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Abstract

In March 2020, safe asset markets experienced surprising and unprecedented price crashes. We explain how strategic investor behavior can create such market fragility in a model with investors valuing safety, investors valuing liquidity, and constrained dealers. While safety investors and liquidity investors can interact symbiotically with offsetting trades in times of stress, liquidity investors' strategic interaction harbors the potential for self-fulfilling fragility. When the market is fragile, standard flight-to-safety can have a destabilizing effect and trigger a "dash-for-cash" by liquidity investors. Well-designed policy interventions can reduce market fragility ex ante and restore orderly functioning ex post.

Keywords: safe assets, liquidity shocks, global games, treasury securities, COVID-19

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1 Introduction

In March 2020, the market for U.S. Treasuries behaved precisely the opposite of how economists expect it to behave during a crisis: investors flooded the market with an unprecedented level of sales, and the market experienced sudden and significant price drops. This extraordinary behavior of investors and prices requires a new model for safe asset markets. We show in a model of regime shifts that a safe asset market functions as expected as long as the market is sufficiently deep. However, under certain conditions the market can break down, with investors rushing to sell and prices falling precipitously, if trade imbalances have to be absorbed by dealers that are subject to balance sheet constraints. Surprisingly, an increase in the demand for safe assets from a standard flight-to-safety can be destabilizing: When the market is relatively fragile, the flight-to-safety among certain investors can trigger the dash-for-cash among other investors. Our model helps understand the unprecedented events in March 2020 and highlights the risks of such events repeating in the future in safe asset markets more broadly.

Our analysis is motivated by the following facts. First, the prices of Treasuries suddenly collapsed in mid-March 2020, in sharp contrast to previous crisis episodes (Panel A of Figure 1). Until the beginning of March, Treasury prices did increase and the S&P 500 decreased with the gradual realization of the severity of the COVID-19 outbreak, consistent with the usual negative correlation between safe and risky assets during a flight-to-safety episode (Nagel, 2016; Adrian, Crump, and Vogt, 2019). However, starting the week of March 9, prices of Treasury notes and bonds declined together with stock prices as investors moved into cash or ultra-short maturity Treasury bills (i.e. the equivalent of cash). It is well-documented, that dealer balance sheet constraints played an important role in the Treasury price declines (He, Nagel, and Song, 2022; Duffie et al., 2023). Panel B of Figure 1 illustrates how dealer balance sheets were filling up with Treasuries through both the run-up in Treasury prices and their crash, and the recovery of Treasury prices after March 18 coincided with the receding of dealer balance sheet pressure as purchases by the Federal Reserve ramped up.

However, the existing literature takes as given the Treasury sales in March 2020 and does not address why this episode featured sales so large that they reversed the typical appreciation of safe assets during times of stress and required Fed intervention to "support the smooth functioning of markets" (FOMC statement on March 15).[1] Panel C of Figure 1 illustrates that the dash-for-cash featured Treasury sales on a historically unprecedented

[1] The statement by the Fed's Federal Open Market Committee (FOMC) on March 15 is available at https://www.federalreserve.gov/newsevents/pressreleases/monetary20200315a.htm

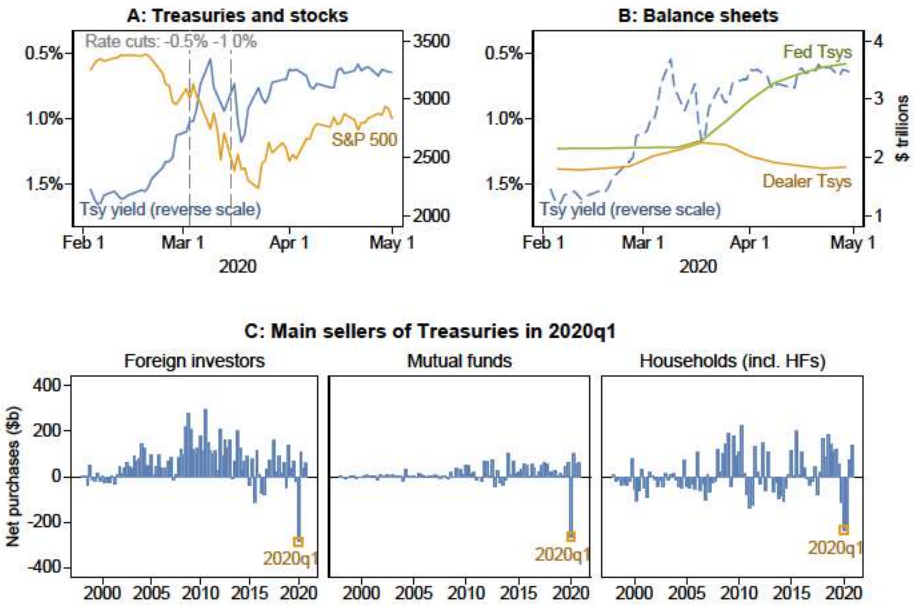

Figure 1: The market for U.S. Treasuries in early 2020. Panel A: market yield on 10-year Treasuries, Federal Reserve rate cuts, and S&P 500 index. Panel B: Treasuries on the balance sheets of the Federal Reserve (outright holdings) and of Primary Dealers (net positions and gross reverse repo). Panel C: net purchases of Treasuries. For details on the data see Appendix A.

scale: Foreign investors and mutual funds, the main sellers of Treasuries in 2020q1, each sold roughly $250 billion, an order of magnitude more than in any previous quarter. In the post-2008 period these sales equal three standard deviations for foreign investors and five standard deviations for mutual funds.[2] Sales of such magnitude appear inconsistent with a smooth response to continuously varying underlying fundamentals, suggesting the need for a model of regime shifts.

Finally, a significant part of these sales appear to have been preemptive, i.e. not due to genuine liquidity needs. Foreign official agencies (a subset of foreign investors) sold $196 billion of Treasury bonds but reduced their total U.S. Dollar assets by only $48 billion, suggesting that 76% of their sales were preemptive.[3] Among mutual funds, those in the CRSP dataset sold $157 billion of Treasuries but used only $103 billion to satisfy outflows, suggesting that 34% of their sales were preemptive.[4] Consistent with this evidence, the

[2]While hedge funds unwinding the cash-futures basis trade were also net sellers of Treasuries (Barth and Kahn, 2021), panel C of Figure 1 shows that their sales (included in the "household" sector in the Financial Accounts) were more in line with historical experience.

[3]From the international transactions data of the Bureau of Economic Analysis, Table 9.1, International Financial Transactions for Liabilities to Foreign Official Agencies. Reported similarly in Panel B of Table 9 in Vissing-Jørgensen (2021). See Weiss (2022) for additional detail on foreign sales of Treasuries in 2020.

[4]From Table 6 in Vissing-Jørgensen (2021).

Inter-Agency Working Group for Treasury Market Surveillance (2021) reports that "some Treasury holders appeared to react to the decline in market liquidity by selling securities for precautionary reasons lest conditions worsen further, and these sales only added to the stress on the market."

In sum, investors sold safe assets on an unprecedented scale for what appear to be preemptive reasons. In the language of Haddad, Moreira, and Muir (2021), "selling became viral," with investors selling safe assets — whether Treasuries or investment grade corporate bonds — akin to depositors running on a bank, leading to more severe dislocations for safer and more liquid assets in a reversal of the usual liquidity hierarchy.

In contrast to the existing literature showing how dealer constraints can lead to price dislocations given exogenous sales from investors (e.g. He, Nagel, and Song, 2022), we show how dealer constraints can endogenously induce certain investors to sell, and especially so when there is concurrent flight-to-safety demand from other investors. Our model can generate strategic complementarities leading to regime shifts in which continuous changes in fundamentals trigger discontinuous jumps in investor behavior and, therefore, a sudden precipitous drop in equilibrium prices. Using standard global game techniques, we uniquely link the market outcome to fundamentals including the degree of liquidity risk, the strength of flight-to-safety demand, and the severity of dealer constraints.

Our model captures the key characteristics of safe assets — safety and liquidity — as well as the central role of constrained dealers to intermediate trade and absorb imbalances. First, safe assets in practice are *safe* in the sense that they will pay par at maturity with very high probability so investors hold them as a store of value, useful for diversification and intertemporal smoothing (e.g. Caballero and Farhi, 2017). In our model, such "safety investors" hold the safe asset in a portfolio together with a risky asset. In times of stress, when fundamentals worsen for the risky asset, these investors rebalance their portfolio to demand more of the safe asset (equivalently, markets reprice the value of safe assets to reflect fundamentals, even in the absence of large trade volume). Such flight-to-safety has been the focus of most existing analyses of safe assets in times of stress.

Second, safe assets in practice are *liquid*, meaning that, typically, they can be easily sold when in need of cash and therefore trade at a convenience yield (e.g. Krishnamurthy and Vissing-Jørgensen, 2012). In our model, there are "liquidity investors" who are subject to liquidity shocks (i.e. immediate consumption needs) and therefore hold the safe asset as liquidity insurance. When hit by the liquidity shock, these investors sell the safe asset in order to consume. Importantly, even in times of stress, not all liquidity investors suffer liquidity shocks. This leaves a group of liquidity investors without genuine liquidity needs

who act strategically when deciding whether to sell their assets in the current environment, or whether to hold on and face the risk of a liquidity shock in the near future. An individual investor may sell preemptively if they expect worse market conditions in the future and if their likelihood of having to sell in the future is sufficiently high.

In addition, our model features dealers who buy and sell the safe asset and whose main role is to intermediate over time. Dealers are competitive but subject to balance sheet constraints such as the Supplemental Leverage Ratio rule (SLR), and therefore provide an elastic residual demand for the safe asset. Because dealers' demand in the future is affected by inventory they take on today, they provide an intertemporal link between prices in different periods. In particular, dealers' competitive behavior to bid for a large net supply today means that their bids in the future will necessarily be lower in equilibrium.

With these ingredients, our model yields two main results. The first result is that the liquidity insurance role of safe assets, together with dealer balance sheet constraints, implies that a safe asset market can be *fragile*, featuring sudden regime changes. For low liquidity risk (low probability of facing a liquidity shock), a strategic liquidity investor never finds it optimal to sell preemptively, irrespective of what other investors are doing; the only equilibrium in this case is for all strategic investors to hold on to the safe asset such that the only investors selling are those with a genuine liquidity need. For high liquidity risk, the opposite is true: an individual investor finds it dominant to sell preemptively such that the only equilibrium is for all liquidity investors to sell. In this case, the safe asset market is flooded with sales, including by investors who do not actually have liquidity needs — a "market run." The stability of the market is represented by the global game threshold for liquidity risk around which the equilibrium switches from "hold" to "run" with a higher threshold representing a more stable or, equivalently, less fragile market. The severity of dealer balance sheet costs has two effects on the market. First, higher balance sheet costs increase market fragility such that a market run already occurs for lower liquidity risk. Second, higher balance sheet costs increase the magnitude of the price crash conditional on the run occurring.

Our second and key result is that the safety and liquidity roles of safe assets can interact in such a way that a flight-to-safety can *worsen* fragility, making a dash-for-cash more likely. Recall that in typical models of fire sales, the crucial friction is slow-moving capital: prices can be depressed because potential buyers cannot enter the market to purchase distressed assets. Thus, in these situations, new buyers entering the market would mitigate fire sales and stabilize asset prices. In contrast, we find that the entry of new capital to purchase safe assets can actually amplify fire sales.

How can this occur? In our model, safety investors form a natural partnership with liq-

uidity investors as their trades offset during stress episodes. Demand from safety investors absorbs sales from liquidity investors; all else equal, this leads to higher prices for safe assets than would otherwise occur. However, the timing of safety investor demand is key, as it affects the intertemporal tradeoff of strategic liquidity investors. Safety investor demand early on in a stress episode has an ambiguous effect on fragility as it increases prices both contemporaneously (which is destabilizing) and in the future by relaxing dealer balance sheets (which is stabilizing). Additional demand from safety investors today can *induce* liquidity investors to sell today, precisely because the market today has relatively high capacity to absorb sales; but higher prices in the future imply that being forced to sell in the future is less costly, and this is stabilizing.

Which effect dominates — and therefore whether a flight-to-safety can trigger a dash-for-cash — is ambiguous and depends on the inherent fragility of the market. In a relatively stable market, strategic liquidity investors will sell preemptively only if liquidity risk is very high (i.e. only if they are likely to be forced to sell in the future). In that case, the stabilizing effect of the flight-to-safety dominates: the investors weight more their concern about being forced to sell in the future but the flight-to-safety has relaxed dealer balance sheets and increased the price the investors would face in the future. In a relatively fragile market, however, strategic liquidity investors sell preemptively even when liquidity risk is low (i.e. even when they are unlikely to be forced to sell in the future). In this case, the destabilizing effect of the flight-to-safety dominates: the investors put a greater weight on the ability to sell assets at a higher price today even though the fire-sale price tomorrow is also less severe. This means that safety investors have an *amplification effect* on market fragility: When the market is already relatively stable, they stabilize it further (flight-to-safety prevents a dash-for-cash); but if the market is already relatively fragile, they destabilize it even more (flight-to-safety triggers a dash-for-cash). Whether flight-to-safety will trigger a dash-for-cash is unclear unconditionally, but the answer is clear conditional on the degree of market fragility.

The behavior of Treasury markets in March 2020 is particularly striking in contrast to the great financial crisis of 2007–2009 (GFC), during which Treasuries rallied. Our model helps to understand the differences between these two episodes that led to such dramatically different outcomes. First, our model highlights the central role of dealer balance sheet constraints, which are a potentially unintended result of post-GFC regulations, such as the SLR. During the GFC, dealers' activities in Treasury markets were relatively unconstrained and thus investors did not worry about dealers running out of balance sheet space and Treasury prices collapsing. Second, the size of the liquidity shock during the COVID-19 crisis appears to have been much larger than during the GFC. As our analysis shows,

very large increases in liquidity risk and flight-to-safety can tilt the system into a region in which investors sell preemptively. Because the GFC did not feature dealers constrained by balance sheet costs, and because the shock to liquidity needs was arguably smaller, the Treasury market remained in the relatively stable region in which flight-to-safety prevents a dash-for-cash, which is why the market behaved as usual despite the tremendous stress in the financial sector. In contrast, in March 2020, the liquidity shock was larger and dealers were more constrained, so much so that the Treasury market suffered a regime change, and flight-to-safety triggered a dash-for-cash. In sum, our analysis suggests that these two episodes did not feature fundamentally different shocks or shocks of different direction, but rather shocks that differed in degree within different regulatory environments.

Our analysis has policy implications, in particular for asset purchase facilities, dealer balance sheet regulation, and market structure. Fragility in our model hinges on the intertemporal considerations of strategic liquidity investors who compare prices today to prices in the near future. In general, there is scope for policy interventions that increase prices both in the present and in the future. However, due to the intertemporal considerations and the coordination effects, the timing of policy interventions is important and announcements can have large effects well before the interventions are executed. We show that an asset purchase facility can have a large effect upon announcement, even if it does not become active until a future date, by shifting strategic investors from the run equilibrium to the hold equilibrium, consistent with the evidence of Haddad, Moreira, and Muir (2021). Similarly, policy interventions that relax dealer balance sheet constraints can be stabilizing. However, because the strategic incentive to sell is caused by fear of low prices in the future, effective policy has to relax balance sheet constraints in the future as well.

Finally, our model shows that markets where trading occurs in a decentralized, sequential way and where dealers play a large role intermediating flow imbalances over time harbor the potential for fragility. These elements generate a strategic tradeoff where an investor can hope to receive the average in-run price when selling preemptively but has to worry about bearing the full impact of dealer inventory when being forced to sell in the future. Changes to market structure that lead to more pooling of trades and that reduce the role of dealers as a bottleneck for trade flow can therefore reduce the fragility of safe asset markets. The growth of the Treasury market since the GFC has greatly outpaced the capacity of dealers' balance sheets, and that trend is expected to continue (Duffie, 2020). The strategic mechanism in our model will therefore become increasingly relevant unless balance sheet constraints are relaxed. Episodes like March 2020 are likely to become more frequent as dash-for-cash motivations become more pronounced.

After discussing related literature, the rest of the paper proceeds as follows. In Section 2, we present and analyze the baseline model of the strategic interaction among liquidity investors. In Section 3, we add safety investors and derive the ambiguous effect of flight-to-safety on market fragility. We then discuss policy implications in Section 4 and conclude in Section 5.

Related Literature. The market turmoil in the spring of 2020 has been documented in detail by Vissing-Jørgensen (2021) and He, Nagel, and Song (2022) for Treasuries, and by Haddad, Moreira, and Muir (2021) and Boyarchenko, Kovner, and Shachar (2022) for corporate bonds.[5] In particular, Duffie et al. (2023) show that the typically linear relation between yield volatility and Treasury market liquidity broke down in March 2020 and that the residuals are well explained by the shadow cost of dealer balance sheets.

In a literature that focuses on empirically studying the events, He, Nagel, and Song (2022) stand out as also providing a formal theoretical analysis to understand the implications. Using a model based on Greenwood and Vayanos (2014) but incorporating frictions between dealers and hedge funds, they illustrate how large net sales can generate an "inconvenience yield" for Treasuries. Specifically, He, Nagel, and Song (2022) show that, given large exogenous sales, the presence of regulatory constraints can lead to pricing distortions measured as the spread between Treasuries and overnight-index swap rates, as well as spreads between dealers' reverse repo and repo rates. Importantly, He, Nagel, and Song (2022) take net flows as given and consider in detail the equilibrium pricing consequences. In contrast, our paper shows how, in a strategic environment, the same regulatory constraints can lead to run behavior, thus endogenizing the large net flows. Our focus is on the determinants of large net sales of safe assets during a crisis — the unusual behavior not typically observed — and on the policy implications that can be derived in such a model of regime change.

In contrast to market runs, bank runs have received much greater attention because of the common pool problem inherent with liquidity transformation (e.g., Diamond and Dybvig, 1983 and Goldstein and Pauzner, 2005). In the case of a market run, there is no common pool threatened by illiquidity. The seminal papers on market runs by Bernardo and Welch (2004) and Morris and Shin (2004) highlight how market frictions can cre-

[5]See also D'Amico, Kurakula, and Lee (2020), Fleming et al. (2021), Nozawa and Qiu (2021), Aramonte, Schrimpf, and Shin (2022), and Haughwout, Hyman, and Shachar (2022). For detailed analysis of market liquidity conditions, see Fleming and Ruela (2020), Kargar et al. (2021), O'Hara and Zhou (2021). Ahmed and Rebucci (2022) find sizable estimates of the price impact of foreign officials' sales of Treasuries. The role of mutual funds in particular as large sellers of safe assets has been studied by Falato, Goldstein, and Hortaçsu (2021) and Ma, Xiao, and Zeng (2022). On the role of hedge funds, see e.g. Barth and Kahn (2021).

ate incentives to front-run other investors by selling assets preemptively. Bernardo and Welch (2004) introduce the intertemporal tradeoff our model relies on, but their model does not feature strategic complementarities and therefore cannot generate regime shifts. Our model with strategic complementarities can generate regime shifts and allows for continuous comparative statics in the analysis of flight-to-safety demand and policy implications.[6] Morris and Shin (2004) consider a static model in which strategic complementarities arise because investors have "stop-loss rules" and will be forced to liquidate if prices fall sufficiently low. The preponderance of sales in March 2020 were from investors subject to liquidity shocks, suggesting that a stop-loss mechanism did not drive preemptive sales during this episode. Allen, Morris, and Shin (2006) show how higher-order beliefs can generate "beauty contests" à la Keynes (1936) in asset markets with short-lived investors and imperfect information.

The literature on safe assets is large; see e.g. Gorton (2017) for an overview. Krishnamurthy and Vissing-Jørgensen (2012) show that Treasuries are valued both for their safety and their liquidity by documenting yield spreads both with respect to assets similarly liquid but not safe and assets similarly safe but not liquid (see also Duffee, 1998, Longstaff, 2004, and Greenwood and Vayanos, 2010, 2014). Caballero and Farhi (2017) consider a model where the "specialness" of public debt is its safety during bad aggregate states and where safe assets have "negative beta," as they tend to appreciate in times of aggregate market downturns, providing investors diversification against aggregate macroeconomic risks (see also Maggiori, 2017, Adrian, Crump, and Vogt, 2019, and Brunnermeier, Merkel, and Sannikov, 2022). Safe assets valued for their safety appear in a model of limited participation and risk sharing in Gomes and Michaelides (2007) and through special investors who need safe assets to match liability cash flows in Greenwood and Vayanos (2010). More generally, Treasury bonds have had negative beta over longer horizons in recent decades, rising in price when stock prices fall apart from market turmoil (Baele et al., 2019; Campbell, Sunderam, and Viceira, 2017; Cieslak and Vissing-Jørgensen, 2020). Our paper focuses on the correlation in times of crisis or market turmoil in which the typical correlation (flight-to-safety) has been otherwise clear (Nagel, 2016; Adrian et al., 2019).

Safe assets' liquidity is intimately linked to their safety: when payoffs are (nearly) risk free, assets are information-insensitive and thus easily traded "no questions asked" (Gorton and Pennacchi, 1990, Holmström, 2015, Dang, Gorton, and Holmström, 2015). Holmström and Tirole (1998) model the use of safe assets as a store of value and as insurance against liquidity shocks. Safe assets valued for their liquidity appear in Vayanos and Vila

[6] We discuss the differences between our model and Bernardo and Welch (2004) in Appendix B

(1999) and Rocheteau (2011) as well as in the monetarist literature surveyed by Lagos, Rocheteau, and Wright (2017). The premium for moneyness has been studied empirically, e.g. by Greenwood, Hanson, and Stein (2015), Carlson et al. (2016), and Cipriani and La Spada (2021) (see also Nagel, 2016, and d'Avernas and Vandeweyer, 2021).

Gorton and Ordoñez (2022) study the interaction of public and private provision of safe assets used as store of value and as collateral (see also Holmström and Tirole, 2011, Stein, 2012, Gorton, Lewellen, and Metrick, 2012, Sunderam, 2014 and Krishnamurthy and Vissing-Jørgensen, 2015). Caballero and Krishnamurthy (2008) study flight to quality episodes triggered by uncertainty shocks. He, Krishnamurthy, and Milbradt (2019) study the roles of strategic complementarities and substitutes among investors in determining which asset becomes the safe asset via coordination (see also Farhi and Maggiori, 2017). For recent empirical analysis of safe assets, both current and historical, see Chen et al. (2022) and Choi, Kirpalani, and Perez (2022).

The role of dealers and slow-moving capital more generally in short-term price dislocations is introduced, e.g. in Duffie (2010). Fontaine and Garcia (2012) and Hu, Pan, and Wang (2013) show the effects on liquidity in Treasury markets (see also Vayanos and Vila, 2021). Adrian, Boyarchenko, and Shachar (2017) specifically consider the effects of dealer balance sheet constraints on bond market liquidity. Goldberg and Nozawa (2021) show that dealer inventory capacity is a key driver of liquidity in corporate bond markets (see also Bruche and Kuong, 2021).

2 Baseline Model

The model is set in two periods $t = 0, 1$ and has three types of agents and two types of assets, a safe asset and a risky asset. The safe asset, which is the focus of the analysis, has a fundamental value of 1 and is traded among the agents in both periods. Among the agents, there are risk-averse investors who hold portfolios of the safe asset and the risky asset ("safety investors"), risk-neutral investors who hold the safe asset as protection against liquidity shocks ("liquidity investors"), and risk-neutral dealers who participate in the safe asset market and are subject to balance sheet costs. All agents have a discount rate of zero and act competitively, and there is a measure one of each type. All asset prices are determined in equilibrium. We defer discussion of the safety investors until Section 3.

2.1 Liquidity Investors and their Strategic Interaction

Liquidity investors start out holding one unit of the safe asset and are subject to i.i.d. liquidity shocks, i.e. preference shocks in the style of Diamond and Dybvig (1983), in both periods. If a liquidity investor is hit by the shock, they need to consume immediately and sell their entire holdings of the safe asset. The probability of a liquidity shock at date 0 is $s \in (0,1)$ so, by the law of large numbers, a fraction s of liquidity investors are forced to sell at date 0 at price p_0. Among the remaining fraction $1-s$, each investor has to decide whether to also sell at date 0, receiving p_0 for sure, or to hold on to the safe asset and face liquidity risk at date 1, again with probability s.[7] Investors who hold on to the safe asset at date 0 and then suffer a liquidity shock at date 1 are forced to sell at price p_1. Investors who don't suffer a shock at either date receive a continuation value $v > 1$ akin to a "convenience yield" that reflects the benefit of the safe asset as a liquid store of value for future investment opportunities (e.g., Holmström and Tirole, 1998, 2001).[8]

The liquidity shock probability s is drawn at the beginning of date 0 from a distribution F on $(0,1)$. There is imperfect information about s, and each individual investor i observes an idiosyncratic signal $\widehat{s}_i = s + \sigma_\varepsilon \varepsilon_i$, where the mean-zero signal noise ε_i is i.i.d. across all i with distribution G_ε and $\sigma_\varepsilon > 0$. We focus on the limit of vanishing signal noise, $\sigma_\varepsilon \to 0$, and therefore treat s as non-random in the exposition except when deriving the global game equilibrium.

Examples of real-world liquidity investors we have in mind include foreign official agencies that may face sudden liquidity needs to conduct foreign exchange interventions or mutual funds that may face sudden liquidity needs due to investor withdrawals. Both were among the largest sellers of Treasuries in 2020q1, and their sales were historically unprecedented (Figure 1, Panel C). The consumption good in our model therefore stands in for cash and cash-like instruments, such as bank deposits or short-maturity Treasury bills. Due to its stylized nature, our model cannot not provide a theory of the exact maturity cut-off between short-maturity bills, treated as cash, and longer-maturity notes and bonds, which were not treated as cash during March 2020 and represent the safe asset in our model. While we focus on the strategic interaction among liquidity investors, there are potential additional layers of strategic interaction underlying the liquidity shocks, both in the foreign exchange context (Morris and Shin, 1998) and in the mutual fund context

[7] We can also allow for different liquidity risk s_0 and s_1 at dates 0 and 1, respectively, where s_0 is the global game fundamental that investors observe imperfectly. As this does not meaningfully affect the analysis, we focus on the case of one single liquidity risk s for expositional clarity.

[8] Joslin, Li, and Song (2021) use a conceptually similar model of liquidity investors and show that its comparative statics match well the empirical features of the Treasury liquidity premium.

(Chen, Goldstein, and Jiang, 2010).

Strategic Interaction. Denote by $\lambda \in [0,1]$ the fraction of strategic liquidity investors who decide to sell at date 0. Together with the non-strategic sales s from investors who receive a liquidity shock, total sales of safe assets at date 0 are

$$q_0 = s + (1-s)\,\lambda.$$

At date 1, only the remaining strategic investors who receive a liquidity shock sell their safe assets, resulting in total sales

$$q_1 = s\,(1-s)\,(1-\lambda)\,.$$

Given a fraction λ of strategic investors preemptively sell at date 0, we denote by $p_0^e(\lambda)$ the price an investor expects to receive at date 0 from also selling preemptively and by $p_1^e(\lambda)$ the price the investor expects to receive at date 1 if forced to sell by a liquidity shock (we will derive the relevant expressions for $p_0^e(\lambda)$ and $p_1^e(\lambda)$ in Section 2.2). A strategic liquidity investor compares the payoff from selling early, $p_0^e(\lambda)$, to the expected payoff from holding, $sp_1^e(\lambda) + (1-s)\,v$.

The equilibria of the game among strategic investors are governed by the payoff gain from preemptively selling at date 0:

$$\pi(\lambda) = p_0^e(\lambda) - \left(sp_1^e(\lambda) + (1-s)\,v\right).$$

Under complete information, there are three candidates for Bayesian Nash equilibria:

Hold equilibrium: If the incentive to sell is negative when no other strategic investors sell, that is if $\pi(0) < 0$, then it is a pure-strategy equilibrium for no strategic investors to sell ($\lambda^* = 0$).

Run equilibrium: If the incentive to sell is positive when all other strategic investors sell, that is if $\pi(1) > 0$, then it is a pure-strategy equilibrium for all strategic investors to sell ($\lambda^* = 1$).

Mixed equilibrium: If the incentive to sell is zero when a fraction of strategic investors sell, that is if $\pi(\lambda^*) = 0$ for $\lambda^* \in (0,1)$, then it is a mixed-strategy equilibrium for all strategic investors to sell with probability λ^*.

Recall that safe asset markets typically function smoothly — they are considered the most deep and liquid markets in the world. Thus, any empirically realistic model should include the potential for hold equilibria, even as we seek out a candidate run equilibrium. The hold equilibrium exists if

$$p_0^e(0) < s p_1^e(0) + (1-s)\, v.$$

Since liquidity investors' continuation value v is greater than 1 and the safe asset's fundamental value is 1, the hold equilibrium exists as long as, without any strategic sales, the price at date 1 is not considerably lower than the price at date 0 and liquidity risk at date 1 is sufficiently low. Such conditions are representative of normal times, when not many investors have liquidity needs and expected prices are not very different between date 0 and date 1. In the hold equilibrium, the safe asset market features only those investors selling who have a genuine need for liquidity and dealers taking the net supply into inventory. This is an important difference to the model of Bernardo and Welch (2004), in which a pure-strategy hold equilibrium never exists (Appendix B).

The run equilibrium exists if

$$p_0^e(1) > s p_1^e(1) + (1-s)\, v.$$

In this case, strategic investors prefer to sell early rather than risk having to sell at a worse price in case they suffer a liquidity shock at date 1. Compared to the condition for the hold equilibrium, more is needed for a run equilibrium to exist. First, the expected price at date 1 has to be considerably lower than the price at date 0. Second, liquidity risk at date 1 has to be sufficiently high. The analysis of our paper shows how frictions can lead prices at date 1 to be lower than at date 0 and therefore how a run equilibrium can arise in times of stress.

The identifying feature of a run equilibrium are the preemptive sales by investors who do not face a genuine liquidity need and who therefore do not "consume" the proceeds of their sales. As noted in the introduction, the detailed analysis of Treasury markets in March 2020 by Vissing-Jørgensen (2021) provides evidence of such preemptive sales: Among the largest sellers, foreign official agencies sold $196 billion of Treasury bonds but "consumed" only 24% of the proceeds in the form of a $48 billion reduction in their total U.S. Dollar assets. This evidence, and the viral selling behavior noted by Haddad, Moreira, and Muir (2021) with greater dislocations in the typically safer and more liquid assets, point to a need for a model in which sales reflect strategic, self-fulling decisions.

There is the potential for both pure-strategy equilibria to exist if the incentive to sell

$\pi(\lambda)$ is increasing in the fraction of strategic investors who sell. In such a situation of strategic complementarities, the safe asset market can break down due to self-fulfilling beliefs. Each individual strategic investor sells early only because they expect other strategic investors to sell early, and the run on the safe asset market could be avoided if beliefs were coordinated instead on the hold equilibrium.

2.2 Dealers and Market Clearing

Dealers consume at the end of date 1 and are forward looking and risk neutral. They value the safe asset at its fundamental value of 1 but face convex balance sheet costs for any inventory q, given by cq^2 with $c > 0$. Dealers start out with no inventory and compete for sales à la Bertrand by quoting prices in each period. We solve for the demand that results from the Subgame Perfect Nash Equilibrium among dealers.

A dealer's final payoff from purchasing quantities q_0, q_1 at prices p_0, p_1 is

$$(1 - p_0) q_0 + (1 - p_1) q_1 - c (q_0 + q_1)^2 .$$

Using backward induction, suppose dealers enter period 1 with q_0 units of inventory purchased at a price p_0, which would yield a payoff of $(1 - p_0) q_0 - cq_0^2$. Dealers will quote prices to purchase additional q_1 units of supply, bidding up prices until they earn zero profits on the additional units. Bertrand competition results in a price p_1 such that

$$\underbrace{(1 - p_0) q_0 + (1 - p_1) q_1 - c (q_0 + q_1)^2}_{\text{payoff after taking on } q_1} = \underbrace{(1 - p_0) q_0 - cq_0^2}_{\text{payoff without } q_1}. \tag{1}$$

Solving for p_1, this implies that the equilibrium price at date 1, given pre-existing inventory q_0, is

$$p_1(q_0, q_1) = 1 - 2cq_0 - cq_1 \tag{2}$$

which can be rewritten as a demand from dealers given by[9]

$$q_1^D = \frac{1}{c} (1 - p_1) - 2q_0. \tag{3}$$

Now consider the pricing behavior at date 0. Dealers will compete for the supply q_0

[9]Note that our framework does not restrict dealer demand at date 1 to be positive. If there is additional demand at date 1 such as from an asset purchase facility discussed in Section 4, we can have dealers sell part of their date-0 inventory such that $q_1^D < 0$. Since the quadratic balance sheet costs are symmetric around zero, we can also consider negative dealer demand at date 0 (e.g. if they start with an initial endowment of inventory or if they are able to go short the safe asset).

anticipating their behavior at date 1, as derived above. Bertrand competition means they will bid up prices until their total payoff is zero:

$$(1 - p_0)\, q_0 + (1 - p_1)\, q_1 - c\,(q_0 + q_1)^2 = 0. \tag{4}$$

Substituting condition (1) that follows from Bertrand competition at date 1 into the zero-profit condition (4) implies that forward-looking dealers will bid up prices at date 0 until

$$(1 - p_0)\, q_0 - c q_0^2 = 0.$$

Solving for p_0, this yields an equilibrium date-0 price

$$p_0(q_0) = 1 - c q_0, \tag{5}$$

which implies a demand from dealers given by

$$q_0^D = \frac{1}{c}\,(1 - p_0)\,. \tag{6}$$

Comparing the equilibrium prices p_1 in (2) and p_0 in (5), note that date-0 sales q_0 have twice the impact on p_1 as on p_0, which is one element that can lead to p_1 being sufficiently lower than p_0, which is necessary for a run equilibrium to exist. Even though dealers are forward looking, competing away their profits at date 0 leads them to pay relatively high prices at date 0. Once the market opens on date 1, the inventory they took on at date 0 reduces their willingness to pay and prices are much lower.

Role of Regulatory Constraints. Our modeling of balance sheet costs captures the effects of the Supplementary Leverage Ratio (SLR), an unweighted capital requirement for banks that was introduced as part of the Basel III reforms after the GFC as a backstop to risk-weighted capital regulation and became effective in 2014. Since the largest dealers in the U.S. are part of bank holding companies, the SLR constrains their activity, including in the Treasury market, a potentially unintended consequence of the regulatory reform. Importantly, both the direct holdings of Treasuries and reverse repo positions take up dealers' balance sheet space and are subject to the SLR (for more details, see, e.g. Duffie, 2016). Boyarchenko et al. (2020) show that the constraints pass through to unregulated arbitrageurs who rely on the balance sheet of regulated dealers (see also Du, Hébert, and Li, 2022 and Siriwardane, Sunderam, and Wallen, 2022).

The balance sheet costs matter in markets for safe assets such as Treasuries, as they

rely heavily on dealers for intermediating trades. Brain et al. (2019) document that Treasury market trading volume is split roughly evenly between dealer-to-client trades and inter-dealer trades; this suggests that, on average, a trade originating with one investor and ending with another investor passes through two dealers. The effects of balance sheet constraints are also quantitatively meaningful. For example, He, Nagel, and Song (2022) show that Treasury and repo spreads are significantly wider in the post-SLR period. In March 2020, the ability of dealers to provide liquidity in Treasuries was severely impaired as market depth dropped by a factor of more than 10 in the inter-dealer market (Duffie, 2020) while trading volume roughly doubled, reaching historically unprecedented levels (Fleming and Ruela, 2020). Duffie et al. (2023) show strong explanatory power of dealer balance sheet utilization for Treasury market illiquidity after controlling for yield volatility. Furthermore, the SLR constraint was initially not alleviated by the Fed's purchases of Treasuries because they were exchanged for reserves which, though perfectly liquid and safe, are treated the same under the SLR. Only on April 14 did the Fed temporarily exempt both Treasuries and reserves from the SLR rule (announced on April 1). Infante, Favara, and Rezende (2022) document the effect of the SLR and its temporary relaxation on dealers' Treasury market activity. We return to these issues in our discussion of policy implications in Section 4.

For tractability, we model balance sheet constraints as a convex function of *net* dealer demand and abstract from bid-ask spreads. In reality, dealers can rarely net out offsetting trades instantaneously, and so sales or purchases that are not perfectly synchronized at the same dealer will increase balance sheet costs across the financial system, making the role of balance sheet constraints more pronounced. While we model balance sheet costs as convex, in reality the SLR may at times impose hard quantity constraints with effectively infinite costs of expanding balance sheet further (Duffie, 2020). To the extent that regulatory constraints at times become totally binding, our results would be further strengthened. In sum, our modeling decisions bias the analysis toward less significant balance sheet costs. Note that we abstract from the intended benefits of the SLR for the stability of the banking system as these are outside the scope of our model.[10]

Expected Prices. Similar to Morris and Shin (2004) and consistent with the decentralized nature of the Treasury dealer-to-client market (Brain et al., 2019), we assume that

[10]The macro-finance literature shows how leverage regulation can have macroprudential benefits, decreasing the probability and severity of crises and fire sales (e.g. Phelan, 2016, Dávila and Korinek, 2017). While the SLR is intended as a "non-risk based backstop measure" (Basel Committee on Banking Supervision, 2014), it's potential to interfere with Treasury market functioning had been anticipated, e.g. by Duffie (2016)

trades are executed sequentially. For aggregate sales q_0, each seller's position in the queue is uniformly distributed on $[0, q_0]$ such that the expected position in the queue is $q_0/2$ and each investor expects to sell at the expected price $p_0^e = 1 - c\,(q_0/2)$.[11] Substituting in total supply $q_0 = s + (1-s)\,\lambda$, we have an expected payoff from selling at date 0 given by

$$p_0^e(\lambda) = 1 - \frac{c}{2}\left(s + (1-s)\,\lambda\right). \tag{7}$$

The expected price at date 0 is decreasing in the share of strategic investors λ who sell but also in the non-strategic sales s which directly reflect the severity of the liquidity risk at date 0.

At date 1, a seller expects to receive the average price $p_1^e = 1 - 2cq_0 - cq_1/2$. Substituting in total date 1 supply $q_1 = s\,(1-s)\,(1-\lambda)$ as well as total date 0 supply q_0, which is now in dealers' inventory, we have an expected payoff from selling at date 1 given by

$$p_1^e(\lambda) = 1 - 2c\underbrace{\left(s + (1-s)\,\lambda\right)}_{\text{date 0 inventory}} - \frac{c}{2}\underbrace{s\,(1-s)\,(1-\lambda)}_{\text{date 1 sales}}. \tag{8}$$

Because strategic sales λ move sales from date 1 to date 0, they have a direct negative effect on p_0^e with a coefficient $-\frac{1}{2}c\,(1-s)$ and a direct positive effect on p_1^e with a coefficient $\frac{1}{2}c\,(1-s)\,s$. These direct effects are stabilizing since they make selling at date 0 less attractive and selling at date 1 more attractive. However, strategic sales λ also affect p_1^e indirectly with a coefficient $-2c\,(1-s)$ through inventory on dealer balance sheets. This indirect effect is destabilizing since it makes selling at date 1 less attractive.

Why is the destabilizing indirect effect of strategic sales so much stronger than the stabilizing direct effect, at a ratio of 2 to 1/2? There are two reasons: First, existing inventory q_0 has twice the price impact on dealer demand at date 1 as new inventory q_1 has. Second, while investors anticipate the full effect of existing inventory in case they have to sell at date 1, they internalize only half the effect of sales on price at date 0 since they expect to sell at the average in-run price.

2.3 Incentive to Sell Preemptively

Using the expressions for p_0^e and p_1^e, we can derive the payoff gain $\pi(\lambda)$, which captures the incentive of an individual strategic liquidity investor to sell at date 0 if a fraction λ of

[11] We show in Appendix D that our results maintain if all trades are pooled and executed jointly as long as balance sheet constraints are sufficiently tight.

other strategic investors sells:

$$\pi(\lambda) = \overbrace{1 - \frac{c}{2}\left(s + (1-s)\,\lambda\right)}^{p_0^e(\lambda)} - \underbrace{s\left(1 - 2c\left(s + (1-s)\,\lambda\right) - \frac{c}{2}\,s\,(1-s)\,(1-\lambda)\right)}_{p_1^e(\lambda)} - (1-s)\,v. \tag{9}$$

As discussed in Section 2.1, the level and slope of the payoff gain determine the equilibrium (or equilibria) of the strategic interaction among liquidity investors.

Proposition 1 (Strategic liquidity investors' incentive to sell preemptively at date 0)**.**

- *There are strategic complementarities if and only if liquidity risk is sufficiently high:*

$$\begin{aligned} \pi'(\lambda) > 0 \quad &\Leftrightarrow \quad (4-s)\,s > 1 \\ &\Leftrightarrow \quad s > \widetilde{s} \equiv 2 - \sqrt{3} \approx 0.27. \end{aligned}$$

- *Higher liquidity risk increases the incentive to sell whenever there are strategic complementarities,* $\pi'(\lambda) > 0 \Rightarrow \partial\pi/\partial s > 0$.

- *Greater dealer balance sheet costs increase the incentive to sell whenever there are strategic complementarities,* $\pi'(\lambda) > 0 \Rightarrow \partial\pi/\partial c > 0$.

- *A greater continuation value uniformly decreases the incentive to sell,* $\partial\pi/\partial v < 0$.

Proof. See Appendix C. □

As discussed in Section 2.2, strategic sales λ have a stabilizing direct effect that decreases p_0^e and increases p_1^e, and they have a destabilizing indirect effect through dealer balance sheets that decreases p_1^e — the latter effect is considerably stronger, with a ratio of 2 to 1/2. When evaluating the effects of higher λ on the incentive to sell π in (9), we have to account for the fact that effects on p_1^e are discounted by the liquidity shock probability s since they are only relevant if the investor actually suffers a liquidity shock at date 1. We therefore have a destabilizing indirect effect of λ on the incentive to sell π with a coefficient

$$2c\,(1-s)\,s,$$

and a stabilizing total direct effect with a coefficient (in absolute value)

$$\frac{1}{2}c\,(1-s)\,\left(1+s^2\right),$$

resulting in a ratio of

$$\frac{2s}{\frac{1}{2}\left(1+s^2\right)}.$$

Consider the relative strength of the two effects and how they depend on the magnitude of liquidity risk s. The stabilizing effect in the denominator is present whether or not there is liquidity risk, i.e. the coefficient is non-zero even for $s = 0$ and then increases slowly with liquidity risk, as it is quadratic in s — combining the individual investor's date-1 liquidity risk and the aggregate date-1 liquidity risk. In contrast, the destabilizing effect through dealer balance sheets is linear in s — reflecting only the individual investor's risk of facing the constrained dealers — and it increases faster due to the stronger effect of strategic sales on p_1^e than on p_0^e. For sufficiently high s, the destabilizing effect dominates, resulting in strategic complementarities. This is in contrast to the model of Bernardo and Welch (2004) which only features strategic substitutes (Appendix B).

Besides increasing the slope $\pi'(\lambda)$, higher liquidity risk also increases the level of the incentive to sell. This is intuitive, as higher s for given λ means additional non-strategic sales as well as strategic sales, which load up dealer balance sheets at date 0 and destabilize the market. Consistent with the important role dealer balance sheets play for market fragility, they tend to increase the incentive to sell preemptively and do so for sure if strategic complementarities are present. In sum, the inventive to sell at date 0 and therefore the potential for market fragility is increasing in how much liquidity risk investors face and in the balance sheet constraints faced by dealers who absorb sales at both dates.

Figure 2 illustrates the incentive to sell and the resulting equilibria of the complete information game for different levels of liquidity risk. For low s, $\pi(\lambda)$ is uniformly negative and decreasing, and the unique equilibrium is the hold equilibrium ($\lambda^* = 0$). As s increases, the level and slope of $\pi(\lambda)$ increase, until it first becomes flat at $s = \widetilde{s} \equiv 2 - \sqrt{3}$ and then intersects the horizontal axis, at which point the game has multiple equilibria (hold, sell and mixed). For sufficiently high s, $\pi(\lambda)$ is uniformly positive and the unique equilibrium is for everyone to sell ($\lambda^* = 1$). Note that Figure 2 shows strategic complementarities arising at a point where the payoff gain is negative, that is

$$\pi(\lambda \mid s = \widetilde{s}) = \frac{c}{2}\widetilde{s}^2 - (1-\widetilde{s})\,(v-1) < 0,$$

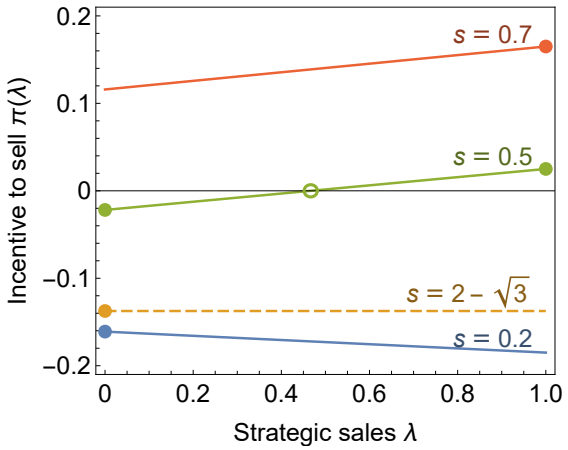

Figure 2: Incentive to sell and equilibria. The figure shows the payoff gain $\pi(\lambda)$ for different values of liquidity risk s. Circles indicate equilibria of the game under complete information. Parameters: $c = 0.25,\ v = 1.2$.

so the dashed horizontal line is below the horizontal axis. In the following, we will focus on this case by imposing the following assumption on c and v.

Assumption 1. *We assume that* $\frac{c}{2}\widetilde{s}^2 - (1-\widetilde{s})(v-1) < 0$.

Consistent with the comparative statics in Proposition 1, the point at which the payoff gain π changes from decreasing to increasing is more likely to be below the horizontal axis if v is larger or c is smaller. What happens if Assumption 1 is not satisfied? In that case, the unique equilibrium is still to hold for sufficiently small s and to sell for sufficiently large s. However, for an intermediate range of s, the unique equilibrium is in mixed strategies since the payoff gain crosses the horizontal axis with negative slope. Since our emphasis is on the potential for fragility, we focus the analysis on the case where multiple pure-strategy equilibria arise in an intermediate range of s and we can have regime shifts. This allows for the use of global game techniques and results in a unique equilibrium for every $s \in [0,1]$ with the switch from the hold to the run equilibrium at an endogenous threshold. Since the threshold is a continuous function of other model parameters, comparative statics and policy analysis follow naturally.

2.4 Global Game and Unique Equilibrium

Under complete information, there can be multiple equilibria in the strategic interaction among liquidity investors — a hold equilibrium and a run equilibrium (and a mixed equilibrium). We now introduce noise into investors' payoffs to break the common knowledge

underpinning the multiplicity and use global game techniques to derive a unique equilibrium. In particular, we assume that investor i does not observe the degree of liquidity risk s perfectly, instead receiving a signal $\widehat{s}_i = s + \sigma_\varepsilon \varepsilon_i$ with ε_i i.i.d. across all i and σ_ε positive but arbitrarily small. As a result, a strategic investor faces fundamental uncertainty about the likelihood of a liquidity shock, s, as well as strategic uncertainty about the fraction of other strategic investors who sell preemptively, λ.[12]

We can write the payoff gain explicitly as a function of the fundamental s as well as the fraction of strategic investors who sell, $\pi(\lambda, s)$. Making use of standard global game results (e.g. Morris and Shin, 2003), we can derive a unique Bayesian Nash equilibrium for the game among strategic investors.

Proposition 2 (Unique global game equilibrium)**.** *For signal noise $\sigma_\varepsilon \to 0$, the unique Bayesian Nash equilibrium among strategic investors is in switching strategies around a threshold s^* defined by*

$$\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0.$$

For liquidity risk below the threshold, $s < s^*$, all strategic investors hold on to their safe assets and only investors with genuine liquidity needs sell. For liquidity risk above the threshold, $s > s^*$, all strategic investors sell their safe assets and the market suffers a run.

Proof. See Appendix C. □

While Appendix C contains the full proof, we provide the following outline for intuition. An investor who receives a signal exactly equal to the switching point has to be indifferent between holding and selling,

$$E\left[\pi(\lambda, s) \,\middle|\, \widehat{s}_i = s^*\right] = 0, \tag{10}$$

where the expectation is with respect to both λ and s. Note from equation (9) that $\pi(\lambda, s)$ is linear in λ and cubic in s. We have $E[s \mid \widehat{s}_i = s^*] = s^*$, and, in the limit $\sigma_\varepsilon \to 0$, we have $E\left[s^2 \mid \widehat{s}_i = s^*\right] \to (s^*)^2$ and $E\left[s^3 \mid \widehat{s}_i = s^*\right] \to (s^*)^3$, so fundamental uncertainty vanishes, and strategic uncertainty in the form of the distribution of λ becomes uniform on $[0, 1]$. We therefore have

$$\lim_{\sigma_\varepsilon \to 0} E\left[\pi(\lambda, s) \,\middle|\, \widehat{s}_i = s^*\right] = \int_0^1 \pi(\lambda, s^*)\, d\lambda,$$

[12] We acknowledge that the uniqueness of the global game equilibrium is potentially not robust in the presence of public information (Angeletos and Werning, 2006). However, the fact that prices in the Treasury market are not as transparent to the non-dealer investors as, e.g. in the equity market, alleviates these concerns in our setting.

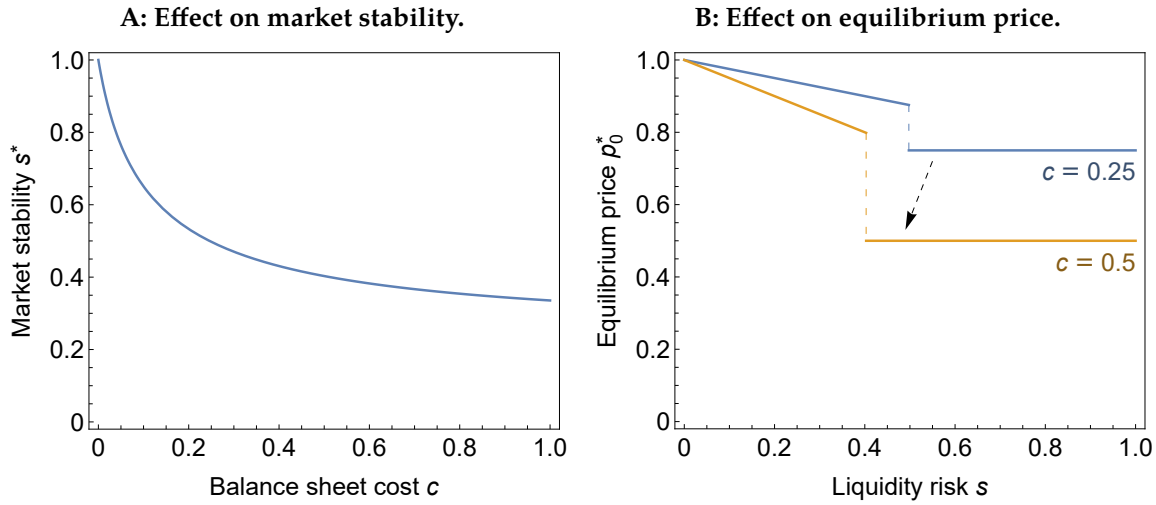

Figure 3: Effect of balance sheet costs on market stability and equilibrium price. Panel A shows market stability measured by the equilibrium threshold s^* as a function of the dealer balance sheet cost c. Panel B shows the equilibrium price at date 0 p_0^* as a function of liquidity risk s for different values of dealer balance sheet cost c. Parameters: $v = 1.2$.

where $\int_0^1 \pi(\lambda, s)\, d\lambda$ is a cubic polynomial in s. We show in the proof of Proposition 2 that $\frac{\partial}{\partial s}\int_0^1 \pi(\lambda, s)\, d\lambda > 0$ with $\int_0^1 \pi(\lambda, 0)\, d\lambda < 0$ and $\int_0^1 \pi(\lambda, 1)\, d\lambda > 0$ so there is a unique threshold s^* that satisfies the indifference condition $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$.

The equilibrium switches from hold to sell when liquidity risk s crosses the threshold s^* and a higher threshold implies a larger range of liquidity risk $[0, s^*]$ where the market remains in the hold equilibrium. Given the distribution F of liquidity risk s, the ex-ante probability of the hold equilibrium is therefore $\Pr[s \leq s^*] = F(s^*)$ and the ex-ante probability that the market suffers a run is $\Pr[s > s^*] = 1 - F(s^*)$. The threshold s^* is therefore a well-defined measure of market stability or $1 - s^*$ a measure of fragility, and we can refer to a market with higher s^* as more stable or, equivalently, less fragile.

Corollary 1. *Market stability as measured by the global game threshold s^* is decreasing in dealer balance sheet costs, $\partial s^*/\partial c < 0$ and increasing in liquidity investors' continuation value, $\partial s^*/\partial v > 0$.*

Proof. See Appendix C. □

Market stability naturally inherits the properties of the incentive to sell listed in Proposition 1. Consider the effect of dealer balance sheet costs c on market stability s^* illustrated in Figure 3A. If dealers faced no balance sheet costs ($c = 0$), the market would be perfectly stable ($s^* = 1$) and strategic investors would never sell preemptively, even for very high

liquidity risk s. However, as balance sheet costs c increase from zero, market stability s^* decreases rapidly and then levels off at higher values of c. In contrast, preemptive sales in the model of Bernardo and Welch (2004) are *decreasing* in dealer balance sheet costs such that market runs in their model are less prevalent for higher balance sheet costs (Appendix B).

The threshold equilibrium implies that the behavior of strategic liquidity investors and therefore the equilibrium price drops precipitously around the threshold s^*. In particular, total supply at date 0 increases from s to 1 as s crosses the thresholds^*, so the equilibrium price from equation (5) becomes[13]

$$p_0^*(s) = \begin{cases} 1 - cs & \text{for} \quad s < s^*, \\ 1 - c & \text{for} \quad s > s^*. \end{cases} \tag{11}$$

Figure 3B illustrates the equilibrium price p_0^*. When liquidity risk is very low, all strategic investors hold on to their safe assets and only investors who receive a liquidity shock sell — the equilibrium price is therefore steadily decreasing in s, representing the sales of non-strategic investors. However, once liquidity risk crosses the threshold s^*, all strategic investors preemptively sell their safe assets — the market is flooded and the equilibrium price drops precipitously. Figure 3B further illustrates the equilibrium price for two different levels of dealer balance sheet costs c. As balance sheet costs increase, the threshold s^* and therefore market stability decreases (Corollary 1). In addition, the drop in market prices at the discontinuity is much larger for higher balance sheet costs. This is due to the fact that the drop in equation (11) is given by $c\,(1 - s^*)$, where c and s^* interact multiplicatively.

2.5 Investor Welfare and Inefficient Runs

Because liquidity investors value the safe asset at $v > 1$ when held to maturity, selling the asset without a genuine liquidity need is generally inefficient. As a result, our model features panic-based run equilibria in which liquidity investors would be better off if they could coordinate to hold instead. However, the welfare consequences of the hold and run equilibria are somewhat subtle because investors who would have suffered liquidity shocks at date 1 can be better off if they sell in a panic-based run equilibrium at date 0. Nonetheless, our model predicts that the market will always feature run equilibria at times when investors would be better off if all investors could coordinate to hold.

[13]The expression in (11) represents the zero-noise limit case ($\sigma_\varepsilon \to 0$) where the price drops discontinuously at s^*. For small but positive σ_ε, the price drop would be continuous but very steep in a small neighborhood around s^*.

If all strategic investors sell at date 0, they (and the non-strategic investors) receive an expected payoff $p_0^e(\lambda = 1) = 1 - c/2$. Expected payoffs if all strategic investors hold are more complicated: investors with a liquidity shock at date 0 receive $p_0^e(\lambda = 0) = 1 - (c/2)\, s$; investors with a liquidity shock at date 1 receive $p_1^e(\lambda = 0) = 1 - 2cs - (c/2)\, s\,(1-s)$; and investors who do not receive a liquidity shock at either date receive v. Altogether, the difference in investor welfare between the hold allocation and the run allocation is a function of liquidity risk s and given by

$$\Delta(s) \equiv s\left(1 - \frac{c}{2}s\right) + (1-s)\, s\left(1 - 2cs - \frac{c}{2}s\,(1-s)\right) + (1-s)^2\, v - \left(1 - \frac{c}{2}\right). \tag{12}$$

The first three terms are the payoffs in the hold allocation ($\lambda = 0$) and the last term is the payoff in the run allocation ($\lambda = 1$).

With no liquidity shocks, we have $\Delta(0) = v - 1 + c/2 > 0$, which includes the elevated value of the asset plus the saved balance sheet cost. With guaranteed liquidity shocks, we have $\Delta(1) = 0$ since all investors are forced to sell early — there is no one who could hold. However, $\Delta(s)$ is a polynomial of degree 4 and not necessarily strictly positive for $s \in [0, 1]$. Figure 4A plots $\Delta(s)$ for different values of the balance sheet cost c and illustrates that $\Delta(s)$ has a root $s^{**} \in (0, 1)$ for $c > 0$, so there is a range of s near 1 where $\Delta(s) < 0$, i.e. where investor welfare is higher in the run allocation.

How can the run allocation welfare-dominate if liquidity investors value the safe asset at $v > 1$ and the price they sell at is strictly less than 1? The issue is precisely the main feature of our model: investors' fear of being forced to liquidate at date 1 at depressed prices. In a hold allocation, a fraction $(1-s)\, s$ of investors will be forced to sell at date 1, after a fraction s have already sold at date 0. We have already noted in Proposition 1 that, for high s, investors would prefer to sell early, even if the price p_0^e is depressed by strategic sales from other investors — and this is all the more so in a hold allocation with no strategic sales. Thus, with sufficient liquidity risk, agents are better off selling early, when prices are high, rather than in the future, when dealers' balance sheets would be bloated.

While the welfare difference $\Delta(s)$ compares the hold and run allocations, only one of the two is an equilibrium for any level of s: hold for s below the global game threshold s^* and sell for s above s^*. An important question therefore is whether our model features inefficient run equilibria, with investors selling strategically when the hold allocation would have yielded higher welfare. This amounts to determining whether the welfare difference $\Delta(s)$ is positive for s at or above the threshold s^* or equivalently, whether $s^* < s^{**}$.

Proposition 3 (Inefficient Runs)**.** *The model features inefficient run equilibria in which investors would be better off coordinating on the hold allocation. In particular, we have $s^* < s^{**}$ and therefore*

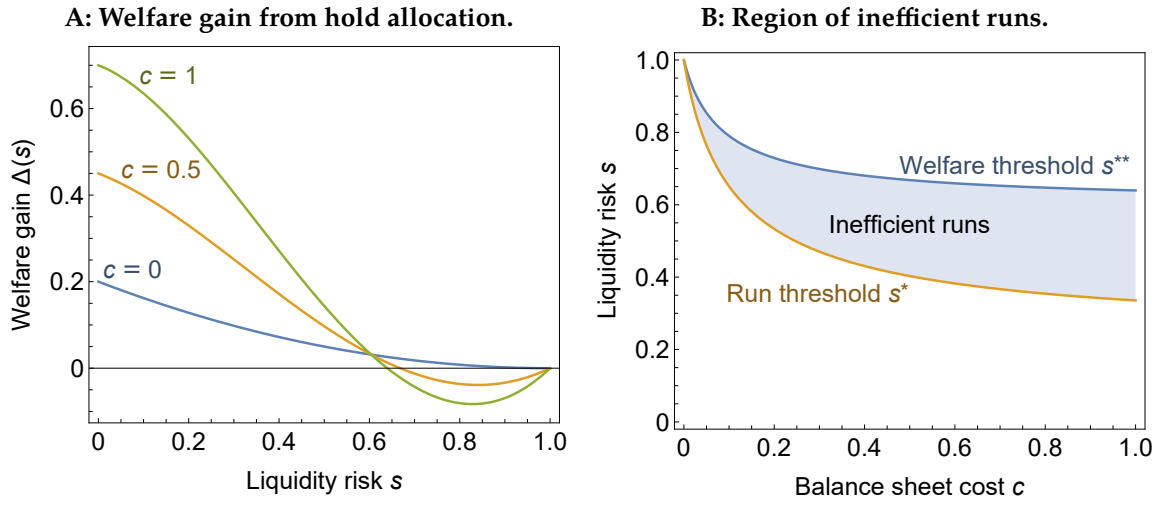

Figure 4: Investor welfare and inefficient runs. Panel A shows the difference in welfare between the hold and run allocations as a function of liquidity risk s for different values of dealer balance sheet cost c. Panel B shows the welfare threshold s^{**} and the equilibrium threshold s^* as a function of dealer balance sheet cost c. The shaded region indicates the "panic region": the values of liquidity risk s such that the global game features a run equilibrium but the hold equilibrium leads to higher welfare. Parameters: $v = 1.2$.

$\Delta(s) > 0$ *for* $s \in (s^*, s^{**})$.

Proof. See Appendix C. □

Figure 4B illustrates the result. The figure plots the equilibrium threshold s^* and the welfare cutoff s^{**} as functions of the dealer balance sheet cost c. The shaded region plots the "panic region": the values of liquidity risk s in which the global game features a run equilibrium but the hold allocation would lead to higher investor welfare. Outside the shaded region, agents coordinate on the equilibrium that leads to the highest welfare. Our results imply that there is scope for policy in order to shrink the inefficient (shaded) region by increasing s^*. By decreasing the frequency of the run equilibrium, policy could tilt outcomes in favor of higher welfare (the hold equilibrium) whenever liquidity risk is not too high.

3 Model with Safety Investors

We now introduce a second type of investors who are risk averse and hold a portfolio of the safe asset and the risky asset. These "safety investors" are subject to aggregate shocks

to the expected payoff of the risky asset which lead them to shift their desired portfolio composition. We are interested in the situation where, in a bad state of the world, safety investors increase their demand for the safe asset in a flight-to-safety, offsetting the flow of sales from liquidity investors or leading to repricing even in the absence of large trade volume. Examples of real-world safety investors we have in mind include pension funds who face a traditional risk–return tradeoff and were among the largest net buyers of Treasuries in 2020q1 (Financial Accounts Table FU.210).

Although we impose general equilibrium through market clearing for both the safe and risky asset, our modeling of safety investors is deliberately simple in order to integrate them into the model of strategic interaction among liquidity investors. In addition, while safety investors could be active both at date 0 and at date 1, we focus attention on the case where safety investors have interesting effects. Additional safe asset demand at date 1 unambiguously increases the price at date 1, which reduces the incentive to sell preemptively and has a natural stabilizing effect on the strategic interaction at date 0. In contrast, additional demand at date 0 increases both the price at date 0 as well as the price at date 1 — by reducing dealer inventory — with an ambiguous overall effect on market stability at date 0. We therefore restrict attention to the case in which safety investors are active only at date 0. Appendix E discusses the general case.

3.1 Safety Investors' Safe Asset Demand

Safety investors' utility is linear in consumption at date 0 and quadratic in future wealth,

$$u(c_0, w) = c_0 + w - \frac{1}{2}\kappa w^2,$$

where the curvature parameter $\kappa > 0$ commingles risk aversion and intertemporal substitution and we assume $w < 1/\kappa$. In addition to the safe asset with future payoff 1, there is a risky asset with future payoff z distributed according to H_z, where we denote the expected payoff as $\mu_z = \int z\, dH_z(z)$ and the variance as $\sigma_z^2 = \int z^2\, dH_z(z) - \mu_z^2$.

Given initial wealth w_0, safety investors choose consumption c_0 and a portfolio with holdings q_0^S of the safe asset and q_z of the risky asset subject to the budget constraint $c_0 + p_0 q_0^S + p_z q_z \leq w_0$ to maximize $E[u(c_0, w)]$, where future wealth is given by $w = q_0^S + z q_z$. After substituting in for c_0 using the budget constraint, we have first-order conditions for

q_0^S and q_z given by

$$\begin{aligned} 0 &= E\left[1 - \kappa\left(q_0^S + zq_z\right)\right] - p_0 \\ &= 1 - \kappa\left(q_0^S + zq_z\right) - p_0, \end{aligned}$$

and

$$\begin{aligned} 0 &= E\left[z - \kappa\left(q_0^S + zq_z\right)z\right] - p_z \\ &= \mu_z - \kappa\left(\mu_z q_0^S + \left(\mu_z^2 + \sigma_z^2\right)q_z\right) - p_z, \end{aligned}$$

which are both linear in q_0^S and q_z. Solving, we arrive at safety investors' demand for the safe asset and the risky asset given by

$$\begin{aligned} q_0^S &= \frac{1}{\kappa\sigma_z^2}\left(\sigma_z^2 + \mu_z p_z - \left(\mu_z^2 + \sigma_z^2\right)p_0\right) \\ q_z &= \frac{1}{\kappa\sigma_z^2}\left(\mu_z p_0 - p_z\right), \end{aligned}$$

while their consumption at date 0 is given as the residual $c_0 = w_0 - \left(p_z q_z + p_0 q_0^S\right)$.

To close the model and impose general equilibrium, we assume that safety investors have to hold the entire supply $Z > 0$ of the risky asset, i.e. $q_z = Z$. In this case, the risky asset price is $p_z = \mu_z p_0 - \kappa\sigma_z^2 Z$ and drops after a negative shock to the risky asset's expected payoff μ_z (as the S&P 500 did in March 2020). Substituting in the equilibrium p_z, safety investors' demand for the safe asset simplifies to

$$q_0^S = \frac{1}{\kappa}\left(1 - \kappa\mu_z Z - p_0\right), \tag{13}$$

which is linear in p_0 and has a similar structure to dealers' demand in equation (6). For ease of exposition, we write safety investors' demand as

$$q_0^S = a - bp_0,$$

with $a = 1/\kappa - \mu_z Z$ and $b = 1/\kappa$. We are interested in shocks to the risky asset's expected payoff μ_z, which enter safety investors' safe asset demand only through the intercept a. A decrease in μ_z is therefore equivalent to an increase in a and implies a flight-to-safety as a level shift in safety investors' demand for the safe asset.

3.2 Effect of Safety Investors on Market Stability

Combining the demand from dealers, $q_0^D = \frac{1}{c}(1 - p_0)$, with the demand from safety investors, $q_0^S = a - bp_0$, total demand for safe assets at date 0, $q_0^D + q_0^S$, can be rewritten as

$$p_0(q_0) = \frac{1 + ac}{1 + bc} - \frac{c}{1 + bc} q_0. \tag{14}$$

With total supply of $q_0 = s + (1 - s)\lambda$, a liquidity investor who sells at date 0 expects to receive

$$p_0^e(\lambda) = \frac{1 + ac}{1 + bc} - \frac{1}{2}\frac{c}{1 + bc}\left(s + (1 - s)\lambda\right).$$

An increase in the additional demand from safety investors (higher a) therefore uniformly increases the expected price at date 0.

At date 1, only dealers buy the safe asset so demand is unchanged from equation (2) in Section 2. However, dealer inventory is no longer the entire date-0 supply q_0 as some of these sales have been absorbed by safety investors. Specifically, dealer inventory is given by

$$q_0^D = \frac{1}{c}\left(1 - p_0(q_0)\right) = \frac{\frac{1}{c}q_0 - \frac{b}{c}\left(1 - \frac{a}{b}\right)}{\frac{1}{c} + b}.$$

Combining dealer demand at date 1 from equation (2) with inventory q_0^D and date-1 supply $q_1 = s(1 - s)(1 - \lambda)$, a liquidity investor who sells at date 1 expects to receive

$$p_1^e(\lambda) = 1 - 2c \underbrace{\frac{s + (1 - s)\lambda + b - a}{1 + bc}}_{\text{date-0 inventory}} - \frac{c}{2} \underbrace{s(1 - s)(1 - \lambda)}_{\text{date-1 sales}}.$$

Our focus now is how changes in additional sales a affect the two expected prices and the strategic interaction of liquidity investors captured by the payoff gain:

$$\pi(\lambda, s) = \overbrace{\frac{1 + ac}{1 + bc} - \frac{1}{2}\frac{c}{1 + bc}\left(s + (1 - s)\lambda\right)}^{p_0^e(\lambda)} - \underbrace{s\left(1 - 2c\frac{s + (1 - s)\lambda + b - a}{1 + bc} - \frac{c}{2}s(1 - s)(1 - \lambda)\right)}_{p_1^e(\lambda)} - (1 - s)v.$$

The question is if (or when) additional demand from safety investors is stabilizing (decreases π) or destabilizing (increases π).

Proposition 4. *Flight-to-safety demand at date 0 increases the incentive to sell preemptively if and*

only if liquidity risk is low, $\partial\pi/\partial a > 0 \Leftrightarrow s < 1/2$*. The effect of additional demand is monotonic in liquidity risk,* $\partial^2\pi/(\partial s \partial a) < 0$*.*

Proof. See Appendix C. □

Where does the ambiguous effect of a on π originate? Similar to strategic sales by liquidity investors, purchases from safety investors have a direct effect and an indirect effect on the payoff gain π. The direct effect of an increase in demand a is an increase in the date-0 price p_0^e and therefore an increase in the payoff gain π with a coefficient

$$\frac{c}{1+bc}.$$

This effect is destabilizing since a higher price at date 0 incentivizes strategic investors to sell preemptively.

The indirect effect works through relaxing dealer balance sheet constraints, which increases the date-1 price p_1^e and therefore reduces the payoff gain π with a coefficient (in absolute value)

$$s\,\frac{2c}{1+bc}.$$

This stabilizing effect on p_1^e is twice as high as the destabilizing effect on p_0^e because of the larger effect of existing date-0 inventory on dealer demand than of new date-1 inventory. However, the effect on p_1^e is discounted by the liquidity shock probability s since it is only relevant if the investor actually suffers a liquidity shock at date 1. For low liquidity risk, $s < 1/2$, the destabilizing effect of a higher date-0 price dominates the stabilizing effect of a higher date-1 price, such that flight-to-safety increases the incentive to sell preemptively. Vice versa for high liquidity risk, $s > 1/2$, the stabilizing effect dominates such that flight-to-safety decreases the incentive to sell.

The payoff gain with safety investor demand retains the standard global game conditions of Morris and Shin (2003) so, for vanishing signal noise, the unique equilibrium remains in switching strategies around a threshold s^* defined by the indifference condition $\int_0^1 \pi(\lambda, s^*)\,d\lambda = 0$ as in Proposition 2. In particular, recall that π is increasing in s so an exogenous decrease in π leads to a higher threshold s^*, capturing higher market stability. The ambiguous effect of safety investor demand on the payoff gain π (Proposition 4) therefore directly translates into an analogous effect on market stability.

Corollary 2. *Flight-to-safety demand at date 0 is stabilizing if the market is relatively stable and destabilizing if the market is relatively fragile,* $ds^*/da > 0 \Leftrightarrow s^* > 1/2$*.*

Proof. See Appendix C. □

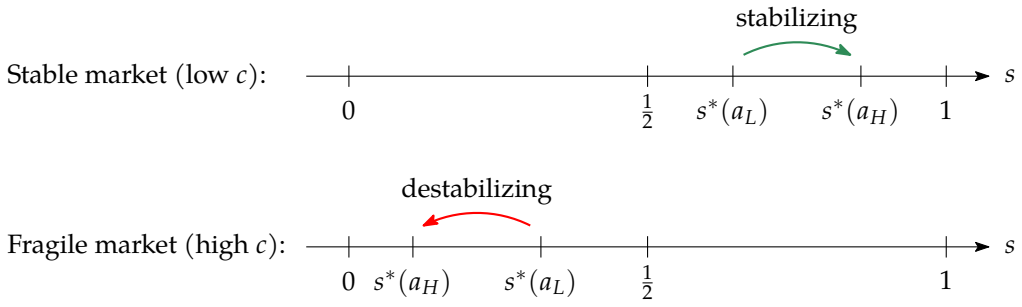

Figure 5: Effect of flight-to-safety on equilibrium market stability. The figure shows the effect of an increase in safety investor demand from a_L to a_H on market stability s^* for different levels of dealer balance sheet cost c.

Figure 5 illustrates the ambiguous effect of flight-to-safety demand on market stability by comparing two markets with different levels of dealer balance sheet cost c. When balance sheet costs are low, the market is relatively stable: the threshold s^* where the price drops precipitously is above $1/2$. In this case, liquidity investors sell preemptively at date 0 only if liquidity risk s is very high (i.e. only if they are very likely to be forced to sell at date 1). In this environment of high liquidity risk, the stabilizing effect of flight-to-safety demand increasing the price at date 1 dominates and the threshold s^* is increasing in a, so that runs become less likely as safety demand increases from a_L to a_H.

When balance sheet costs are high, in contrast, the market is relatively fragile with the threshold s^* below $1/2$. In this case, liquidity investors already sell preemptively when liquidity risk s is still low (i.e. when they are unlikely to be forced to sell at date 1). In this environment of low liquidity risk, the destabilizing effect of flight-to-safety demand increasing the price at date 0 dominates and the run threshold s^* is decreasing in a so higher safety demand is destabilizing. In fact, for a given level of liquidity risk that is close to but below the run threshold, an increase in safety investor demand can reduce the threshold sufficiently to tilt the market into the run equilibrium such that flight-to-safety triggers a dash-for-cash.

The interaction of liquidity investors and safety investors therefore results in a feedback effect in market stability. If the market is resilient to begin with (e.g. as with low balance sheet costs before the GFC), then liquidity investors and safety investors interact symbiotically: In times of stress, the additional demand for safe assets from safety investors has a stabilizing effect on the strategic interaction of liquidity investors and attenuates the risk of market breakdown. However, if the market is relatively fragile (e.g. due to the post-GFC

increase in dealer balance sheet costs), the relationship reverses: Additional demand from safety investors in times of stress further destabilizes the strategic interaction of liquidity investors, increasing their incentive to sell preemptively and thereby increasing the risk of market breakdown.

3.3 Correlated Liquidity and Safety Shocks

Now suppose the risks faced by liquidity investors and safety investors are correlated. In times of stress, liquidity investors face a higher risk of suffering a liquidity shock (i.e. s is high), and safety investors face a low payoff of the risky asset (i.e. μ_z is low and therefore a is high). To understand the net effect of increases in s and a on the safe asset market, we can derive the equilibrium price at date 0 as a function of s and a. As before, total supply in the global game equilibrium is s for $s < s^*$ (all strategic investors hold) and 1 for $s > s^*$ (all strategic investors sell). Substituting into the price with demand from safety investors in equation (14), the equilibrium price becomes

$$p_0^*(s,a) = \begin{cases} \frac{1}{1+bc}\left(1 - c\left(s-a\right)\right) & \text{for} \quad s < s^*(a), \\ \frac{1}{1+bc}\left(1 - c\left(1-a\right)\right) & \text{for} \quad s > s^*(a). \end{cases} \tag{15}$$

Figure 6 illustrates the equilibrium price for combinations of s and a with a contour plot. The figure shows a case in which the market is relatively fragile: The threshold s^* is always below 1/2, so the cliff where the price drops as the equilibrium switches from hold to run is decreasing in (s,a)-space: for liquidity risk s close to s^*, an increase in safety investor demand a can push the market over the cliff and trigger a price crash. In the hold equilibrium (i.e. for $s < s^*$), the expression in equation (15) shows that equal-sized increases in s and a exactly offset each other and leave the price unchanged so the contour lines in Figure 6 have a slope of 1. This implies that whenever safety demand a increases more than 1:1 with liquidity risk s and liquidity risk remains below the threshold s^*, we observe a classic flight-to-safety with p_0^* increasing (i.e. safe assets appreciating). This corresponds to the period from mid-February to early March 2020, where stock prices decreased and Treasury prices increased (Figure 1, Panel A). However, if the balance shifts and the increase in liquidity risk s outweighs the increase in safety demand a, the price p_0^* can decrease and suddenly drop as s crosses the threshold s^* and the equilibrium shifts to a dash-for-cash. This corresponds to the period in mid-March 2020 when Treasury prices reversed their increase and dropped together with stock prices.

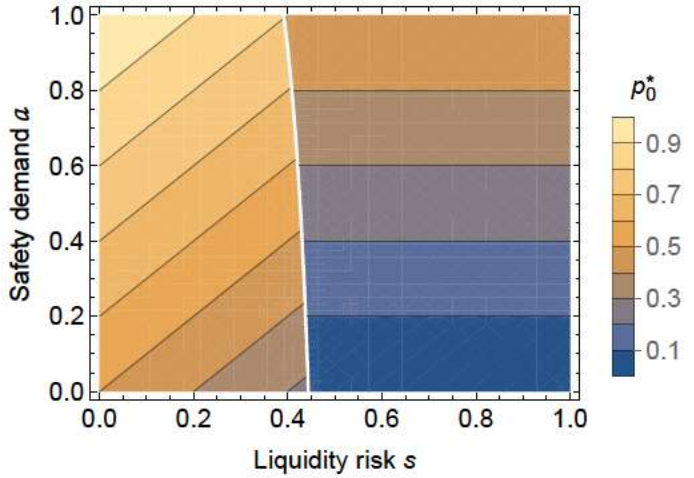

Figure 6: Equilibrium price for combinations of liquidity and safety shocks. The figure shows a contour plot of the equilibrium price p_0^* as a function of liquidity risk s and safety demand a. Parameters: $v = 1.2,\ b = 1, c = 1$.

4 Policy Implications

The events of March 2020 triggered an immediate, short-term policy response and have sparked a lively debate about longer-term policy implications. In this section, we consider three main policy implications through the lens of our model: market structure, dealer balance sheet constraints, and asset purchase facilities. In particular, we highlight the implications that follow from the strategic, regime-change nature of our model where policy tools can have a large effects, both expected and unexpected, by switching the equilibrium.

Market Structure. One focus of the current policy debate is whether to reform the market structure in the U.S. Treasury market. Trading between investors and dealers occurs largely in an over-the-counter (OTC) structure and separate from inter-dealer trading which involves brokers and limit-order books (Brain et al., 2019). Potential reforms include expanded central clearing (Securities and Exchange Commission, 2022) and all-to-all trading (Chaboud et al., 2022).

Our model implicitly already incorporates central clearing and all-to-all trading since purchases by safety investors directly offset sales by liquidity investors at date 0. Only the *net* sales at date 0 end up on dealer balance sheets and cause the potential for fragility at date 0 by depressing prices at date 1. However, all-to-all trading could allow easier market access to potential buyers at date 1, supporting prices and thereby stabilizing the market at date 0 (Kutai et al., 2022).

A further destabilizing element of our model is the sequential trade execution in the spirit of the OTC market structure between investors and dealers. As a result, a strategic investor expects to receive the average in-run price when selling preemptively at date 0 but bears the full impact of dealer inventory from date 0 when being forced to sell at date 1. Changes to market structure that lead to more pooling of trades and that reduce the role of dealers as a bottleneck for inter-temporal flow imbalances can therefore reduce the fragility of safe asset markets. Note, however, that our model features a run equilibrium even in the absence of sequential trade execution if dealer constraints are sufficiently tight (see Appendix D). Pooling of trades therefore attenuates incentives to sell preemptively but is not sufficient to rule out fragility.

Dealer Constraints. Dealer balance sheet costs play a crucial role in the strategic interaction of liquidity investors, since dealer inventory is the key link between the price at date 0 and the price at date 1. When considering only the interaction of liquidity investors, higher dealer balance sheet costs result in a more fragile safe asset market (i.e. a market that is more prone to runs and sudden price crashes; Figure 3B). Also taking into account the effect of additional demand from safety investors, an increase in dealer balance sheet costs can tip the market from a relatively stable region in which risk agents have a stabilizing effect to a relatively fragile region in which risk agents have a destabilizing effect (Figure 5). However, a policy that aims to relax dealer balance sheet constraints in times of stress has to be designed with care due to the subtleties of the strategic interaction. For example, if the policy relaxes dealer constraints only at date 0 (or relatively more at date 0), then it can increase the incentive to sell preemptively at date 0. If the market is in a run equilibrium, such a policy will appear to not have an effect, and if it is in the hold equilibrium then a short-run relaxation of constraints can precipitate a run. In addition, policy has to target the constraint that is actually binding.

Figure 7 illustrates these subtleties by showing the sequence of Fed interventions aimed at the Treasury market in the spring of 2020. Going into March, the Fed was conducting limited Treasury repo operations (lending against Treasuries) as part of its regular monetary policy implementation and was actually shrinking the offering size of these operations.[14] As conditions deteriorated starting March 9, repo offering sizes were increased to over $1 trillion by March 12 but, as shown in Figure 7, take-up by dealers was only moderate at around $100 billion and the liquidity provision through repos was not effective

[14]On February 4 the New York Fed's Open Market Trading Desk decreased term repo operation offering size from $35 billion to $30 billion and again on February 13 from $30 billion to $25 billion, concurrent with a reduction in overnight repo from $120 billion to $100 billion (Federal Reserve Bank of New York, 2021).

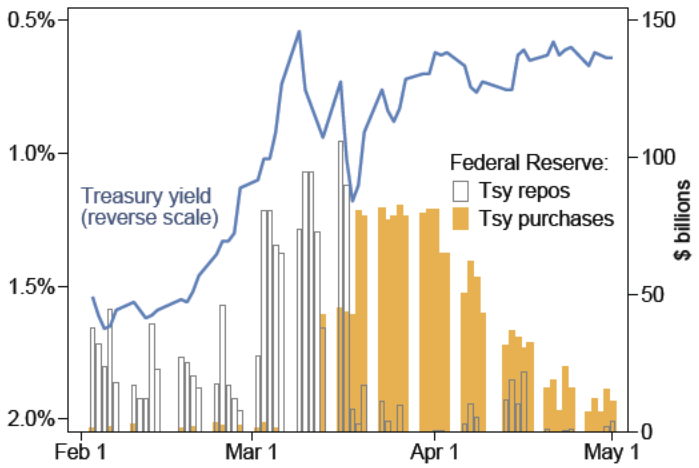

Figure 7: Federal Reserve interventions in Treasury markets. The figure shows the market yield on 10-year Treasuries and two types of Federal Reserve interventions: Treasury repos (lending against Treasury securities) and outright Treasury purchases. For details on the data see Appendix A.

against the drop in Treasury prices. This is consistent with the SLR being the binding constraint on dealers, as the SLR is not relaxed by funding a Treasury position with a loan from the Fed (Duffie, 2020).[15] Figure 7 shows that the recovery in Treasury prices in mid-March coincided with a switch by the Fed — from lending against Treasuries to purchasing them outright — and that the Fed was able to scale back purchases once Treasury holdings were exempted from the SLR on April 14 (announced on April 1). If the SLR had been relaxed earlier, including for Treasury repos, the switch from the hold to the run equilibrium and the resulting market collapse could potentially have been avoided.

Of course, the SLR and other post-GFC regulatory constraints were introduced for good reason. However, the fact that these constraints interfered with intermediation in safe assets during times of stress seems like an unintended consequence. For future stress episodes, a temporary relaxation of the SLR would therefore improve market stability in our model. Indeed, relaxing the SLR in times of stress would be a way to mitigate the fragility emphasized in our paper while maintaining the stabilizing macroprudential consequences for which the regulation was designed.

Asset Purchase Facilities. Finally, our model provides a clear lens to understand how asset purchase facilities can be used to stabilize safe asset markets. Suppose the policy

[15] See Infante, Favara, and Rezende (2022) for a detailed study of the effect of the SLR on dealer's activity in the Treasury market.

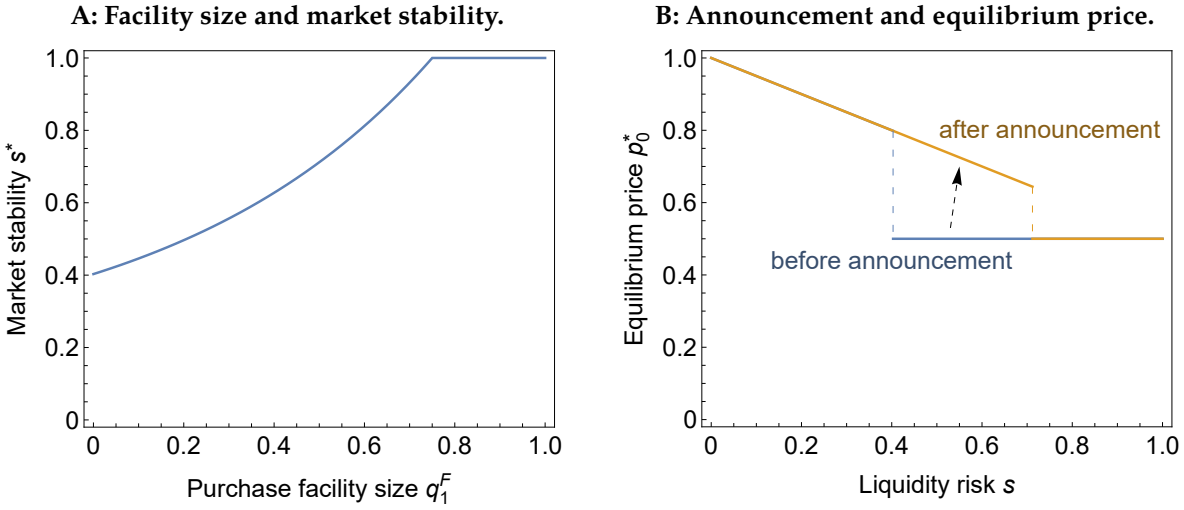

Figure 8: Effects of date-1 purchase facility announced at date 0. Panel A shows the effect of the facility size q_1^F on date-0 market stability as measured by the equilibrium threshold s^*. Panel B shows the effect of the announcement of a facility with $q_1^F = 0.5$ on the equilibrium price at date 0. Parameters: $v = 1.2,\ c = 0.5$.

maker announces at date 0 an asset purchase facility that will purchase a quantity q_1^F of the safe asset at date 1. This leaves demand at date 0 unchanged but adds to dealer demand at date 1, such that the payoff gain becomes

$$\pi(\lambda) = 1 - \frac{c}{2}\left(s + (1-s)\,\lambda\right) - \underbrace{s\left(cq_1^F + 1 - 2c\left(s + (1-s)\,\lambda\right) - \frac{c}{2}\,s\,(1-s)\,(1-\lambda)\right)}_{p_1^e \text{ affected by } q_1^F} - (1-s)\,v.$$

The payoff gain is uniformly decreasing in the size of the facility q_1^F, and this stabilizing effect, given by $\partial\pi/\partial q_1^F = -sc$, is larger (in absolute value) for both higher degrees of liquidity risk s and higher dealer balance sheet costs c.

Figure 8 illustrates the effects of the purchase facility. Panel A shows the effect of the facility size q_1^F on market stability at date 0 as measured by the equilibrium threshold s^*. Consistent with the stabilizing effect of q_1^F being increasing in liquidity risk s, we see that market stability is increasing and convex in q_1^F until s^* reaches 1 and the market is perfectly stable. Panel B of Figure 8 shows the announcement effect of a facility on the date-0 price p_0^*. Upon announcement, the equilibrium threshold s^* increases from the value without a facility, s^*_{pre}, to the value with a facility, $s^*_{\text{post}} > s^*_{\text{pre}}$. For intermediate levels of liquidity risk, $s \in \left[s^*_{\text{pre}}, s^*_{\text{post}}\right]$, the announcement leads to a switch from the run equilibrium to the

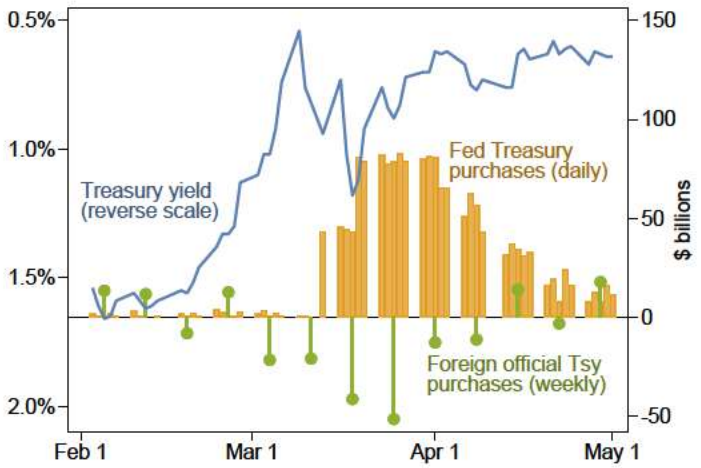

Figure 9: Federal Reserve purchases and foreign official sales of Treasuries. The figure shows the market yield on 10-year Treasuries as well as Federal Reserve Treasury purchases and foreign official agencies' net Treasury purchases inferred from changes in custody holdings. For details on the data see Appendix A.

hold equilibrium and therefore a discrete jump in the date-0 price.

Our theoretical results suggest that what matters for stabilizing a fragile market is the announcement more than the purchases directly. However, this result should be interpreted with some care especially when there is little time between the announcement and execution of purchases as was the case for the Treasury market in March 2020 (Vissing-Jørgensen, 2021). In this case, the purchases can be interpreted as falling into period 0 or into period 1 with potentially opposite effects as official sector purchases in period 0 can be destabilizing and trigger strategic sales in the same way that purchases from safety investors can (Section 3).

Figure 9 shows purchases of Treasuries by the Fed as well as net purchases of foreign official agencies, among the largest sellers of Treasuries in 2020q1 (Figure 1, Panel C). In early March, foreign net purchases started turning moderately negative for two weeks just as Treasury prices peaked, consistent with only non-strategic sales from investors with genuine liquidity needs. Then prices dropped and foreign sales accelerated as the Fed started purchasing Treasuries, consistent with switching to the run equilibrium, in which strategic investors without genuine liquidity needs preemptively sell, potentially amplified by the initial Fed purchases that may have been interpreted as limited in time and scale.[16]

[16]For its first purchase of roughly $40 billion on Friday, March 13, the Fed "brought forward about half of the Treasury purchases previously scheduled for the mid-March to mid-April period into one day" (Federal Reserve Bank of New York, 2021).

Only after the Fed roughly doubled its daily purchases on March 19 and then committed to maintaining them as long as necessary on March 23 did prices recover and foreign sales subside.[17] This is consistent with the market switching back to the hold equilibrium once investors were confident that they would not face worse prices in the future.

The corporate bond market provides a clean illustration of the announcement effects implied by our model. While corporate bonds are not considered as safe (or liquid) as Treasuries, highly rated ones are on a spectrum of relative safety slightly below agency MBS (He and Song, 2022) and also feature flight-to-safety (Baele et al., 2019). Haddad, Moreira, and Muir (2021) document that, in March 2020, prices of corporate bonds suffered a crash similar to that in Treasuries. Surprisingly, the dislocations were *worse* for bonds considered safer, which is consistent with safe asset fragility as shown in our model. Further, Haddad, Moreira, and Muir (2021) show in detail that the Fed's purchase facilities for corporate bonds had large positive effect on prices *at the time they were announced* in March even though purchases would not start until June (see also Boyarchenko, Kovner, and Shachar, 2022).

5 Conclusion

We focus on three key features of safe asset markets: investors who value the assets' safety, investors who value the assets' liquidity, and dealers who face balance sheet constraints. Combining these features, we show that safe asset markets can be fragile in that they are susceptible to sudden price crashes due to coordination effects among investors valuing liquidity that are amplified by investors valuing safety.

Our model helps us understand the unprecedented events in the U.S. Treasury market at the onset of the COVID-19 pandemic in March 2020 as a "perfect storm" of the three features: First, financial regulation in the wake of the GFC had significantly tightened dealer balance sheet constraints, increasing the inherent fragility of the market. Second, the pandemic threatened a global economic slowdown, leading to a powerful flight-to-safety demand, further destabilizing the market. Third, lockdowns created unprecedented liquidity needs among consumers and official agencies. The result, according to our model, was a market run that featured indiscriminate sales by liquidity investors, including those

[17] The statement by the Fed's Federal Open Market Committee (FOMC) on March 23 reads "The Federal Reserve will continue to purchase Treasury securities and agency mortgage-backed securities in the amounts needed to support smooth market functioning and effective transmission of monetary policy to broader financial conditions." Available at https://www.federalreserve.gov/newsevents/pressreleases/monetary20200323a.htm.

without genuine liquidity needs who feared having to sell at even worse conditions in the future.

The issues of dealer balance sheet constraints is almost surely only going to get worse over time as the federal deficit grows and Treasury supply increases. So long as dealers' balance sheet capacity grows more slowly than the stock of Treasuries, the market relying on dealer balance sheet capacity will have insufficient ability to intermediate trades (Duffie, 2020). Our model implies that this will exacerbate preemptive selling and increase the frequency of dash-for-cash episodes.

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Appendix

A Data

Treasury yield: The market yield on U.S. Treasury securities at 10-year constant maturity, daily frequency, from the Federal Reserve's H.15 via FRED series `DGS10`.

S&P 500: The S&P 500 index, daily frequency, from Standard & Poors via FRED series `SP500`.

Fed holdings of Treasuries: Federal Reserve outright holdings of Treasury notes and bonds (both nominal and TIPS), weekly frequency as of Wednesday, from the Federal Reserve's H.4.1 via FRED series `WSHONBNL` and `WSHONBIIL`.

Dealer net positions of Treasuries: Primary Dealers' net position in Treasuries (both nominal and TIPS) from the New York Fed's Primary Dealer statistics available at https://www.newyorkfed.org/markets/counterparties/primary-dealers-statistics.

Dealer reverse repo against Treasuries: Primary Dealers' gross reverse repurchase agreements against Treasuries (both nominal and TIPS), including other financing activity and securities borrowed, from the New York Fed's Primary Dealer statistics available at https://www.newyorkfed.org/markets/counterparties/primary-dealers-statistics.

Net purchases of Treasuries: Net purchases of Treasuries (all types), quarterly frequency (not seasonally adjusted), from the Federal Reserve's Financial Accounts Table FU.210 available in the CSV files at https://www.federalreserve.gov/releases/z1. The label "foreign investors" refers to the sector "rest of the world" in the original table.

Fed Treasury Purchases: Federal Reserve Treasury purchases (all types), daily frequency, from the New York Fed's Treasury securities operations, available at https://www.newyorkfed.org/markets/desk-operations/treasury-securities.

Foreign Official Treasury Purchases: Net Treasury purchases inferred from changes in Treasury securities held in custody for foreign officials and international accounts, weekly frequency as of Wednesday, from the Federal Reserve's H.4.1 via FRED series `WMTSECL1`.

Fed Treasury Repos: Federal Reserve Treasury repurchase agreements (overnight and term) in temporary open market operations, daily frequency, from the New York Fed via FRED series `RPTSYD`.

B Comparison to Bernardo and Welch (2004)

Our model deviates from Bernardo and Welch (2004), hereafter BW, in a few important details which allow us to apply global game methods in a model of regime shifts and to derive a state-contingent interaction between flight-to-safety and dash-for-cash.

The first difference between the two models is as follows: While dealers and investors both value the (risky) asset in BW at its expected value μ, dealers value the (safe) asset in our model at its par value of 1 while liquidity investors value the asset at $v > 1$ due to it's insurance properties. Such a difference in valuations is natural when thinking of a safe asset that conveys specific benefits to certain investors. In terms of modeling, the difference allows for the existence of a hold equilibrium under complete information in our model, while BW only have a mixed or a run equilibrium. The possibility of a hold equilibrium is both empirically plausible and technically important: Empirically plausible because we do not think that investors routinely sell assets preemptively during normal times as the BW model implies; technically important because it provides the second pure-strategy equilibrium that is necessary for a true model of regime shifts.

The second difference is that the BW model features only strategic substitutes while our model allows for strategic complementarities. The possibility of strategic complementarities is another necessary ingredient for a model of regime shifts as it allows for multiplicity of equilibria under complete information.

As a result of these differences, the unique equilibrium in the BW model features preemptive sales $\lambda^*(s)$ that are continuously increasing in the degree of liquidity risk s from $\lambda^*(0) = 0$ to $\lambda^*(\bar{s}) = 1$ at some $\bar{s} \leq 1$; therefore the date 0 price is continuously decreasing in s until $\bar{s}$ (and then constant). In contrast, the unique global game equilibrium in our model features preemptive sales that jump discontinuously from $\lambda^*(s) = 0$ for s below the switching point s^* to $\lambda^*(s) = 1$ for s above s^*; therefore the date 0 price drops precipitously as s crosses the threshold. Furthermore, preemptive sales in BW are *decreasing* in dealer balance sheet costs (due to the strategic substitutability) while fragility and therefore preemptive sales in our model are *increasing* in dealer balance sheet costs.

Finally, our model features safety investors whose demand for the asset has an ambiguous effect on liquidity investors' strategic sales. In the BW model, market depth at date 0 is destabilizing and market depth at date 1 is stabilizing but the net effect of more market depth at both dates does not vary with the degree of liquidity risk — it is either uniformly positive or uniformly negative. In contrast, our modeling of safety investors — who increase market depth both at date 0 and at date 1 — combined with our regime shift model of liquidity investors generates one of our key results: more market depth at both dates

is stabilizing if the market is relatively stable (high s^*) but destabilizing if the market is relatively unstable (low s^*).

C Proofs

Proof of Proposition 1. We can rewrite the payoff gain $\pi(\lambda)$ as

$$\pi(\lambda) = \frac{c}{2}s^2 + \frac{c}{2}s\left(s\left(4-s\right)-1\right) - \left(1-s\right)\left(v-1\right) + \frac{c}{2}\left(1-s\right)\left(s\left(4-s\right)-1\right)\lambda,$$

and differentiate with respect to λ to get

$$\pi'(\lambda) = \frac{c}{2}\left(1-s\right)\left(s\left(4-s\right)-1\right),$$

with $c > 0$, $s \in (0,1)$, $v > 1$, and $\lambda \in [0,1]$, which imply the following comparative statics:

- We have $\pi'(\lambda) > 0$ if and only if $s\left(4-s\right)-1 > 0$ which has one root in the unit interval given by $\widetilde{s} \equiv 2-\sqrt{3}$.
- Differentiating π with respect to s, we have

  $$\frac{\partial \pi}{\partial s} = \frac{c}{2}\left(\left(10-3s\right)\left(1-\lambda\right)s + \left(5\lambda-1\right)\right) + v - 1,$$

  which is positive unless λ is small. For $s > \frac{1}{3}\left(5-\sqrt{22}\right) \approx 0.103$, it is positive for all λ and therefore also for $s > 2-\sqrt{3}$.
- Differentiating π with respect to c, we have

  $$\frac{\partial \pi}{\partial c} = \frac{1}{2}s^2 + \frac{1}{2}\left(s\left(4-s\right)-1\right)\left(s+\left(1-s\right)\lambda\right),$$

  which is positive if $s\left(4-s\right)-1 > 0$.
- Differentiating π with respect to v, we have

  $$\frac{\partial \pi}{\partial v} = -\left(1-s\right) < 0$$

  which is negative. □

Proof of Proposition 2. In order to apply the standard global game result that there is a unique equilibrium and that it is in switching strategies, we have to show that the payoff gain $\pi(\lambda, s)$ satisfies certain properties (Morris and Shin, 2003). Proposition 1 establishes State Monotonicity and Action Monotonicity, that is $\pi(\lambda, s)$ is increasing in s and increasing in λ for $s > \widetilde{s}$, which is satisfied if there are multiple equilibria of the complete-information game. The payoff gain satisfies Strict Laplacian State Monotonicity since we have

$$\int_0^1 \pi(\lambda, s)\, d\lambda = \frac{c}{2}s^2 + \frac{c}{2}s\left(s\left(4-s\right)-1\right) - \left(1-s\right)\left(v-1\right) + \frac{c}{4}\left(1-s\right)\left(s\left(4-s\right)-1\right), \tag{16}$$

which satisfies

$$\int_0^1 \pi(\lambda, 0)\, d\lambda = -\left(v-1\right) - \frac{c}{4} < 0,$$

and

$$\int_0^1 \pi(\lambda, 1)\, d\lambda = \frac{3c}{2} > 0,$$

as well as

$$\frac{\partial}{\partial s}\int_0^1 \pi(\lambda, s)\, d\lambda = cs + \frac{c}{4}\left(\left(s\left(4-s\right)-1\right) + \left(1+s\right)\left(4-2s\right)\right) + \left(v-1\right) > 0,$$

for $s > \widetilde{s}$ and therefore a unique $s^* \in (\widetilde{s}, 1)$ solves $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$. Finally, $\pi(\lambda, s)$ satisfies Uniform Limit Dominance since we have

$$\pi(\lambda, 0) = -\left(v-1\right) - \frac{c}{2}\lambda < 0,$$

and

$$\pi(\lambda, 1) = \frac{3c}{2} > 0.$$

Under these properties, Morris and Shin (2003) show that, in the limit $\sigma_\varepsilon \to 0$, the global game has a unique equilibrium and that the equilibrium is in switching strategies around a threshold s^* defined by the indifference condition $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$ where distribution of λ conditional on signal $\widehat{s}_i = s^*$ is uniform on $[0, 1]$. □

Proof of Corollary 1. Implicit differentiation of the equilibrium condition $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$ using (16) yields

$$\frac{ds^*}{dc} = -\frac{\frac{1}{2}\left(s^*\right)^2 + \frac{1}{2}s^*\left(s^*\left(4-s^*\right)-1\right) + \frac{1}{4}\left(1-s^*\right)\left(s^*\left(4-s^*\right)-1\right)}{cs^* + \frac{c}{4}\left(\left(s^*\left(4-s^*\right)-1\right) + \left(1+s^*\right)\left(4-2s^*\right)\right) + \left(v-1\right)} < 0$$

and

$$\frac{ds^*}{dv} = \frac{1-s^*}{cs^* + \frac{c}{4}\left(\left(s^*\left(4-s^*\right)-1\right)+\left(1+s^*\right)\left(4-2s^*\right)\right)+\left(v-1\right)} > 0$$

as stated in the corollary. □

Proof of Proposition 3 First, the equilibrium threshold s^* satisfies $\int_0^1 \pi(\lambda, s^*)\, d\lambda = 0$. We can rewrite equation (16) as

$$\int_0^1 \pi(\lambda, s)\, d\lambda = \frac{c}{4}\left(-s^3 + 5s^2 + 3s - 1\right) - (1-s)\,(v-1)\,. \tag{17}$$

Second, we can rewrite the difference between welfare in the hold and run allocation in equation (12) as

$$\Delta(s) = (1-s)\left(\frac{c}{2}\left(s^3 - 5s^2 + s + 1\right) + (1-s)\,(v-1)\right), \tag{18}$$

which satisfies $\Delta(0) > 0$ and $\Delta(1) = 0$, and has one root in $(0,1)$ for $c > 0$ which we denote s^{**}.

We want to show that $\Delta(s^*) > 0$, which means that $s^* < s^{**}$ and therefore the hold allocation is better than the run allocation and yet investors play the run equilibrium in the global game for $s \in (s^*, s^{**})$. From equation (17), the equilibrium threshold s^* satisfies (writing s without the star for simplicity)

$$\frac{c}{4}\left(-s^3 + 5s^2 + 3s - 1\right) = (1-s)\,(v-1)\,.$$

Substituting this into (18), we can calculate $\Delta(s^*)$ and we have (writing s without the star for simplicity)

$$\begin{aligned}
\Delta(s^*) &\propto (1-s)\left(\frac{c}{2}\left(s^3 - 5s^2 + s + 1\right) + \frac{c}{4}\left(-s^3 + 5s^2 + 3s - 1\right)\right), \\
&\propto s^3 - 5s^2 + 5s + 1, \\
&= (1-s)\left(-s^2 + 4s - 1\right) + 2.
\end{aligned}$$

The first term has one root in $(0,1)$ given by $2 - \sqrt{3} \equiv \widetilde{s}$ and is strictly positive for $s \in (\widetilde{s}, 1)$. We have $s^* > \widetilde{s}$ from the proof of Proposition 2 and thus $\Delta(s^*) > 0$ and therefore $\Delta(s) > 0$ for all $s \in (s^*, s^{**})$. Thus, we have an inefficient region. □

Proof of Proposition 4. We can rewrite the payoff gain with additional demand as

$$\pi(\lambda) = \frac{c}{2}s^2 + \frac{c\,(a-b)}{1+bc}\,(1-2s) - (1-s)\,(v-1) + \frac{c}{2}\left(\frac{1}{1+bc}\,(4s-1) - s^2\right)(s+(1-s)\,\lambda)\,,$$

and differentiate with respect to a to obtain

$$\frac{\partial \pi}{\partial a} = \frac{c}{1+bc}\,(1-2s)\,,$$

and

$$\frac{\partial^2 \pi}{\partial s \partial a} = -\frac{2c}{1+bc}.$$

We therefore have $\partial\pi/\partial a > 0$ if and only if $s < 1/2$ as well as $\partial^2\pi/(\partial s \partial a) < 0$. □

Proof of Corollary 2. The global game threshold is defined by $\int_0^1 \pi(\lambda, s^*)\,d\lambda = 0$ and implicit differentiation yields

$$\frac{ds^*}{da} = -\frac{\int_0^1 \frac{\partial}{\partial a}\pi(\lambda, s^*)\,d\lambda}{\int_0^1 \frac{\partial}{\partial s^*}\pi(\lambda, s^*)\,d\lambda},$$

and therefore $ds^*/da > 0$ if and only if $s^* > 1/2$. □

D Pooled Trade Execution and Cubic Balance Sheet Costs

Suppose that instead of the quadratic balance sheet costs of the main text, we consider cubic balance sheet costs cq^3. With inventory q_0 the equilibrium condition at date 1 is given by

$$(1-p_0)\,q_0 + (1-p_1)\,q_1 - c\,(q_0+q_1)^3 = (1-p_0)\,q_0 - cq_0^3.$$

which results in demand

$$p_1(q_0, q_1) = 1 - c\left(3q_0^2 + 3q_0q_1 + q_1^2\right).$$

At date 0, the price p_0 at which dealers take on inventory q_0, anticipating the competition at date 1, is given by the equilibrium condition

$$(1-p_0)\,q_0 - cq_0^3 = 0,$$

which implies a demand given by

$$p_0(q_0) = 1 - cq_0^2.$$

With pooled trade execution, the expected price does not have the factor 1/2 of the average in-run price of the main text. Substituting in date-0 supply $q_0 = s + (1-s)\lambda$ and date-1 supply $q_1 = s(1-s)(1-\lambda)$, we have expected prices

$$\begin{aligned}
p_0^e(\lambda) &= 1 - c\left(s + (1-s)\lambda\right)^2 \\
p_1^e(\lambda) &= 1 - c\left(3\left(s + (1-s)\lambda\right)^2 + 3\left(s + (1-s)\lambda\right)\left(s(1-s)(1-\lambda)\right) + \left(s(1-s)(1-\lambda)\right)^2\right).
\end{aligned}$$

The payoff gain then is, as before

$$\pi(\lambda) = p_0^e(\lambda) - sp_1^e(\lambda) - (1-s)v$$

with derivatives given by

$$\begin{aligned}
\pi'(\lambda) &= c(1-s)\left(2s^4 - 8s^3 + 9s^2 - 2s - 2(1-s)^4\lambda\right) \\
\pi''(\lambda) &= -2c(1-s)^5
\end{aligned}$$

Since $\pi''(\lambda) < 0$ we have $\pi'(\lambda) > 0$ for all λ if $\pi'(1) > 0$. With

$$\pi'(1) = c(1-s)\left(-3s^2 + 6s - 2\right),$$

we have $\pi'(1) > 0$ iff $s > 1 - 1/\sqrt{3} \approx 0.42$.

In sum, with pooled trade execution and cubic balance sheet costs, we have strategic complementarities for $s > 1 - 1/\sqrt{3}$ (i.e. a threshold slightly higher than the threshold $\widetilde{s} = 2 - \sqrt{3}$ in the main text). Figure 10 illustrates the payoff gain and the resulting equilibria of the complete information game for different levels of liquidity risk (analogous to Figure 2 in the main text).

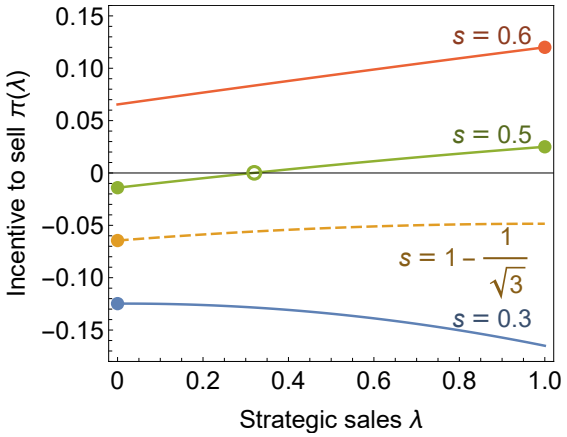

Figure 10: Pooled trade execution and cubic balance sheet costs. The figure shows the payoff gain $\pi(\lambda)$ for different values of liquidity risk s. Circles indicate equilibria of the game under complete information. Parameters: $v = 1.2,\ c = 0.25$.

E Case with Safety Investors Active at Both Dates

Suppose we have additional demand $q_0^S = a_0 - b_0 p_0$ at date 0 and $q_1^S = a_1 - b_1 p_1$ at date 1. Things are unchanged at date 0 with expected price

$$p_0^e(\lambda) = \frac{1 + a_0 c}{1 + b_0 c} - \frac{1}{2}\frac{c}{1 + b_0 c}\left(s + (1 - s)\,\lambda\right).$$

At date 1, dealers demand $q_1^D = \frac{1}{c}\left(1 - p_1\right) - 2q_0^D$ with inventory q_0^D as in the main text. With additional demand, total demand at date 1 can be written as

$$p_1(q_1) = \frac{1 + a_1 c - 2c q_0^D}{1 + b_1 c} - \frac{c}{1 + b_1 c} q_1.$$

With total supply $q_1 = s\,(1 - s)\,(1 - \lambda)$ substituting in dealer inventory

$$q_0^D = \frac{s + (1 - s)\,\lambda + b_0 - a_0}{1 + b_0 c}$$

we have an expected price

$$p_1^e(\lambda) = \frac{1 + a_1 c - 2c\left(\frac{s + (1-s)\lambda}{1 + b_0 c} - \frac{a_0 - b_0}{1 + b_0 c}\right)}{1 + b_1 c} - \frac{1}{2}\frac{c}{1 + b_1 c} s\,(1 - s)\,(1 - \lambda)$$

Collecting terms, we have expected prices given by

$$p_0^e(\lambda) = \frac{1 + \left(a_0 - \frac{1}{2}s\right)c}{1 + b_0 c} - \frac{1}{2}\frac{c}{1 + b_0 c}(1 - s)\lambda$$

$$p_1^e(\lambda) = \frac{1 + \left(a_1 - \frac{1}{2}s(1 - s)\right)c}{1 + b_1 c} + \frac{2c(a_0 - b_0 - s)}{(1 + b_1 c)(1 + b_0 c)} - \frac{1}{1 + b_1 c}\left(\frac{2c}{1 + b_0 c} - \frac{c}{2}s\right)(1 - s)\lambda$$

As before, a_0 has twice the effect on p_1^e as on p_0^e but p_1^e is discounted by s, so for a_0 to be stabilizing, we need $s > 1/2$. In contrast, a_1 only affects p_1^e, so it is always stabilizing.