "success" or "failure". If the project fails, the merchant cannot participate to the platform. The project's initial outlay is $I > 0$. Merchant j has wealth A_j distributed according to the cumulative distribution function $F^m(.)$ over $(0, A^{max})$. Assume that both distribution functions have a monotone hazard rate: $\frac{1-F^b(.)}{f^b(.)}$ and $\frac{1-F^m(.)}{f^m(.)}$ are both decreasing. We assume that A_j is not observable, that is, a merchant can always claim to have less funds than what he actually possesses.

The investment project is subject to moral hazard. Following Holmstrom and Tirole (1997), we assume that each merchant can pick one of three types of projects. Project choice is not observable. The good project succeeds with probability $p_h > 0$ and yields no private benefit to the merchant. One bad project succeeds with probability $p_l \equiv p_h - \Delta_p$ with $\Delta_p > 0$ and yields a small private benefit $b > 0$ to the merchant. Another bad project also succeeds with probability p_l, and yields a large private benefit $B > b$ to the merchant. The three projects are summarized in the table below:

Project	1	2	3
Private Benefit	0	b	B
Prob. of Success	p_h	p_l	p_l

To satisfy his investment need, a merchant can borrow money from investors, which can be banks, or the platform itself. Both the bank and the platform can monitor the merchant to prevent him from choosing the large private benefit project. The monitoring cost is, respectively, $\gamma_p \geq 0$ for the platform, and $\gamma_b \geq 0$ for the bank.

The project's value depends on the number of buyers on the platform N^b, as well as on the price per transaction P^m charged by the platform. The following assumptions ensure that only the good project can be profitable.

Assumption 1. $p_h V^m - \gamma_b > I$.

Assumption 1 implies that it can be profitable for a merchant to obtain financing with monitoring. It states that choosing the good project generates a strictly positive NPV when the bank monitors the merchant, when the number of buyers is maximal ($N^b = 1$) and the price paid per transaction is null ($P^m = 0$),

Assumption 2. $p_l(V^m + \overline{V}^b) + B < I$.

Assumption 2 states that the bad project yields a negative profit, for any platform fee P^m and number of buyers N^b.

The timing of the model is as follows. At date 1, the platform sets fees P^m and P^b. At date 2, investors make financial offers to merchants, specifying an amount of money lent $(I - A)$, a repayment R, and whether monitoring takes place. At date 3, the investor, i.e. the bank or the platform, exerts monitoring or not, depending on the contract accepted. At date 4, each merchant chooses the project type. Finally, transactions occur on the platform for successful projects.

3 Equilibrium with bank financing only

Let us first determine the equilibrium outcome on the platform when only banks provide financing to merchants. Denote N^m the number of merchants who invest in the project.

3.1 Buyers

If the platform charges a price P^b per buyer transaction, buyer i's utility from transacting on the platform is $p_h N^m(V_i^b - P^b)$. Therefore, buyer i transacts on the platform if and only if $V_i^b > P^b$, which pins down the number of buyers present on the platform

$$N^b(P^b) = 1 - F^b(P^b). \tag{1}$$

Equation (1) implies that the number of buyers only depends on the price P^b charged for each buyer transaction. This is because buyers attribute the same value to each transaction with a merchant, irrespective of the number of transactions they perform: In that sense, there is no network effect on the buyer's side. This will not be the case on the merchant side as the number of buyers will determine the value of the investment project.

3.2 Merchants

The project's payoff upon success is $N^b(V^m - P^m)$, net of the platform transaction fee. Suppose first that the bank offers a contract without monitoring. The merchant chooses the good project if and only if

$$\begin{aligned} & p_h(N^b(V^m - P^m) - R) \geq p_l(N^b(V^m - P^m) - R) + B \\ \Leftrightarrow \quad & R \leq N^b(V^m - P^m) - \tfrac{B}{\Delta_p}. \end{aligned} \tag{2}$$

The right hand side of Condition (2) represents the pledgeable income, i.e. the maximum payoff that the bank can obtain while ensuring that the merchant chooses the good project. Assume next that the bank sector is competitive. The bank's participation constraint is $p_h R = I - A$, which, together with (2) implies that the merchant can only raise funds without monitoring if

$$A \geq I - p_h \left(N^b(V^m - P^m) - \frac{B}{\Delta_p} \right) \equiv \overline{A}(P^m, P^b). \tag{3}$$

Note that the minimum level of wealth required, $\overline{A}(P^m, P^b)$, depends on the number of buyers on the platform. From Equation (1), this number depends on the transaction fee P^b set by the platform. We therefore directly write that $\overline{A}$ depends on P^b. To make the financing problem non trivial, we further assume that merchants cannot obtain financing without investing some of their wealth, which is ensured by the following assumption.

Assumption 3. $p_h\left(V^m - \frac{B}{\Delta_p}\right) < I$.

Following Holmström and Tirole (1997), Assumption 3 implies that $\overline{A}(P^m, P^b) > 0$, whatever the number of buyers and platform fees.

Because the banking sector is competitive, the merchant's expected payoff is equal to the project's NPV, net of transaction fees, that is, if the bank does not monitor,

$$p_h N^b(V^m - P^m) - I. \tag{4}$$

Suppose next that the bank offers a contract with monitoring. Following the same reasoning as before, the pledgeable income is then equal to $N^b(V^m - P^m) - \frac{b}{\Delta_p}$. The bank's participation constraint is now

$$p_h R - \gamma_b \geq I - A. \tag{5}$$

Therefore, the merchant can only raise funds with monitoring if

$$A \geq I + \gamma_b - p_h\left(N^b(V^m - P^m) - \frac{b}{\Delta_p}\right) \equiv \underline{A}(P^m, P^b). \tag{6}$$

The merchant's expected payoff with monitoring is then

$$p_h N^b(V^m - P^m) - I - \gamma_b. \tag{7}$$

Comparing (7) and (4), it follows that only merchants with initial wealth strictly lower than $\overline{A}(P^m, P^b)$ accept a contract with monitoring. Next, the following assumption ensures that $\overline{A}(P^m, P^b) > \underline{A}(P^m, P^b)$, that is, monitoring expands the range of firms that can be funded.

Assumption 4. $\gamma_b < \frac{p_h}{\Delta_p}(B - b)$.

Assumption 4 states that the cost of monitoring is lower than the increase in the pledgeable income so that merchants who are not wealthy enough to obtain financing without monitoring opt for financing with monitoring when $A \geq \underline{A}(P^m, P^b)$. To summarize, when only banks provide financing, we obtain the standard result of Holmström and Tirole (1997):

- Merchants with $A \geq \overline{A}(P^m, P^b)$ get funding from the bank without monitoring;

- Merchants with $\underline{A}(P^m, P^b) \leq A < \overline{A}(P^m, P^b)$ get funding from the bank with monitoring;
- Merchants with $A < \underline{A}(P^m, P^b)$ do not get funding.

It follows that the number of merchants on the platform is

$$N^m(P^m, P^b) = 1 - F^m(\underline{A}(P^m, P^b)) \tag{8}$$

In principle, there exist many different contracts that grant merchants the project's NPV. To fix idea, we assume that the bank offers only two contracts. The first one includes monitoring, and requires an investment $\underline{A} = \underline{A}(P^b, P^m)$ from the merchant, and sets a repayment $\overline{R} = \frac{1}{p_h}(I + \gamma_b - \underline{A})$. The second one does not include monitoring, and requires an investment $\overline{A} = \overline{A}(P^b, P^m)$ and sets a repayment $\underline{R} = \frac{1}{p_h}(I - \overline{A})$.

3.3 Platform's optimal pricing strategy

We now derive the optimal transaction fees (P^b, P^m) charged by the platform, in the case when financing is only provided by banks.

Denote π the platform's profit. The platform solves the following program:

$$\max_{P^b, P^m} \pi = p_h N^m(P^m, P^b) N^b(P^b)(P^m + P^b - c), \tag{9}$$

where $N^b(P^b)$ and $N^m(P^m, P^b)$ are defined in Equations (1) and (8) respectively. The first order condition with respect to P^m yields

$$\big(1 - F^m(\underline{A}(P^m, P^b))\big) - f^m(\underline{A}(P^m, P^b))p_h(1 - F^b(P^b))(P^m + P^b - c) = 0. \tag{10}$$

The first term represents the increase in profit when the platform charges a high price P^m to all merchants who access the platform. The second term represents the decrease in profit when the platform charges a higher price P^m and worsens financial frictions. As P^m increases, the minimal level of wealth necessary to obtain financing $\underline{A}(P^m, P^b)$ increases. Some merchants become credit rationed and cannot offer services through the platform, which reduces the latter's profit. Equation (10) reflects this tension and illustrates how the platform's pricing strategy interacts with financial frictions. If there was no moral hazard, the second term would not be there, and the platform would set P^m at its maximal value (i.e. V^m).

The first order condition with respect to P^b yields

$$(1 - F^b(P^b))(1 - F^m(\underline{A}(P^m, P^b))) - f^b(P^b)(1 - F^m(\underline{A}(P^m, P^b)))(P^m + P^b - c)$$
$$-p_h f^m(\underline{A}(P^m, P^b)) f^b(P^b)(V^m - P^m)(1 - F^b(P^b))(P^m + P^b - c) = 0 \tag{11}$$

Rearranging (11) and (10) leads to the following proposition.

Proposition 1. *The optimal fee charged to buyers P^{b*} is defined by*

$$\frac{1 - F^b(P^{b*})}{f^b(P^{b*})} = V^m + P^{b*} - c. \tag{12}$$

The optimal fee charged to merchants P^{m*} is defined by

$$P^{m*} = \frac{1 - F^m(\underline{A}(P^{m*}, P^{b*}))}{p_h N^{b*} f^m(\underline{A}(P^{m*}, P^{b*}))} - P^{b*} + c, \tag{13}$$

where $N^{b*} \equiv 1 - F^b(P^{b*})$.

Equation (12) implicitly defines the optimal transaction fee for buyers, P^{b*}. Note that P^{b*} does not depend on the number of merchants. This is because the buyers' transaction value does not depend on the number of transactions they perform. In other words, the platform cannot induce more transactions from each buyer by modifying P^b. It only depends on the distribution of buyers' valuation per transaction. Next, using (12), Equation (13) implicitly defines the optimal transaction fee for merchants, P^{m*}.

From Equations (13) and (9), we obtain the platform's profit under bank financing π^*:

$$\pi^* = [1 - F^m(\underline{A}(P^{m*}, P^{b*}))] \frac{1 - F^m(\underline{A}(P^{m*}, P^{b*}))}{f^m(\underline{A}(P^{m*}, P^{b*}))}. \tag{14}$$

Through the financing constraint, the platform faces the familiar monopoly problem. On the one hand, increasing the price P^m raises the margin on merchants but tightens the financing constraints, hence merchants' demand. This can give an incentive to the platform to enter the credit market and offer financing to merchants, in order to increase its merchant base. We explore in the next section whether it is optimal for the platform to offer financing and compete with regular banks.

4 Equilibrium with bank and platform financing

Now consider the case in which the platform can also provide financing to merchants. Note that the competitive banking sector is still offering the contracts described in the previous subsection: $(\overline{R}, \underline{A}(P^m, P^b))$ with monitoring, and $(\underline{R}, \overline{A}(P^m, P^b))$ without monitoring. Since the bank observes platform fees before offering financing contracts, it can adjust its offers accordingly. Without loss of generality, the platform's offer is a contract $\mathcal{C}_p = (\mathcal{R}, \mathcal{A})$ where $\mathcal{R}$ is a repayment to the platform in case of success and $\mathcal{A}$ is a minimum investment by the merchant. The platform now optimizes jointly on the fees charged to buyers and merchants to access the platform, P^m and P^b, and on the contract $\mathcal{C}_p$. Assume as a tie-breaking rule that if a merchant is indifferent between bank financing and platform financing, he chooses the latter.

This assumption is immaterial for the results but simplifies the exposition. A first step is to simplify this optimization problem by narrowing down the space of contracts that can be optimal for the platform.

4.1 The platform's optimization problem

We first show in the next lemma that the platform does not have an incentive to offer financing without monitoring.

Lemma 1. *The platform does not gain at offering financing without monitoring.*

Proof. See Appendix. □

Clearly, the platform cannot gain at offering a contract such that $\mathcal{A} \geq \overline{A}$. To be accepted by merchants, any such contract would need to grant merchants more than the corresponding project's NPV, i.e., it would need to subsidize merchants. So the platform would make losses on this contract, without increasing the number of merchants financed, and the platform's profit π would decrease. Lemma 1 shows that the platform never gains at offering a contract without monitoring such that $\mathcal{A} < \overline{A}$. Indeed, if $\underline{A} \leq \mathcal{A} < \overline{A}$, the platform makes losses on its financial contract without expanding the merchant base, and is better off not offering this contract. If $\mathcal{A} < \underline{A}$, so that the platform aims at increasing the number of merchants, it will attract all merchants with an initial wealth larger than $\mathcal{A}$. Since the platform will then subsidize all merchants, it is equivalent to lowering the merchant's fee P^{m*}: offering financing does not increase the platform's profit.

Suppose now the platform offers a contract that requires monitoring to merchants with wealth at least equal to $\mathcal{A}$. A first observation we formalize in the next Lemma is that if offered, this contract is available for all merchants that can borrow from banks.

Lemma 2. *If in equilibrium some merchants are financed by the platform, then amount lent by the platform, $I - \mathcal{A}$ satisfies $\mathcal{A} < \underline{A}(P^m, P^b)$.*

To understand Lemma 2, consider the financial contract offered by the platform $\mathcal{C}_p = \{\mathcal{R}, \mathcal{A}\}$. For a given $\mathcal{A}$ (and given P^m and P^b), the contract that maximizes the platform's financial income sets

$$\mathcal{R} = (1 - F(P^b))(V^m - P^m) - \frac{b}{\Delta_p},$$

and pays off the agency rent $p_h \frac{b}{\Delta_p}$ to the agent. In that case the platform's per-merchant financial payoff (i.e., gross of the monitoring cost γ_p) is

$$\varphi(\mathcal{A}, P^m, P^b) \equiv p_h \mathcal{R} - (I - \mathcal{A}) = p_h(1 - F^b(P^b))(V^m - P^m) - p_h \frac{b}{\Delta_p} - (I - \mathcal{A}). \tag{15}$$

Note that this reduces the platform's financial contracting problem to the choice of a threshold $\mathcal{A}$. Consider now a merchant who takes the platform's contract. Its expected payoff is

$$p_h \frac{b}{\Delta_p} - \mathcal{A},$$

while its payoff in a bank's contract that requires monitoring is

$$p_h(1 - F(P^b))(V^m - P^m) - I - \gamma_b = p_h \frac{b}{\Delta_p} - \underline{A}(P^m, P^b),$$

where the equality comes from equation (6). Comparing both payoffs, it is immediate that the merchant prefers the platform's contract with monitoring iff $\mathcal{A} < \underline{A}(P^m, P^b)$. So if the platform wants to provide financing, it has to attract merchants rationed by banks.

On the other hand, the platform never finds it optimal to attract *all* merchants when offering a contract with monitoring.

Lemma 3. *If the platform offers a contract that requires monitoring, this contract is such that merchants with wealth higher than $\overline{A}(P^m, P^b)$ prefer being financed by banks.*

To understand the logic of Lemma 3, remember next that firms with $A \geq \overline{A}(P^m, P^b)$ can borrow from banks and secure an expected payoff equal to

$$p_h(1 - F(P^b))(V^m - P^m) - I.$$

It follows that if $\varphi(\mathcal{A}, P^m, P^b) < 0$, the platform's contract would dominate banks' contracts for firms with $A \geq \overline{A}(P^m, P^b)$. As a result, all merchants would take the platform contract. To see why this is suboptimal for the platform, suppose the platform starts lowering P^m keeping $\mathcal{A}$ constant. As long as $\varphi(\mathcal{A}, P^m, P^b) < 0$, this has no effect on the platform profit because what it loses by charging a lower price is exactly offset by a decrease in the financing subsidy necessary to preserve incentives for merchants above $\mathcal{A}$ to work. However, at the point where $\varphi(\mathcal{A}, P^m, P^b)$ turns positive, the platform makes a strict gain: total revenue (fees net of funding costs) from merchants who borrow with monitoring is still unchanged, but the plaform economizes the monitoring cost γ^p on all the merchants who borrow with monitoring and now turn to banks.

Using Lemmas 1, 2 and 3 we can write the platform's optimization problem when it can offer financing: the platform needs to optimize on fees P^m and P^b and a funding threshold $\mathcal{A}$, subject to the constraints ensuring that only merchants that need to be monitored take the contract, and to the constraint that the platform's profit is larger than with bank financing only.

The platform picks fees P^m, P^b and $\mathcal{A}$ to solve

$$\begin{aligned} \max_{P^m, P^b, \mathcal{A}} \pi(P^m, P^b, \mathcal{A}) \quad &= \quad [1 - F^b(P^b)](1 - F^m(\mathcal{A}))p_h(P^m + P^b - c) \\ &\quad + [F^m(\overline{A}(P^m, P^b))) - F^m(\mathcal{A})](\varphi(\mathcal{A}, P^m, P^b) - \gamma_p), \end{aligned} \tag{16}$$

$$\text{s.t.} \qquad \varphi(\mathcal{A}, P^m, P^b) \geq 0 \tag{17}$$

$$\varphi(\mathcal{A}, P^m, P^b) \leq \gamma_b \tag{18}$$

$$\pi(P^m, P^b, \mathcal{A}) \geq \pi^* \tag{19}$$

Condition (17) ensures that merchants who can obtain bank financing without monitoring (i.e. with wealth $A \geq \overline{A}(P^m, P^b)$) prefer to accept the bank's offer rather than the platform's contract with monitoring. Condition (18) ensures that merchants who need to be monitored (i.e. with wealth $A < \overline{A}(P^m, P^b)$) prefer to borrow from the platform. Last, Condition (19) ensures that the platform's profit increases compared to the case when only banks provide financing.

Denote by λ_φ, $\lambda_\mathcal{A}$ and λ_π the multipliers associated to the constraints (17), (18), and (19) respectively. We focus on the solution when $\lambda_\mathcal{A} = 0$ and the constraint (18) never binds.[1] The first order conditions of the above defined Lagrangian yield

$$1 - F^m(\overline{A}(P^m, P^b)) + f^m(\overline{A}(P^m, P^b))(\varphi(\mathcal{A}, P^m, P^b) - \gamma_p) = \frac{\lambda_\varphi - \lambda_\mathcal{A}}{1 + \lambda_\pi} \tag{20}$$

[1] If (18) was binding, the platform's expected profit π would write slightly differently as we assumed that when merchants are indifferent between the bank's and the platform's offer, they choose the bank's offer.

$$f^m(\mathcal{A})\left[p_h(1-F^b(P^b))(P^m+P^b-c)+\varphi(\mathcal{A},P^m,P^b)-\gamma_p\right]+F^m(\mathcal{A})-F^m(\overline{A}(P^m,P^b))=\frac{\lambda_\varphi-\lambda_{\mathcal{A}}}{1+\lambda_\pi} \tag{21}$$

$$\frac{1-F^b(P^b)}{f^b(P^b)}=P^b+V^m-c \tag{22}$$

$$\mathcal{A}=I-p_h(1-F^b(P^b))(V^m-P^m)-p_h\frac{b}{\Delta_p} \tag{23}$$

$$\mathcal{A}<I+\gamma_b-p_h(1-F^b(P^b))(V^m-P^m)-p_h\frac{b}{\Delta_p} \tag{24}$$

$$\begin{aligned}&p_h(1-F^b(P^b))\left[(1-F^m(\mathcal{A}))(P^m+P^b-c)-(1-F^m(\underline{A}^*))(P^{m*}+P^b-c)\right]\\&=[F^m(\overline{A}(P^m,P^b)))-F^m(\mathcal{A})](\varphi(\mathcal{A},P^m,P^b)-\gamma_p)\end{aligned} \tag{25}$$

Corollary 1. *The optimal price set for buyers is the same when the platform offers financing as when the bank offers financing.*

The proof of Corollary 1 is straightforward when comparing Equations (12) and (22). Intuitively, this result illustrates the fact that the price charged to buyers does not affect the number of transactions each buyer undertakes.

The optimal price set for merchants, as well as the platform financial contract, depend on which constraints are binding. If (19) is binding, there is no platform financing, and the optimal pricing strategy is defined as in Proposition 1. In the following, we assume that (19) is not binding, and consider three cases.

Consider first that constraints (17) and (18) are both not binding. We then have $\lambda_\varphi=0$, $\lambda_{\mathcal{A}}=0$. The first order condition for P^m, (20), writes

$$1-F^m(\overline{A}(P^m,P^b))+f^m(\overline{A}(P^m,P^b))(\varphi(\mathcal{A},P^m,P^b)-\gamma_p)=0.$$

The trade-off faced by the platform when setting P^m is the following: by increasing P^m, it extracts more profit from all merchants who obtain financing without monitoring $(1-F^m(\overline{A}(P^m,P^b)))$. At the same time, the threshold $\overline{A}(P^m,P^b)$ increases and some merchants who previously obtained financing without monitoring now turn to the platform's financial contract. The platform loses $\varphi(\mathcal{A},P^m,P^b)-\gamma_p$ on each of these merchants. Depending on the distribution of merchants, it can be that the latter effect dominates the former, which prevents the platform from increasing P^m.

When $\lambda_\varphi = 0$, $\lambda_\mathcal{A} = 0$, the first order condition for $\mathcal{A}$, (21), writes

$$f^m(\mathcal{A})\left[p_h(1-F^b(P^b))(P^m+P^b-c)+\varphi(\mathcal{A},P^m,P^b)-\gamma_p\right]+F^m(\mathcal{A})-F^m(\overline{A}(P^m,P^b))=0$$

The trade-off faced by the platform when setting $\mathcal{A}$ is the following: when decreasing $\mathcal{A}$, the platform increases the subsidy provided to each merchant who accepts the platform's offer. At the same time, more merchants borrow from the platform, which increases platform fees. So for these additional merchants, the platform loses $\varphi(\mathcal{A},P^m,P^b)-\gamma_p$ but gains $p_h(1-F^b(P^b))(P^m+P^b-c)$. The optimal $\mathcal{A}$ is such that the net gain from attracting new merchants is exactly offset by the loss from providing the subsidy to all merchants $F^m(\mathcal{A})-F^m(\overline{A}(P^m,P^b))$.

Rearranging (20) and (21), we obtain

$$P^m=\frac{1-F^m(\mathcal{A})}{p_h(1-F(P^b))f^m(\mathcal{A})}+\frac{1-F^m(\overline{A}(P^m,P^b))}{p_h(1-F(P^b))f^m(\overline{A}(P^m,P^b))}(1-\frac{f^m(\overline{A}(P^m,P^b))}{f^m(\mathcal{A})})-P^b+c,$$

and

$$\mathcal{A}=\frac{F^m(\overline{A}(P^m,P^b))-F^m(\mathcal{A})}{f^m(\mathcal{A})}-p_h(1-F^b(P^b))(V^m+P^b-c)+p_h\frac{b}{\Delta_p}+I+\gamma_p,$$

where P^b is defined by (22).

Constraints (17) and (18) can never bind at the same time. Consider next that (17) is binding while (18) is not, i.e. $\varphi(\mathcal{A},P^m,P^b)=0$ and $\lambda_\mathcal{A}=0$. The optimal price set for merchants is implicitly defined by

$$P^m=\frac{1-F^m(\mathcal{A})}{p_h(1-F(P^b))f^m(\mathcal{A})}+\frac{\gamma_p}{p_h(1-F(P^b))}(1-\frac{f^m(\overline{A}(P^m,P^b))}{f^m(\mathcal{A})})-P^b+c,$$

and $\mathcal{A}$ is given by Equation (23).

Consider finally that (18) is binding while (17) is not, i.e. $\underline{A}(P^m,P^b)=\mathcal{A}$ and $\lambda_\varphi=0$. The optimal price set for merchants and the platform's financial contract are defined implicitly as follows:

$$P^m=\frac{1-F^m(\mathcal{A})}{p_h(1-F(P^b))f^m(\mathcal{A})}+\frac{(\gamma_p-\gamma_b)}{p_h(1-F(P^b))}(1-\frac{f^m(\overline{A}(P^m,P^b))}{f^m(\mathcal{A})})-P^b+c$$

$$\mathcal{A}=I+\gamma_b-p_h(1-F^b(P^b))(V^m-P^m)-p_h\frac{b}{\Delta_p}$$

if $\gamma_p=\gamma_b$, we return back to the bank financing case, i.e. see equation (13).

4.2 Linear Merchant Demand

We first consider that A follows a uniform distribution: $A \sim \mathcal{U}[0, A^{max}]$. This generates a linear demand from merchants, which allows us to obtain close-form solutions. We further assume here that A^{max} is large enough, in a sense we make precise in the Appendix. This will ensure that solutions for P^m and $\mathcal{A}$ are interior.[2]

Using (13), the platform's optimal merchant fee under bank financing writes

$$P^{m*} = \frac{1}{2}(V^m - P^{b*} + c) - \frac{1}{2p_h N^{b*}}\left(p_h \frac{b}{\Delta_p} + I + \gamma_b - A^{max}\right) \tag{26}$$

where P^{b*} is still defined by (12). Next, the minimal wealth required by banks to provide financing is

$$\underline{A}(P^{m*}, P^{b*}) = \frac{1}{2}\left[I + \gamma_b + p_h \frac{b}{\Delta_p} + A^{max} - p_h N^{b*}(V^m + P^{b*} - c)\right]. \tag{27}$$

Last, using Equation (14), the platform's profit under bank financing now writes

$$\pi^* = \frac{\left(A^{max} - \underline{A}(P^{m*}, P^{b*})\right)^2}{A^{max}}. \tag{28}$$

Before turning to bank financing, it worth noting that the impact of an improvement in banks' monitoring technology has an ambiguous impact on merchants' welfare. From (26), a decrease in γ_b leads to an increase in P_m: intuitively reducing financial friction makes merchants' demand less price-elastic. This price increase harms merchants who borrow without monitoring. On the other hand, a lower γ_b has an overall positive effect for the less constrained merchants:

$$\frac{\partial}{\partial \gamma_b}[p_h N^b (V^m - P^m) - \gamma_b] = -p_h N^b \frac{\partial P^m}{\partial \gamma_b} - 1 = -\frac{\gamma_b}{2}.$$

Turn now to platform financing. The platform maximizes

$$\begin{aligned}
\max_{P^m, P^b, \mathcal{A}} \pi(P^m, P^b, \mathcal{A}) \quad &= \quad \frac{1}{A^{max}}[(A^{max} - \mathcal{A})p_h(1 - F^b(P^b))(P^m + P^b - c) && (29)\\
&\quad\quad +(\overline{A}(P^m, P^b) - \mathcal{A})(\varphi(\mathcal{A}, P^m, P^b) - \gamma_p)] && \\
\text{s.t.} \quad &\quad\quad \varphi(\mathcal{A}, P^m, P^b) \geq 0 && (30)\\
&\quad\quad \varphi(\mathcal{A}, P^m, P^b) \leq \gamma_b && (31)\\
&\quad\quad \pi(\mathcal{A}, P^m, P^b) \geq \pi^*. && (32)
\end{aligned}$$

We denote P^{m**}, P^{b**} and $\mathcal{A}^{**}$ the solutions to the above program.[3]

[2] See Proof of Proposition 2.

[3] From Corollary 1, we know that $P^{b**} = P^{b*}$ and we will use both notations interchangeably.

Proposition 2. *Suppose A is uniformly distributed. There exists $\overline{\gamma}_p > \gamma_b$ such that the platform offers financing if and only if $\gamma_p \leq \overline{\gamma}_p$. When the platform offers financing, it charges a higher fee to merchants and expands the range of merchants who receive funding relative to the benchmark case where only banks can provide funding: $P^{m**} > P^{m*}$ and $\mathcal{A}^{**} < \underline{A}(P^{m*}, P^{b*})$.*

Proof. See Appendix. □

Proposition 2 combines two motives for platforms to enter the credit market. The first one is straightworward: when platforms are more efficient than banks at monitoring, $\gamma_p < \gamma_b$, they can capture the corresponding efficiency gain ($\gamma_b - \gamma_p$ per funded merchant) while at the same time offering credit to more merchants. In particular, the monitoring cost $\overline{\gamma}_p$ below which the platform is willing to provide funding is increasing in γ_b: when the banking system is more inefficient, the platform is more likely to step in.

In addition, entering the credit market allows the platform to engage in the equivalent of price discrimination. This second benefit explains why the platform is provide credit even in cases where it is *less* efficient than the banking sector at monitoring creditors: $\overline{\gamma}_p > \gamma_p > \gamma_b$. Indeed, we show that the equilibrium financial contract offered by the platform is loss-making when incorporating the monitoring cost:

$$\varphi(\mathcal{A}, P^{m**}, P^{b**}) < \gamma_p.$$

That is, the platform uses subsidized credit to lower the overall charge $p_h N^b P^b + \varphi(\mathcal{A}, P^{m**}, P^{b**})$ supported by the more financially, thereby expanding equilibrium demand: $\mathcal{A}^{**} < \underline{A}(P^{m*}, P^{b*})$. Importantly, this subsidy only benefits merchants who need monitoring, which gives the platform an incentive to increase P^m in order to extract more surplus from less financially constrained merchants, who borrow from the banking sector without monitoring. Note that these more wealthy merchants are still better off than the more financially constrained merchants who need monitoring: their financing cost is zero while it is positive (equal to $\varphi(\mathcal{A}, P^{m**}, P^{b**})$) for merchants with $A < \overline{A}(P^{m**}, P^{b**})$. However they are worse off than if only banks could provide funding. We formalize this intuition in the following corollary.

Corollary 2. *Relative to the benchmark case where only banks provide funding, when the platform provides funding,*

- *merchants with wealth $A_j > \overline{A}(P^{m**}, P^{b**})$ are strictly worse-off,*

- *merchants with wealth* $\overline{A}(P^{m**}, P^{b**}) > A_j > \mathcal{A}^{**}$ *are strictly better-off,*

- *buyers are strictly better off.*

There are four categories of merchants. From Lemma 3, merchants with wealth $A_j > \overline{A}(P^{m**}, P^{b**})$ still borrow from the bank and their welfare decreases because of the price hike. Merchants with wealth $\overline{A}(P^{m**}, P^{b**}) > A_j > \overline{A}(P^{m*}, P^{b*})$ could borrow without monitoring from banks if the platform did not offer credit. However, once the platform enters the credit market and raises P^m, they cannot borrow from banks anymore and turn to the platform. The combination of the price hike and the higher cost of funding leads to a net loss of

$$p_h(1 - F^b(P^{b**}))(P^{m**} - P^{m*}) + \varphi(\mathcal{A}, P^m, P^b).$$

Note that this loss is not just a transfer from merchants to the platform: it entails an additional monitoring cost which is deadweight loss.

Merchants with wealth $\overline{A}(P^{m*}, P^{b*}) > A_j > \underline{A}(P^{m*}, P^{b*})$ move from borrowing from banks with monitoring to borrowing from the platform. They then face a higher price but also subsidized funding which add up to a strictly positive net gain:

$$-p_h(1 - F^b(P^{b**}))(P^{m**} - P^{m*}) - \varphi(\mathcal{A}, P^m, P^b) + \gamma_b] = [\underline{A}(P^{m*}, P^{b*}) - \mathcal{A}^{**}] > 0$$

Finally, merchants with wealth $\underline{A}(P^{m*}, P^{b*}) > A_j > \mathcal{A}^{**}$ who could not get funded without the platform can now borrow and become active and are therefore strictly better off.

On the other side of the platform buyers face the same per-transaction price P^{b*} whether the platform provides credit or not, but because the number of merchants N^m expand, their overall payoff,

$$p_h N^m (V^b - P^{b*}),$$

goes up.

Our analysis also suggests that not only the range of merchants that are funded but also pricing is sensitive to the platform's monitoring efficiency.

Proposition 3. *When the platform becomes more efficient at monitoring (when* γ_p *goes down),*

- *it provides more credit to merchant,* $\overline{A}(P^m, P^b) - \mathcal{A}$ *increases,*
- *it charges a higher fee* P^{m**} *to merchants,*

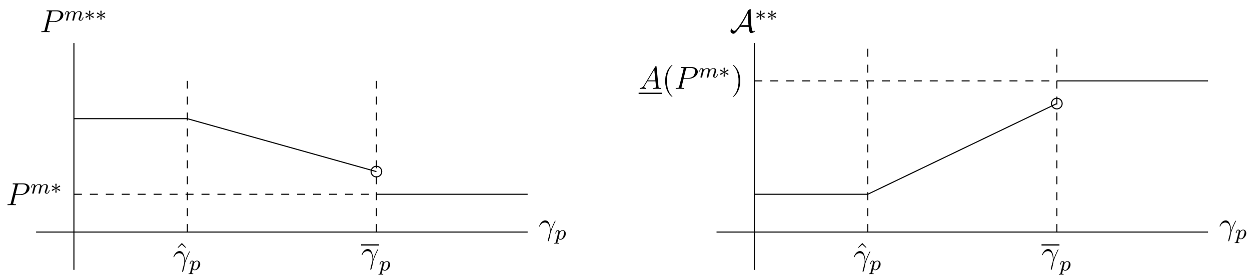

Figure 1: Comparative statics

That the platform provides more credit when its cost of doing so decreases is straightforward. On the pricing side, the intuition can be seen from the platform first-order condition with respect to the price P^m,

$$\frac{\partial \pi}{\partial P^m} = 1 - F^m(\overline{A}(P^m, P^b)) + f^m(\overline{A}(P^m, P^b))(\varphi(\mathcal{A}, P^m, P^b) - \gamma_p).$$

Keeping $\mathcal{A}$ constant, increasing P^m has a benefit that is directly proportional to the mass of unconstrained merchants, $1 - F^m(\overline{A}(P^m, P^b))$, who can borrow from the banking sector and end up paying a higher fee. That benefit is independent from γ_p. On the other hand, increasing P^m has generates a cost $\varphi(\mathcal{A}, P^m, P^b) - \gamma_p$ corresponding to the marginal merchant with $A = \overline{A}(P^m, P^b)$ becoming unable to borrow from the banking system given a higher fee P^m and turning to the platform. That loss $\varphi(\mathcal{A}, P^m, P^b) - \gamma_p$ becomes more severe when γ_p is higher, hence a platform less efficient at monitoring has lower incentives to increase its fee.

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Appendix

Proof of Lemma 1

Suppose the platform offers a contract without monitoring such that $\mathcal{A} < \overline{A}$ and $R = \underline{R}$, where $\overline{A}$ is the minimum merchant's investment required by the bank without monitoring, for given transaction fees P^m and P^b. In that case, the loss incurred by the platform for each merchant accepting the contract is

$$L(\mathcal{A}, P^b, P^m) \equiv \overline{A} - \mathcal{A}.$$

Clearly, all merchants with $A \geq \overline{A}$ accept the platform contract, as they obtain the project's NPV, $p_h N^b(V^m - P^m) - I$, plus $\overline{A} - \mathcal{A}$.

Consider now merchants with $\mathcal{A} \leq A < \overline{A}$. All these merchants are better off accepting the platform contract. We now show that the platform is worse off by providing financing. We need to distinguish two cases.

- If $\mathcal{A} \geq \underline{A}$, where $\underline{A}$ is the minimum merchant's investment required by the bank with monitoring, for given transaction fees P^m and P^b. Then, the platform's contract does not increase the merchant base, so that the platform's profit is strictly lower. To see this, consider the platform's profit:

$$\begin{aligned} \pi &= p_h N^m N^b (P^m + P^b - c) - (1 - F^m(\mathcal{A}))L(\mathcal{A}, P^m, P^b) \\ &= p_h[1 - F^b(P^b)][1 - F^m(\underline{A})](P^m + P^b - c) - (1 - F^m(\mathcal{A}))L(\mathcal{A}, P^m, P^b) \end{aligned} \tag{33}$$

  The first term in (33) is the same as with bank financing, while the second term is strictly decreasing in $\mathcal{A}$.

- If $\mathcal{A} \leq \underline{A}$, then the platform contract increases the number of merchants who can obtain financing. In that case, the platform ends up financing all merchants.

$$\begin{aligned} \pi &= p_h[1 - F^m(\mathcal{A})]N^b(P^m + P^b - c) - (1 - F^m(\mathcal{A}))L(\mathcal{A}, P^m, P^b) \\ &= p_h[1 - F^b(P^b)][1 - F^m(\mathcal{A})](P^m + P^b - c) - (1 - F^m(\mathcal{A}))L(\mathcal{A}, P^m, P^b) \end{aligned} \tag{34}$$

  First order conditions

$$\frac{\partial \pi}{\partial P^m} = p_h[1 - F^b(P^b)][1 - F^m(\mathcal{A})] - [1 - F^m(\mathcal{A})]p_h[1 - F^b(P^b)] = 0 \tag{35}$$

$$\frac{\partial\pi}{\partial\mathcal{A}} = p_h[1-F^b(P^b)](-f^m(\mathcal{A}))(P^m+P^b-c)+f^m(\mathcal{A})L(\mathcal{A},P^m,P^b)+[1-F^m(\mathcal{A})]=0$$
$$\Leftrightarrow P^m = \frac{1-F^m(\mathcal{A})+f^m(\mathcal{A})L(\mathcal{A},P^m,P^b)}{p_h[1-F^b(P^b)]f^m(\mathcal{A})} - P^b + c \tag{36}$$

$$\frac{\partial\pi}{\partial P^b} = 0 \Leftrightarrow P^b = \frac{1-F^b(P^b)}{f^b(P^b)} - V^m + c \tag{37}$$

Equation (35) always holds, thus as long as the relationship between P^m and $\mathcal{A}$ satisfies equation (36), the platform's profit reaches its maximum. Let $\mathcal{A} = \underline{A}^*$, and get the value of P^{m**} from equation (36)

$$P^{m**} = \frac{1-F^m(\underline{A}^*)+f^m(\underline{A}^*)L(\underline{A}^*,P^{m*},P^b)}{p_h[1-F^b(P^b)]f^m(\underline{A}^*)} - P^b + c$$

Rewrite equation (34)

$$\Leftrightarrow \pi = [1-F^m(\underline{A}^*)]\frac{1-F^m(\underline{A}^*)}{f^m(\underline{A}^*)}$$

which is the same as the platform's profit under the bank financing case (i.e. equation (14)). Since the profit doesn't increase, the platform has no incentive to provide funding.

□

Proof of Lemma 2

Suppose the platform offers a contract $\mathcal{C}^p = (R^p, \mathcal{A})$ where R^p is the merchant repayment in case of success and $\mathcal{A}$ is the minimum investment from the merchant. The platform can only benefit from offering $\mathcal{C}^p$ if the contract is more attractive to some merchants than the contract $\mathcal{C}^b = (R^b, A^*(P^m, P^b))$ that banks offer:

$$\begin{aligned}
& p_h(N^b(V^m-P^m)-R^b) - \underline{A}(P^m,P^b) < p_h(N^b(V^m-P^m)-R^p) - \mathcal{A} \\
\Leftrightarrow\ & p_hR^P + \mathcal{A} \le p_hR^b + \underline{A}(P^m,P^b) \\
\Leftrightarrow\ & p_hR^P + \mathcal{A} \le I + \gamma_b
\end{aligned} \tag{38}$$

where the last inequality follows from banks breaking even. If it is optimal for the platform to set $\mathcal{A} > \underline{A}(P^m, P^b)$, then the platform should maximize its revenue from financial contracts since offering funding does not affect the mass of merchant that join the platform, $1 - F(\underline{A}(P^m, P^b)$, therefore does not affect the platform's revenues from charging fees (P^m, P^b). It follows that (38) is binding, i.e., the platform's revenu from offering $\mathcal{C}^p$ is then

$$[F(\overline{A}(P^m,P^b)) - F(\mathcal{A})](p_hR^P - (I-\mathcal{A}) - \gamma_p) = [F(\overline{A}(P^m,P^b)) - F(\mathcal{A})](\gamma_b - \gamma_p).$$

Therefore if $\gamma_b \leq \gamma_p$, offering a financial contract with $\mathcal{A} > \underline{A}(P^m, P^b)$ does not improve the platform's payoff, and if $\gamma_b > \gamma_p$, $\mathcal{A} > \underline{A}(P^m, P^b)$ is strictly dominated by $\mathcal{A} = \underline{A}(P^m, P^b)$. □

Proof of Lemma 3

Suppose the platform offers a contract such that $\varphi(\mathcal{A}, P^m, P^b) < 0$. The platform overall profit is then

$$\begin{aligned} & [1 - F^m(\mathcal{A})] \left[(1 - F^b(P^b))p_h(P^m + P^b - c) + \varphi(\mathcal{A}, P^m, P^b) - \gamma_p\right] && (39) \\ = \; & [1 - F^m(\mathcal{A})] \left[(1 - F^b(P^b))p_h(V^m + P^b - c) - p_h \frac{b}{\Delta p} - (I - \mathcal{A}) - \gamma_p\right] && (40) \end{aligned}$$

Note (40) does not depend on P^m and consider two cases. First, suppose $\varphi(\mathcal{A}, 0, P^b) < 0$. Then the platform strategy is akin to charging 0 to merchants and getting a strictly negative profit from providing credit which cannot be optimal. Second, suppose $\varphi(\mathcal{A}, 0, P^b) \geq 0$. Then since $\varphi(\mathcal{A}, ., P^b)$ is decreasing, there exists $P^{m'} < P^m$ such that $\varphi(\mathcal{A}, P^{m'}, P^b) = 0$ and (40) (and therefore (39)) is unchanged. But given $(\mathcal{A}, P^{m'}, P^b)$ merchants with $A > \overline{A}(P^{m'}, P^b)$ borrow from banks, which yields a profit for the platform equal to

$$\begin{aligned} & [1 - F^m(\mathcal{A})](1 - F^b(P^b))p_h(P^{m'} + P^b - c) - [F^m(\overline{A}(P^{m'}, P^b)) - F^m(\mathcal{A})]\gamma_p \\ > \; & [1 - F^m(\mathcal{A})] \left[(1 - F^b(P^b))p_h(P^{m'} + P^b - c) - \gamma_p\right] \\ = \; & [1 - F^m(\mathcal{A})] \left[(1 - F^b(P^b))p_h(P^m + P^b - c) + \varphi(\mathcal{A}, P^m, P^b) - \gamma_p\right], \end{aligned}$$

where the last expression is (39). This shows $\varphi(\mathcal{A}, P^m, P^b) < 0$ cannot be optimal for the platform. □

Proof of Proposition 2

As mentioned in the main text, we assume A^{max} is large enough that we get interior solutions for $\mathcal{A}$ and P^m. Specifically,

$$A^{max} \geq \max\{\underline{A}_1^{max}, \underline{A}_2^{max}, \underline{A}_3^{max}, \underline{A}_4^{max}\}$$

where $\underline{A}_1^{max}$, $\underline{A}_2^{max}$, $\underline{A}_3^{max}$, $\underline{A}_4^{max}$ are defined as follows:

$$\underline{A}_1^{max} \equiv p_h \frac{b}{\Delta_p} + \gamma_b - \left(p_h[1 - F^b(P^{b*})](V^m - P^{b*} + c) - I\right), \tag{41}$$

$$\underline{A}_2^{max} \equiv -p_h \frac{b}{\Delta_p} + p_h[1 - F^b(P^{b*})](V^m + P^{b*} - c) - I, \tag{42}$$

$$\underline{A}_3^{max} \equiv -p_h \frac{B+b}{\Delta_p} - \gamma_p + 2p_h[1 - F^b(P^{b*})](V^m + P^{b*} - c) - 2I, \tag{43}$$

$$\underline{A}_4^{max} \equiv p_h \frac{2B-b}{\Delta_p} - \gamma_b - \left(p_h[1 - F^b(P^{b*})](V^m + P^{b*} - c) - I\right), \tag{44}$$

We already know from Section 4, Corollary 1 that pricing on the buyers' side does not change, i.e., the platform charges $P^{b**} = P^{b*}$ where P^{b*} is the unique solution to

$$\frac{1 - F^b(P^b)}{f^b(P^b)} = V^m + P^b - c. \tag{45}$$

Consider next the optimization program (29) and ignore constraints (31) and (32) for the moment. First-order conditions with respect to P^m and $\mathcal{A}$ are respectively

$$\frac{p_h(1 - F^b(P^b))}{A^{max}}[A^{max} - \overline{A}(P^m, P^b) + \varphi(\mathcal{A}, P^m, P^b) - \gamma_p] + \lambda p_h[1 - F(P^b)] = 0, \tag{46}$$

$$-\frac{1}{A^{max}}[p_h(1 - F^b(P^b))(P^m + P^b - c) + \varphi(\mathcal{A}, P^m, P^b) - \gamma_p - \overline{A}(P^m, P^b) + \mathcal{A}] - \lambda = 0, \tag{47}$$

where λ is the Lagrange multiplier associated with constraint (30).

Note that second-order condition are satisfied,

$$\frac{\partial^2 \hat{\pi}}{\partial P^{m2}} = -2\frac{p_h(1 - F^b(P^{b**}))}{A^{max}}\left(p_h(1 - F^b(P^{b**}))\right) < 0,$$

and

$$\frac{\partial^2 \hat{\pi}}{\partial \mathcal{A}^2} = -\frac{2}{A^{max}} < 0,$$

i.e., $\hat{\pi}(., P^{b**}, .)$ is strictly concave.

We then delineate two cases that depend on a threshold

$$\hat{\gamma}_p \equiv \frac{p_h}{2}\left[(1 - F^b(P^{b**}))(V^m + P^{b**} - c) - \frac{2B-b}{\Delta_p} - \frac{I - A^{max}}{p_h}\right] \geq 0,$$

where the positivity of $\hat{\gamma}_p$ follows from (44).

Case 1: The platform monitoring cost γ_p is large: $\gamma_p \geq \hat{\gamma}_p$

We first show that if $\gamma_p \geq \hat{\gamma}_p$, then (30) is not binding. To see this note that if (30) does not bind, solutions to the platform's optimization problem are given by (46) and (47) with $\lambda = 0$, which yields

$$P^{m**} = \frac{1}{3}(2V^m - P^{b**} + c) - \frac{1}{3p_h(1 - F^b(P^{b**}))}\left(p_h \frac{B+b}{\Delta_p} + \gamma_p + 2I - 2A^{max}\right), \tag{48}$$

$$\mathcal{A}^{**} = A^{max} - \frac{p_h}{3}\left[2(1 - F^b(P^{b**}))(V^m + P^{b**} - c) - \left(\frac{B+b}{\Delta_p} + \frac{\gamma_p}{p_h} + 2\frac{I - A^{max}}{p_h}\right)\right]. \tag{49}$$

Pluging these expressions into (15) yields

$$\varphi(\mathcal{A}, P^m, P^{b*}) = \frac{2}{3}(\gamma_p - \hat{\gamma}_p), \tag{50}$$

which is positive if $\gamma_p \geq \hat{\gamma}_p$. That is, the solutions to the unconstrained optimization problem satisfy (30), which is therefore not binding.

Next, combine (26), (48) and (50) to show

$$P^{m**} - P^{m*} = \frac{1}{2}(1 - F^b(P^{b**}))(\gamma_b - \varphi(\mathcal{A}, P^{m**}, P^{b*})). \tag{51}$$

Similarly, combine (27), (49) and (50) to show

$$\underline{A}(P^{m*}, P^{b*}) - \mathcal{A}^{**} = \frac{1}{2}(\gamma_b - \varphi(\mathcal{A}, P^{m**}, P^{b*})). \tag{52}$$

Suppose $\gamma_p = \frac{3}{2}\gamma_b + \hat{\gamma}_p > \gamma_b$, then $\varphi(\mathcal{A}, P^m, P^{b*}) = \gamma_b$ and

$$\hat{\pi}(P^{m**}, P^{b**}, \mathcal{A}^{**})\big|_{\gamma_p = \frac{3}{2}\gamma_b + \hat{\gamma}_p} = \pi^* - [\overline{A}(P^{m**}, P^{b**}) - \underline{A}(P^{m**}, P^{b**})](\gamma_p - \gamma_b) < \pi^*.$$

Suppose $\gamma_p = \gamma_b$, then $\varphi(\mathcal{A}, P^m, P^{b*}) = \frac{2}{3}(\gamma_b - \hat{\gamma}_p) < \gamma_b$. Furthermore, since $\hat{\pi}(., P^{b**}, .)$ reaches a maximum at $\hat{\pi}(P^{m**}, P^{b**}, \mathcal{A}^{**})$, $P^{m**} > P^{m*}$ and $\mathcal{A}^{**} < \underline{A}(P^{m*}, P^{b*})$, the strict concavity of $\hat{\pi}(., P^{b**}, .)$ implies

$$\hat{\pi}(P^{m**}, P^{b**}, \mathcal{A}^{**})\big|_{\gamma_p = \gamma_b} > \hat{\pi}(P^{m*}, P^{b**}, \underline{A}(P^{m*}, P^{b*})\big|_{\gamma_p = \gamma_b} = \pi^*$$

Finally, using the envelope theorem,

$$\frac{\partial \hat{\pi}(P^{m**}, P^{b**}, \mathcal{A}^{**})}{\partial \gamma_p} = \mathcal{A}^{**} - \overline{A}(P^{m**}, P^{b**}) < 0.$$

It follows there is a unique $\overline{\gamma}_p$ such that

$$\hat{\pi}(P^{m**}, P^{b**}, \mathcal{A}^{**})\big|_{\gamma_p=\overline{\gamma}_p} = \pi^*,$$

and $\gamma_b < \overline{\gamma}_p < \frac{3}{2}\gamma_b + \hat{\gamma}_p$, which implies (31) never binds. Therefore if $\gamma_p > \overline{\gamma}_p$, the optimization problem has no solution, i.e., the platform gives up financing. If $\gamma_p \leq \overline{\gamma}_p$, (31) does not bind, the optimum is given by (48) and (49), and (51) and (52) imply $P^{m**} > P^{m*}$ and $\mathcal{A}^{**} < \underline{A}(P^{m*}, P^{b*})$.

Case 2: The platform monitoring cost γ_p is small: $\gamma_p < \hat{\gamma}_p$

Then (30) is binding. It follows that $\varphi(\mathcal{A}, P^m, P^b) = 0$ in equilibrium. Using (46) and (47) with $\varphi(\mathcal{A}, P^m, P^b) = 0$, we get

$$A^{max} - \mathcal{A} = p_h(1 - F^b(P^b))(P^m + P^b - c),$$

$$p_h(1 - F^b(P^b))(V^m - P^m) - p_h\frac{b}{\Delta_p} - (I - \mathcal{A}) = 0,$$

which yields

$$P^{m**} = \frac{1}{2}(V^m - P^{b**} + c) - \frac{1}{2(1 - F^b(P^{b**}))}\left(\frac{b}{\Delta_p} + \frac{I - A^{max}}{p_h}\right) > P^{m*} \tag{53}$$

$$\mathcal{A}^{**} = I - \frac{p_h}{2}\left[(1 - F^b(P^{b**}))(V^m + P^{b*} - c) - \frac{b}{\Delta_p} + \frac{I - A^{max}}{p_h}\right] < \underline{A}(P^{m*}, P^{b*}) \tag{54}$$

□