Monetary Policy and Wealth Effects: The Role of Risk and Heterogeneity*

NICOLAS CARAMP
UC Davis

DEJANIR H. SILVA
Purdue University

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Abstract

We study the role of wealth effects, i.e. the revaluation of real and financial assets, in the monetary policy transmission mechanism. We build an analytical heterogeneous-agents model with three main ingredients: i) rare disasters; ii) heterogeneous portfolios; and iii) heterogeneous MPCs. The model captures time-varying risk premia and precautionary savings in a linearized setting that nests the textbook New Keynesian model. Quantitatively, the model matches the empirical response of asset prices. The effect of risk premia on the wealth effect is subtle: it affects households' net asset positions as well as the cost of the consumption bundle. The net effect depends on the government's liabilities but not on the equity premium. Thus, the transmission of monetary policy through asset prices crucially depends on the composition of government debt.

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*Caramp: UC Davis. Email: ncaramp@ucdavis.edu. Silva: Krannert School of Management, Purdue University. Email: dejanir@purdue.edu. We would like to thank Francesco Bianchi, Francisco Buera, Colin Cameron, James Cloyne, Bill Dupor, Gauti Eggertsson, Òscar Jorda, Rohan Kekre (discussant), Narayana Kocherlakota, Steffan Lewellen (discussant), Dmitry Mukhin, Benjamin Moll, Sanjay Singh, Jón Steinsson, Gianluca Violante (discussant) and seminar participants at EFA Annual Meeting, NBER Summer Institute, UC Berkeley, UC Davis, the Federal Reserve Bank of St. Louis, SED, and WFA Annual Meeting for their helpful comments and suggestions.

1 Introduction

A long tradition in monetary economics emphasizes the role of wealth effects, i.e. the revaluation of real and financial assets, in the economy's response to changes in monetary policy. Its importance can be traced back to both classical and Keynesian economists, such as Pigou, Patinkin, Metzler, and Tobin.[1] Keynes himself described the effects of interest rate changes as follows:

> There are not many people who will alter their way of living because the rate of interest has fallen from 5 to 4 per cent, if their aggregate income is the same as before. [...] Perhaps the most important influence, operating through changes in the rate of interest, on the readiness to spend out of a given income, *depends on the effect of these changes on the appreciation or depreciation in the price of securities and other assets*.
> - John Maynard Keynes, The General Theory of Employment, Interest, and Money (emphasis added).

A large empirical literature documents the impact of monetary policy on asset prices. Bernanke and Kuttner (2005) study the effects of monetary shocks on stock prices. Gertler and Karadi (2015) and Hanson and Stein (2015) consider the effects on bonds. Cieslak and Vissing-Jorgensen (2020) show that policymakers track the behavior of stock markets because of their impact on households' consumption, while Chodorow-Reich et al. (2021) establish the importance of this channel empirically.

Despite the evidence, relatively little work has been done to theoretically study the role of asset price fluctuations on the monetary transmission mechanism.[2] An important reason for this is that incorporating these channels represents a challenge to standard monetary models. A robust finding of the empirical literature is that changes in asset prices can be explained mainly by fluctuations in future excess returns, related to changes in the risk premia, rather than changes in the risk-free rate. However, the standard approach abstracts from risk premia and generates counterfactual asset-pricing dynamics. Moreover, models that feature richer asset-pricing dynamics require the use of complex global or high-order perturbation methods, which lack the insights of the role of the different channels of transmission provided by analytically tractable models.

In this paper, we propose a new framework that generates rich asset-pricing dynamics and heterogeneous portfolios while preserving the simplicity of the textbook New Keynesian model. We consider an economy populated by borrowers and savers with three

[1] The revaluation of government liabilities was central to Pigou (1943) and Patinkin (1965), while Metzler (1951) considered stocks and money. Tobin (1969) focused on how monetary policy interacted with the value of real assets.

[2] Important recent exceptions are Caballero and Simsek (2020) and Kekre and Lenel (2020).

main ingredients: i) rare disasters, ii) heterogeneous beliefs, and iii) heterogeneous MPCs. Rare disasters enable us to capture both a precautionary savings motive and realistic risk premia. Barro (2009) and Gabaix (2012) argue that the risk of a rare disaster can successfully explain major asset-pricing facts.[3] Savers invest in stocks, government bonds, and household debt, and have heterogeneous beliefs, as in Caballero and Simsek (2020). As a consequence, they hold heterogeneous portfolios in equilibrium. This allows us to capture *time-variation* in risk premia in response to monetary shocks. Borrowers are constrained in equilibrium, so borrowers and savers have heterogeneous MPCs. Borrowers play an important role, given that they account for the bulk of the response of aggregate consumption to changes in interest rates, as recently shown by Cloyne et al. (2020). Despite being stylized, the model captures quantitatively central aspects of the monetary transmission mechanism, including the term premium, the equity premium, and corporate spreads, as well as the differential responses of borrowers and savers to monetary shocks observed in the data.

Our first contribution is methodological and consists of an aggregation result. Given investor heterogeneity, we must characterize not only the dynamics of aggregate output and inflation, but also the behavior of portfolios, asset prices, net worth, and individual consumption. This increases the dimensionality of the problem and typically makes deriving analytical results infeasible. We show that our economy satisfies an *as if* result: the economy with heterogeneous savers behaves as an economy with a representative saver, but the probability of disaster, as implied by market prices, is time-varying and responds to monetary policy. This *market-implied disaster probability* is a key determinant of asset prices, and it is the main channel through which investor heterogeneity affects the real economy.

Our second contribution identifies conditions under which time-varying risk premia plays a role in the transmission of monetary policy to the real economy. Consistent with the empirical evidence, a contractionary monetary shock leads to an increase in risk premia and a reduction in the price of risky assets. One could then conclude that this reduction in households' wealth leads to a reduction in consumption. However, as the discount rate increases, the present discounted value of consumption decreases as well. The net effect of changes in risk premium is ambiguous and depends on whether households are net buyers or net sellers of risky assets. As recently articulated by Cochrane (2020) and Krugman (2021), a household who just consumes the dividends from their financial assets can still afford the same level of consumption after a change in discount rates. The

[3] Rare disasters have been widely used to explain a range of asset-pricing "puzzles"; see Tsai and Wachter (2015) for a review.

wealth effect should then be zero in this case.

Formally, we show that the aggregate wealth corresponds to the sum of all households' wealth *net* of the change in the cost of the original consumption bundle. Naturally, the aggregate wealth effect does not depend on private debt. While private debt matters for individual households' consumption, the gross positions cancel out when we aggregate at the household sector level. More interestingly, the aggregate wealth effect does not depend on the equity premium either. It turns out that the difference between the revaluation of the households' assets and liabilities (including consumption) is given by the government's liabilities. The intuition is simple: in a closed economy, only the government is a counterpart to the household sector taken as a whole.[4] Thus, whether risk affects the *aggregate* wealth effect depends on the characteristics of government debt. We show that, in the absence of a precautionary motive, there are three cases in which risk has no impact on aggregate wealth: *i*) when government debt is zero, *ii*) when government debt is short term, and *iii*) when government debt is a consol. In these cases, either the households' net revaluation effect is zero or it is independent of risk premia.

The presence of risk also affects the households' precautionary motives. This effect arises from the redistribution among savers after a monetary shock. Because optimists hold a larger fraction of their wealth in risky assets (long-term bonds and equity), an increase in the interest rate disproportionately reduces their wealth. Holding the *aggregate* wealth effect constant, this redistribution of wealth is then reflected in the market-implied probability of disaster, which increases after the monetary shock as pessimist savers increase their holdings of risky assets. This is the "as-if" result in action: redistribution between optimists and pessimists is akin to an increase in the "objective" probability of disaster risk in a model with a representative agent. Note that the precautionary savings channel changes the *timing* of consumption but not the households' aggregate wealth.

Putting together all these results, we obtain a complete characterization of the consumption channel of monetary policy in this model. We show that the transmission of monetary policy to aggregate consumption has two components, one that affects its *present value* and one that affects its *timing*. The present value of consumption is given by the aggregate wealth effect. The timing of consumption depends in a prominent way on private debt and aggregate risk. For private debt, the intuition is that monetary policy redistributes between borrowers and savers. Because borrowers and savers have different MPCs with respect to transitory income shocks, a contractionary monetary policy reduces aggregate consumption on impact. However, because all households in the economy have an MPC of one for *permanent* changes in their income, savers eventually increase

[4] In an open economy, the foreign sector would be an additional counterpart.

their consumption so that the present value of the changes cancel out. For aggregate risk, while precautionary savings increase on impact, they gradually decrease as the market-implied risk in the economy transitions back to its steady-state level. The present value of this effect is also zero.

In the absence of an aggregate wealth effect, monetary policy has then only a limited effect on the economy. A reduction in interest rates stimulates the economy in the present at the expense of a more depressed economy in the future. We also show that the central bank is unable to affect inflation when the wealth effect is zero. Moreover, future inflation rates respond *positively* to changes in nominal interest rates in this case. Therefore, the central bank's ability to stimulate the economy and control inflation is tightly connected to its ability to generate aggregate wealth effects.

Finally, our solution method allows us to obtain time-varying risk premia in a linearized setting and provide a complete analytical characterization of the channels involved. The method consists on perturbing the economy around a stationary equilibrium with *positive aggregate risk* instead of adopting the more common approach of approximating around a non-stochastic steady state. By perturbing around the stochastic stationary equilibrium, we are able to obtain time variation in precautionary motives and risk premia using a first-order approximation, while the standard approach would require a third-order approximation (see e.g. Andreasen 2012).[5] This hybrid approach can prove useful in other settings where capturing risk premia is important. It is well known that business cycle fluctuations in TFP cannot generate large risk premia without assuming implausible large risk aversion (see Mehra and Prescott, 1985). Disaster risk has been successful on this front, and our method shows how to incorporate it into rich macroeconomic models without sacrificing tractability.

Our calibration departs from the standard practice in three important ways. First, we set the households' intertemporal elasticity of substitution to 0.25 (which implies a risk aversion coefficient of 4 given our CRRA specification). This choice is lower than the usual value of 1 or 0.5. However, our choice is closer to recent studies using microdata, such as Best et al. (2020) who find a value of 0.1. Second, we need to calibrate the parameters associated with the disaster risk. For the parameters governing the steady-state levels, we follow Barro (2009). This implies an annual probability of a disaster of 1.7%. For the time-varying component of the risk premium, we calibrate the elasticity of the disaster shock to monetary policy to match the initial response of the term premium in Gertler and

[5]Moreover, by linearizing around an economy with zero monetary risk, we are able to solve for the stochastic stationary equilibrium in closed form, avoiding the need to compute the risky steady state numerically, as in Coeurdacier et al. (2011).

Karadi (2015). We show that this calibration generates a conditional equity premium and corporate spread that is consistent with the literature. Finally, for the fiscal response to a monetary shock, we augment the procedure in Christiano et al. (1999) to incorporate fiscal variables. We use the yield on the 5-year government bond to compute the government's intertemporal budget constraint.

To quantify the importance of the channels present in the model, we start with the standard RANK model and add risk and household debt one at a time. We find that the forces in RANK explain less than 20% of the consumption response on impact to a monetary shock, risk explain around 50%, household slightly more than 20%, and the interaction of the two slightly less than 10% Thus, risk and household debt are crucial components of the monetary transmission mechanism.

Literature review. Wealth effects have a long tradition in monetary economics. Pigou (1943) relied on a wealth effect to argue that full employment could be reached even in a liquidity trap. Kalecki (1944) argued that these effects apply only to government liabilities, as inside assets cancel out in the aggregate, while Tobin highlighted the role of private assets and high-MPC borrowers.[6] Recently, wealth effects have regained relevance. In an influential paper, Kaplan et al. (2018) build a quantitative HANK model and find only a minor role for the standard intertemporal-substitution channel, leading the way to a more important role for wealth effects. Much of the literature has focused on the role of heterogeneous marginal propensities to consume (MPCs) in settings with idiosyncratic income risk. Instead, our focus is on aggregate risk and private debt.

Our work is closely related to two strands of literature. First, it relates to the analytical HANK literature, such as Werning (2015), Debortoli and Galí (2017), and Bilbiie (2018). While this literature focuses primarily on how the cyclicality of income interacts with differences in MPCs, we focus instead on how heterogeneous asset positions interact with differences in MPCs. We see these two channels as mostly complementary: even though Cloyne et al. (2020) does not find significant differences in income sensitivity across borrowers and savers, Patterson (2019) finds a positive covariance between MPCs and the sensitivity of earnings to GDP across different demographic groups, suggesting that the income-sensitivity channel is operative for a different cut of the data. We share with Eggertsson and Krugman (2012) and Benigno et al. (2020) the emphasis on private debt,

[6] Tobin (1982) describes the role of inside assets: "The gross amount of these 'inside' assets was and is orders of magnitude larger than the net amount of the base. Aggregation would not matter if we could be sure that the marginal propensities to spend from wealth were the same for creditors and debtors. But if the spending propensity were systematically greater for debtors, even by a small amount, the Pigou effect would be swamped by this Fisher effect."

but they abstract from a precautionary motive and focus instead on the implications of deleveraging. Iacoviello (2005) also considers a monetary economy with private debt but focuses instead on the role of housing as collateral. Our work is also related to Auclert (2019), which studies the redistribution channel of monetary policy arising from portfolio heterogeneity. Our paper emphasizes the redistribution channel in the context of a general equilibrium setting with aggregate risk.

The paper is also closely related to work on how monetary policy affects the economy through changes in asset prices, including models with sticky prices, such as Caballero and Simsek (2020), and models with financial frictions, such as Brunnermeier and Sannikov (2016) and Drechsler et al. (2018). In recent contributions, Kekre and Lenel (2020) consider the role of the marginal propensity to take risk in determining the risk premium and shaping the response of the economy to monetary policy, and Campbell et al. (2020) use a habit model to study the role of monetary policy in determining bond and equity premia. Our model highlights instead the role of heterogeneous MPCs, positive private liquidity, and disaster risk in an analytical framework that preserves the tractability of standard New Keynesian models.

Finally, a recent literature studies rare disasters and business cycles. Gabaix (2011) and Gourio (2012) consider a real business cycle model with rare disasters, while Andreasen (2012) and Isoré and Szczerbowicz (2017) allow for sticky prices. They focus on the effect of changes in disaster probability while we study monetary shocks in an analytical HANK model with rare disasters.

2 D-HANK: A Rare Disasters Analytical HANK Model

In this section, we consider an analytical HANK model with three main ingredients: i) the possibility of rare disasters, ii) heterogeneous portfolios, and iii) heterogeneous MPCs. First, we describe the non-linear model and later consider a log-linear approximation around a stochastic stationary equilibrium.

2.1 The Model

Environment. Time is continuous and denoted by $t \in \mathbb{R}_+$. The economy is populated by households, firms, and a government. There is a continuum of households which can be of three types: *borrowers*, *optimistic savers*, and *pessimistic savers*, who differ in their discount rates and their beliefs about the probability of aggregate shocks. Households can borrow or lend at a riskless rate, but they are subject to a borrowing constraint.

Firms can produce final or intermediate goods. Final-goods producers operate competitively and combine intermediate goods using a CES aggregator with elasticity $\epsilon > 1$. Intermediate-goods producers use labor as their only input and face quadratic (Rotemberg 1982) pricing adjustment costs. Intermediate-goods producers are subject to an aggregate productivity shock: with Poisson intensity $\overline{\lambda} \geq 0$, they receive a shock that permanently reduces their productivity. This shock is meant to capture the possibility of rare disasters: low-probability, large drops in productivity and output, as in the work of Barro (2006, 2009). We say that periods that predate the realization of the shock are in the *no-disaster state*, and periods that follow the shock are in the *disaster state*. The disaster state is absorbing, and there are no further shocks after the disaster is realized. Assuming an absorbing disaster state simplifies the presentation, but it can be easily relaxed, as shown in Appendix **??**.[7]

The government sets fiscal policy, comprising a corporate profit tax and transfers to borrowers, and monetary policy, specified by an interest rate rule subject to a sequence of monetary shocks. The government issues long-term nominal bonds that pay exponentially decaying coupons. We denote by $Q_{L,t}e^{-\psi_L t}$ the nominal price of the bond in the no-disaster state, which pays coupons $e^{-\psi_L s}$ for all dates $s \geq t$. We denote by $Q^*_{L,t}$ the corresponding (normalized) price of the bond in the disaster state, where the star superscript is used throughout the paper to denote variables in the disaster state. The rate of decay ψ_L is inversely related to the bond's duration, where a perpetuity corresponds to $\psi_L = 0$ and the limit $\psi_L \to \infty$ corresponds to the case of short-term bonds.

Savers' problem. Savers face a portfolio problem where they choose how much to invest in short-term bonds, long-term bonds, and corporate equity. In this section, we assume that households issue only short-term risk-free bonds and the government issues only long-term bonds. We study the case of defaultable long-term household debt in Section 5. The nominal return on the long-term bond is given by[8]

$$dR_{L,t} = \left[\frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L\right] dt + \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} d\mathcal{N}_t,$$

where $\mathcal{N}_t$ is a Poisson process with arrival rate $\overline{\lambda}$ (under the objective measure).

The price of a claim on real aggregate corporate profits is denoted by $Q_{E,t}$ and the

[7] Allowing for partial recovery, as in Barro et al. (2013) and Gourio (2012), introduces dynamics in the disaster state, but it does not change the main implications for the no-disaster state, which is our focus.

[8] This expression follows from $dR_{L,t} = \frac{e^{-\psi_L t}}{Q_{L,t}e^{-\psi_L t}}dt + \frac{d(Q_{L,t}e^{-\psi_L t})}{Q_{L,t}e^{-\psi_L t}}$ and $dQ_{L,t} = \dot{Q}_{L,t}dt + (Q^*_{L,t} - Q_{L,t})d\mathcal{N}_t$.

cumulative (real) return on equities evolves according to

$$dR_{E,t} = \left[\frac{\Pi_t}{Q_{E,t}} + \frac{\dot{Q}_{E,t}}{Q_{E,t}}\right] dt + \frac{Q^*_{E,t} - Q_{E,t}}{Q_{E,t}} d\mathcal{N}_t,$$

where Π_t denotes real profits and $Q^*_{E,t}$ is the equity price in the disaster state.

Importantly, savers have heterogeneous beliefs regarding the probability of a disaster. Savers' subjective beliefs about the arrival rate of the aggregate productivity shock are given by λ_j, for $j \in \{o, p\}$, where we assume that $\lambda_o \leq \lambda_p$. We follow e.g. Chen et al. (2012) and assume that savers are dogmatic in their beliefs about disaster risk, so we abstract from any learning process. We also assume that $\rho_o - \rho_p = \lambda_p - \lambda_o$, which ensures that both types of savers are unconstrained in the long run.

Savers face a constant hazard rate of death $\xi \geq 0$. Newborn savers inherit the wealth from parents and they are optimistic with probability $\frac{\mu_o}{\mu_o+\mu_p}$ and pessimistic with probability $\frac{\mu_p}{\mu_o+\mu_p}$. Let $C_{j,t}(s)$ denote the time t consumption of a type-j saver born at date $s \leq t$ and $\overline{C}_{j,t} = \int_{-\infty}^{t} \xi e^{-\xi(t-s)} C_{j,t}(s) ds$ denote average consumption of type-j savers, where similar notation applies to other variables. For ease of notation, we often drop the dependence on s and simply write $C_{j,t}$ instead of $C_{j,t}(s)$.[9]

Let $B_{j,t} = B^S_{j,t} + B^L_{j,t} + B^E_{j,t}$ denote the net worth of a type-j saver, the sum of short-term bonds ($B^S_{j,t}$), long-term bonds ($B^L_{j,t}$), and equity holdings ($B^E_{j,t}$). A type-j saver chooses consumption $C_{j,t}$, long-term bonds $B^L_{j,t}$, and equity holdings $B^E_{j,t}$, given an initial net worth $B_{j,t} > 0$, to solve the following problem:

$$V_{j,t}(B_{j,t}) = \max_{[C_{j,z}, B^L_{j,z}, B^E_{j,z}]_{z \geq t}} \mathbb{E}_{j,t}\left[\int_t^{t^*} e^{-\rho_j(z-t)} \frac{C_{j,z}^{1-\sigma}}{1-\sigma} dz + e^{-\rho_j(t^*-t)} V^*_{j,t^*}(B^*_{j,t^*})\right],$$

subject to the flow budget constraint

$$dB_{j,t} = \left[(i_t - \pi_t) B_{j,t} + r_{L,t} B^L_{j,t} + r_{E,t} B^E_{j,t} - C_{j,t}\right] dt + \left[B^L_{j,t} \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} + B^E_{j,t} \frac{Q^*_{E,t} - Q_{E,t}}{Q_{E,t}}\right] d\mathcal{N}_t,$$

as well as borrowing and short-selling constraints

$$B_{j,t} \geq -D_P, \qquad B^L_{j,t} \geq 0, \qquad B^E_{j,t} \geq 0,$$

where i_t is the nominal interest rate, π_t is the inflation rate, $r_{L,t} \equiv \frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L - i_t$ is the

[9] The perpetual youth assumption pins down the long-run wealth distribution among optimistic and pessimistic savers, but it is otherwise not central to our results.

excess return on long-term bonds conditional on no disasters, and $r_{E,t} \equiv \frac{\Pi_t}{Q_{E,t}} + \frac{\dot{Q}_{E,t}}{Q_{E,t}} - (i_t - \pi_t)$ is the excess return on equities conditional on no disasters. The random (stopping) time t^* represents the period in which the aggregate shock hits the economy. $V^*_{j,t^*}(\cdot)$ and B^*_{j,t^*} denote, respectively, the value function and net worth in the disaster state. The savers' problem in the disaster state corresponds to a deterministic version of the problem above, as the disaster happens only once. The non-negativity constraint on $B^L_{j,t}$ captures the assumption that only the government can issue long-term bonds. The discount rate for savers can be written as $\rho_j \equiv \tilde{\rho}_j + \xi$, where $\tilde{\rho}_j$ captures subjective discounting and ξ captures the effect of mortality risk.

Savers are unconstrained at all times in equilibrium. The Euler equation for short-term bonds is given by

$$\frac{\dot{C}_{j,t}}{C_{j,t}} = \sigma^{-1}(i_t - \pi_t - \rho_j) + \frac{\lambda_j}{\sigma}\left[\left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma} - 1\right], \tag{1}$$

where $C^*_{j,t}$ is the consumption of a type-j saver in the disaster state. The first term captures the usual intertemporal-substitution force present in RANK models. The second term captures the *precautionary savings motive* generated by the disaster risk, and it is analogous to the precautionary motive that emerges in HANK models with idiosyncratic risk.

The Euler equation for long-term bonds, if $B^L_{j,t} > 0$, is given by

$$r_{L,t} = \underbrace{\lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma}}_{\substack{\text{price of} \\ \text{disaster risk}}} \underbrace{\frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}}}_{\substack{\text{quantity of} \\ \text{risk}}}. \tag{2}$$

This expression captures a risk premium on long-term bonds, which pins down the level of long-term interest rates in equilibrium. The premium on long-term bonds is given by the product of the *price of disaster risk*, the compensation for a unit exposure to the risk factor, and the *quantity of risk*, the loss the asset suffers conditional on switching to the disaster state.

Similarly, the Euler equation for equities, if $B^E_{j,t} > 0$, is given by

$$r_{E,t} = \lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma} \frac{Q_{E,t} - Q^*_{E,t}}{Q_{E,t}}. \tag{3}$$

The expression above pins down the (conditional) equity premium in equilibrium. As stocks and long-term bonds are exposed to the same aggregate shock, average returns by unit of risk (the price of risk) is the same for both assets. Differences in expected returns are then driven by differences in the quantity of risk.

Borrowers' problem. In contrast to savers, borrowers supply labor and have GHH preferences (Greenwood et al., 1988) over consumption and labor. Their problem is given by

$$V_{b,t}(B_{b,t}) = \max_{[C_{b,z},N_{b,z}]_{z\geq t}} \mathbb{E}_{b,t}\left[\int_t^{t^*} \frac{e^{-\rho_b(z-t)}}{1-\sigma}\left(C_{b,z} - \frac{N_{b,z}^{1+\phi}}{1+\phi}\right)^{1-\sigma} dz + e^{-\rho_b(t^*-t)} V^*_{b,t^*}(B_{b,t^*})\right],$$

subject to the flow budget constraint

$$dB_{b,t} = \left[(i_t - \pi_t)B_{b,t} + \frac{W_t}{P_t}N_{b,t} + \widetilde{T}_{b,t} - C_{b,t}\right]dt,$$

and the borrowing constraints $B_{b,t} \geq -D_P$, where W_t is the nominal wage, P_t is the price level, and $\widetilde{T}_{b,t}$ denote fiscal transfers to borrowers.

We focus on the case where the initial condition is $B_{b,0} = -D_P$ and ρ_b is sufficiently large, so borrowers are constrained at all periods. For simplicity, we have already imposed that $B^S_{b,t} = B^E_{b,t} = 0$, given that short-selling constraints would otherwise be binding if borrowers could choose $B^S_{b,t}$ and $B^E_{b,t}$. As borrowers are constrained, their beliefs about the disaster probability play no role in the determination of equilibrium.

The labor supply is determined by the standard condition:

$$\frac{W_t}{P_t} = N^{\phi}_{j,t}.$$

GHH preferences imply that there is no income effect on labor supply, roughly in line with the evidence (see e.g. Auclert et al., 2021), and simplifies the model aggregation.[10]

Market-implied probabilities and the SDF. From Equations (2) and (3), we can see that, even though savers disagree on the probability of a disaster, they agree on the *value* of a unit of consumption in that state. We can then price any cash flow using the beliefs and marginal utility of either optimistic or pessimistic savers. Instead of using the beliefs of a specific saver, it will be convenient to define the economy's stochastic discount factor

[10]GHH preferences also avoid the counterfactual implications caused by income effects on labor supply in heterogeneous-agent models with sticky prices emphasized by Broer et al. (2020).

(SDF) using the aggregate consumption of savers, $C_{s,t} \equiv \frac{\mu_o}{\mu_o+\mu_p}\overline{C}_{o,t} + \frac{\mu_p}{\mu_o+\mu_p}\overline{C}_{p,t}$, and the corresponding disaster probability implied by asset prices, as shown in Proposition 1.[11]

Proposition 1 (Disaster probability). *Define the market-implied disaster probability λ_t as follows:*

$$\lambda_t \equiv \left[\frac{\mu_o\overline{C}_{o,t}}{\mu_o\overline{C}_{o,t}+\mu_p\overline{C}_{p,t}}\lambda_o^{\frac{1}{\sigma}} + \frac{\mu_p\overline{C}_{p,t}}{\mu_o\overline{C}_{o,t}+\mu_p\overline{C}_{p,t}}\lambda_p^{\frac{1}{\sigma}}\right]^{\sigma}, \tag{4}$$

and let $\mathbb{E}_t[\cdot]$ denote the expectation operator associated with the arrival rate λ_t for the disaster shock. Then, $\eta_t = e^{-\int_0^t \rho_{s,z}dz}C_{s,t}^{-\sigma}$ is a valid stochastic discount factor, i.e., η_t correctly prices all tradeable assets given the disaster probability λ_t and the process $\rho_{s,t}$.

Proof. To ensure that η_t correctly prices long-term bonds and equities, consistent with Equations (2) and (3), the market-implied disaster probability must satisfy the condition:

$$\lambda_t\left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma} = \lambda_j\left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma} \Rightarrow C^*_{j,t} = \left(\frac{\lambda_j}{\lambda_t}\right)^{\frac{1}{\sigma}}\frac{C^*_{s,t}}{C_{s,t}}C_{j,t}.$$

Plugging $C^*_{j,t}$ into the definition of savers' average consumption in the disaster state, $C^*_{s,t} \equiv \frac{\mu_o}{\mu_o+\mu_p}\overline{C}^*_{o,t} + \frac{\mu_p}{\mu_o+\mu_p}\overline{C}^*_{p,t}$, and rearranging gives Equation (4). By setting $\rho_{s,t} = \rho_j + \lambda_j - \lambda_t$, we ensure that η_t correctly prices risk-free bonds, i.e., $\mathbb{E}_t[d\eta_t]/\eta_t = -(i_t - \pi_t)dt$. □

The market-implied probability is a CES aggregator of individual probabilities, weighted by the corresponding consumption share. Expression (4) is reminiscent of the complete-markets formula with heterogeneous beliefs in e.g. Varian (1985). Under complete markets, individual probabilities are weighted by (constant) Pareto weights. In contrast, consumption shares can potentially move over time in our setting which leads to endogenous *time-variation* in the perceived probability of disaster, even though the objective probability of disaster is constant in this economy.

Firms' problem. Intermediate-goods producers are indexed by $i \in [0,1]$ and operate in monopolistically competitive markets. Final good producers are price takers and combine intermediate goods to produce the final good. Their demand for variety i is given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon}Y_t$, and the equilibrium price level is given $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon}di\right)^{\frac{1}{1-\epsilon}}$.

Intermediate-goods producers operate the linear technology $Y_{i,t} = A_tN_{i,t}$. Productivity in the no-disaster state is given by $A_t = A$, and productivity in the disaster state is

[11] A long tradition in asset-pricing relates the consumption of stockholders, savers in our economy, and asset prices. See e.g. Mankiw and Zeldes (1991) and Parker (2001).

given by $A_t = A^*$, where $0 < A^* < A$. Intermediate-goods producers choose the *rate-of-change* of prices $\pi_{i,t} = \dot{P}_{i,t}/P_{i,t}$, given the initial price $P_{i,0}$, to maximize the expected discounted value of real (after-tax) profits subject to Rotemberg quadratic adjustment costs:

$$Q_{i,t}(P_{i,t}) = \max_{[\pi_{i,z}]_{z\geq t}} \mathbb{E}_t \left[\int_t^{t^*} \frac{\eta_z}{\eta_t}(1-\tau_t) \left(\frac{P_{i,z}}{P_z} Y_{i,z} - \frac{W_z}{P_z}\frac{Y_{i,z}}{A} - \frac{\varphi}{2}\pi_{i,t}^2 \right) dz + \frac{\eta_{t^*}}{\eta_t} Q^*_{i,t^*}(P_{i,t^*}) \right], \tag{5}$$

subject to the demand $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, where $Q^*_{i,t}(P_i)$ denotes the firms' value function in the disaster state. The price $P_{i,t}$ is a state variable in the firms' problem and $\pi_{i,t}$ is a control variable. The parameter φ controls the magnitude of the pricing adjustment costs. We assume that these costs are rebated to households, so they do not represent real resource costs. Profits are discounted using the economy's SDF and expectations are computed using the market-implied probability λ_t, which is consistent with savers' own valuation.

Combining the first-order condition and the envelope condition for problem (5), we obtain the non-linear New Keynesian Phillips curve:

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t} \right) \pi_t - \frac{\epsilon}{\varphi A} \left(\frac{W_t}{P_t} - (1-\epsilon^{-1})A \right) Y_t, \tag{6}$$

assuming a symmetric initial condition $P_{i,0} = P_0$, for all $i \in [0,1]$, and $\pi^*_{i,t} = 0$.

Government. The government is subject to a flow budget constraint

$$\dot{D}_{G,t} = (i_t - \pi_t + r_{L,t}) D_{G,t} + \mu_b \widetilde{T}_{b,t} - \tau_t \left(Y_t - \frac{W_t}{P_t} N_t \right),$$

and a No-Ponzi condition $\lim_{t\to\infty} \mathbb{E}_0[\eta_t D_{G,t}] \leq 0$, where $D_{G,t}$ denotes the real value of government debt, $D_{G,0} = D_G$ is given, and analogous conditions hold in the disaster state. Transfers to borrowers are determined by the policy rule $\widetilde{T}_{b,t} = \widetilde{T}_b(Y_t)$, where the elasticity of $\widetilde{T}_b(\cdot)$ determines the cyclicality of government transfers to borrowers.

In the no-disaster state, monetary policy is determined by the policy rule

$$i_t = r_n + \phi_\pi \pi_t + u_t, \tag{7}$$

where $\phi_\pi > 1$, u_t is a monetary shock, and r_n denotes the real rate when $\pi_t = u_t = 0$ at all periods. We assume that in the disaster state there are no monetary shocks, that is, $i^*_t = r^*_n + \phi_\pi \pi^*_t$. By abstracting from the policy response after a disaster, we isolate the impact of changes in monetary policy during "normal times."

Market clearing. The market-clearing conditions for goods, bonds, and equities are given by

$$\sum_{j\in\{b,o,p\}} \mu_j \overline{C}_{j,t} = Y_t, \qquad \sum_{j\in\{b,o,p\}} \mu_j \overline{B}^S_{j,t} = 0, \qquad \sum_{j\in\{b,o,p\}} \mu_j \overline{B}^L_{j,t} = D_{G,t}, \qquad \sum_{j\in\{b,o,p\}} \mu_j \overline{B}^E_{j,t} = Q_{E,t},$$

and labor market clearing is $\mu_b N_{b,t} = N_t$, where $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon-1}} di\right)^{\frac{\epsilon-1}{\epsilon}}$ and $N_t = \int_0^1 N_{i,t} di$.

2.2 Equilibrium dynamics

Stationary equilibrium. We define a stationary equilibrium as an equilibrium in which all variables are constant in each aggregate state. In particular, the economy will be in a stationary equilibrium in the absence of monetary shocks, that is, $u_t = 0$ for all $t \geq 0$. Since variables are constant in each state, we drop time subscripts and write, for instance, $C_{j,t} = C_j$ and $C^*_{j,t} = C^*_j$. For ease of exposition, we follow Bilbiie (2018) and assume that $\tilde{T}_b$ implements $C_b = Y$ and $C^*_b = Y^*$, and discuss the general case in the appendix.

The natural interest rate, the real rate in the stationary equilibrium, is given by

$$r_n = \rho_s - \lambda \left[\left(\frac{C_s}{C^*_s} \right)^\sigma - 1 \right],$$

where $0 < C^*_s < C_s$, $\rho_s \equiv \rho_j + \lambda_j - \lambda$, and we assume that $r_n > 0$.

The precautionary motive depresses the natural interest rate relative to the one that would prevail in a non-stochastic economy, and the magnitude depends on the extent to which savers can self-insure. In particular, everything else constant, a higher level of private debt D_P implies a weaker precautionary motive and a higher natural interest rate.

From Equation (2), we obtain the term spread, the difference between the yield on the long-term bond and the short-term rate,

$$i_L - r_n = \lambda \left(\frac{C_s}{C^*_s} \right)^\sigma \frac{Q_L - Q^*_L}{Q_L},$$

where i_L is the yield on the long-term bond in the stationary equilibrium.[12] We show in Appendix **??** that the term spread $i_L - r_n$ is strictly positive. Thus, our model generates an upward-sloping yield curve, where the yield on the long-term bond exceeds the natural (short-term) rate, consistent with the data.[13]

[12]Note that the yield on the bond is given by $i_{L,t} = Q^{-1}_{L,t} - \psi_L$ and, in a stationary equilibrium, the expected excess return conditional on no disaster r_L equals the term spread $i_L - r_n$.

[13]The upward-sloping yield curve is caused by the lack of precautionary savings in the disaster state.

Similarly, the equity premium (conditional on no-disaster) is given by[14]

$$r_E = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_E - Q_E^*}{Q_E},$$

where $Q_E^* < Q_E$. Therefore, the equity premium is positive in the stationary equilibrium.

Households have heterogeneous portfolios in equilibrium. Borrowers are against the borrowing constraint and hold no equities or long-term bonds. Optimistic savers are more exposed to disaster risk than pessimist investors. The exact composition of the portfolio of each saver is indeterminate, as we have one redundant asset. For concreteness, we focus on the case $B_o^E = B_p^E$, so optimists are more exposed to long-term bonds i.e. $B_o^L > B_p^L$. This leads to a simpler presentation in the analysis that follows.

Log-linear dynamics. Following the practice in the literature on monetary policy, we focus on a log-linear approximation of the equilibrium conditions. However, instead of linearizing around the non-stochastic steady state, we linearize the equilibrium conditions around the (stochastic) symmetric stationary equilibrium described above. Formally, we perturb the allocation around the economy where $u_t = 0$ and $\lambda > 0$, while the standard approach would perturb around the economy where $u_t = \lambda_t = 0$. This enables us to capture the effects of (time-varying) precautionary savings and risk premia in a linear setting, as shown below.[15]

Let lower-case variables denote log-deviations from the stationary equilibrium, e.g., $y_t \equiv \log Y_t / Y$ and $c_{j,t} \equiv \log \overline{C}_{j,t} / \overline{C}_j$. Borrowers' consumption is given by

$$c_{b,t} = (1-\alpha)(w_t - p_t + n_{b,t}) + T_{b,t} - (i_t - \pi_t - r_n)d_P, \tag{8}$$

where $1 - \alpha \equiv \frac{WN_b}{PY}$, $T_{b,t} \equiv \frac{\widetilde{T}_{b,t} - \widetilde{T}_b}{Y}$, and $d_P \equiv \frac{D_P}{Y}$. Given $T_{b,t} = T_b'(Y)y_t$ and using the facts $w_t - p_t = \phi y_t$ and $n_{b,t} = y_t$, we obtain

$$c_{b,t} = \chi_y y_t - (i_t - \pi_t - r_n)d_P, \tag{9}$$

where $\chi_y \equiv T_b'(Y) + (1-\alpha)(1+\phi)$. The coefficient χ_y controls the cyclicality of income

We would obtain similar results by introducing expropriation and inflation in a disaster, as in Barro (2006).

[14] The unconditional equity premium equals r_E minus the expected loss on a disaster. Using λ to compute the expected loss, the (unconditional) equity premium would be given by $\lambda \left[\left(\frac{C_s}{C_s^*}\right)^{\sigma} - 1\right] \frac{Q_E - Q_E^*}{Q_E}$.

[15] This method differs from the procedure considered by Coeurdacier et al. (2011) or Fernández-Villaverde and Levintal (2018), as we linearize around a stochastic steady state of an economy with no monetary shocks, instead of the stochastic steady state of the economy with both shocks.

inequality among borrowers and savers and its role has been extensively studied by the literature on analytical HANK models. We focus throughout the paper on the case in which $0 < \chi_y < \mu_b^{-1}$, such that the consumption of both agents increases with y_t.[16] The second term is not present in the commonly studied case of zero private liquidity, $d_P = 0$, and it captures the impact of monetary policy on the consumption of borrowers through changes in borrowing costs. This term plays an important role in the analysis that follows.

Next, consider the savers' problem. Linearizing Equation (1) and aggregating across savers, we obtain

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^{\sigma}\left[\hat{\lambda}_t + \sigma c_{s,t}\right]. \tag{10}$$

where $\hat{\lambda}_t \equiv \log \frac{\lambda_t}{\lambda}$. Importantly, time-varying disaster risk introduces a new precautionary savings channel, which ultimately shapes how nominal rates impact consumption.

Combining condition (9) for borrowers' consumption, Equation (10) for savers' Euler equation, and the market-clearing condition for goods, we obtain the evolution of aggregate output. Proposition 2 characterizes the dynamics of aggregate output and inflation, given the paths of $i_t - r_n$ and $\hat{\lambda}_t$. Proofs omitted in the text are provided in Appendix A.

Proposition 2 (Aggregate dynamics)**.** *The dynamics of output and inflation is described by the conditions:*

i. ***Aggregate Euler equation:***

$$\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + \delta y_t + v_t, \tag{11}$$

where $\tilde{\sigma}^{-1} \equiv \frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1} - \frac{\mu_b d_P r_n}{1-\mu_b\chi_y}$, $\delta \equiv \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} - \frac{\mu_b d_P \kappa}{1-\mu_b\chi_y}$ *and* $v_t \equiv \frac{\mu_b d_P}{1-\mu_b\chi_y}(\rho(i_t - r_n) - \dot{i}_t) + \frac{1-\mu_b}{1-\mu_b\chi_y}\frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^{\sigma}\hat{\lambda}_t$.

ii. ***New Keynesian Phillips curve:***

$$\dot{\pi}_t = \rho\pi_t - \kappa y_t, \tag{12}$$

where $\rho \equiv \rho_s + \lambda$ *and* $\kappa \equiv \varphi^{-1}(\epsilon - 1)\phi Y$.

Condition (11) represents the *aggregate Euler equation* for this economy. The aggregate Euler equation has three terms. The first term is the product of the aggregate elasticity of intertemporal substitution (EIS) and the real interest rate. The aggregate EIS depends on

[16]The role of χ_y, including the case where $\chi_y > \mu_b^{-1}$, was originally considered by Bilbiie (2008).

the cyclicality of inequality among borrowers and savers, as captured by χ_y.[17] Positive private liquidity, as captured by $d_P > 0$, reduces the aggregate EIS.

The second term, δy_t, captures how future real interest rates affect output. Assuming $v_t = 0$, we can write output as $y_t = -\tilde{\sigma}^{-1} \int_t^\infty e^{-\delta(s-t)}(i_s - \pi_s - r_n)ds$. Hence, a positive value of δ tends to dampen (or discount) the effect of future real interest rates, while a negative value of δ tends to amplify (or compound) the effect of future interest rate. The case $\delta > 0$ corresponds to the discounted Euler equation of McKay et al. (2017), where aggregate disaster risk plays the role of idiosyncratic income risk. In contrast, if d_P is sufficiently large, then $\delta < 0$ and the effect of future interest rate changes is amplified. The aggregate Euler equation (11) can in general feature either compounding or discounting, depending on the relative magnitude of λ and d_P.

The third term in the aggregate Euler equation, v_t, captures a direct effect of monetary policy on borrowers and savers. First, in an economy with household debt, monetary policy directly affects borrowers' disposable income, the high-MPC agents in this economy. Second, the time-varying component of the disaster risk directly impacts the savers' precautionary savings motive. Therefore, monetary policy has real effects even in the absence of intertemporal-substitution forces.

Finally, Proposition 2 derives the New Keynesian Phillips curve. The linearized Phillips curve coincides with the one obtained from models with Calvo pricing. As in a textbook New Keynesian model, inflation is given by the present discounted value of future output gaps, $\pi_t = \kappa \int_t^\infty e^{-\rho(s-t)} y_s ds$. One distinction relative to the standard formulation is that future output gaps are not discounted by the natural rate r_n but by a higher rate $\rho > r_n$. This is a consequence of the riskiness of the firm's value, so the appropriate discount rate incorporates an adjustment for risk.

Asset prices. The response of asset prices to monetary policy depends crucially on the behavior of the price of disaster risk, as shown in Equations (2) and (3). In its log-linear form, the price of disaster risk is given by

$$p_{d,t} \equiv \sigma c_{s,t} + \hat{\lambda}_t. \tag{13}$$

Note that this expression has two terms. The first term captures the change in the effective size of the shock, represented by the increase in the savers' marginal utility of consumption if the disaster shock is realized. The second term represents the change in the market-implied disaster probability after a monetary shock.

[17] This is a well-known result in the literature, see e.g. Werning (2015) and Bilbiie (2017).

Given the price of risk, we can price any financial asset in this economy. For example, the (linearized) price of the long-term bond in period zero is given by[18]

$$q_{L,0} = -\underbrace{\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n)dt}_{\text{path of nominal interest rates}} - \underbrace{\int_0^\infty e^{-(\rho+\psi_L)t} r_L p_{d,t} dt}_{\text{term premium}}. \tag{14}$$

The yield on the long-term bond, expressed as deviations from the stationary equilibrium, is given by $-Q_L^{-1} q_{L,0}$, which can be decomposed into two terms: the path of nominal interest rates, as in the expectations hypothesis, and a *term premium*, capturing variations in the compensation for holding long-term bonds. The term premium depends on the price of risk, $p_{d,t}$, and the asset-specific loading r_L. Because the term premium responds to monetary shocks, the expectation hypothesis does not hold in this economy.

The pricing condition for equity is analogous to the one for bonds:

$$q_{E,0} = \underbrace{\frac{Y}{Q_E}\int_0^\infty e^{-\rho t}\hat{\Pi}_t dt}_{\text{dividends}} - \underbrace{\int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + r_E p_{d,t}\right] dt}_{\text{discount rate}}, \tag{15}$$

where $\hat{\Pi}_t = -\hat{\tau}_t \Pi / Y + (1-\tau)[y_t - (1-\alpha)(w_t - p_t + n_t)]$. This expression shows that equity prices respond to changes in monetary policy through two channels: a *dividend channel*, capturing changes in firms' profits, and a *discount rate channel*, capturing changes in real interest rates and risk premia. Risk premia depends on the price of risk, $p_{d,t}$, like in the expression for the long-term bond, but it has a loading r_E rather than r_L, capturing the different exposure to risk of the two assets.

Fiscal policy. The government's fiscal instruments are $T_t \equiv \mu_b T_{b,t}$ and $\hat{\tau}_t$. Given that T_t is pinned down by aggregate output, the government must choose $\hat{\tau}_t$ such that its No-Ponzi condition is satisfied. Therefore, we will often refer to $\hat{\tau}_t$ as the *fiscal backing* corresponding to a given monetary shock. Having a corporate profit tax enable us to indirectly have a lump-sum tax on savers, while keeping their portfolio problem as simple as possible.

Market-implied disaster probability. Log-linearizing Equation (4), we obtain

$$\frac{1}{\sigma}\lambda^{\frac{1}{\sigma}}\hat{\lambda}_t = \mu_{c,o}\mu_{c,p}\left(\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}\right)\left[c_{p,t} - c_{o,t}\right], \tag{16}$$

[18] Linearizing Equation (2) and rearranging, we obtain $\dot{q}_{L,t} - (\rho+\psi_L)q_{L,t} = i_t - r_n + r_L p_{d,t}$. Integrating this condition forward gives us Equation (14).

where $\mu_{c,j} \equiv \frac{\mu_j \overline{C}_j}{\mu_o \overline{C}_o + \mu_p \overline{C}_p}$.

The market-implied disaster probability increases when the monetary shock redistributes consumption towards pessimistic investors. We show in Appendix B.3 that the relative aggregate consumption of the two types of savers evolves according to

$$\dot{c}_{p,t} - \dot{c}_{o,t} = -\xi(b_{p,t} - b_{o,t}), \tag{17}$$

where $\xi \geq 0$ is the mortality parameter.

Hence, relative consumption and ultimately $\hat{\lambda}_t$ depends on relative net worth $b_{p,t} - b_{o,t}$. In turn, the dynamics of $b_{p,t} - b_{o,t}$ depends on the portfolio of optimistic and pessimistic savers, asset prices, and aggregate variables, as shown in Appendix B.3. In general, these variables must be simultaneously solved for, which involves solving a potentially large dynamic system. In this case, obtaining analytical results would be infeasible. In the next proposition, we show that this system actually satisfies a form of *approximate block recursivity* property, where we are able to solve for $\hat{\lambda}_t$ and $b_{p,t} - b_{o,t}$ independently of (y_t, π_t), provided that the effect of $c_{s,t}$ on risk premia is small.

Proposition 3 (Approximate block recursivity)**.** *Suppose the term $r_k \sigma c_{s,t}$ is small for $k \in \{L, E\}$, i.e. $r_k \sigma c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$. Then, the market-implied probability of disaster $\hat{\lambda}_t$ can be solved independently of the aggregate variables (y_t, π_t), and it is given by*

$$\hat{\lambda}_t = e^{-\psi_\lambda t} \hat{\lambda}_0, \tag{18}$$

where $\psi_\lambda \geq 0$ is strictly increasing in ξ, it is equal to zero if $\xi = 0$ and it approaches infinity if $\xi \to \infty$. If $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, then the initial value of $\hat{\lambda}_t$ is given by

$$\hat{\lambda}_0 = \epsilon_\lambda (i_0 - r_n), \tag{19}$$

where $\epsilon_\lambda \geq 0$ and the inequality is strict if $\lambda_p > \lambda_o$.

Note that the effect of changes in the price of risk on risk premia is given by $r_k(\sigma c_{s,t} + \hat{\lambda}_t)$. If the term $r_k \sigma c_{s,t}$ is second-order on the size of the monetary shock, then this term can be ignored and Proposition 3 shows that we can solve for $\hat{\lambda}_t$ independently of (y_t, π_t). As the dynamics of (y_t, π_t) depends on $\hat{\lambda}_t$, but $\hat{\lambda}_t$ does not depend on (y_t, π_t), we say the system is (approximately) block recursive. We show in the appendix that the solution ignoring the terms $r_k \sigma c_{s,t}$ tracks very closely the numerical solution where these terms are taken into account. This is intuitive as it is exactly because changes in $c_{s,t}$ have a small effect on risk premia that introducing heterogeneity among savers is important.

Equations (18) and (19) imply that *market incompleteness* is important for monetary policy to affect $\hat{\lambda}_t$ and ultimately risk premia in our setting. The first form of market incompleteness refers lack of hedge instruments against monetary shocks. An increase in nominal interest rates will then redistribute away from investors more exposed to long-term assets, which is why $\hat{\lambda}_0$ is increasing in $i_0 - r_n$. The second form of market incompleteness comes from the presence of mortality risk. If $\xi = 0$, then $\psi_\lambda = 0$, and monetary shocks would have a permanent effect on $\hat{\lambda}_t$. It can be shown that $b_{p,t} - b_{o,t} = e^{-\psi_\lambda t}(b_{p,0} - b_{o,0})$, so the net worth of optimistic investors would be permanently affected if $\xi = 0$, regardless of how temporary the monetary shocks are. Mortality risk pins down the long-run wealth distribution and ξ effectively controls how fast the net worth of optimistic investors relative to pessimistic investors mean revert after the shock.[19]

Having solved for the dynamics of the market-implied disaster probability $\hat{\lambda}_t$ and the relative net worth of the two types of savers $b_{p,t} - b_{o,t}$, it remains to determine the dynamics of aggregate ouput and inflation, which we consider next.

3 Monetary Policy and Wealth Effects

In this section, we study how monetary policy affects the economy through the revaluation of real and financial assets. The main result presents a decomposition that isolates the role of intertemporal substitution, precautionary savings, and wealth effects in the transmission of monetary shocks. To derive this decomposition, we proceed in two steps. First, we express the evolution of output and inflation in terms of equilibrium policy variables, that is, the path of nominal interest rates $\{i_t\}$ and the corresponding fiscal backing $\{\hat{\tau}_t\}$. Second, we derive an *implementability result* that shows how to map the path of policy variables to the underlying monetary shock u_t in the interest rate rule (7).

3.1 The dynamic system

We can express output and inflation in terms of policy variables by solving the system of differential equations described in Proposition 2:

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \delta & -\tilde{\sigma}^{-1} \\ -\kappa & \rho \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \nu_t \\ 0 \end{bmatrix}, \tag{20}$$

[19]See Gârleanu and Panageas (2015) for a discussion of how introducing overlapping-generation (OLG) leads to a well-defined long-run wealth distribution.

where $\nu_t \equiv \tilde{\sigma}^{-1}(i_t - r_n) + v_t$ depends only on the path of nominal interest rates. The eigenvalues of the system are given by

$$\overline{\omega} = \frac{\rho + \delta + \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}, \qquad \underline{\omega} = \frac{\rho + \delta - \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}.$$

The following assumption, which we assume holds for all subsequent analysis, guarantees that the eigenvalues are real-valued and have opposite signs, i.e., $\overline{\omega} > 0$ and $\underline{\omega} < 0$.

Assumption 1. *The following condition holds: $\rho\delta < \tilde{\sigma}^{-1}\kappa$.*

Assumption 1 implies that the system lacks exactly one boundary condition.[20] Next, we show that the missing boundary condition can be provided by an aggregate intertemporal budget constraint (IBC). From the savers' transversality condition, and using market-implied probabilities to compute expectations, we obtain the individual IBC:

$$\mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0} C_{j,t} dt\right] = B_{j,0}.$$

Aggregating across savers and borrowers, we obtain the (non-linear) aggregate IBC:

$$\mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0} C_t dt\right] = D_{G,0} + Q_{E,0} + \mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}\left(\frac{W_t}{P_t} N_t + \mu_b \tilde{T}_{b,t}\right) dt\right],$$

using $C_t \equiv \mu_b C_{b,t} + (1-\mu_b) C_{s,t}$ and the market clearing condition for bonds and equities.

The expression above says that the value of the stream of aggregate consumption equals the net value of assets held by the household sector: government bonds, stocks, and human wealth (the value of labor income after transfers). Private debt does not appear in the right-hand side, as it does not represent net wealth for the household sector.

To linearize the expression above, it is convenient to define $Q_{C,t} \equiv \mathbb{E}_t\left[\int_t^\infty \frac{\eta_z}{\eta_t} C_z dz\right]$, the value of the consumption claim, and $Q_{H,t} \equiv \mathbb{E}_t\left[\int_t^\infty \frac{\eta_z}{\eta_t}\left(\frac{W_z}{P_z} N_z + \mu_b \tilde{T}_{b,z}\right) dz\right]$, the value of human wealth. We can solve for the price of these two claims in the same way as we priced stocks and bonds (see Equations 14 and 15). The linearized intertemporal budget constraint can then be written as follows:

$$Q_C q_{C,0} = Q_H q_{H,0} + D_G q_{L,0} + Q_E q_{E,0}, \tag{21}$$

[20] Assumption 1 implies that the equilibrium is indeterminate under an interest-rate peg, as local determinacy requires $\phi_\pi \geq \frac{\tilde{\sigma}^{-1}\kappa - \rho\delta}{\tilde{\sigma}^{-1}\kappa + \frac{\mu_p d_p \rho}{1-\mu_b \chi_y}\kappa} \equiv \overline{\phi}_\pi$, and $\overline{\phi}_\pi > 0$ under Assumption 1. It can also be shown that $\overline{\phi}_\pi < 1$ due to a precautionary motive, in line with findings by Bilbiie (2018) and Acharya and Dogra (2020).

where $q_{C,0} \equiv \log Q_{C,0}/Q_C$ and $q_{H,0} \equiv Q_{H,0}/Q_H$.

From equation (21), we have that the aggregate IBC is a necessary equilibrium condition. The next lemma establishes the sufficiency of the aggregate IBC for pinning down the equilibrium. That is, it shows that if $[y_t, \pi_t]_0^\infty$ satisfies system (20) and the IBC (in its log-linear form), then we can determine the consumption, portfolio, and labor supply of households as well as wages and prices such that all equilibrium conditions are satisfied.

Lemma 1. *Suppose that, given a path for the nominal interest rate and fiscal backing $[i_t, \tau_t]_0^\infty$, $[y_t, \pi_t]_0^\infty$ satisfy system (20) and the aggregate intertemporal budget constraint (21). Then, $[y_t, \pi_t]_0^\infty$ can be supported as part of a competitive equilibrium.*

Therefore, the equilibrium dynamics can be characterized as the solution to the dynamic system (20), subject to the boundary condition (21). Notice that, for a given $\lambda > 0$, heterogeneous beliefs affect the aggregate dynamics only through $\hat{\lambda}_t$. Hence, the economy with heterogeneous savers behaves as an economy with a representative saver, but the probability of disaster is time varying and respond to monetary shocks.

3.2 Aggregate wealth effect and risk-premium neutrality

Monetary policy affects system (20) directly through ν_t, capturing intertemporal substitution and precautionary effects, and indirectly through the boundary condition (21), capturing the revaluation of real and financial assets. Lemma 2 shows that asset revaluation is a key determinant of the average consumption response to monetary shocks.

Lemma 2. *The present discounted value of aggregate consumption is given by*

$$\int_0^\infty e^{-\rho t} c_t dt = \Omega_0, \tag{22}$$

where Ω_0 denotes the aggregate wealth effect, given by

$$\Omega_0 \equiv \int_0^\infty e^{-\rho t} \left[\hat{\Pi}_t + (1-\alpha)(w_t - p_t + n_t) + T_t\right] dt + \overline{d}_G q_{L,0} + \overline{d}_G \int_0^\infty e^{-\rho t} \left(i_t - \pi_t - r_n + r_L p_{d,t}\right) dt.$$

Proof. Using the pricing condition for $q_{k,0}$, $k \in \{C, H, E\}$, and Equation (21), we obtain

$$\int_0^\infty e^{-\rho t} c_t dt - \frac{Q_C}{Y} \int_0^\infty e^{-\rho t} \left[i_t - \pi_t - r_n + r_C p_{d,t}\right] dt = \int_0^\infty e^{-\rho t} \left[\hat{\Pi}_t + (1-\alpha)(w_t - p_t + n_t) + T_t\right] dt$$
$$- \frac{Q_H + Q_E}{Y} \int_0^\infty e^{-\rho t} \left[i_t - \pi_t - r_n\right] dt - \left[\frac{Q_H}{Y} r_H + \frac{Q_E}{Y} r_E\right] \int_0^\infty e^{-\rho t} p_{d,t} dt + \frac{D_G}{Y} q_{L,0}.$$

Using the fact that $Q_C = Q_H + D_G + Q_E$ and $Q_C^* = Q_H^* + D_G \frac{Q_L^*}{Q_L} + Q_E^*$, we obtain $\frac{Q_C}{Y} - \frac{Q_H + Q_E}{Y} = \frac{D_G}{Y} \equiv \overline{d}_G$ and $\frac{Q_C}{Y} r_C - \frac{Q_H r_H + Q_E r_E}{Y} = \overline{d}_G r_L$, given $r_k = \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_k - Q_k^*}{Q_k}$. Combining these expressions with the equation above, we obtain (22) after some rearrangement. $\square$

Lemma 2 implies that the present discounted value of aggregate consumption equals Ω_0, which we refer to as the *aggregate wealth effect*. We show in Appendix C.4 that Ω_0 corresponds to (minus) the amount of wealth required to compensate all households for the price and income changes caused by the monetary shock, so their consumption bundle in the stationary equilibrium is just affordable. This result justifies referring to Ω_0 as a wealth effect, given it corresponds to the Slutsky wealth compensation (see Mas-Colell et al. 1995) associated with the monetary shock.

The first term in Ω_0 captures the effect of changes in cash flows, namely (after-tax) profits and wages. Naturally, households become wealthier if profits and wages increase in response to a monetary shock, everything else constant. The last two terms capture the net effect of changes in discount rates, i.e. interest rates and risk premia. Importantly, the effect of discount rates depend on the level and riskiness of government debt.

Risk-premium neutrality. Asset revaluations caused by monetary policy have received significant attention recently. For instance, Cieslak and Vissing-Jorgensen (2020) show that policymakers pay attention to the stock market due to its potential (consumption) wealth effect. In contrast, Cochrane (2020) and Krugman (2021) argue that wealth gains on "paper" are not relevant for households who simply consume their dividends. The next proposition isolate the necessary conditions under which the latter view is correct.

Proposition 4 (Risk-premium neutrality)**.** *Suppose the government uses a consumption tax to neutralize the precautionary motive induced by $\hat{\lambda}_t$, that is, consider τ_t^c satisfying $\dot{\hat{\tau}}_t^c = \lambda \left(\frac{C_s}{C_s^*}\right) \hat{\lambda}_t$, where $\hat{\tau}_t^c \equiv \log(1 + \tau_t^c)$ and $\tau_t^c = \tau_t^{c,*}$. Then, $[y_t, \pi_t]_0^\infty$ is independent of $\hat{\lambda}_t$ if one of the following conditions are satisfied: i) $\overline{d}_G = 0$; ii) $\overline{d}_G > 0$ and $\psi_L = \infty$; iii) $\overline{d}_G > 0$ and $\psi_L = 0$.*

Proof. Savers' Euler equation for the riskless bond is now given by $\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n - \dot{\hat{\tau}}_t^c) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^\sigma \left[\hat{\lambda}_t + \sigma c_{s,t}\right]$, which is independent of $\hat{\lambda}_t$ if $\dot{\hat{\tau}}_t^c = \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \hat{\lambda}_t$. As $\tau_t^c = \tau_t^{c,*}$, Euler equations for risky assets are not affected. Assuming the revenue is rebated back to households, borrowers are not affected. The dynamic system (20) is then unaffected, except ν_t is independent of $\hat{\lambda}_t$. If $\overline{d}_G = 0$, the last two terms in Ω_0 are equal to zero. If $\overline{d}_G > 0$ and $\psi_L = 0$, $\hat{\lambda}_t$ in the last two terms in Ω_0 exactly cancel out. If $\psi_L = \infty$, government bonds are safe and $r_L = 0$. Ω_0 is independent of $\hat{\lambda}_t$ in all three cases. $\square$

Proposition 4 provides conditions under which time variation in the market-implied disaster probability λ_t does not impact the monetary transmission mechanism. Under such conditions, heterogeneity in portfolios among savers may help to improve the model's asset-pricing implications, but they have no bearing on how monetary shocks ultimately affect the real economy. In particular, it is possible that a monetary contraction leads to a substantial drop in assets prices, due to the increase in risk premium, but the impact on output and inflation would be the same as in an economy with a representative saver and no such impact on risk premium.

As it is often the case with neutrality results, the purpose of Proposition 4 is not to argue that the conditions for neutrality are realistic, but to provide a clear benchmark that help us understand when neutrality does or does not hold. For instance, even in the absence of a direct impact on the aggregate wealth effect, time variation in $\hat{\lambda}_t$ would affect output through the precautionary savings motive. This precautionary motive is the result of the reallocation of risk among investors and, as seen in the next section, can have potentially large real effects. Proposition 4 assumes that such effects are neutralized by tax policy, which is unlikely to happen in practice.[21]

The second set of conditions in Proposition 4 refer to characteristics of the government liability. But why does the presence of government debt matters to determine how changes in stocks prices and human wealth ultimately affect the economy? The reason goes back to the intuition provided by Cochrane (2020) and Krugman (2021). If $\overline{d}_G = 0$, then the household sector simply consumes the "dividends" from stocks and human wealth. An increase in risk premium depresses the value of real assets, but it also reduces the value of the consumption claim by the same amount, so no change in average consumption is required for the IBC to be satisfied. If $\overline{d}_G > 0$ and $\psi_L = 0$, a similar logic applies: dividends received by the household sector now includes the coupons from government bonds, but again there is no need to trade at any period and changes in risk premium do not affect Ω_0. Finally, if $\overline{d}_G = 0$ and $\psi_L = \infty$, the household sector as a whole only trades on safe bonds, so changes in risk premium have again no effect on Ω_0.

The logic above is reminiscent of the one recently provided by Fagereng et al. (2022). They show that the welfare effect of changes in discount rates equals the present discounted value of *trades* times the change in prices. Proposition 4 says that changes in risk premium do not affect Ω_0 directly when the household sector do not trade risky assets. They also show that such effects represent pure redistribution, i.e., they exactly cancel out when the counterparts to the household sector, foreigners and the government, are

[21] The precautionary motive is present with CRRA preferences as the prudence coefficient, $-\frac{u'''}{u''}$, is positive in this case (see Kimball 1990). This adjustment would not be needed for preferences satisfying $u''' = 0$.

taken into account. In our closed economy, the government is the only counterpart to the household sector as a whole, which explains the special role played by government liablities.

Finally, notice that we have considered the impact of changes in discount rates on Ω_0 for a given path of cash flows. If policy variables $[i_t, \hat{\tau}_t]_0^\infty$ react directly to $\hat{\lambda}_t$, then it is possible to obtain a substantial effect on Ω_0 through its first term, even if $\overline{d}_G = 0$.

3.3 Intertemporal substitution, risk, and wealth effects

The next proposition characterizes the output response to a sequence of monetary policy shocks for a given value of the aggregate wealth effect Ω_0. We provide a full characterization of Ω_0 in Section 3.4. For ease of exposition, we focus on the case of exponentially decaying nominal interest rates; that is, we assume $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where ψ_m determines the persistence of the path of interest rates.

Proposition 5 (Aggregate output in D-HANK)**.** *Suppose that $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$. The path of aggregate output is then given by*

$$y_t = \underbrace{\sigma^{-1}\hat{y}_{m,t}}_{\text{ISE}} + \underbrace{\chi_p \hat{y}_{\lambda,t}}_{\substack{\text{time-varying}\\ \text{risk}}} + \underbrace{\frac{\mu_b \overline{d}_P}{1-\mu_b}\tilde{\psi}_m \hat{y}_{m,t}}_{\text{inside wealth effect}} + \underbrace{(\rho - \underline{\omega})e^{\underline{\omega}t}\Omega_0}_{\substack{\text{GE multiplier}\times\\ \text{aggregate wealth effect}}}, \tag{23}$$

where $\chi_p \equiv \frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^\sigma \epsilon_\lambda$, $\tilde{\psi}_m \equiv \rho - r_n + \psi_m$, and $\hat{y}_{k,t}$ is given by

$$\hat{y}_{k,t} = \frac{1-\mu_b}{1-\mu_b\chi_y}\frac{(\rho - \underline{\omega})\, e^{\underline{\omega}t} - (\rho + \psi_k)\, e^{-\psi_k t}}{(\overline{\omega} + \psi_k)\,(\underline{\omega} + \psi_k)}(i_0 - r_n), \tag{24}$$

and satisfies $\int_0^\infty e^{-\rho t}\hat{y}_{k,t}dt = 0$, $\frac{\partial \hat{y}_{k,0}}{\partial i_0} < 0$, for $k \in \{m, \lambda\}$.

Proposition 5 shows that output can be decomposed into four terms: an intertemporal-substitution effect (ISE), a time-varying risk channel, a revaluation of assets in zero net supply (the inside wealth effect), and a revaluation of assets in positive net supply (the aggregate wealth effect). These effects encompass some of main channels of transmission considered by the literature. By setting $\lambda_p = \lambda_o = \overline{d}_P = 0$, the model behaves as a TANK model with zero liquidity, as in Bilbiie (2019) and Broer et al. (2020). The case $\overline{d}_P > 0$ captures the implications of household debt, as in e.g. Benigno et al. (2020). Positive disaster probability $\lambda_p = \lambda_o > 0$ introduces a precautionary motive, analogous to HANK

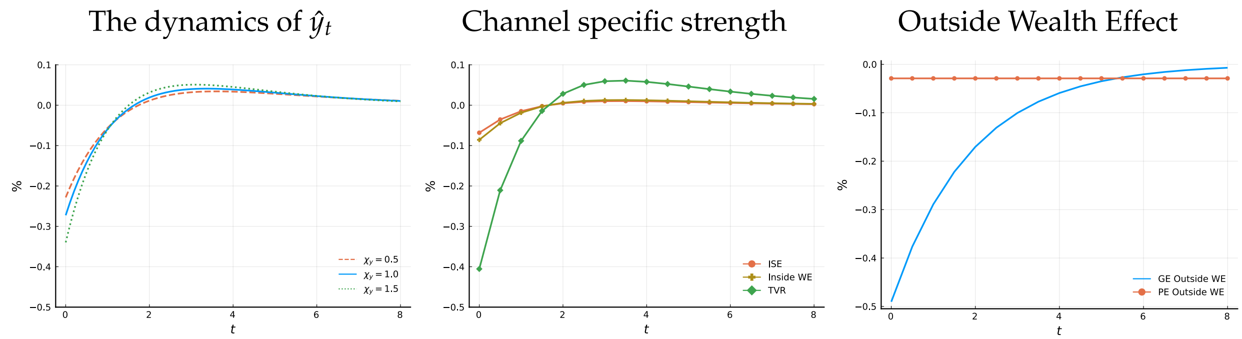

Figure 1: Aggregate output decomposition in D-HANK

models (Kaplan et al. 2018), while $\lambda_p > \lambda_o > 0$ allow us to capture the effect of time-varying risk premia, as in Caballero and Simsek (2020) and Kekre and Lenel (2020).

The first term captures the standard intertemporal substitution channel present in RANK models. It depends on the EIS σ^{-1} and $\hat{y}_{m,t}$ given in (24). Notice that, even though only a fraction $1 - \mu_b$ of agents substitute consumption intertemporally, the ISE does not necessarily gets weaker as we reduce the mass of savers in the economy. As we reduce $1 - \mu_b$, less agents are capable of intertemporal substitution, but the amplification from hand-to-mouth agents gets stronger. The two effects exactly cancel out when $\chi_y = 1$. Another important property of the ISE is that it is equal to zero on average, i.e. $\int_0^\infty e^{-\rho t} \hat{y}_{m,t} = 0$. An increase in interest rates shifts demand from the present to the future, but it does not change by itself the overall level of aggregate demand.

The second term captures the effect of time-varying risk. It is equal to zero in the absence of belief heterogeneity, i.e. $\lambda_o = \lambda_p$. As risk gets reallocated after the monetary shock, a disaster leads to a larger increase in the marginal utility of both types of savers, which strengthens the precautionary motive. As with the EIS, the precautionary motive is amplified by the presence of hand-to-mouth agents and it shifts demand from the present to the future without changing its overall level, that is, $\int_0^\infty e^{-\rho t} \hat{y}_{\lambda,t} dt = 0$. In contrast to the EIS, the persistence of the precautionary effects is controlled by ψ_λ instead of ψ_m.

The third term captures the effect of assets in zero net supply, namely household debt. A temporary increase in interest rates tends to reduce the consumption of borrowers and increase the consumption of savers. These two effects cancel out exactly when considering average consumption over time. However, as savers smooth consumption, they increase their consumption in the short run by less than borrowers reduce their consumption. Through this channel, monetary policy again shifts demand from the present to the future. As with the ISE, the persistence of such effects depend on ψ_m, which controls the persistence of the monetary shock.

The fourth term in expression (23) plays an important role, as the aggregate wealth effect determines the average response of output to the monetary shock. Everything else constant, an increase in Ω_0 would tend to raise output in all periods by $\rho\Omega_0$, creating a parallel shift in output over time. In general equilibrium, a positive aggregate wealth effect leads to inflation, which reduces the real rate and shift consumption to the present. Therefore, an increase in Ω_0 disproportionately affect the initial level of output. The GE multiplier in period 0, $\frac{\rho-\underline{\omega}}{\rho}$, is more than 15 in our calibration, leading to a substantial amplification, as shown in the right panel of Figure 1.

Inflation. The next proposition characterizes the response of inflation to monetary policy shocks in the context of our heterogeneous-agent economy.

Proposition 6 (Inflation in D-HANK)**.** *Suppose $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$. The path of inflation is given by*

$$\pi_t = \sigma^{-1}\hat{\pi}_{m,t} + \chi_p\hat{\pi}_{\lambda,t} + \frac{\mu_b \overline{d}_P}{1-\mu_b}\tilde{\psi}_m\hat{\pi}_{m,t} + \kappa e^{\underline{\omega}t}\Omega_0, \tag{25}$$

where $\hat{\pi}_{k,t} = \frac{1-\mu_b}{1-\mu_b\chi_y}\frac{\kappa(e^{\underline{\omega}t}-e^{-\psi_k t})}{(\underline{\omega}+\psi_k)(\overline{\omega}+\psi_k)}(i_0 - r_n)$*,* $\hat{\pi}_{k,0} = 0$ *and* $\frac{\partial\hat{\pi}_{k,t}}{\partial i_0} \geq 0$*, for* $k \in \{m, \lambda\}$*.*

Inflation can be analogously decomposed into four terms. The first three terms capture the impact of the ISE, the time-varying risk, and the inside wealth effect, while the last term captures the impact of the aggregate wealth effect. Because $\hat{\pi}_{k,0} = 0$, the first three terms are initially zero. This implies that initial inflation is determined entirely by the aggregate wealth effect. Moreover, π_t is actually *increasing* in i_0 if $\Omega_0 = 0$.

In a nutshell, Proposition 5 and 6 imply that monetary policy has a very limited impact on the economy in the absence of an aggregate wealth effect, i.e. if $\Omega_0 = 0$. In this case, a stimulus on output in the short run would come at the expense of a more depressed economy in the future, while the central bank would lose its ability to affect initial inflation. Therefore, the aggregate wealth effect plays a key role in the central bank's ability to control inflation or stimulate the economy.

3.4 The determination of the aggregate wealth effect

We consider next the determination of the aggregate wealth effect Ω_0. The aggregate wealth effect can be written as

$$\Omega_0 \equiv \int_0^\infty e^{-\rho t}\left[-\hat{\tau}_t\frac{\Pi}{Y} + (1-\chi_\tau)y_t\right]dt + \overline{d}_G q_{L,0} + \overline{d}_G\int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_L p_{d,t}\right)dt,$$

where $\chi_\tau \equiv \tau[1-(1-\alpha)(1+\phi)] - \mu_b T_b'(Y)$ is the cyclicality of tax revenues.

The expression above shows that Ω_0 depends on the path of policy variables $[i_t, \hat{\tau}_t]_0^\infty$ as well as the path of output and inflation $[y_t, \pi_t]_0^\infty$. Propositions 5 and 6 show that y_t and π_t depend on both policy variables and Ω_0. By combining Equations (23) and (25) with the expression for Ω_0, we can solve for Ω_0 in terms of policy variables.

Proposition 7. *Suppose $\chi_\tau + \frac{\bar{d}_G \kappa}{\rho - \underline{\omega}} > 0$. Then, Ω_0 is a function of $[i_t, \hat{\tau}_t]_0^\infty$ given by*

$$\Omega_0 = \frac{\rho - \underline{\omega}}{(\rho - \underline{\omega})\chi_\tau + \bar{d}_G \kappa} \left[-\frac{\Pi}{Y} \int_0^\infty e^{-\rho t} \hat{\tau}_t dt + \bar{d}_G \left(q_{L,0} + \int_0^\infty e^{-\rho t} (i_t - \tilde{\pi}_t - r_n + r_L \hat{\lambda}_t) dt \right) \right], \quad (26)$$

where $\tilde{\pi}_t \equiv \sigma^{-1} \hat{\pi}_{m,t} + \chi_p \hat{\pi}_{\lambda,t} + \frac{\mu_b \bar{d}_p}{1 - \mu_b} \tilde{\psi}_m \hat{\pi}_{m,t}$ is a function of $[i_t]_0^\infty$.

Proof. Using $\int_0^\infty e^{-\rho t} y_t dt = \Omega_0$ and $\int_0^\infty e^{-\rho t} \pi_t dt = \int_0^\infty e^{-\rho t} \tilde{\pi}_t dt + \frac{\kappa}{\rho - \underline{\omega}} \Omega_0$, we obtain

$$\left(\chi_\tau + \frac{\bar{d}_G \kappa}{\rho - \underline{\omega}} \right) \Omega_0 = -\frac{\Pi}{Y} \int_0^\infty e^{-\rho t} \hat{\tau}_t dt + \bar{d}_G q_{L,0} + \bar{d}_G \int_0^\infty e^{-\rho t} \left(i_t - \tilde{\pi}_t - r_n + r_L p_{d,t} \right) dt,$$

after rearranging the expression for Ω_0. Given our assumption, we can divide both sides by $\chi_\tau + \frac{\bar{d}_G \kappa}{\rho - \underline{\omega}}$. This gives Equation (26), using the fact that $r_L p_{d,t} = r_L \hat{\lambda}_t$ up to a first-order approximation. From Equation (14) and $r_L p_{d,t} = r_L \hat{\lambda}_t$, $q_{L,0}$ is a function of only $[i_t]_0^\infty$. □

Proposition 7 shows that Ω_0 is uniquely pin down by $[i_t, \hat{\tau}_t]_0^\infty$, given $\chi_\tau + \frac{\bar{d}_G \kappa}{\rho - \underline{\omega}} > 0$. This assumption simply states that monetary policy affects the fiscal authority either through tax revenues or through the cost of servicing the debt (or both). This proposition has an important implication: there is only *two* ways through which monetary policy impacts the aggregate wealth effect. First, monetary policy affects Ω_0 through its fiscal backing. Second, monetary policy affects Ω_0 through a *net discount rate effect*, similar to the one discussed in the context of Proposition 4. Importantly, this net revaluation effect is only present when $\bar{d}_G > 0$.

Net discount rate effect. Suppose $\int_0^\infty e^{-\rho t} \hat{\tau}_t dt = 0$. If we also assume that $\bar{d}_G = 0$, then the household sector consumes the value of profits and wages, i.e the dividends on stocks and human wealth, every period. As there is no trade at the aggregate level, the intuition in Cochrane (2020) and Krugman (2021) applies in this case and changes in discount rates (interest rates and risk premia) generate no aggregate wealth effect. As shown in Propositions 5 and 6, this implies that monetary policy has only a limited impact in the economy. While Cochrane (2020) and Krugman (2021) focused on the implications of changes in discount rates on inequality, Proposition 7 shows that this basic intuition has also important implications on how monetary policy affects the economy. However, this logic does not apply when $\overline{d}_G > 0$. It can be shown that, in this case, $\frac{\partial \Omega_0}{\partial i_0} < 0$ if the maturity of the government debt is sufficient long. Moreover, this effect gets stronger with ϵ_λ, the response of the price of risk to changes in interest rates.

Fiscal backing. Suppose $\overline{d}_G = 0$. In this case, a monetary tightening creates a negative wealth effect, and ultimately reduces π_0, if and only if $\int_0^\infty e^{-\rho t}\hat{\tau}_t dt > 0$. A monetary tightening must necessarily be followed by a fiscal tightening. Moreover, given the fiscal backing and χ_τ, the aggregate wealth effect is independent of the path nominal interest rates i_t or the market-implied probability of disaster $\hat{\lambda}_t$. Hence, the strength of the inflation response to monetary shocks depends crucially on the expectation of the response of the fiscal authority. This illustrates the importance of disciplining monetary policy's fiscal backing empirically, as otherwise the model can generate a response an arbitrarily large response impact of monetary shocks based on a (potentially counterfactual) fiscal response.

The case $\chi_\tau + \frac{\overline{d}_G \kappa}{\rho - \underline{\omega}} = 0$. The analysis above relied on the assumption $\chi_\tau + \frac{\overline{d}_G \kappa}{\rho - \underline{\omega}} > 0$. If we relax this assumption, then the fiscal backing must satisfy $\frac{\Pi}{Y}\int_0^\infty e^{-\rho t}\hat{\tau}_t dt = \overline{d}_G \left(q_{L,0} + \int_0^\infty e^{-\rho t}(i_t - \tilde{\pi}_t - r_n + r_L \hat{\lambda}_t)dt\right)$. In the commonly assumed case $\chi_\tau = \overline{d}_G = 0$, this implies that the fiscal backing is zero. Moreover, Ω_0 would be independent of $[i_t, \hat{\tau}_t]_0^\infty$. In the case $\chi_\tau = \overline{d}_G = 0$, monetary policy can effectively choose Ω_0 in the absence of a net discount rate effect even without the help of the fiscal authority. Proposition 7 shows this is possible *only* in this knife-edge case. If $\chi_\tau > 0$ and/or $\overline{d}_G > 0$, the empirically relevant case, then monetary policy requires the help of fiscal policy in the absence of a net discount rate effect.

3.5 Implementability condition

Propositions 5, 6, and 7 demonstrate how policy variables $[i_t, \hat{\tau}_t]_0^\infty$ affect output and inflation. However, both the nominal interest rate and the associated fiscal backing are endogenous variables. The next proposition shows how we can choose the monetary rule that implements a particular equilibrium path of nominal interest rates and fiscal backing by appropriately choosing the exogenous process for the monetary shock u_t.

Proposition 8 (Implementability). *Let y_t be given by (23) and π_t be given by (25), for a given path of nominal interest rates $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $\psi_m \neq -\underline{\omega}$, and the associated fiscal backing $\hat{\tau}_t$. Let $[\bar{i}_t, \overline{y}_t, \overline{\pi}_t]_0^\infty$ denote the (bounded) solution to the system comprising the Taylor rule*

(7), *the aggregate Euler equation* (11), *and the New Keynesian Phillips curve* (12), *and suppose the monetary shock* u_t *is given by*

$$u_t = \vartheta e^{-\psi_m t}(i_0 - r_n) + \theta e^{\underline{\omega} t}. \tag{27}$$

Then, there exists parameters ϑ *and* θ *such that* $\bar{i}_t = i_t$, $\overline{y}_t = y_t$, *and* $\overline{\pi}_t = \pi_t$.

Proposition 8 shows that there exists a sequence of monetary shocks that implements any given path $[i_t, \hat{\tau}_t]_0^\infty$ in equilibrium, so one can equivalently express the solution either in terms of policy variables or in terms of the underlying process for u_t. Equivalence results like Proposition 8 are well-known in the literature on fiscal-monetary interactions (see e.g. Cochrane 2019). This approach is useful in our context because, as shown in Proposition 7, the fiscal backing can amplify or dampen the impact of the net revaluation of real and financial assets. To isolate the role of asset revaluations, one must control for the response of fiscal policy, which we do so by expressing the solution directly in terms of policy variables.[22] Proposition 8 shows that this approach is without loss of generality, as one can always choose u_t to uniquely determine i_t and the fiscal backing.

The formulation in Equation (27) generalizes the process for monetary shocks frequently used in the literature, where the parameter θ is usually set to zero. While ϑ simply scales the shock such that the initial nominal interest rate equals a given i_0, θ pins down the outside wealth effect Ω_0 and the underlying fiscal backing.[23] The extra degree of freedom given by the parameter θ is important to discipline the aggregate wealth effect empirically, as it allow us to simultaneously match the persistence of the equilibrium interest rate and the corresponding fiscal backing.

4 The Quantitative Importance of Wealth Effects

In this section, we study the quantitative importance of wealth effects in the transmission of monetary shocks. We calibrate the model to match key unconditional and conditional moments, including asset-pricing dynamics and the fiscal response to a monetary shock. We find that household heterogeneity and time-varying risk are the predominant channels of transmission of monetary policy.

[22] Another advantage of this approach comes from the approximate block recursivity discussed in Proposition 3. Using the Taylor rule to replace i_t, we must then solve simultaneously for $[y_t, \pi_t, \hat{\lambda}_t, b_{p,t} - b_{o,t}]$.

[23] Note that a contractionary shock increases nominal rates if $\psi_m < |\underline{\omega}|$, while it reduces nominal rates if $\psi_m > |\underline{\omega}|$. Thus, the nominal interest rate do not react to a monetary shock when $\psi_m = |\underline{\omega}|$.

4.1 Calibration

The parameter values are chosen as follows. The discount rate of savers is chosen to match a natural interest rate of $r_n = 1\%$. We assume a Frisch elasticity of one, $\phi = 1$, and set the elasticity of substitution between intermediate goods to $\epsilon = 6$, common values adopted in the literature. The fraction of borrowers is set to $\mu_b = 30\%$, and the parameter $\overline{d}_P$ is chosen to match a household debt-to-disposable income ratio of 1 (consistent with the U.S. Financial Accounts). The parameter $\overline{d}_G$ is chosen to match a public debt-to-GDP ratio of 66%, and we assume a duration of five years, consistent with the historical average for the United States. The tax rate is set to $\tau = 0.27$ and the parameter $T_b'(Y)$ is chosen such that $\chi_y = 1$, which requires countercyclical transfers to balance the procyclical wage income. A value of $\chi_y = 1$ is consistent with the evidence in Cloyne et al. (2020) that the net income of mortgagors and non-mortgagors reacts similarly to monetary shocks. The pricing cost parameter φ is chosen such that κ coincides with its corresponding value under Calvo pricing and an average period between price adjustments of three quarters. The half-life of the monetary shock is set to three and a half months to roughly match what we estimate in the data, and we set $\phi_\pi = 1.5$.

We calibrate the disaster risk parameters in two steps. For the stationary equilibrium, we choose a calibration mostly based on the parameters adopted by Barro (2009). We set λ (the steady-state disaster intensity) to match an annual disaster probability of 1.7%, and A^* to match a drop in output of $1 - \frac{Y}{Y^*} = 0.39$.[24] The risk-aversion coefficient is set to $\sigma = 4$, a value within the range of reasonable values according to Mehra and Prescott (1985), but substantially larger than $\sigma = 1$, a value often adopted in macroeconomic models. Our calibration implies an equity premium in the stationary equilibrium of 6.1%, in line with the observed equity premium of 6.5%. Moreover, by setting $\sigma = 4$ we obtain a micro EIS of $\sigma^{-1} = 0.25$, in the ballpark of an EIS of 0.1 as recently estimated by Best et al. (2020). We discuss the calibration of ϵ_λ, which determines the elasticity of asset prices to monetary shocks, in the next subsection.

For the policy variables, we estimate a standard VAR augmented to incorporate fiscal variables and compute empirical IRFs applying the recursiveness assumption of Christiano et al. (1999). From the estimation, we obtain the path of monetary and fiscal variables: the path of the nominal interest rate, the change in the initial value of government bonds, and the path of fiscal transfers. We provide the details of the estimation in Appendix D. Figure 2 shows the dynamics of fiscal variables in the estimated VAR in re-

[24] As discussed in Barro (2006), it is not appropriate to calibrate A^*/A to the average magnitude of a disaster, given that empirically the size of a disaster is stochastic. We instead calibrate A^*/A to match $\mathbb{E}[(C_s/C_s^*)^\sigma]$ using the empirical distribution of disasters reported in Barro (2009).

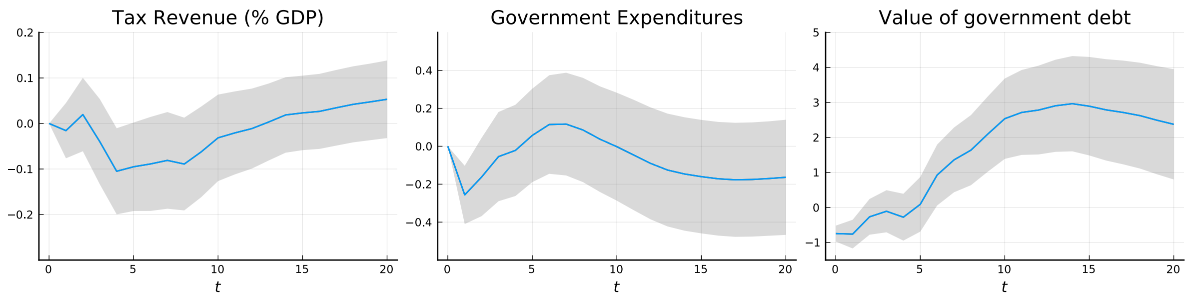

Figure 2: Estimated fiscal response to a monetary policy shock

Note: IRFs computed from a VAR identified by a recursiveness assumption, as in Christiano et al. (1999). Variables included: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, federal funds rate, 5-year constant maturity rate and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The real value of government debt and the 5-year rate are ordered last, and the fed funds rate is ordered third to last. Gray areas are bootstrapped 95% confidence bands. See Appendix D for the details.

sponse to a contractionary monetary shock. Government revenues fall in response to the contractionary shock, while government expenditures fall on impact and then turn positive, likely driven by the automatic stabilizer mechanisms embedded in the government accounts. The present value of interest payments increases by 69 bps and the initial value of government debt drops by 50 bps.[25] In contrast, the present value of transfers T_t drops by 12 bps.[26] Moreover, we cannot, at the 95% confidence level, reject the possibility that the present discounted value of the primary surplus does not change in response to monetary shocks and that the increase in interest payments is entirely compensated by the initial reaction in the value of government bonds.

4.2 Asset-pricing implications of time-varying risk

Recall that the price of the long-term government bond is given by

$$q_{L,0} = -\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n + r_L p_{d,t})dt,$$

where $p_{d,t} = \sigma c_{s,t} + \epsilon_\lambda (i_t - r_n)$ is the price of the disaster risk. We use this expression and calibrate ϵ_λ to match the initial response of the 5-year yield on government bonds.

[25] The present discounted value of interest payments is calculated as $\sum_{t=0}^{\mathcal{T}} \left(\frac{1-\lambda}{1+\rho_s}\right)^{\frac{t}{4}} \left[\overline{d}_t^g (\hat{i}_{L,t} - \hat{\pi}_t)\right]$, where $\mathcal{T}$ is the truncation period, $\hat{i}_{L,t}$ is the IRF of the 5-year rate estimated in the data, and $\hat{\pi}_t$ is the IRF of inflation. We choose $\mathcal{T} = 60$ quarters, when the main macroeconomic variables, including government debt, are back to their pre-shock values. Other present value calculations follow a similar logic.

[26] In the data, expenditures also include the response of government consumption and investment. When run separately, however, we cannot reject the possibility that the sum of these two components is equal to zero in response to monetary shocks.

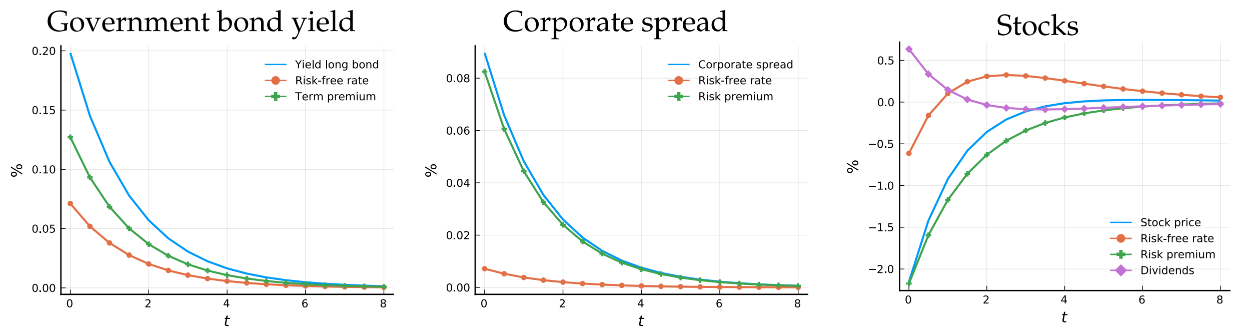

Figure 3: Asset-pricing response to monetary shocks with time-varying risk.

Consistent with Gertler and Karadi (2015) and our own estimates reported in Appendix D, we find that a 100 bps increase in the nominal interest rate leads to an increase in the 5-year yield of roughly 20 bps. This procedure leads to a calibration of ϵ_λ of 2.25, which implies an annual increase in the probability of disaster of roughly 95 bps after a 100 bps increase in the nominal interest rate. Figure 3 shows the response of the yield on the long bond and the contributions of the path of future interest rates and the term premium. We find that the bulk of the reaction of the 5-year yield reflects movements in the term premium, a finding that is consistent with the evidence.

The model is also able to capture the responses of asset prices that were not directly targeted in the calibration. Consider first the response of the *corporate spread*, the difference between the yield on a corporate bond and the yield on a government bond (without risk of default) with the same promised cash flow. This corresponds to how the GZ spread is computed in the data by Gilchrist and Zakrajšek (2012). Let $e^{-\psi_F t}$ denote the coupon paid by the corporate bond. We assume that the monetary shock is too small to trigger a corporate default, but the corporate bond defaults if a disaster occurs, where lenders recover the amount $1-\zeta_F$ in case of default. We calibrated ψ_F and ζ_F to match a duration of 6.5 years and a credit spread of 200 bps in the stationary equilibrium, which is consistent with the estimates reported by Gilchrist and Zakrajšek (2012). Note that the calibration targets the *unconditional* level of the credit spread. We evaluate the model on its ability to generate an empirically plausible *conditional* response to monetary shocks.

The price of the corporate bond can be computed analogously to the computation of the long-term government bond:

$$q_{F,0} = -\int_0^\infty e^{-(\rho+\psi_F)t}(i_t - r_n)dt - \int_0^\infty e^{-(\rho+\psi_F)t}\left[\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_F - Q_F^*}{Q_F} p_{d,t}\right]dt,$$

where Q_F and Q_F^* denote the price of the corporate bond in the stationary equilibrium in

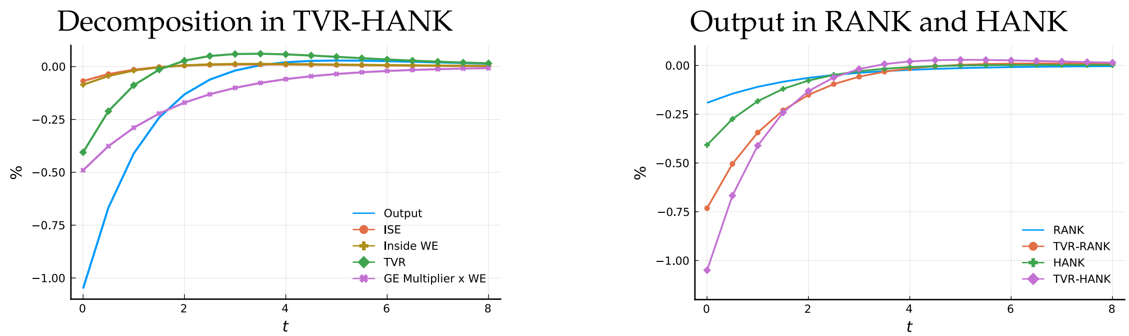

Figure 4: Output in RANK and HANK.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and the fiscal backing corresponds to the value estimated in Section 4.1.

the no-disaster and disaster states, respectively. Given the price of the corporate bond, we can compute the corporate spread. Figure 3 shows that the corporate spread responds to monetary shocks by 8.9 bps. We introduce the excess bond premium (EBP) in our VAR and find an increase in the EBP of 6.5 bps and an upper bound of the confidence interval of 10.9 bps, consistent with the model's prediction. Thus, even though this was not a targeted moment, time-varying risk is able to produce quantitatively plausible movements in the corporate spread.

Another moment that is not targeted by the calibration is the response of stocks to monetary shocks. We find a substantial response of stocks to changes in interest rates, which is explained mostly by movements in the risk premium. In contrast to the empirical evidence, we find a *positive* response of dividends to a contractionary monetary shock. This is the result of the well-known feature of sticky-prices models that profits are strongly countercyclical. This counterfactual prediction could be easily solved by introducing some form of wage stickiness. Despite the positive response of dividends, the model generates a decline in stocks of 2.15% in response to a 100 bps increase in interest rates, which is smaller than the point estimate of Bernanke and Kuttner (2005) but is still within their confidence interval.[27] Fixing the degree of countercyclicality of profits would likely bring the response of stocks closer to their point estimate.

4.3 Wealth effects in the monetary transmission mechanism

Figure 4 (left) presents the response of output and its components to a monetary shock in the New Keynesian model with heterogeneous agents and time-varying risk. We find

[27]We follow standard practice in the asset-pricing literature and report the response of a levered claim on firms' profits, using a debt-to-equity ratio of 0.5, as in Barro (2006).

that output reacts by -1.05% to a 100 bp increase in the nominal interest rate, which is consistent with the empirical estimates of e.g. Miranda-Agrippino and Ricco (2021). In terms of its components, time-varying risk (TVR) and the outside wealth effect are the two main components determining the output dynamics, representing 39% and 47% of the output response, respectively. In contrast, the ISE accounts for only 6.5% of the output response, indicating that intertemporal substitution plays only a minor role in the monetary transmission mechanism.

These findings stand in sharp contrast to the dynamics in the absence of heterogeneity and time-varying risk. Figure 4 (right) plots the response of output for different combinations of heterogeneity ($\mu_b > 0$ and $\mu_b = 0$) and time-varying risk ($\epsilon_\lambda > 0$ and $\epsilon_\lambda = 0$). By shutting down the two channels, denoted by "RANK" in the figure, the initial response of output would be -0.14%, a more than a sevenfold reduction in the impact of monetary policy. There are two reasons for this result. First, our calibration of $\sigma = 4$ implies an EIS that is one fourth of the standard calibration. This significantly reduces the quantitative importance of the ISE, even if the intertemporal substitution channel represents a large fraction of the output response in the RANK model. Second, our estimate of the fiscal response is substantially lower than the one implied by a standard Taylor equilibrium that imposes an AR(1) process for the monetary shock. We discuss the role of fiscal backing and the implications for the New Keynesian model in Section 4.5 below.

Figure 4 (right) also plots the response of output when there is household heterogeneity but not time-varying risk ("HANK" in the figure), and the response of output when there is time-varying risk but not household heterogeneity ("TVR-RANK" in the figure). We find that heterogeneity increases the response of output by 22 bps while time-varying risk increases it by 54 bps. Notably, by combining both features, we get an increase in the response of output of 86 bps, which is 10 bps larger than the sum of the individual effects. Thus, heterogeneity and time-varying risk reinforce each other. In terms of the fraction of the response of output that can be attributed to each channel, we find that 20.5% can be attributed to household heterogeneity, 51.5% corresponds to time-varying risk, and 9.7% is the amplification effect of heterogeneity together with time-varying risk (which is around 50% larger than the contribution of the ISE), while the remainder represents the channels in the RANK model.

Finally, time-varying risk is essential for properly capturing the heterogeneous response of borrowers and savers to monetary policy. Figure 5 shows that borrowers are disproportionately affected by monetary shocks. However, the magnitude of the relative response of borrowers and savers is too large in the economy without time-varying risk. The drop in borrowers' consumption is 7 times greater than the decline in savers' con-

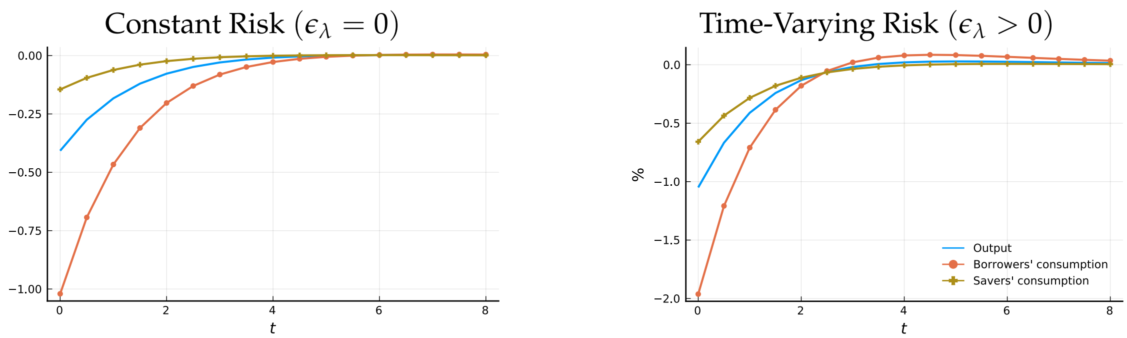

Figure 5: Consumption of borrowers and savers with constant risk and time-varying risk.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and the fiscal backing corresponds to the value estimated in Section 4.1.

sumption with a constant disaster probability, while it is 3 times greater in the economy with time-varying risk. Cloyne et al. (2020) estimate a relative peak response of mortgagors and homeowners of roughly 3.6. Therefore, allowing for time-varying risk is also important if we want to capture the heterogeneous impact of monetary policy.

4.4 The limitations of the constant disaster risk model

Consider the response of asset prices to a monetary shock in an economy that features constant disaster risk (i.e. $\lambda > 0$ but $\epsilon_\lambda = 0$). Figure 6 (left) shows that the yield on the long bond increases by 6.5 bps, which implies a decline of the value of the bond of 32 bps (given a 5-year duration), less than half of the response estimated by the VAR in Section 4.1. Moreover, movements in the long bond yield are almost entirely explained by the path of nominal interest rates, while the term premium is indistinguishable from zero. This stands in sharp contrast to the evidence reported in Gertler and Karadi (2015) and Hanson and Stein (2015). Similarly, it can be shown that most of the response of stocks in the model is explained by movements in interest rates instead of changes in risk premia, a finding that is inconsistent with the evidence documented in e.g. Bernanke and Kuttner (2005).

Figure 6 (right) shows how the presence of constant disaster risk affects the response of output to monetary shocks for the HANK and RANK economies. We find that risk has only a minor impact on the response of output. Aggregate risk increases the value of the discounting parameter δ, which reduces the GE multiplier and dampens the initial impact of the monetary shock. Given that the term premium barely moves, disaster risk plays only a small role in determining the outside wealth effect. In contrast, the important role of heterogeneity can be seen by comparing the response of the D-HANK and D-RANK

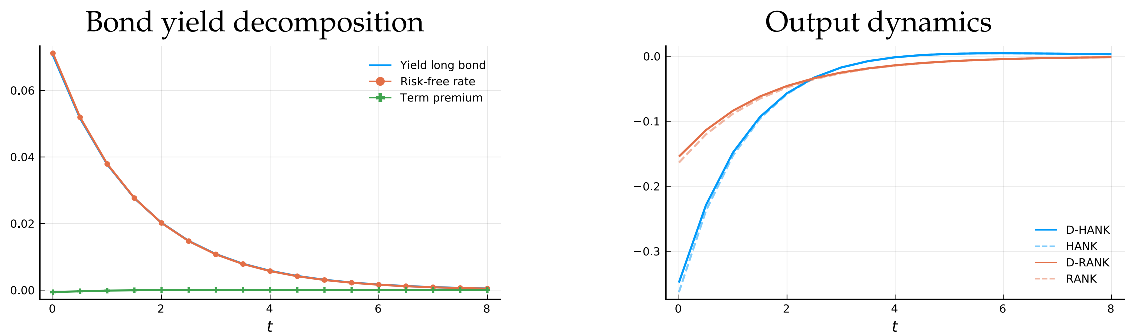

Figure 6: Long-term bond yields and output for economies with and without risk.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and the fiscal backing corresponds to the value estimated in Section 4.1. D-HANK and D-RANK correspond to heterogeneous-agent and representative agent economies with constant disaster risk (i.e. $\lambda > 0$ and $\epsilon_\lambda = 0$). HANK and RANK correspond to economies with no disaster risk (i.e. $\lambda = 0$).

economies.

Therefore, while introducing a constant disaster probability allows the model to capture important *unconditional* asset-pricing moments, such as the (average) risk premium or the upward-sloping yield curve, the model is unable to match key *conditional* moments, in particular, the response of asset prices to monetary policy. The limitations of the model with constant disaster probability in matching conditional asset-pricing moments were recognized early on in the literature, leading to an assessment of the implications of *time-varying disaster risk*, as in Gabaix (2012) and Gourio (2012). This justifies our focus on time-varying disaster risk and how it affects the asset-pricing response to monetary shocks and, ultimately, its impact on real economic variables.

4.5 The role of fiscal backing and the EIS

We have found that time-varying risk and heterogeneity substantially amplify the impact of monetary policy on the economy. To properly assess the importance of these two channels, however, it was crucial to control for the implicit fiscal backing, as discussed in Section 3.4.

Figure 7 illustrates this point. In the three panels, we show the impact of a monetary shock that leads to an increase in nominal interest rates on impact of 100 bps. In the left panel, we consider a RANK economy ($\mu_b = \lambda = 0$) with the standard value for the EIS ($\sigma^{-1} = 1$) and fiscal backing implicitly determined by a Taylor rule with a monetary shock that follows a standard AR(1) process, corresponding to the textbook New Keynesian model. In the middle panel, we consider the same economy but the fiscal backing is set to the value estimated in the data, corresponding to a Taylor equilibrium with a

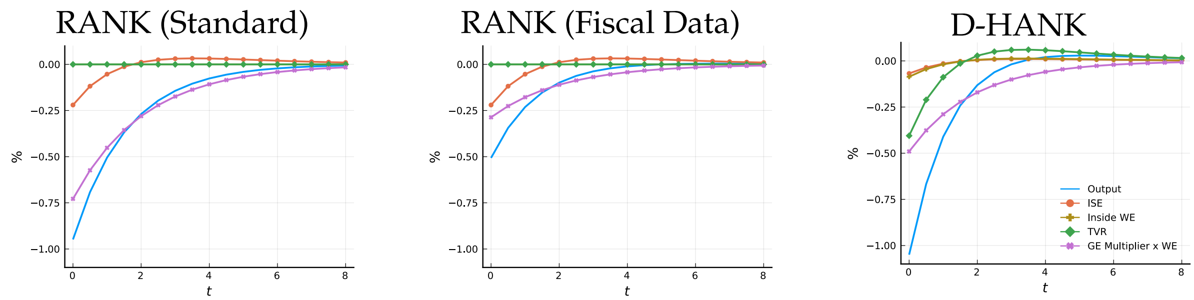

Figure 7: Output in RANK vs D-HANK with time-varying risk.

Note: The first two panels show output in RANK ($\mu_b = \lambda = 0$) with unit EIS ($\sigma^{-1} = 1$). In the left panel, fiscal backing is determined by a Taylor rule, while in the middle panel fiscal backing corresponds to the value estimated in the data. The right panel corresponds to the D-HANK economy with time-varying risk and the estimated fiscal backing.

monetary shock that follows the more general specification from equation (27). The right panel shows our D-HANK model with time-varying risk and the calibrated value of the EIS, $\sigma^{-1} = 0.25$.

The response of the textbook economy is only slightly smaller than that of our D-HANK economy despite the lack of time-varying risk or heterogeneous agents. An important reason for this is the difference in the value of the (implicit) fiscal backing, which is almost ten times larger in the textbook economy compared with the one we estimated in the data. When the fiscal backing is the same as in the data, the response of output drops by almost half. The EIS also plays an important role. Even with fiscal backing directly from the data, the response of output is still significant, only slightly less than that in our D-RANK with time-varying risk (see Figure 4). But this same response comes from very different channels. In the RANK economy, the ISE accounts for roughly 40% of the output response, while in our D-RANK the ISE accounts for less than 7% of that response.

These results suggest that the quantitative success of the RANK model is likely the result of a counterfactually large fiscal backing in response to monetary shocks and a strong intertemporal-substitution channel, which compensate for missing heterogeneous agents and risk channels. Once we discipline the fiscal backing with data and calibrate the EIS to the estimates obtained from microdata, our model suggests that heterogeneous agents and, in particular, time-varying risk are crucial for generating quantitatively plausible output dynamics. However, it is important to note that our model made several simplifications to incorporate indebted agents and time-varying aggregate risk without sacrificing the tractability of standard macro models. A natural extension would be to incorporate these channels into a medium-sized DSGE model to better assess the quantitative properties of the New Keynesian model.

5 The Effect of Risk and Maturity of Household Debt

We have assumed so far that households borrow using short-term riskless debt. In practice, however, most household debt takes the form of long-term risky debt. In this case, the effect of monetary policy on borrowers depends on how the term spread and credit spread, the compensation for holding interest rate and default risk, respond to changes in the short-term interest rate. In this section, we extend the baseline model to allow for default risk and long-term maturities on household debt and show how these two features affect the transmission of monetary policy shocks to the real economy.

5.1 The model with long-term risky household debt

We describe next the model with long-term risky household debt. We highlight the main differences with the model described in Section 2 and present a detailed description in Appendix **??**. Households issue long-term debt that promises to pay exponentially decaying coupons given by $e^{-\psi_P t}$ at period $t \geq 0$, where $\psi_P \geq 0$. Importantly, households cannot commit to always repay their debts. In response to a large shock, i.e. the occurrence of a disaster, households default and lenders receive a fraction $1 - \zeta_P$ of the promised coupons, where $0 \leq \zeta_P \leq 1$. We assume that fluctuations in the no-disaster state are small enough such that they do not trigger a default. Thus, households default only in the disaster state.

We denote the price of household debt in the no-disaster (disaster) state by $Q_{P,t}$ ($Q^*_{P,t}$), so the nominal return on household debt is given by

$$dR_{P,t} = \left[\frac{1}{Q_{P,t}} + \frac{\dot{Q}_{P,t}}{Q_{P,t}} - \psi_P\right] dt + \frac{Q^*_{P,t} - Q_{P,t}}{Q_{P,t}} d\mathcal{N}_t,$$

where $i_{P,t} \equiv \frac{1}{Q_{P,t}} - \psi_P$ is the yield on the bond. In a stationary equilibrium, the spread between the interest rate on household debt and the short-term interest rate controlled by the central bank is given by

$$r_P = \lambda \left(\frac{C_s}{C^*_s}\right)^{\sigma} \frac{Q^*_P - Q_P}{Q_P}.$$

Note that the interest rate on household debt incorporates both a credit and a term spread.[28] We assume that households can borrow up to $\overline{D}_{P,t} = Q_{P,t}\overline{F}$, which effectively

[28] Let $i^{ND}_{P,t}$ denote the yield on a non-defaultable bond with coupons decaying at rate ψ_p. The term spread corresponds to $i^{ND}_{P,t} - i_t$ and the credit spread to $i_{P,t} - i^{ND}_{P,t}$, so $r_P = (i^{ND}_P - r_n) + (i_P - i^{ND}_P)$.

puts a limit on the face value of household debt $\overline{F}$.[29] In a log-linear approximation of the economy around a zero-inflation stationary equilibrium, borrowers are constrained at all periods, and their consumption is given by

$$c_{b,t} = (1-\alpha)(w_t - p_t + n_{b,t}) + T_{b,t} - \left(\frac{\psi_P}{i_P + \psi_P}(i_{P,t} - i_P) - \pi_t\right)\overline{d}_P. \tag{28}$$

Equation (28) generalizes the expression for borrowers' consumption given in Section 2. Monetary policy affects borrowers indirectly through its effect on the yield on household debt $i_{P,t}$. If we assume that debt is short-term, $\psi_P \to \infty$, and riskless, $\zeta_P = 0$, we obtain $i_{P,t} = i_t$ and the expression above boils down to equation (8). At the other extreme, we have the case of a perpetuity, $\psi_P = 0$. In this case, households simply pay the coupon every period and there is no need to issue new debt. Therefore, they are completely insulated from movements in nominal interest rates.

The price of household debt evolves according to

$$\dot{q}_{P,t} = (\rho + \psi_P)q_{P,t} + i_t - r_n + r_P p_{d,t}.$$

The price of the bond depends on the future path of short-term interest rates and the risk premium. Quantitatively, the fluctuations in the risk premium are dominated by the time-varying risk component, while the term $\sigma c_{s,t}$ gives a negligible contribution, as shown in Figure 6. Motivated by this fact, we assume that $r_P \sigma c_{s,t}$ is negligible in a first-order approximation, such that we can write the price of the bond as follows[30]

$$q_{P,t} = -\frac{1 + r_P \epsilon_\lambda}{\rho + \psi_P + \psi_m}(i_t - r_n), \tag{29}$$

where ψ_m is the decaying rate of nominal interest rates.

Combining the behavior of borrowers' consumption with savers' Euler equation (10) and the Phillips curve (12), we can derive the response of aggregate output to monetary shocks. In particular, we can extend the decomposition in Proposition 5 to the case of long-term risky debt.

Proposition 9 (Aggregate output with long-term risky debt)**.** *Suppose that $i_t - r_n = e^{-\psi_m t}(i_0 -$*

[29] This formulation guarantees that, after an increase in nominal rates, the value of household debt and the borrowing limit decline by the same amount. This specification of the borrowing constraint, combined with the assumption of impatient borrowers, guarantees that borrowers are constrained at all periods.

[30] Formally, we assume that the parameter $r_P \sigma$ is of the same order as the (small) monetary shock, $r_P \sigma = \mathcal{O}(i_0 - r_n)$. Therefore, the term $r_P \sigma c_{s,t}$ is second-order in $i_0 - r_n$ and it can be ignored in a first-order approximation. The solution in the absence of this assumption is available upon request.

r_n) *and* $r_P\sigma = \mathcal{O}(i_0 - r_n)$. *The path of aggregate output is then given by*

$$y_t = \underbrace{\sigma^{-1}\hat{y}_t}_{\textit{ISE}} + \underbrace{\chi_d\epsilon_\lambda\hat{y}_t}_{\substack{\textit{time-varying}\\ \textit{risk}}} + \underbrace{\frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\frac{\psi_P(1+r_P\epsilon_\lambda)}{\rho+\psi_P+\psi_m}\tilde{\psi}_m\hat{y}_t}_{\textit{inside wealth effect}} + \underbrace{(\rho-\underline{\omega})e^{\underline{\omega}t}\Omega_0,}_{\substack{\textit{GE multiplier}\times\\ \textit{outside wealth effect}}}. \tag{30}$$

Proposition 9 shows that default risk and debt maturity have opposite effects on the magnitude of the inside wealth effect. For instance, in the case of short-term debt, the term $\frac{\psi_P(1+r_P\epsilon_\lambda)}{\rho+\psi_P+\psi_m}$ simplifies to $1 + r_P\epsilon_\lambda > 1$, so the inside wealth effect is amplified relative to the case of riskless debt. The interest rate on household debt now moves in response to changes in the short-term interest rate as well as changes in the risk premium. In contrast, the inside wealth effect is dampened for long-term bonds. In the limit case of a perpetuity, $\psi_P = 0$, the inside wealth goes to zero. Given that households do not issue new debt, they are not affected by the change in interest rates, which eliminates the (inside) wealth effect.

5.2 Quantitative implications

We consider next the quantitative implications of default risk and maturity on household debt. As shown in Proposition 9, these two features have opposing effects on the response of output to monetary policy. To assess the quantitative impact of risk and maturity, we show in Figure 8 the inside wealth effect (left panel) and aggregate output (right panel) as a function of the duration of household debt for different values of the haircut parameter ζ_P. Greenwald et al. (2021) estimate the duration of mortgage debt as 5.2 years, the duration of student debt as 4.50, and the duration of consumer debt as 1.0 year. Therefore, we focus on values of duration up to five years in Figure 8. We consider three different values for the haircut parameter: riskless debt ($\zeta_P = 0$); risky debt with a spread in the stationary equilibrium of roughly 4.0% with a 5-year duration ($\zeta_P = 0.10$); risky debt with a spread of 5.0% with a 5-year duration ($\zeta_P = 0.25$).

Default risk substantially amplifies the effect of monetary policy on output when debt is short term. The inside wealth effect is almost three times larger in the case of $\zeta_P = 0.25$ compared to $\zeta_P = 0.0$, which corresponds to an increase in the initial response of output of almost 25%. However, this effect is strongly attenuated when household debt is long term. For even relatively small values of duration, the inside wealth effect is smaller than in the case of short-term riskless debt. For instance, in the case of a five-year duration, the response of output is roughly 10% smaller than the response in the case of short-term riskless debt.

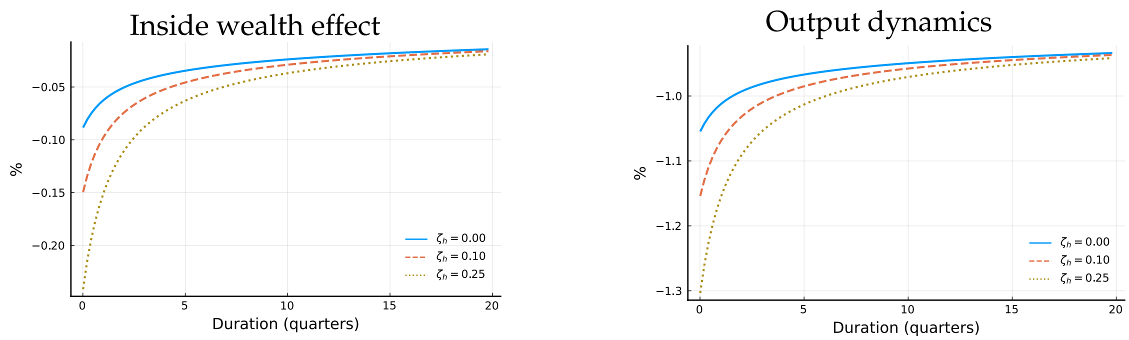

Figure 8: Inside wealth effect and output as a function of duration and haircut ζ_P.

It is important to note that even though the inside wealth effect is substantially dampened with long-term debt, the presence of household debt still generates amplification through its impact on the compounding parameter δ, as shown in Proposition 2. The response of output when household debt is zero is roughly 35% smaller than in the economy with (positive) riskless debt, a much larger drop relative to the one caused by introducing long-term bonds.

6 Conclusion

In this paper, we provide a novel unified framework to analyze the role of heterogeneity and risk in a tractable linearized New Keynesian model. The methods introduced can be applied beyond the current model. For instance, they can be applied to a full quantitative HANK model with idiosyncratic risk, extending the results of Ahn et al. (2018) to allow for time-varying risk premia. Alternatively, one could introduce a richer capital structure for firms and study the pass-through of monetary policy to households and firms. These methods may enable us to bridge the gap between the extensive existing work on heterogeneous agents and monetary policy and the emerging literature on the role of asset prices in the transmission of monetary shocks.

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Appendix: For Online Publication

A Proofs

Proof of Proposition 2. Consider first the New Keynesian Phillips curve

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t}\right)\pi_t - \frac{\epsilon}{\varphi A}\left(\frac{W}{P}e^{w_t - p_t} - (1-\epsilon^{-1})A\right)Ye^{y_t}.$$

Linearizing the above expression, and using $\frac{W}{P} = (1-\epsilon^{-1})A$, we obtain

$$\dot{\pi}_t = \left(r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\right)\pi_t - \varphi^{-1}(\epsilon - 1)Y(w_t - p_t).$$

Using the fact that $w_t - p_t = \phi y_t$, we obtain $\dot{\pi}_t = (\rho_s + \lambda)\,\pi_t - \kappa y_t$, where $\kappa \equiv \varphi^{-1}(\epsilon-1)\phi Y$ and we used that $r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} = \rho_s + \lambda$.

Consider next the generalized Euler equation. From the market-clearing condition for goods and borrowers' consumption, we obtain $c_{s,t} = \frac{1-\mu_b\chi_y}{1-\mu_b}y_t + \frac{\mu_b \overline{d}_P}{1-\mu_b}(i_t - \pi_t - r_n)$. Combining this condition with the Phillips Curve and savers' Euler equation, and using the fact that $r_n = \rho - \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}$, we obtain $\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi - r_n) + \delta y_t + v_t$, where the constants $\tilde{\sigma}^{-1}$, δ, and v_t are defined in the proposition.

$\square$

Proof of Lemma 1. Suppose $[y_t, \pi_t]_0^{\infty}$ satisfies system (20) and the intertemporal budget constraint (21) in the no-disaster state. We will show that $[y_t, \pi_t]_0^{\infty}$ can be supported as an equilibrium. Consider first the disaster state. The savers' budget constraint implies $T_{s,t} = -\rho_s b_s b_{s,t^*}$. All the remaining variables are equal to zero in the disaster state.

Consider now the no-disaster state. The real wage is given by $w_t - p_t = (\phi + \sigma)y_t$. Borrowers' consumption is given by $c_{b,t} = \chi_y y_t - \chi_r \overline{d}_P(i_t - \pi_t - r - n)$, while savers' consumption are given by $c_{s,t} = \frac{1-\mu_b\chi_y}{1-\mu_b}y_t + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}(i_t - \pi_t - r_n)$, and the labor supply is given by $n_{j,t} = \phi^{-1}(w_t - p_t) - \phi^{-1}\sigma c_{j,t}$.

By construction, the market-clearing condition for goods and labor are all satisfied. Because y_t satisfies the aggregate Euler equation, the savers' Euler equation is also satisfied. Because π_t satisfies the New Keynesian Phillips curve, the optimality condition for firms is satisfied. Bond holdings by savers and government debt evolve according to

$$b_s\dot{b}_{s,t} = r_n b_s b_{s,t} + (1-\alpha)(w_t - p_t + n_{s,t}) + T_{s,t} + \frac{(1-\tau)y_t - (1-\alpha)(w_t - p_t + n_t)}{1-\mu_b} + \\ (i - \pi_t - r_n)b_s + (r_{L,t} - r_L)b_s^L + r_L b_s^L b_{s,t}^L - c_{s,t},$$

$$\overline{d}_G\dot{d}_{G,t} = \overline{d}_G(r_n + r_L)d_{G,t} + T_t - \tau y_t + (i_t - \pi_t - r_n + r_{L,t} - r_L)\overline{d}_G,$$

where $b_{s,0} = \frac{b_s^L}{b_s} q_{L,0}$ and $d_{G,0} = \overline{d}_G q_{L,0}$. The value of $c_{b,t}$ is such that the flow budget constraint for borrowers also holds.

Aggregating the budget constraint of borrowers and savers and using the market clearing condition for goods and labor, we obtain

$$(1-\mu_b) b_s \dot{b}_{s,t} = r_n (1-\mu_b) b_s b_{s,t} + T_t - \tau y_t + (i - \pi_t - r_n) \left[(1-\mu_b) b_s - \mu_b \overline{d}_P \right] + (r_{L,t} - r_L + r_L b_{s,t}^L)(1-\mu_b) b_s^L.$$

Note that $b_s b_{s,t} = b_s^S b_{s,t}^S + b_s^L b_{s,t}^L$. We set $b_{s,t}^S = 0$, so the market for short-term bonds clear at all periods. It remains to show that the market for long-term bonds also clears. Subtracting the government's flow budget constraint from the condition above, we obtain

$$(1-\mu_b) b_s^L \dot{b}_{s,t}^L - \overline{d}_G \dot{d}_{G,t} = (r_n + r_L)((1-\mu_b) b_s^L b_{s,t}^L - \overline{d}_G d_{G,t}),$$

using $b_s b_{s,t} = b_{s,t}^L b_{s,t}^L$ and $(1-\mu_b) b_s - \mu_b \overline{d}_P = (1-\mu_b) b_s^L = \overline{d}_G$. Integrating this expression, we obtain $(1-\mu_b) b_s^L b_{s,t}^L - \overline{d}_G d_{G,t} = e^{(r_n + r_L)t} \left[(1-\mu_b) b_s^L b_{s,0}^L - \overline{d}_G d_{G,0} \right] = 0$, where the equality uses the market clearing condition in period 0. Therefore, the market clearing condition for long-term bonds is satisfied in all periods. The only condition that remains to be checked is the No-Ponzi condition for the government or, equivalently, the aggregate intertemporal budget constraint. Because condition (21) is satisfied, the No-Ponzi condition for the government is also satisfied. □

Proof of Propositions 5 and 6. We can write dynamic system (20) in matrix form as $\dot{Z}_t = A Z_t + B \nu_t$, where $B = [1, 0]'$. Applying the spectral decomposition to matrix A, we obtain $A = V \Omega V^{-1}$ where $V = \begin{bmatrix} \frac{\rho - \overline{\omega}}{\kappa} & \frac{\rho - \underline{\omega}}{\kappa} \\ 1 & 1 \end{bmatrix}$, $V^{-1} = \frac{\kappa}{\overline{\omega} - \underline{\omega}} \begin{bmatrix} -1 & \frac{\rho - \underline{\omega}}{\kappa} \\ 1 & -\frac{\rho - \overline{\omega}}{\kappa} \end{bmatrix}$, and $\Omega = \begin{bmatrix} \overline{\omega} & 0 \\ 0 & \underline{\omega} \end{bmatrix}$. Decoupling the system, we obtain $\dot{z}_t = \Omega z_t + b \nu_t$, where $z_t = V^{-1} Z_t$ and $b = V^{-1} B$.

Solving the equation with a positive eigenvalue forward and the one with a negative eigenvalue backward, and rotating the system back to the original coordinates, we obtain

$$y_t = V_{12} \left(V^{21} y_0 + V^{22} \pi_0 \right) e^{\underline{\omega} t} - V_{11} V^{11} \int_t^\infty e^{-\overline{\omega}(z-t)} \nu_z dz + V_{12} V^{21} \int_0^t e^{\underline{\omega}(t-z)} \nu_z dz$$
$$\pi_t = V_{22} \left(V^{21} y_0 + V^{22} \pi_0 \right) e^{\underline{\omega} t} - V_{21} V^{11} \int_t^\infty e^{-\overline{\omega}(z-t)} \nu_z dz + V_{22} V^{21} \int_0^t e^{\underline{\omega}(t-z)} \nu_z dz,$$

where $V^{i,j}$ is the (i,j) entry of matrix V^{-1}. Integrating $e^{-\rho t} y_t$ and using the intertemporal budget constraint,

$$\Omega_0 = V_{12} \left(V^{21} y_0 + V^{22} \pi_0 \right) \frac{1}{\rho - \underline{\omega}} - \frac{1}{\rho - \overline{\omega}} V_{11} V^{11} \int_0^\infty \left(e^{-\overline{\omega} t} - e^{-\rho t} \right) \nu_t dt + \frac{1}{\rho - \underline{\omega}} V_{12} V^{21} \int_0^\infty e^{-\rho t} \nu_t dt.$$

Rearranging the above expression, we obtain

$$V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right) = (\rho - \underline{\omega})\Omega_0 + \frac{\rho-\underline{\omega}}{\rho-\overline{\omega}} V_{11}V^{11}\int_0^\infty e^{-\overline{\omega}t}\nu_t dt,$$

where we used the fact $\frac{V_{11}V^{11}}{\rho-\overline{\omega}} + \frac{V_{12}V^{21}}{\rho-\underline{\omega}} = 0$. Output is then given by $y_t = \tilde{y}_t + (\rho-\underline{\omega})e^{\underline{\omega}t}\Omega_0$, where $\tilde{y}_t = -\frac{\overline{\omega}-\rho}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\overline{\omega}-\delta}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\rho-\underline{\omega}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$. Inflation is given by $\pi_t = \tilde{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0$, where $\tilde{\pi}_t = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\kappa}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$.

If $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, then $\dot{i}_t = -\psi_m(i_t - r_n)$. This allows us to write $\tilde{y}_t = \sigma^{-1}\hat{y}_{m,t} + \frac{\mu_b\chi_r\overline{d}_p}{1-\mu_b}\tilde{\psi}_m\hat{y}_{m,t} + \chi_p\epsilon_\lambda\hat{y}_{\lambda,t}$ and $\tilde{\pi}_t = \sigma^{-1}\hat{\pi}_{m,t} + \chi_p\epsilon_\lambda\hat{\pi}_{\lambda,t} + \frac{\mu_b\chi_r\overline{d}_p}{1-\mu_b}\tilde{\psi}_m\hat{\pi}_{m,t}$, where $\tilde{\psi}_m \equiv \rho - r_n + \psi_m$, $\hat{y}_{k,t} = \frac{1-\mu_b}{1-\mu_b\chi_y}\left[-\frac{\psi_k+\rho}{(\psi_k+\underline{\omega})(\psi_k+\overline{\omega})}e^{-\psi_k t} + \frac{\rho-\underline{\omega}}{(\psi_k+\overline{\omega})(\psi_k+\underline{\omega})}e^{\underline{\omega}t}\right](i_0 - r_n)$, and $\hat{\pi}_{k,t} = \frac{\kappa(e^{\underline{\omega}t}-e^{-\psi_k t})}{(\underline{\omega}+\psi_k)(\overline{\omega}+\psi_k)}(i_0 - r_n)$. Note that $\int_0^\infty e^{-\rho t}\hat{y}_{k,t}dt = 0$, $\frac{\partial\hat{y}_{k,0}}{\partial i_0} = -\frac{1}{\psi_k+\overline{\omega}} < 0$, and $\lim_{t\to\infty}\hat{y}_{k,t} = 0$. Moreover, $\hat{\pi}_0 = 0$, $\frac{\partial\hat{\pi}_{k,t}}{\partial i_0} \geq 0$ with strict inequality if $t > 0$.

□

Proof of Proposition 7. From Propositions 5 and 6, we have $y_t = \chi\hat{y}_t + (\overline{\omega} - \delta)e^{\underline{\omega}t}\Omega_0$, and $\pi_t = \chi\hat{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0$, where $\chi \equiv \sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r}{1-\mu_b}\tilde{\psi}_m\overline{d}_P$. Moreover, we can rewrite the price of disaster risk as $p_{d,t} = \hat{p}_{d,t} + \chi_{p_d,\Omega}e^{\underline{\omega}t}\Omega_0$, where $\hat{p}_{d,t} \equiv \sigma\frac{1-\mu_b\chi_y}{1-\mu_b}\chi\hat{y}_t + \sigma\frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}(i_t - \chi\hat{\pi}_t - r_n) + \epsilon_\lambda(i_t - r_n)$ and $\chi_{p_d,\Omega} \equiv \sigma\left(\frac{1-\mu_b\chi_y}{1-\mu_b}(\rho-\underline{\omega}) - \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\kappa\right)$. Finally, the price of the long-term bond in period 0 can be written as $q_{L,0} = \hat{q}_{L,0} + \chi_{q_L,\Omega}\Omega_0$, where $\hat{q}_{L,0} \equiv -\int_0^\infty e^{-(\rho+\psi_L)t}\left[i_t - r_n + r_L\hat{p}_{d,t}\right]$ and $\chi_{q_L,\Omega} \equiv -\frac{r_L}{\rho+\psi_L-\underline{\omega}}\chi_{p_d,\Omega}$. Introducing these expressions into (**??**), and using that $\int_0^\infty e^{-\rho t}y_t dt = \Omega_0$, we obtain

$$\Omega_0 = (1-\chi_\Omega)\Omega_0 + \overline{d}_G\hat{q}_{L,0} + \int_0^\infty e^{-\rho t}\left[T_t + \overline{d}_G(i_t - \chi\hat{\pi}_t - r_n + r_L\hat{p}_{d,t})\right]dt,$$

where $\chi_\Omega \equiv \tau + \frac{\kappa\overline{d}_G}{\rho-\underline{\omega}} - \frac{\psi_L r_L\overline{d}_G}{(\rho-\underline{\omega})(\rho+\psi_L-\underline{\omega})}\chi_{p_d,\Omega}$.

□

Proof of Proposition 8. We divide this proof in three steps. First, we derive the condition for local uniqueness of the solution under the policy rule (7). Second, we derive the path of $[y_t, \pi_t, i_t]_0^\infty$ for a given path of monetary shocks. Third, we show how to implement a given path of nominal interest rates $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and a given value of Ω_0, which maps to a given value of fiscal backing $\int_0^\infty e^{-\rho t}T_t dt$.

Equilibrium determinacy. First, note that we can write ν_t as $\nu_t = \tilde{\sigma}^{-1}\left[1 + \frac{1-\mu_b}{1-\mu_b\chi_y}\tilde{\sigma}\chi_d\epsilon_\lambda\right]\phi_\pi\pi_t + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b\chi_y}\phi_\pi\kappa y_t + \tilde{\nu}_t$, where $\tilde{\nu}_t \equiv \left[\frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1} + \frac{1-\mu_b}{1-\mu_b\chi_y}\chi_d\epsilon_\lambda\right]u_t - \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b\chi_y}((r_n-\rho)u_t + \dot{u}_t)$.

The dynamic system for y_t and π_t can now be written as

$$\begin{bmatrix}\dot{y}_t\\ \dot{\pi}_t\end{bmatrix} = \begin{bmatrix}\tilde{\delta} & -\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi)\\ -\kappa & \rho\end{bmatrix}\begin{bmatrix}y_t\\ \pi_t\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}\tilde{\nu}_t,$$

where $\tilde{\delta} \equiv \delta + \frac{\mu_b \chi_r \overline{d}_P}{1-\mu_b \chi_y}\phi_\pi \kappa$ and $\tilde{\phi}_\pi \equiv \left[1 + \frac{1-\mu_b}{1-\mu_b\chi_y}\tilde{\sigma}\chi_d \epsilon_\lambda\right]\phi_\pi$. The eigenvalues of the system are given by $\overline{\omega}_T = \frac{\rho+\tilde{\delta}+\sqrt{(\rho+\tilde{\delta})^2+4(\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi)\kappa-\rho\tilde{\delta})}}{2}$ and $\underline{\omega}_T = \frac{\rho+\tilde{\delta}-\sqrt{(\rho+\tilde{\delta})^2+4(\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi)\kappa-\rho\tilde{\delta})}}{2}$. The system has a unique bounded solution if both eigenvalues have positive real parts. A necessary condition for the eigenvalues to have positive real parts is

$$\rho + \delta + \frac{\mu_b \chi_r \overline{d}_P}{1-\mu_b\chi_y}\phi_\pi\kappa > 0 \iff \phi_\pi > -(\rho+\delta)\left(\frac{\mu_b\chi_r\overline{d}_P\kappa}{1-\mu_b\chi_y}\right)^{-1} = 1 - \left(\rho + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma\right)\frac{1-\mu_b\chi_y}{\mu_b\chi_r\overline{d}_P\kappa}.$$

If the condition above is violated, then the real part of $\underline{\omega}_T$ is negative. Another necessary condition for the eigenvalues to have positive real parts is

$$\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi)\kappa < \rho\left[\delta + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b\chi_y}\phi_\pi\kappa\right] \iff \phi_\pi > 1 - \frac{\chi_d\epsilon_\lambda + \frac{\rho\lambda}{\kappa}\frac{1-\mu_b\chi_y}{1-\mu_b}\left(\frac{C_s}{C_s^*}\right)^\sigma}{\chi_d\epsilon_\lambda + \sigma^{-1}}.$$

If this condition is violated, then the eigenvalues are real-valued and $\underline{\omega}_T < 0$. This establishes the necessity of the condition

$$\phi_\pi > \max\left\{1 - \frac{\chi_d\epsilon_\lambda + \frac{\rho\lambda}{\kappa}\frac{1-\mu_b\chi_y}{1-\mu_b}\left(\frac{C_s}{C_s^*}\right)^\sigma}{\chi_d\epsilon_\lambda + \sigma^{-1}}, 1 - \left(\rho + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma\right)\frac{1-\mu_b\chi_y}{\mu_b\chi_r\overline{d}_P\kappa}\right\}.$$

If $\mu_b\chi_y < 1$, then $\phi_\pi > 1$ is sufficient to guarantee the local uniqueness of the solution.

Solution to the dynamic system. The dynamic system for $[y_t, \pi_t]_{t=0}^\infty$ is given by

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \tilde{\delta} & -\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi) \\ -\kappa & \rho \end{bmatrix}\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\tilde{v}_t.$$

In matrix form, the system is given by $\dot{\tilde{Z}}_t = \tilde{A}\tilde{Z}_t + B\tilde{v}_t$, where $B = [1, 0]'$. Applying the spectral decomposition to matrix $\tilde{A}$, we obtain $\tilde{A} = \tilde{V}\Omega_T\tilde{V}^{-1}$ where $\tilde{V} = \begin{bmatrix} \frac{\rho-\overline{\omega}_T}{\kappa} & \frac{\rho-\underline{\omega}_T}{\kappa} \\ 1 & 1 \end{bmatrix}$, $\tilde{V}^{-1} = \frac{\kappa}{\overline{\omega}_T - \underline{\omega}_T}\begin{bmatrix} -1 & \frac{\rho-\underline{\omega}_T}{\kappa} \\ 1 & -\frac{\rho-\overline{\omega}_T}{\kappa} \end{bmatrix}$, and $\Omega_T = \begin{bmatrix} \overline{\omega}_T & 0 \\ 0 & \underline{\omega}_T \end{bmatrix}$. Decoupling the system, we obtain $\dot{\tilde{z}}_t = \Omega_T\tilde{z}_t + \tilde{b}\tilde{v}_t$, where $\tilde{z}_t = \tilde{V}^{-1}\tilde{Z}_t$ and $\tilde{b} = \tilde{V}^{-1}B$. Solving the system forward, and rotating the system back to the original coordinates, we obtain

$$\begin{aligned} y_t &= -\tilde{V}_{11}\tilde{V}^{11}\int_t^\infty e^{-\overline{\omega}_T(z-t)}\tilde{v}_z dz - \tilde{V}_{12}\tilde{V}^{21}\int_t^\infty e^{-\underline{\omega}_T(z-t)}\tilde{v}_z dz \\ \pi_t &= -\tilde{V}_{21}\tilde{V}^{11}\int_t^\infty e^{-\overline{\omega}_T(z-t)}\tilde{v}_z dz - \tilde{V}_{22}\tilde{V}^{21}\int_t^\infty e^{-\underline{\omega}_T(z-t)}\tilde{v}_z dz. \end{aligned}$$

We rewrite the above expression as follows:

$$y_t = -\frac{\overline{\omega}_T - \rho}{\overline{\omega}_T - \underline{\omega}_T}\int_t^\infty e^{-\overline{\omega}_T(z-t)}\tilde{v}_z dz + \frac{\underline{\omega}_T - \rho}{\overline{\omega}_T - \underline{\omega}_T}\int_t^\infty e^{-\underline{\omega}_T(z-t)}\tilde{v}_z dz$$
$$\pi_t = -\frac{\kappa}{\overline{\omega}_T - \underline{\omega}_T}\int_t^\infty \left(e^{-\underline{\omega}_T(z-t)} - e^{-\overline{\omega}_T(z-t)}\right)\tilde{v}_z dz,$$

where $\tilde{v}_t \equiv \left[\frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1} + \frac{1-\mu_b}{1-\mu_b\chi_y}\chi_d\epsilon_\lambda\right]u_t - \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b\chi_y}((r_n-\rho)u_t + \dot{u}_t)$. Using that $u_t = e^{-\psi_m t}u_0$, we obtain

$$y_t = -\frac{\rho+\psi_m}{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\tilde{\psi}_m\right)u_t$$
$$\pi_t = -\frac{\kappa}{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\tilde{\psi}_m\right)u_t,$$

where $(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m) = \tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi - 1) + (\tilde{\delta}+\psi_m)(\rho+\psi_m) > 0$.

The wealth effect is given by $\Omega_0 = -\frac{1}{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\tilde{\psi}_m\right)u_0$.

The nominal interest rate is given by $i_t = r_n + \frac{(\delta+\psi_m)(\rho+\psi_m)-\tilde{\sigma}^{-1}\kappa}{(\tilde{\delta}+\psi_m)(\rho+\psi_m)+\tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi-1)}u_t$. Note that if $\psi_m = -\underline{\omega} > 0$, then $i_t - r_n$, using the fact that $\overline{\omega}\underline{\omega} = \rho\delta - \tilde{\sigma}^{-1}\kappa$ and $\overline{\omega} + \underline{\omega} = \rho + \delta$. Despite the zero interest rate, the impact on output and inflation is non-zero. In particular, the outside wealth effect is given by $\Omega_0 = -\frac{1}{(\overline{\omega}_T-\underline{\omega})(\underline{\omega}_T-\underline{\omega})}\left(\frac{1-\mu_b}{1-\mu_b\chi_y}(\sigma^{-1}+\chi_d\epsilon_\lambda) + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b\chi_y}(\rho - r_n - \underline{\omega})\right)u_0$.

Implementability condition. Suppose $u_t = \vartheta e^{-\psi_m t}(i_0 - r_n) + \theta e^{\underline{\omega}t}$ and denote by $(\overline{i}_t, \overline{y}_t, \overline{\pi}_t)$ the value of the nominal interest rate, output, and inflation under the Taylor rule. Given the linearity of the system, the solution will be sum of the solutions for $u_{1,t} = \vartheta e^{-\psi_m t}(i_0 - r_n)$ and $u_{2,t} = \theta e^{\underline{\omega}t}$. After some algebra, we get that the nominal interest rate is given by $\overline{i}_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, using the fact that the nominal interest rate is zero under $u_{2,t}$, and choosing $\vartheta = \frac{(\tilde{\delta}+\psi_m)(\rho+\psi_m)+\tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi-1)}{(\delta+\psi_m)(\rho+\psi_m)-\tilde{\sigma}^{-1}\kappa} = \frac{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}{(\overline{\omega}+\psi_m)(\underline{\omega}+\psi_m)}$.

The outside wealth effect, $\overline{\Omega}_0 = \int_0^\infty e^{-\rho t}\overline{y}_t dt$, is given by

$$\overline{\Omega}_0 = -\frac{\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\tilde{\psi}_m\right)}{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}\vartheta(i_0 - r_n) - \frac{\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b\chi_y}(\rho - r_n - \underline{\omega})\right)}{(\overline{\omega}_T - \underline{\omega})(\underline{\omega}_T - \underline{\omega})}\theta$$

To implement a $\overline{\Omega}_0 = \Omega_0$, we must choose $\theta = \frac{(\overline{\omega}_T+|\underline{\omega}|)(\underline{\omega}_T+|\underline{\omega}|)}{\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1}+\chi_d\epsilon_\lambda+\frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}(\rho-r_n+|\underline{\omega}|)\right)}(\Omega_0^{AR(1)} - \Omega_0)$, where $\Omega_0^{AR(1)} = -\frac{1}{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r}{1-\mu_b}\tilde{\psi}_m\right)\vartheta(i_0 - r_n)$.

Given the process for u_t and the values of ϑ and θ, output can be written as $\overline{y}_t = \sigma^{-1}\hat{y}_t + \chi_d\epsilon_\lambda\hat{y}_t + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\tilde{\psi}_m\hat{y}_t + (\overline{\omega} - \delta)e^{\underline{\omega}t}\Omega_0$, which coincides with (23), where we used $\rho - \underline{\omega} = \overline{\omega} - \delta$. This result also implies that $\overline{\pi}_t = \pi_t$, as $\overline{\pi}_t = \kappa\int_0^\infty e^{-\rho t}\overline{y}_t dt = \kappa\int_0^\infty e^{-\rho t}y_t dt = \pi_t$.

Fiscal transfers. Given that y_t and π_t are proportional to u_t, we can write $i_t - r_n - \pi_t = \chi_{r,u}u_t$, $p_{d,t} = \chi_{p,u}u_t$, and $q_{L,0} = \chi_{q_L,u}u_0$. Rearranging the aggregate intertemporal budget constraint, we obtain the present discounted value of transfers: $\int_0^\infty e^{-\rho t}\mu_s T_{s,t}dt = (\tau - \mu_b T_b'(Y))\Omega_0 - \overline{d}_G\chi_{q_L,u}(\vartheta(i_0 - r_n) + \theta) - \overline{d}_g(\chi_{r,u} + r_L\chi_{p,u})\left[\frac{\vartheta(i_0 - r_n)}{\rho+\psi_m} + \frac{\theta}{\rho - \underline{\omega}}\right]$. □

Proof of Proposition 9. From the market clearing condition for goods, we obtain the consumption of savers: $c_{s,t} = \frac{1-\mu_b\chi_y}{1-\mu_b}y_t + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\left(\frac{\psi_P}{i_P+\psi_P}(i_{P,t} - i_P) - \pi_t\right)\overline{d}_P$. Assuming exponentially decaying interest rates, we can write the expression above as

$$c_{s,t} = \frac{1-\mu_b\chi_y}{1-\mu_b}y_t + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\left[\frac{\psi_P(1+r_P\epsilon_\lambda)}{\rho+\psi_P+\psi_m}(i_t - r_n) - \pi_t\right]. \tag{A.1}$$

The Euler equation for savers can be written as

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma c_{s,t} + \chi_d\epsilon_\lambda(i_t - r_n). \tag{A.2}$$

Combining equations (A.1) and (A.2), and using equation (**??**), we obtain

$$\dot{y}_t = \left[\frac{1-\mu_b}{1-\chi_y\mu_b}\sigma^{-1} - \frac{\mu_b\chi_r\overline{d}_P}{1-\chi_y\mu_b}r_n\right](i_t - \pi_t - r_n) + \left[\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma - \frac{\mu_b\chi_r\overline{d}_P}{1-\chi_y\mu_b}\kappa\right]y_t$$
$$+ \left[\frac{1-\mu_b}{1-\chi_y\mu_b}\chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\chi_y\mu_b}\left(r_n + \frac{\psi_P(1+r_P\epsilon_\lambda)}{\rho+\psi_P+\psi_m}(\rho - r_n + \psi_m)\right)\right](i_t - r_n),$$

We can then write the aggregate Euler equation as $\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + \delta y_t + v_t$, where $\tilde{\sigma}^{-1} \equiv \frac{1-\mu_b}{1-\chi_y\mu_b}\sigma^{-1} - \frac{\mu_b\chi_r\overline{d}_P}{1-\chi_y\mu_b}r_n$, $\delta \equiv \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma - \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\kappa$, and $v_t \equiv \frac{\mu_b\chi_r\overline{d}_P}{1-\chi_y\mu_b}\left[r_n + \frac{\psi_P(1+r_P\epsilon_\lambda)}{\rho+\psi_P+\psi_m}\right.$
$\left.(\rho - r_n + \psi_m)\right](i_t - r_n) + \frac{1-\mu_b}{1-\chi_y\mu_b}\chi_d\epsilon_\lambda(i_t - r_n)$. Therefore, output is given by

$$y_t = \sigma^{-1}\hat{y}_t + \chi_d\epsilon_\lambda\hat{y}_t + \frac{\mu_b\chi_r}{1-\mu_b}\frac{\psi_P(1+r_P\epsilon_\lambda)}{\rho+\psi_P+\psi_m}\tilde{\psi}_m\overline{d}_P\hat{y}_t + (\rho - \underline{\omega})e^{\underline{\omega}t}\Omega_0,$$

where $\tilde{\psi}_m \equiv \psi_m + \rho - r_n$. □

B Derivations for Section 2

B.1 The non-linear model

Savers' problem. Let $C_{j,t}(s)$ denote the consumption at time t of a saver of type $j \in \{o, p\}$ and denote the aggregate consumption of a type-j saver by $\overline{C}_{j,t} = \int_{-\infty}^t \xi e^{-\xi(t-s)}C_{j,t}(s)ds$, where a similar notation applies to the other variables. Given that the problem of all type-j savers is the same, except for the value of net worth, we drop the dependence on s and write $C_{j,t}$ instead of

$C_{j,t}(s)$ to ease notation.

The HJB for the savers' problem is given by

$$\tilde{\rho}_j V_{j,t} = \max_{C_{j,t},B^L_{j,t},B^E_{j,t}} \frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \xi V_{j,t} + \frac{\partial V_{j,t}}{\partial t} + \lambda_j \left[V^*_{j,t} - V_{j,t}\right] + \frac{\partial V_{j,t}}{\partial B_{j,t}} \left[(i_t - \pi_t)B_{j,t} + r_{L,t}B^L_{j,t} + r_{E,t}B^E_{j,t} - C_{j,t}\right].$$

where $V^*_{j,t}$ is evaluated at $B^*_{j,t} = B_{j,t} + B^L_{j,t}\frac{Q^*_{L,t}-Q_{L,t}}{Q_{L,t}} + B^E_{j,t}\frac{Q^*_{E,t}-Q_{E,t}}{Q_{E,t}}$ and $B_{j,t} > -\overline{D}_p$.

The corresponding HJB in the disaster state is given by

$$\tilde{\rho}^*_j V^*_{j,t} = \max_{C^*_{j,t},B^{L,*}_{j,t},B^{E,*}_{j,t}} \frac{(C^*_{j,t})^{1-\sigma}}{1-\sigma} - \xi V^*_{j,t} + \frac{\partial V^*_{j,t}}{\partial t} + \frac{\partial V^*_{j,t}}{\partial B^*_{j,t}} \left[(i^*_t - \pi^*_t)B_{j,t} - C^*_{j,t}\right],$$

where we imposed that $r^*_{L,t} = r^*_{E,t} = 0$, as there is no risk in the disaster state.

The first-order conditions are given by[1]

$$C_{j,t}^{-\sigma} = \frac{\partial V_{j,t}}{\partial B_{j,t}}, \qquad \frac{\partial V_{j,t}}{\partial B_{j,t}} r_{k,t} = \frac{\partial V^*_{j,t}}{\partial B^*_{j,t}} \frac{Q_{k,t} - Q^*_{k,t}}{Q_{k,t}}, \qquad (C^*_{j,t})^{-\sigma} = \frac{\partial V^*_{j,t}}{\partial B^*_{j,t}}, \tag{B.1}$$

where $k \in \{L, E\}$ and savers are indifferent about the level of long-term bonds in the disaster state.

Combining the expressions above, we obtain Equations (2) and (3). Differentiating the HJB equation in the no-disaster state with respect to $B_{j,t}$, we obtain the envelope condition:[2]

$$\rho_j \frac{\partial V_{j,t}}{\partial B_{j,t}} = \frac{\partial V_{j,t}}{\partial B_{j,t}}(i_t - \pi_t) + \frac{\mathbb{E}_{j,t}[d\left(\frac{\partial V_{j,t}}{\partial B_{j,t}}\right)]}{dt}, \tag{B.2}$$

where the effective discount rate $\rho_j \equiv \tilde{\rho}_j + \xi$ captures subjective discounting and mortality risk.

Using the optimality condition for consumption, we can write the expression above as follows:

$$i_t - \pi_t - \rho_j = -\frac{\mathbb{E}_t[dC_{j,t}^{-\sigma}]}{C_{j,t}^{-\sigma}dt} = \frac{\sigma C_{j,t}^{-\sigma-1}\dot{C}_{j,t} - \lambda_j\left[(C^*_{j,t})^{-\sigma} - C_{j,t}^{-\sigma}\right]}{C_{j,t}^{-\sigma}}, \tag{B.3}$$

using the fact that $dC_{j,t} = \dot{C}_{j,t}dt + [C^*_{j,t} - C_{j,t}]d\mathcal{N}_t$ and Ito's lemma. Rearranging the expression above, we obtain Equation (1). A similar envelope condition holds in the disaster state, which

[1]Formally, the solution is also subject to the state-constraint boundary condition $\frac{\partial V_{j,t}(-\overline{D}_P)}{\partial B} \geq \left(-(i_t-\pi_t)\overline{D}_P + \frac{W_{j,t}}{P_t}N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \widetilde{T}_{j,t}\right)^{-\sigma}$. See Achdou et al. (2017) for a discussion of state-constraint boundary conditions in the context of continuous-time savings problems with borrowing constraints.

[2]Here we used the fact that $\mathbb{E}_{j,t}[dF(B_{j,t},t)] = \left[F_t + \lambda_j[F^* - F] + F_B\left((i-\pi)B_j + r_L B^L_j + r_E B^E_j - C_j\right)\right]dt$ for any function $F(B_{j,t},t)$, according to Ito's lemma.

gives the Euler equation for the disaster state

$$\frac{\dot{C}^*_{j,t}}{C^*_{j,t}} = \sigma^{-1}(i_t - \pi_t - \rho^*_j), \tag{B.4}$$

where $\rho^*_j \equiv \tilde{\rho}^*_j + \xi$.

Savers' aggregate behavior. Denote aggregate consumption and net worth of type-j savers as follows

$$\overline{C}_{j,t} = \int_{-\infty}^{t} \xi e^{-\xi(t-s)} C_{j,t}(s) ds, \qquad \overline{B}_{j,t} = \int_{-\infty}^{t} \xi e^{-\xi(t-s)} B_{j,t}(s) ds, \tag{B.5}$$

From the optimality condition for risky assets, we obtain

$$\frac{C_{j,t}(s)}{C^*_{j,t}(s)} = \frac{C_{j,t}(s')}{C^*_{j,t}(s')} \Rightarrow \frac{C_{j,t}(s)}{C^*_{j,t}(s)} = \frac{\overline{C}_{j,t}}{\overline{C}^*_{j,t}}. \tag{B.6}$$

We can then write the optimality condition for risky assets as follows

$$r_{k,t} = \lambda_j \left(\frac{\overline{C}_{j,t}}{\overline{C}^*_{j,t}} \right)^{\sigma} \frac{Q_{k,t} - Q^*_{k,t}}{Q_{k,t}}, \tag{B.7}$$

where $k \in \{L, E\}$.

The evolution of aggregate consumption of a type-j saver conditional on no-disaster is given by

$$\dot{\overline{C}}_{j,t} = \int_{-\infty}^{t} \xi e^{-\xi(t-s)} \dot{C}_{j,t}(s) ds + \xi(C_{j,t}(t) - \overline{C}_{j,t}) \tag{B.8}$$

$$= \overline{C}_{j,t} \left[\sigma^{-1} \left(i_t - \pi_t - \rho_j \right) + \frac{\lambda_j}{\sigma} \left[\left(\frac{\overline{C}_{j,t}}{\overline{C}^*_{j,t}} \right)^{\sigma} - 1 \right] + \xi \left(\frac{C_{j,t}(t)}{\overline{C}_{j,t}} - 1 \right) \right]. \tag{B.9}$$

The net worth of newborn savers is given by $B_{j,t}(t) = \frac{\mu_o}{\mu_o+\mu_p}\overline{B}_{o,t} + \frac{\mu_p}{\mu_o+\mu_p}\overline{B}_{p,t}$. As the consumption-wealth ratio is the same for all savers of the same type, we obtain

$$\frac{C_{j,t}(t)}{\overline{C}_{j,t}} = \frac{\frac{\mu_o}{\mu_o+\mu_p}\overline{B}_{o,t} + \frac{\mu_p}{\mu_o+\mu_p}\overline{B}_{p,t}}{\overline{B}_{j,t}} \tag{B.10}$$

Combining the expressions above, we can derive the law of motion of aggregate consumption for savers:

$$\frac{\dot{C}_{s,t}}{C_{s,t}} = \sigma^{-1} \left(i_t - \pi_t - \rho_{s,t} \right) + \frac{\lambda_t}{\sigma} \left[\left(\frac{C_{s,t}}{C^*_{s,t}} \right)^{\sigma} - 1 \right]. \tag{B.11}$$

The parameter ξ does not affect the aggregate Euler equation for savers. However, ξ controls the relative consumption of optimistic and pessimistic savers:

$$\frac{\dot{\overline{C}}_{o,t}}{\overline{C}_{o,t}} - \frac{\dot{\overline{C}}_{p,t}}{\overline{C}_{p,t}} = \xi \left[\overline{B}_{p,t} - \overline{B}_{o,t}\right] \frac{\frac{\mu_o}{\mu_o+\mu_p}\overline{B}_{o,t} + \frac{\mu_p}{\mu_o+\mu_p}\overline{B}_{p,t}}{\overline{B}_{o,t}\overline{B}_{p,t}} \tag{B.12}$$

The evolution of aggregate net worth of a type-j saver conditional on no-disaster is given by

$$\dot{\overline{B}}_{j,t} = \int_{-\infty}^{t} \xi e^{-\xi(t-s)} \dot{B}_{j,t}(s) ds + \xi(B_{j,t}(t) - \overline{B}_{j,t}) \tag{B.13}$$

$$= \overline{B}_{j,t}\left[i_t - \pi_t + r_{L,t}\frac{\overline{B}^L_{j,t}}{\overline{B}_{j,t}} + r_{E,t}\frac{\overline{B}^E_{j,t}}{\overline{B}_{j,t}} - \frac{\overline{C}_{j,t}}{\overline{B}_{j,t}} + \xi\left(\frac{B_{j,t}(t)}{\overline{B}_{j,t}} - 1\right)\right]. \tag{B.14}$$

This implies that the aggregate net worth of savers evolves according to

$$\frac{\dot{B}_{s,t}}{B_{s,t}} = i_t - \pi_t + r_{L,t}\frac{B^L_{s,t}}{B_{s,t}} + r_{E,t}\frac{B^E_{s,t}}{B_{s,t}} - \frac{C_{s,t}}{B_{s,t}}. \tag{B.15}$$

The relative net worth of optimistic and pessimistic savers evolves according to

$$\frac{\dot{\overline{B}}_{o,t}}{\overline{B}_{o,t}} - \frac{\dot{\overline{B}}_{p,t}}{\overline{B}_{p,t}} = \sum_{k\in\{L,E\}} r_{k,t}\left(\frac{\overline{B}^L_{o,t}}{\overline{B}_{o,t}} - \frac{\overline{B}^k_{p,t}}{\overline{B}_{p,t}}\right) - \left(\frac{\overline{C}_{o,t}}{\overline{B}_{o,t}} - \frac{\overline{C}_{p,t}}{\overline{B}_{p,t}}\right) + \xi\left(\frac{B_{o,t}(t)}{\overline{B}_{o,t}} - \frac{B_{p,t}(t)}{\overline{B}_{p,t}}\right). \tag{B.16}$$

Borrowers' problem. The HJB for the borrowers' problem is given by

$$\tilde{\rho}_b V_{b,t} = \max_{\tilde{C}_{b,t}, N_{b,t}, B^L_{j,t}} \frac{\tilde{C}^{1-\sigma}_{b,t}}{1-\sigma} + \frac{\partial V_{b,t}}{\partial t} + \lambda_b\left[V^*_{b,t} - V_{b,t}\right] + \frac{\partial V_{b,t}}{\partial B_{b,t}}\left[(i_t - \pi_t)B_{b,t} + r_{L,t}B^L_{b,t} + \frac{W_t}{P_t}N_{b,t} + \widetilde{T}_{b,t} - \tilde{C}_{b,t} - \frac{N^{1+\phi}_{b,t}}{1+\phi}\right].$$

subject to the state-constraint boundary condition

$$\frac{\partial V_{b,t}(-\overline{D}_P)}{\partial B_{b,t}} \geq \left(-(i_t - \pi_t)\overline{D}_P + \frac{W_t}{P_t}N_{b,t} - \frac{N^{1+\phi}_{b,t}}{1+\phi} + \widetilde{T}_{b,t}\right)^{-\sigma}, \tag{B.17}$$

where we adopted the change of variables $\tilde{C}_{b,t} \equiv C_{b,t} - \frac{N^{1+\phi}_{b,t}}{1+\phi}$.

For simplicity, we have already imposed that $B^E_{b,t} = 0$. We will show below that $B^L_{b,t} = 0$ and a similar argument shows that borrowers would be against the short-selling constraint for equities when $B^E_{b,t}$ is a choice variable.

The optimality condition for labor supply is given by

$$N^{\phi}_{b,t} = \frac{W_t}{P_t}. \tag{B.18}$$

We focus on an equilibrium where borrowers are always constrained. To derive the conditions that ensure this is indeed the case, we start by considering a stationary equilibrium where all variables are constant conditional on the state. If borrowers are constrained in the stationary equilibrium, then they will also be constrained if fluctuations are small enough.

In a stationary equilibrium, net consumption $\tilde{C}_b$ in the no-disaster state is given by

$$\tilde{C}_b = -r_n \overline{D}_p + \frac{W}{P} N_b - \frac{N_b^{1+\phi}}{1+\phi} + \tilde{T}_b, \tag{B.19}$$

and an analogous expression holds in the disaster state. Notice that the expression above does not depend on ρ_b or λ_b.

For borrowers to be unconstrained, the following condition would have to hold:

$$\frac{\dot{\tilde{C}}_{b,t}}{\tilde{C}_{b,t}} = \sigma^{-1}(r_n - \rho_b) + \frac{\lambda_b}{\sigma} \left[\left(\frac{\tilde{C}_{b,t}}{\tilde{C}^*_{b,t}} \right)^{\sigma} - 1 \right]. \tag{B.20}$$

For ρ_b sufficiently large, borrowers would want a declining path of consumption, so current consumption would be above $-r_n \overline{D}_p + \frac{W}{P} N_b - \frac{N_b^{1+\phi}}{1+\phi} + \tilde{T}_b$, which would violate the state-constraint. Hence, the constraint must be binding for ρ_b sufficiently large.

If the borrowers hold a positive amount of the long-term bonds, then the following condition must hold

$$r_L = \lambda_b \left(\frac{\tilde{C}_b}{\tilde{C}^*_b} \right)^{\sigma} \frac{Q_L - Q^*_L}{Q_L}. \tag{B.21}$$

As C_b and C^*_b are independent of λ_b, the equation above would hold only if λ_b equals the value $\overline{\lambda}_b \equiv \frac{r_L}{\left(\frac{C_b}{C^*_b}\right)^{\sigma} \frac{Q_L - Q^*_L}{Q_L}}$. For $\lambda_b > \overline{\lambda}_b$, borrowers would want a smaller dispersion between C_b and C^*_b, which requires holding less risky bonds, violating the non-negativity constraint on long-term bonds. Therefore, borrowers will hold zero long-term bonds for λ_b sufficiently large.

Firms' problem. Final goods are produced according to the production function $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon-1}} di \right)^{\frac{\epsilon-1}{\epsilon}}$. The solution to final-good producers problem is a demand for variety i given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t$. The price level is given by $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$.

The intermediate-goods producers' problem is given by

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s\geq t}} \mathbb{E}_t \left[\int_t^{t^*} \frac{\eta_s}{\eta_t}(1-\tau_t)\left(\frac{P_{i,s}}{P_s}Y_{i,s} - \frac{W_s}{P_s}\frac{Y_{i,s}}{A_s} - \frac{\varphi}{2}\pi_s^2(j)\right) ds + \frac{\eta_{t^*}}{\eta_t} Q^*_{i,t^*}(P_{i,t^*}) \right],$$

subject to $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, given $P_{i,t} = P_i$.

The HJB equation for this problem is

$$0 = \max_{\pi_{i,t}} \eta_t(1-\tau_t)\left(\frac{P_{i,t}}{P_t}Y_{i,t} - \frac{W_t}{P_t}\frac{Y_{i,t}}{A} - \frac{\varphi}{2}\pi_{i,t}^2\right) dt + \mathbb{E}_t[d(\eta_t Q_{i,t})], \tag{B.22}$$

where $\frac{\mathbb{E}_t[d(\eta_t Q_{i,t})]}{\eta_t dt} = -(i_t - \pi_t)Q_{i,t} + \frac{\partial Q_{i,t}}{\partial P_{i,t}}\pi_{i,t}P_{i,t} + \frac{\partial Q_{i,t}}{\partial t} + \lambda_t \frac{\eta_t^*}{\eta_t}\left[Q^*_{i,t} - Q_{i,t}\right]$.

The first-order condition is given by

$$\frac{\partial Q_{i,t}}{\partial P_i} P_{i,t} = (1-\tau_t)\varphi \pi_{i,t}.$$

The change in π_t conditional on no disaster is then given by

$$\left(\frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2}\pi_{i,t}P_{i,t}\right) P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i}\pi_{i,t}P_{i,t} = (1-\tau_t)\varphi\dot{\pi}_{i,t}. \tag{B.23}$$

The envelope condition with respect to $P_{i,t}$ is given by

$$\begin{aligned} 0 = (1-\tau_t)\left((1-\epsilon)\frac{P_{i,t}}{P_t} + \epsilon\frac{W_t}{P_t A}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_{i,t}} + \frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2}\pi_{i,t}P_{i,t} + \\ \frac{\partial Q_{i,t}}{\partial P_i}\pi_{i,t} - (i_t - \pi_t)\frac{\partial Q_{i,t}}{\partial P_i} + \lambda_t\frac{\eta_t^*}{\eta_t}\left(\frac{\partial Q^*_{i,t}}{\partial P_i} - \frac{\partial Q_{i,t}}{\partial P_i}\right). \end{aligned} \tag{B.24}$$

Multiplying the expression above by $P_{i,t}$ and using Equation (B.23), we obtain

$$0 = \left((1-\epsilon)\frac{P_{i,t}}{P_t} + \epsilon\frac{W_t}{P_t A}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t + \varphi\dot{\pi}_t - (i_t - \pi_t)\varphi\pi_{i,t} + \lambda_t\varphi\frac{\eta_t^*}{\eta_t}\left(\frac{1-\tau_t^*}{1-\tau_t}\pi^*_{i,t} - \pi_{i,t}\right).$$

Rearranging the expression above, we obtain the non-linear New Keynesian Phillips curve

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t\frac{\eta^*}{\eta_t}\right)\pi_t - \frac{\epsilon\varphi^{-1}}{A}\left(\frac{W_t}{P_t} - (1-\epsilon^{-1})A\right)Y_t,$$

where we have assumed that $P_{i,t} = P_t$ for all $i \in [0,1]$ and that $\pi_t^* = 0$.

B.2 The stationary equilibrium

Aggregate output. Consider a stationary equilibrium with zero inflation. From the New Keynesian Phillips curve, we obtain

$$\frac{W}{P} = (1-\epsilon^{-1})A, \qquad \frac{W^*}{P} = (1-\epsilon^{-1})A^*. \tag{B.25}$$

Combining the expressions above with the labor supply condition, we obtain

$$Y = \mu_b(1-\epsilon^{-1})^{\frac{1}{\phi}} A^{\frac{1+\phi}{\phi}}, \qquad Y^* = \mu_b(1-\epsilon^{-1})^{\frac{1}{\phi}} (A^*)^{\frac{1+\phi}{\phi}}. \tag{B.26}$$

Disaster state. From the Euler equation for short-term bonds, the type-j saver will be unconstrained in a stationary equilibrium only if $r_n^* = \rho_j^*$. To ensure this is the case, we assume that $\rho_j^* = \rho_s$ for $j \in \{o, p\}$, where ρ_s is the effective discount rate implicit in the SDF, so the real interest rate in the disaster state is given by $i_t^* - \pi_t^* = r_n^* = \rho_s$. The excess return on long-terms bonds and equity are equal to zero, $r_L^* = r_E^* = 0$, so the price of the long-term bond is given by

$$Q_L^* = \frac{1}{r_n^* + \psi_L}, \tag{B.27}$$

and the equity price is given by $Q_E^* = \frac{\Pi^*}{r_n^*}$.

The consumption of borrowers is given by

$$C_b^* = \left[(1-\epsilon^{-1})A^*\right]^{\frac{1+\phi}{\phi}} - r_n^* \overline{D}_p + \tilde{T}_b^*. \tag{B.28}$$

We assume that the government chooses fiscal transfers so borrowers have a given share $0 < \mu_{Y,b} < 1$ of output, so $C_b^* = \mu_{Y,b}^*$. The parameter $\mu_{Y,b}$ captures the government preferences for redistribution.

The consumption of savers are given by

$$C_j^* = r_n^* B_j^*, \tag{B.29}$$

where $B_j^* = B_j + B_j^L \frac{Q_L^* - Q_L}{Q_L} + B_j^E \frac{Q_E^* - Q_E}{Q_E}$.

Aggregate consumption of savers is given by

$$C_s^* = r_n^* \frac{\overline{D}_G^* + \mu_b \overline{D}_p}{1-\mu_b} + (1-\tau^*)\epsilon^{-1} \frac{Y^*}{1-\mu_b}, \tag{B.30}$$

using the market clearing condition for bonds and the expression for the price of stocks.

The government chooses τ^* such that $\tau^*\epsilon^{-1}Y^* = r_n^*\overline{D}_G^* + \mu_b \widetilde{T}_b^*$, which guarantees that the government's budget constraint is satisfied. This implies that the aggregate consumption of savers is given by $C_s^* = (1-\mu_{Y,b})Y^*$.

No-disaster state. The consumption of borrowers is given by

$$C_b = -r_n\overline{D}_p + \left[(1-\epsilon^{-1})A\right]^{\frac{1+\phi}{\phi}} + \tilde{T}_b. \tag{B.31}$$

As in the disaster state, the government chooses fiscal transfers so borrowers have a given share $0 < \mu_{Y,b} < 1$ of output, so $C_b = \mu_{Y,b}Y$ and $C_s = (1-\mu_{Y,b})Y$. It remains to determine the relative consumption of optimistic and pessimistic savers.

The consumption of individual savers is given by

$$C_j = r_n B_j + r_L B_j^L + r_E B_j^E. \tag{B.32}$$

The expression above ensures that total bond holdings of each individual saver is constant over time. To ensure that the aggregate bond holdings of optimistic and pessimistic savers is also constant, we must take into account the effect of births and deaths. Each instant a mass $\xi\mu_o$ of optimistic savers dies, which leads to a reduction in wealth for this group of $\xi\mu_o B_o$. Newborns inherit the wealth of their parents and a fraction $\frac{\mu_o}{\mu_o+\mu_p}$ of newborns is optimistic, so the influx of newborns raise the aggregate wealth of optimistic savers by $\frac{\mu_o}{\mu_o+\mu_p}\xi\left[\mu_o B_o + \mu_p B_p\right]$. These two effects cancel each other if the following condition is satisfied

$$\xi\mu_o B_o = \frac{\mu_o}{\mu_o+\mu_p}\xi\left[\mu_o B_o + \mu_p B_p\right] \Rightarrow B_o = B_p, \tag{B.33}$$

so $B_j = B_s$ for $j \in \{o, p\}$, where B_s is the average net worth of savers. We then have $B_j(s) = \overline{B}_j$ and $C_j(s) = \overline{C}_j$.

From the market clearing condition for assets, we obtain

$$B_s = \frac{\overline{D}_G + \mu_b\overline{D}_p + Q_E}{1-\mu_b}, \qquad B_s^L = \frac{\overline{D}_G}{1-\mu_b}, \qquad B_s^E = \frac{Q_E}{1-\mu_b}. \tag{B.34}$$

Using the fact that $B_o = B_p$ in a stationary equilibrium, we can write the consumption of optimistic and pessimistic savers as follows:

$$C_o = C_s + r_L \frac{\mu_p}{\mu_o + \mu_p}(B_o^L - B_p^L) + r_E \frac{\mu_p}{\mu_o + \mu_p}(B_o^E - B_p^E) \tag{B.35}$$

$$C_p = C_s - r_L \frac{\mu_o}{\mu_o + \mu_p}(B_o^L - B_p^L) - r_E \frac{\mu_o}{\mu_o + \mu_p}(B_o^E - B_p^E). \tag{B.36}$$

We can use the Euler equations for risky assets to eliminate r_L and r_E from the expression above, which gives us

$$C_o = C_s \left[1 + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{\mu_p}{\mu_o + \mu_p} \mathcal{R}_o\right], \qquad C_o^* = C_s^* \left[1 - \frac{\mu_p}{\mu_o + \mu_p} \frac{r_n^* \mathcal{R}_o}{1 - \zeta_Y}\right], \tag{B.37}$$

$$C_p = C_s \left[1 - \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{\mu_o}{\mu_o + \mu_p} \mathcal{R}_o\right], \qquad C_p^* = C_s^* \left[1 + \frac{\mu_o}{\mu_o + \mu_p} \frac{r_n^* \mathcal{R}_o}{1 - \zeta_Y}\right], \tag{B.38}$$

where $\mathcal{R}_o \equiv \frac{Q_L - Q_L^*}{Q_L} \frac{B_o^L - B_p^L}{C_s} + \frac{Q_L - Q_L^*}{Q_E} \frac{B_o^E - B_p^E}{C_s}$ represents optimistic relative risk exposure.

Notice that $\mathcal{R}_o$ pins down the share of consumption of optimistic savers:

$$\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p} = \frac{\mu_o}{\mu_o + \mu_p} \left[1 + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{\mu_p}{\mu_o + \mu_p} \mathcal{R}_o\right], \tag{B.39}$$

which is an implicit function of the share of consumption of optimistic savers, as λ is also a function of $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p}$. The left-hand is strictly increasing in $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p}$ and it is zero if $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p} = 0$. For $\mathcal{R}_o > 0$ and $\lambda_o < \lambda_p$, the right-hand side is decreasing in $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p}$ and it is positive if $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p} = 0$. Then, $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p}$ is a strictly increasing function of $\mathcal{R}_0$, in the range $\mathcal{R}_0 > 0$. This implies that λ is decreasing in $\mathcal{R}_o$, but $\lambda \mathcal{R}_o$ is strictly increasing.

From the optimality condition for risky assets, we obtain the condition

$$\left(\frac{1 + \lambda (1 - \zeta_Y)^{-\sigma} \frac{\mu_p}{\mu_o + \mu_p} \mathcal{R}_o}{1 - \frac{\mu_p}{\mu_o + \mu_p} \frac{r_n^* \mathcal{R}_o}{1 - \zeta_Y}}\right)^\sigma = \frac{\lambda_p}{\lambda_o} \left(\frac{1 - \lambda (1 - \zeta_Y)^{-\sigma} \frac{\mu_o}{\mu_o + \mu_p} \mathcal{R}_o}{1 + \frac{\mu_o}{\mu_o + \mu_p} \frac{r_n^* \mathcal{R}_o}{1 - \zeta_Y}}\right)^\sigma, \tag{B.40}$$

where we used the fact that $\frac{C_s^*}{C_s} = 1 - \zeta_Y$.

The left-hand side of the expression above is strictly increasing in $\mathcal{R}_o$ and it is equal to 1 for $\mathcal{R}_o = 0$. The right-hand side is strictly decreasing in $\mathcal{R}_o$ and it is equal to $\frac{\lambda_p}{\lambda_o} \geq 1$. Then, there exists a unique value of $\mathcal{R}_o$ solving the equation above and $\mathcal{R}_o \geq 0$, with strict inequality if $\lambda_p > \lambda_o$.

The value of $\mathcal{R}_o$ pins down $\frac{\mu_o C_o}{\mu_p C_p + \mu_p C_p}$ and the market-implied disaster probability:

$$\lambda = \left[\frac{\mu_o C_o}{\mu_p C_p + \mu_p C_p} \lambda_o^{\frac{1}{\sigma}} + \frac{\mu_p C_p}{\mu_p C_p + \mu_p C_p} \lambda_p^{\frac{1}{\sigma}} \right]^{\sigma}. \tag{B.41}$$

From the Euler equations for short-term and long-term bonds, we obtain

$$r_n = \rho_j - \lambda_j \left[\left(\frac{C_j}{C_j^*} \right)^{\sigma} - 1 \right], \qquad r_k = \lambda_j \left(\frac{C_j}{C_j^*} \right)^{\sigma} \frac{Q_k - Q_k^*}{Q_k}, \tag{B.42}$$

for $k \in \{L, E\}$, where $r_L = \frac{1}{Q_L} - \psi_L - r_n$ and $r_E = \frac{\Pi}{Q_E} - r_n$.

Notice that, given $\lambda_o \left(\frac{C_o}{C_o^*} \right)^{\sigma} = \lambda_p \left(\frac{C_p}{C_p^*} \right)^{\sigma}$, the condition $\rho_o + \lambda_o = \rho_p + \lambda_p$ is necessary for the Euler equation for short-term bonds to be satisfied with constant consumption for both types of savers.

Using the fact that $\lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma} = \lambda_j \left(\frac{C_j}{C_j^*} \right)^{\sigma}$, we can write the Euler equations in terms of aggregate savers' consumption:

$$r_n = \rho_s - \lambda \left[\left(\frac{C_s}{C_s^*} \right)^{\sigma} - 1 \right], \qquad r_k = \lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma} \frac{Q_k - Q_k^*}{Q_k}, \tag{B.43}$$

for $k \in \{L, E\}$, where ρ_s satisfy the condition $\rho_s + \lambda = \rho_j + \lambda_j$ for $j \in \{o, p\}$.

We solve next for the price of risky assets. Given r_L, we can solve for Q_L:

$$\frac{1}{Q_L} - \psi_L - r_n = \lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma} \left(1 - \frac{Q_L^*}{Q_L} \right) \Rightarrow Q_L = Q_L^* \frac{r_n^* + \psi_L + \lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma}}{r_n + \psi_L + \lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma}}, \tag{B.44}$$

where $Q_L > Q_L^*$, as $r_n < r_n^*$ due to the precautionary motive in the no-disaster state.

The loss in long-term bonds in the disaster state is given by

$$\frac{Q_L - Q_L^*}{Q_L} = \frac{r_n^* - r_n}{r_n^* + \psi_L + \lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma}}, \tag{B.45}$$

which is positive as $r_n^* > r_n$. This implies that long-term interest rates are higher than short-term interest rates in the stationary equilibrium, i.e., the yield curve is upward sloping in this economy.

The equity price is given by

$$\frac{\Pi}{Q_E} - r_n = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \left(1 - \frac{Q_E^*}{Q_E}\right) \Rightarrow Q_E = \frac{\Pi + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_E^*}{r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma}}, \tag{B.46}$$

so the loss on equity in the disaster state is given by

$$\frac{Q_E - Q_E^*}{Q_E} = \frac{\Pi - r_n Q_E^*}{\Pi + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_E^*} = \zeta_\Pi + Q_E^* \frac{\frac{\Pi^*}{Q_E^*} - \left(r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \zeta_\Pi\right)}{\Pi + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_E^*}, \tag{B.47}$$

where $\zeta_\Pi \equiv 1 - \frac{\Pi^*}{\Pi}$ is the size of the drop in profits. It can be shown that the second term is positive for $\sigma > 1$. Therefore, the equity premium is positive in the stationary equilibrium. Notice that the drop in the values of equities in the disaster state comes from both the reduction in dividends and the drop in the price-dividend ratio in the disaster state due to higher natural rate.

Finally, given the quantity of risk for stocks and bonds, the value of $\mathcal{R}_o$ pins a linear between of $B_o^L - B_p^L$ and $B_o^E - B_o^E$, but it does not pin down the exact values. For instance, we could assume that $B_o^E = B_p^E$, such that differences in believes translates in differences in bond holdings. Alternatively, we could set $B_o^L - B_p^L = B_o^E - B_p^E$, so the optimistic investors holds more of stocks and long-term bonds. All these configurations are consistent with equilibrium and they do not affect prices or consumption.

B.3 Log-linear approximation

We consider next the effects of an unexpected monetary shock for an economy starting at the stationary equilibrium described above.

Market-based disaster probability. Linearizing Equation (4) around the stationary equilibrium, we obtain

$$\frac{\lambda^{\frac{1}{\sigma}}}{\sigma} \hat{\lambda}_t = \mu_{c,o} \mu_{c,p} \left(\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}\right) \left[c_{p,t} - c_{o,t}\right], \tag{B.48}$$

where $\mu_{c,j} \equiv \frac{\mu_j \overline{C}_j}{\mu_o \overline{C}_o + \mu_p \overline{C}_p}$ and $c_{j,t} \equiv \log \overline{C}_{j,t} / \overline{C}_j$, for $j \in \{o, p\}$. Note that $c_{j,t}$ denote the log-deviation of *average* consumption of type-j savers.

The expression above implies that changes in the relative consumption of optimistic and pessimistic investors affects the market-based probability of disaster. In particular, shocks that redistribute towards pessimistic investors at time t raise $\hat{\lambda}_t$.

Relative consumption. From the optimality condition for risky assets, we obtain

$$\lambda_o^{\frac{1}{\sigma}} \frac{\overline{C}_{o,t}}{\overline{C}^*_{o,t}} = \lambda_p^{\frac{1}{\sigma}} \frac{\overline{C}_{p,t}}{\overline{C}^*_{p,t}} \Rightarrow c_{p,t} - c_{o,t} = c^*_{p,t} - c^*_{o,t} \tag{B.49}$$

Relative consumption, in the no-disaster and disaster states, is given by

$$\dot{c}_{p,t} - \dot{c}_{o,t} = -\xi(b_{p,t} - b_{o,t}), \qquad \dot{c}^*_{p,t} - \dot{c}^*_{o,t} = -\xi(b^*_{p,t} - b^*_{o,t}), \tag{B.50}$$

where we used Equation (B.12) and the analogous condition in the disaster state.

Relative net worth. Linearizing Equation (B.16), we obtain

$$\begin{aligned}\dot{b}_{p,t} - \dot{b}_{o,t} = \sum_{k\in\{L,E\}} r_k \left[\hat{r}_{k,t}\left(\frac{\overline{B}^k_p}{\overline{B}_p} - \frac{\overline{B}^k_o}{\overline{B}_o}\right) + \frac{\overline{B}^k_p}{\overline{B}_p}(b^k_{p,t} - b_{p,t}) - \frac{\overline{B}^k_o}{\overline{B}_o}(b^k_{o,t} - b_{o,t})\right] \\ - \left(\frac{\overline{C}_p}{\overline{B}_p}(c_{p,t} - b_{p,t}) - \frac{\overline{C}_o}{\overline{B}_o}(c_{o,t} - b_{o,t})\right) - \xi\left(b_{p,t} - b_{o,t}\right),\end{aligned} \tag{B.51}$$

where $\hat{r}_{k,t} = \hat{\lambda}_t + \sigma c_{s,t} + \sum_{k\in\{L,E\}} \frac{Q^*_k}{Q_k - Q^*_k} q_{k,t}$.

Using the fact that $\frac{\overline{C}_j}{\overline{B}_j} = r_n + \sum_{k\in\{L,E\}} r_k \frac{B^k_j}{B_j}$, we can write the expression above as follows

$$\begin{aligned}\dot{b}_{p,t} - \dot{b}_{o,t} = \sum_{k\in\{L,E\}} r_k \left[\hat{r}_{L,t}\left(\frac{\overline{B}^k_p}{\overline{B}_p} - \frac{\overline{B}^k_o}{\overline{B}_o}\right) + \frac{\overline{B}^k_p}{\overline{B}_p} b^k_{p,t} - \frac{\overline{B}^k_o}{\overline{B}_o} b^k_{o,t}\right] - \left(\frac{\overline{C}_p}{\overline{B}_p} c_{p,t} - \frac{\overline{C}_o}{\overline{B}_o} c_{o,t}\right) \\ + (r_n - \xi)(b_{p,t} - b_{o,t}).\end{aligned} \tag{B.52}$$

The relative net worth in the disaster state is given by

$$\dot{b}^*_{p,t} - \dot{b}^*_{o,t} = -\xi(b^*_{p,t} - b^*_{o,t}) \Rightarrow b^*_{p,t} - b^*_{o,t} = e^{-\xi(t-t^*)}(b^*_{p,t^*} - b^*_{o,t^*}), \tag{B.53}$$

for $t \geq t^*$, where $b^*_{p,t^*} - b^*_{o,t^*}$ is given by

$$b^*_{p,t^*} - b^*_{o,t^*} = b_{p,t^*} - b_{o,t^*} - \sum_{k\in\{L,E\}} \left[\left(\frac{B^k_p}{B_p} - \frac{B^k_o}{B_o}\right)\frac{Q^*_k}{Q_k} q_{k,t^*} + \left(\frac{B^k_p}{B_p} b^k_{p,t^*} - \frac{B^k_o}{B_o} b^k_{o,t^*}\right)\frac{Q_k - Q^*_k}{Q_k}\right]. \tag{B.54}$$

Relative risk exposure. Given that the consumption-wealth ratio for savers is constant in the disaster state, we have that $c^*_{j,t} = b^*_{j,t}$, so we obtain that $c^*_{p,t} - c^*_{o,t} = b^*_{p,t} - b^*_{o,t}$. Using the expression above and the fact that $b^*_{p,t} - b^*_{o,t} = c^*_{p,t} - c^*_{o,t} = c_{p,t} - c_{o,t}$, we can solve for the relative risk exposure:

$$\sum_{k\in\{L,E\}} \frac{Q_k - Q^*_k}{Q_k}\left(\frac{B^k_p}{B_p}b^k_{p,t} - \frac{B^k_o}{B_o}b^k_{o,t}\right) = b_{p,t} - b_{o,t} - (c_{p,t} - c_{o,t}) - \sum_{k\in\{L,E\}}\left(\frac{B^k_p}{B_p} - \frac{B^k_o}{B_o}\right)\frac{Q^*_k}{Q_k}q_{k,t}. \tag{B.55}$$

The dynamic system. Using the expression above to eliminate the relative risk exposure, the relative net worth at the no-disaster state is given by

$$\begin{aligned}\dot{b}_{p,t} - \dot{b}_{o,t} = {} & (\hat{\lambda}_t + (\sigma-1)c_{s,t})\sum_{k\in\{L,E\}} r_k\left(\frac{B^k_p}{B_p} - \frac{B^k_o}{B_o}\right) + (\rho_s + \lambda - \xi)(b_{p,t} - b_{o,t}) \\ & - (\rho_s+\lambda)(c_{p,t} - c_{o,t}) - \sum_{k\in\{L,E\}} r_k\left(\frac{\overline{B}^k_p}{B_p}(c_{p,t} - c_{s,t}) - \frac{\overline{B}^k_o}{B_o}(c_{o,t} - c_{s,t})\right).\end{aligned} \tag{B.56}$$

The deviation of consumption from average can be written as

$$c_{p,t} - c_{s,t} = \frac{\mu_o\overline{C}_o}{\mu_o\overline{C}_o + \mu_p\overline{C}_p}(c_{p,t} - c_{o,t}), \qquad c_{o,t} - c_{s,t} = -\frac{\mu_p\overline{C}_o}{\mu_o\overline{C}_o + \mu_p\overline{C}_p}(c_{p,t} - c_{o,t}). \tag{B.57}$$

Combining the expressions above, we can write $\dot{b}_{p,t} - \dot{b}_{o,t}$ as follows

$$\dot{b}_{p,t} - \dot{b}_{o,t} = -\chi_{\Delta b,\Delta c}(c_{p,t} - c_{o,t}) + \chi_{\Delta b,\Delta b}(b_{p,t} - b_{o,t}) + \chi_{\Delta b,c_s}c_{s,t}, \tag{B.58}$$

where $\chi_{\Delta b,c_s} \equiv (\sigma-1)\sum_{k\in\{L,E\}} r_k\left(\frac{B^k_p}{B_p} - \frac{B^k_o}{B_o}\right)$, $\chi_{\Delta b,\Delta b} \equiv \rho_s + \lambda - \xi$, and

$$\chi_{\Delta b,\Delta c} \equiv \sigma\mu_{c,o}\mu_{c,p}\left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda}\right)\sum_{k\in\{L,E\}} r_k\left(\frac{B^k_o}{B_o} - \frac{B^k_p}{B_p}\right) + (\rho_s+\lambda) + \sum_{k\in\{L,E\}} r_k\left(\frac{\overline{B}^k_p}{\overline{B}_p}\frac{\mu_o\overline{C}_o}{\mu_o\overline{C}_o + \mu_p\overline{C}_p} + \frac{\overline{B}^k_o}{\overline{B}_o}\frac{\mu_p\overline{C}_p}{\mu_o\overline{C}_o + \mu_p\overline{C}_p}\right).$$

In general, we would have to simultaneously solve for the aggregate variables and the relative net worth and relative consumption of pessimistic savers, which would increase the dimensionality of the problem relative to the standard New Keynesian. We assume that $r_k c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$, so this term is small and can be ignored in our approximate solution. This implies that the system is now *block recursive*, where we can solve for the dynamics of relative consumption and relative net worth before fully characterizing the behavior of other aggregate variables. Under this assumption, we can write the joint dynamics of $b_{p,t} - b_{o,t}$ and $c_{p,t} - c_{o,t}$ as follows:

$$\begin{bmatrix} \dot{c}_{p,t} - \dot{c}_{o,t} \\ \dot{b}_{p,t} - \dot{b}_{o,t} \end{bmatrix} = \begin{bmatrix} 0 & -\xi \\ -\chi_{\Delta b,\Delta c} & \chi_{\Delta b,\Delta b} \end{bmatrix} \begin{bmatrix} c_{p,t} - c_{o,t} \\ b_{p,t} - b_{o,t} \end{bmatrix} \tag{B.59}$$

Persistence of $\hat{\lambda}_t$. Given that $\chi_{\Delta b,\Delta c}$, the system above has a positive and a negative eigenvalue, so there is a unique bounded solution given by

$$\begin{bmatrix} c_{p,t} - c_{o,t} \\ b_{p,t} - b_{o,t} \end{bmatrix} = \begin{bmatrix} \frac{\chi_{\Delta b,\Delta b} + \psi_\lambda}{\chi_{\Delta b,\Delta c}} \\ 1 \end{bmatrix} e^{-\psi_\lambda t}(b_{p,0} - b_{o,0}) \tag{B.60}$$

where

$$\psi_\lambda \equiv \frac{\sqrt{\chi^2_{\Delta b,\Delta b} + 4\xi\chi_{\Delta b,\Delta c}} - \chi_{\Delta b,\Delta b}}{2}, \tag{B.61}$$

where $\psi_\lambda \geq 0$ is strictly increasing in ξ, it is equal to zero if $\xi = 0$ and it approaches infinity as $\xi \to \infty$.

We can then write the market-implied disaster probability as follows:

$$\hat{\lambda}_t = e^{-\psi_\lambda t}\hat{\lambda}_0, \tag{B.62}$$

where

$$\hat{\lambda}_0 \equiv \sigma\mu_{c,o}\mu_{c,p}\left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda}\right)\frac{\chi_{\Delta b,\Delta b} + \psi_\lambda}{\chi_{\Delta b,\Delta c}}(b_{p,0} - b_{o,0}). \tag{B.63}$$

Hence, ψ_λ captures the persistence of $\hat{\lambda}_t$. If $\xi = 0$, then $\psi_\lambda = 0$ and changes in λ_t are permanent. For high values of ψ_λ, the effects on λ_t are transitory and ψ_λ controls the speed of the convergence.

Wealth revaluation and $\hat{\lambda}_0$. The revaluation of the relative net worth is given by

$$b_{p,0} - b_{o,0} = \sum_{k\in\{L,E\}}\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right)q_{k,0}. \tag{B.64}$$

The price of the long-term bond satisfies the condition

$$-\frac{1}{Q_L}q_{L,t} + \dot{q}_{L,t} - (i_t - r_n) = r_L\left[\hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_L^*}{Q_L - Q_L^*}q_{L,t}^*\right] \tag{B.65}$$

Rearranging the expression above, we obtain

$$\dot{q}_{L,t} - (\rho + \psi_L) q_{L,t} = (i_t - r_n) + r_L(\hat{\lambda}_t + \sigma c_{s,t}). \tag{B.66}$$

Solving the differential equation above, we obtain

$$q_{L,0} = -\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n)dt - \int_0^\infty e^{-(\rho+\psi_L)t} r_L(\hat{\lambda}_t + \sigma c_{s,t})dt. \tag{B.67}$$

Suppose $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $r_L \sigma c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$, then

$$q_{L,0} = -\frac{i_0 - r_n}{\rho + \psi_L + \psi_m} - \frac{r_L \hat{\lambda}_0}{\rho + \psi_L + \psi_\lambda}. \tag{B.68}$$

We focus on the case $\frac{B_p^E}{B_p} = \frac{B_o^E}{B_o}$, so the initial relative wealth revaluation is given by

$$b_{p,0} - b_{o,0} = -\left(\frac{B_p^L}{B_p} - \frac{B_o^L}{B_o}\right)\left[\frac{i_0 - r_n}{\rho + \psi_L + \psi_m} + \frac{r_L \hat{\lambda}_0}{\rho + \psi_L + \psi_\lambda}\right]. \tag{B.69}$$

Plugging the expression above into the expression for $\hat{\lambda}_0$

$$\hat{\lambda}_0 \equiv \frac{\sigma \mu_{c,o} \mu_{c,p} \left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda}\right) \frac{\chi_{\Delta b, \Delta b} + \psi_\lambda}{\chi_{\Delta b, \Delta c}} \left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right)}{1 - \sigma \mu_{c,o} \mu_{c,p} \left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda}\right) \frac{\chi_{\Delta b, \Delta b} + \psi_\lambda}{\chi_{\Delta b, \Delta c}} \left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right) \frac{r_L}{\rho + \psi_L + \psi_\lambda}} \frac{i_0 - r_n}{\rho + \psi_L + \psi_m}. \tag{B.70}$$

Notice that there is an amplification mechanism between the price of the long-term bond and the change in disaster probability. A wealth redistribution towards pessimistic investors tends to increase $\hat{\lambda}_0$. An increase in $\hat{\lambda}_0$ depresses the value of long-term bonds, redistributing towards pessimistic investors, further increasing $\hat{\lambda}_t$.

Borrowers' consumption. Log-linearizing borrowers' budget constraint, we obtain

$$c_{b,t} = (1 - \alpha)(w_t - p_t - n_{b,t}) + T_{b,t} - (i_t - \pi_t - r_n)\overline{d}_p, \tag{B.71}$$

where $1 - \alpha \equiv \frac{W N_b}{P C_b}$, $T_{b,t} \equiv \frac{\tilde{T}_{b,t} - \tilde{T}_b}{C_b}$ and $\overline{d}_P \equiv \frac{\overline{D}_P}{C_b}$.

Using the fact that $T_{b,t} = T_b'(Y) y_t$ and $w_t - p_t - n_{b,t} = (1 + \phi) y_t$, we can write the expression above as follows

$$c_{b,t} = \chi_y y_t - \overline{d}_P(i_t - \pi_t - r_n), \tag{B.72}$$

where $\chi_Y \equiv (1-\alpha)(1+\phi) + T_b'(Y)$.

Savers' Euler equation. Linearizing the Euler equation for savers, we obtain

$$\dot{c}_{s,t} = \sigma^{-1}\left(i_t - \pi_t - r_n\right) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^{\sigma}\left[\sigma c_{s,t} + \hat{\lambda}_t\right]. \tag{B.73}$$

Phillips curve. Linearizing the Phillips curver, we obtain

$$\dot{\pi}_t = \rho\pi_t - \kappa y_t, \tag{B.74}$$

where $\kappa \equiv \frac{\phi\epsilon}{\varphi}\frac{WN}{P}$.

B.4 Asset prices

Stock prices. Linearizing the expression for $r_{E,t}$, we obtain

$$\frac{\Pi}{Q_E}(\hat{\Pi}_t - q_{E,t}) + \dot{q}_{E,t} - (i_t - \pi_t - r_n) = r_E\left[\hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_E^*}{Q_E - Q_E^*}q_{E,t}\right]. \tag{B.75}$$

Rearranging the expression above, we obtain

$$\dot{q}_{E,t} - \rho q_{E,t} = -\frac{1}{Q_E}\hat{\Pi}_t + (i_t - \pi_t - r_n) + r_E\left[\hat{\lambda}_t + \sigma c_{s,t}\right], \tag{B.76}$$

where $\hat{\tau}_t = -\log\frac{1-\tau_t}{1-\tau}$ and

$$\hat{\Pi}_t = -\hat{\tau}_t\Pi + (1-\tau)(y_t - (1-\alpha)(w_t - p_t - n_t))Y \tag{B.77}$$

Solving the differential equation above, we obtain

$$q_{E,t} = \frac{1}{Q_E}\int_t^\infty e^{-\rho(s-t)}\hat{\Pi}_s ds - \int_t^\infty e^{-\rho(s-t)}\left[(i_s + \pi_s - r_n) + r_E(\hat{\lambda}_t + \sigma c_{s,t})\right]ds. \tag{B.78}$$

C Derivations for Section 3

C.1 Equilibrium determinacy and the Taylor principle

Combining the dynamics of the output and inflation from Proposition 2 and the Taylor rule $i_t = r_n + \phi_\pi + \epsilon_t$, we obtain the dynamic system

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \tilde{\delta} & -\tilde{\sigma}^{-1}(1-\phi_\pi) \\ -\kappa & \rho \end{bmatrix} + \begin{bmatrix} \tilde{v}_t \\ 0 \end{bmatrix}, \tag{C.1}$$

where $\tilde{\delta} \equiv \delta + \frac{\mu_b d_P \kappa}{1-\mu_b \chi_y}\phi_\pi$ and

$$\tilde{v}_t = \tilde{\sigma}^{-1} u_t + \frac{\mu_b d_P}{1-\mu_b\chi_y}(\rho u_t - \dot{u}_t) + \frac{1-\mu_b}{1-\mu_b\chi_y}\frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^{\sigma} e^{-\psi_\lambda t}\hat{\lambda}_0. \tag{C.2}$$

The eigenvalues of the system incorporating the Taylor rule are given by

$$\overline{\omega}_T = \frac{\rho + \tilde{\delta} + \sqrt{(\rho+\tilde{\delta})^2 + 4(\tilde{\sigma}^{-1}(1-\phi_\pi)\kappa - \rho\tilde{\delta})}}{2}, \qquad \underline{\omega}_T = \frac{\rho + \tilde{\delta} - \sqrt{(\rho+\tilde{\delta})^2 + 4(\tilde{\sigma}^{-1}(1-\phi_\pi)\kappa - \rho\tilde{\delta})}}{2}. \tag{C.3}$$

The two eigenvalues above will be positive, and there will be a unique locally bounded solution, if the following condition is satisfied

$$\tilde{\sigma}^{-1}(1-\phi_\pi)\kappa - \rho\tilde{\delta} < 0 \Rightarrow \phi_\pi \geq \frac{\tilde{\sigma}^{-1}\kappa - \rho\delta}{\tilde{\sigma}^{-1}\kappa + \frac{\mu_b d_P \kappa}{1-\mu_b\chi_y}\rho} \equiv \overline{\phi}_\pi \tag{C.4}$$

Notice that the threshold $\overline{\phi}_\pi$ can be written as

$$\overline{\phi}_\pi = 1 - \frac{\rho\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}}{\tilde{\sigma}^{-1}\kappa + \frac{\mu_b d_P \kappa}{1-\mu_b\chi_y}\rho} < 1, \tag{C.5}$$

and $\overline{\phi}_\pi > 0$ if Assumption 1 holds. As $c_{s,t}$ increases with y_t, given $(\mu_b\chi_y < 1)$, risk is procyclical for savers in our economy. Bilbiie (2018) and Acharya and Dogra (2020) show that procyclical uninsurable idiosyncratic risk reduces the threshold on the response of monetary policy to inflation required to achieve local determinacy. A similar phenomenon happens in our case with aggregate disaster risk. Notice that the jump in marginal utility in the disaster state is given by $\left(\frac{C_{s,t}}{C_{s,t}^*}\right)^{\sigma}$, which in log-linear form is given by $\sigma c_{s,t}$. As $c_{s,t}$ is increasing in y_t if $\mu_b\chi_y < 1$, so the jump in marginal utility is procyclical in our economy.

C.2 Solving the dynamic system

We can write dynamic system (20) in matrix form as $\dot{Z}_t = AZ_t + B\nu_t$, where $B = [1, 0]'$. Applying the spectral decomposition to matrix A, we obtain $A = V\Omega V^{-1}$ where $V = \begin{bmatrix} \frac{\rho-\overline{\omega}}{\kappa} & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & 1 \end{bmatrix}$, $V^{-1} = \frac{\kappa}{\overline{\omega}-\underline{\omega}} \begin{bmatrix} -1 & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & -\frac{\rho-\overline{\omega}}{\kappa} \end{bmatrix}$, and $\Omega = \begin{bmatrix} \overline{\omega} & 0 \\ 0 & \underline{\omega} \end{bmatrix}$. Decoupling the system, we obtain $\dot{z}_t = \Omega z_t + b\nu_t$, where $z_t = V^{-1}Z_t$ and $b = V^{-1}B$.

Solving the equation with a positive eigenvalue forward and the one with a negative eigenvalue backward, and rotating the system back to the original coordinates, we obtain

$$y_t = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{11}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{12}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz$$
$$\pi_t = V_{22}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{21}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{22}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz,$$

where $V^{i,j}$ is the (i,j) entry of matrix V^{-1}. Integrating $e^{-\rho t}y_t$ and using the intertemporal budget constraint,

$$\Omega_0 = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)\frac{1}{\rho-\underline{\omega}} - \frac{1}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty\left(e^{-\overline{\omega}t} - e^{-\rho t}\right)\nu_t dt + \frac{1}{\rho-\underline{\omega}}V_{12}V^{21}\int_0^\infty e^{-\rho t}\nu_t dt.$$

Rearranging the above expression, we obtain

$$V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right) = (\rho-\underline{\omega})\Omega_0 + \frac{\rho-\underline{\omega}}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty\left(e^{-\overline{\omega}t} - e^{-\rho t}\right)\nu_t dt - V_{12}V^{21}\int_0^\infty e^{-\rho t}\nu_t dt.$$

Output is then given by $y_t = \tilde{y}_t + (\rho-\underline{\omega})e^{\underline{\omega}t}\Omega_0$, where $\tilde{y}_t = -\frac{\overline{\omega}-\rho}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\overline{\omega}-\delta}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\rho-\underline{\omega}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$. Inflation is given by $\pi_t = \tilde{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0$, where $\tilde{\pi}_t = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\kappa}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$.

C.3 Intertemporal budget constraint

The following lemma characterizes the intertemporal budget constraint faced by savers.

Lemma 3 (Savers' intertemporal budget constraint)**.** *The intertemporal budget budget constraint (IBC) for individual savers and the aggregate of all savers are given by*

i. Individual IBC:

$$\mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}C_{j,t}(s)\right] = B_{j,t}(s). \tag{C.6}$$

ii. Savers' aggregate IBC:

$$\mathbb{E}_t\left[\int_0^\infty \frac{\eta_t}{\eta_0} C_{s,t} dt\right] = B_{s,t}, \tag{C.7}$$

where $B_{s,t} = \frac{\mu_b D_P + D_{G,t} + Q_{E,t}}{1-\mu_b}$.

Proof. We consider first the derivation of the individual intertemporal budget constraint. The net worth of a type-j saver born at date s evolves according to

$$dB_{j,t}(s) = (i_t - \pi_t)B_{j,t}(s) + r_{L,t}B^L_{j,t}(s) + r_{E,t}B^E_{j,t}(s) - C_{j,t}(s) + \sum_{k\in\{L,E\}} B^k_{s,t}\frac{Q^*_{k,t} - Q_{k,t}}{Q_{k,t}} d\mathcal{N}_t, \tag{C.8}$$

so the expected change in the net worth scaled by SDF is given by

$$\begin{aligned}\frac{\mathbb{E}_t[d(\eta_t B_{j,t}(s))]}{\eta_t dt} &= \left[-(i_t - \pi_t) - \lambda_t\left(\frac{\eta^*_t}{\eta_t} - 1\right)\right] B_{j,t}(s) + (i_t - \pi_t)B_{j,t}(s) + r_{L,t}B^L_{j,t}(s) + r_{E,t}B^E_{j,t}(s) \\ &\quad - C_{j,t(s)} + \lambda_t\left[\frac{\eta^*_t}{\eta_t}B^*_{j,t}(s) - B_{j,t}(s)\right],\end{aligned} \tag{C.9}$$

using Ito's lemma and $\mathbb{E}_t d\eta_t/\eta_t = -(i_t - \pi_t)dt$.

Integrating the expression above and using the fact that $r_{k,t} = \lambda_t \frac{\eta^*_t}{\eta_t}\frac{Q_{k,t} - Q^*_{k,t}}{Q_{k,t}}$, we obtain

$$\frac{\mathbb{E}_t[\eta_T B_{j,T}(s)]}{\eta_t} - B_{j,t}(s) = -\mathbb{E}_t\left[\int_t^T \frac{\eta_z}{\eta_t} C_{j,z}(s) dz\right] \tag{C.10}$$

Given that the household problem with constant mortality rate ξ is identical to the problem of an infinite horizon households with an additional discount ξ, the standard transversality condition holds[3]

$$\lim_{T\to\infty} \mathbb{E}_{j,t}\left[e^{-\rho_j T} C^{-\sigma}_{j,T}(s) B_{j,T}(s)\right] = 0, \tag{C.11}$$

where $\rho_j \equiv \tilde{\rho}_j + \xi$.

We can change measure and price $B_{j,t}(s)$ using the market-implied probabilities:

$$\lim_{T\to\infty} \mathbb{E}_t\left[\eta_T B_{j,T}(s)\right] = 0, \tag{C.12}$$

Combining the expressions above, we obtain the intertemporal budget constraint:

$$\mathbb{E}_t\left[\int_t^\infty \frac{\eta_z}{\eta_t} C_{j,z}(s)\right] = B_{j,t}(s). \tag{C.13}$$

[3] Merton (1992) provides a general proof of this equivalence for stochastic economies (see Chapter 5) and Blanchard (1985) provides a discussion in the context of an otherwise deterministic model.

Notice that $C_{j,z}(s)$ denotes planned consumption for time z for a type-j saver born at date s, conditional on being alive. In particular, this equation implies that, for any date for the household's death $t' \geq t$, we obtain

$$\mathbb{E}_t \left[\int_t^{t'} \frac{\eta_z}{\eta_t} C_{j,z}(s) dz + \frac{\eta_{t'}}{\eta_t} B_{j,t'}(s) \right] = B_{j,t}(s), \tag{C.14}$$

where $B_{j,t'}(s)$ denotes the (involuntary) bequest.

Savers' aggregate IBC. To simplify the aggregation process, it is helpful to index savers in a different way. Let $i \in [\mu_b, 1]$ index the *family* (or dynasty) of a given saver. At each point in time, a family has a single member that derives no utility from bequests and faces mortality risk with intensity $\xi \geq 0$. As the member of the family dies, she is replaced by a new member who inherits the wealth, but may have a different type. Let $C_{i,t}$ denote the consumption of family i's member at time t, $B_{i,t}$ the net worth of family i, $j(i,t) \in \{o, p\}$ the type of the member of the family and $s(i,t)$ the birth date of the current member.

Under this alternative notation, we can write the IBC of family i as follows:

$$\mathbb{E}_t \left[\int_t^{t'} \frac{\eta_z}{\eta_t} C_{i,z} dz + \frac{\eta_{t'}}{\eta_t} B_{i,t'} \right] = B_{i,t}, \tag{C.15}$$

where t' is the time of death and $B_{i,t'}$ is the involuntary bequest. Integrating this forward, the IBC is then given by

$$\mathbb{E}_t \left[\int_t^{\infty} \frac{\eta_z}{\eta_t} C_{i,z} dz \right] = B_{i,t}, \tag{C.16}$$

The aggregate consumption and net worth of savers is given by $C_{s,t} = \frac{1}{1-\mu_b} \int_{\mu_b}^1 C_{i,t} di$ and $B_{s,t} = \frac{1}{1-\mu_b} \int_{\mu_b}^1 B_{i,t} di$. Aggregating the equation above across families, we obtain

$$\mathbb{E}_t \left[\int_t^{\infty} \frac{\eta_z}{\eta_t} C_{s,z} dz \right] = B_{s,t}, \tag{C.17}$$

where $B_{s,t} = \frac{\mu_b D_P + D_{G,t} + Q_{E,t}}{1-\mu_b}$, using the market clearing condition for bonds and equities. $\square$

Applying a similar argument to borrowers, we obtain

$$\mathbb{E}_t \left[\frac{\eta_T}{\eta_t} B_{b,T} \right] - B_{b,t} = \mathbb{E}_t \left[\int_t^T \frac{\eta_z}{\eta_t} \left(\frac{W_z}{P_z} N_{b,z} + \tilde{T}_{b,z} - C_{b,z} \right) dz \right]. \tag{C.18}$$

Using the fact that $B_{b,t} = -D_P$ and $\lim_{T\to\infty} \mathbb{E}_t \left[\frac{\eta_T}{\eta_t} B_{b,T}\right] = 0$, we obtain

$$\mathbb{E}_t \left[\int_t^\infty \frac{\eta_z}{\eta_t} C_{b,z} dz\right] = \mathbb{E}_t \left[\int_t^\infty \frac{\eta_z}{\eta_t} \left(\frac{W_z}{P_z} N_{b,z} + \tilde{T}_{b,z}\right) dz\right] + B_{b,t}. \tag{C.19}$$

Combining the expression above with the IBC for savers, we obtain

$$\mathbb{E}_t \left[\int_t^\infty \frac{\eta_z}{\eta_t} C_z dz\right] = \mathbb{E}_t \left[\int_t^\infty \frac{\eta_z}{\eta_t} \left(\frac{W_z}{P_z} N_z + \mu_b \tilde{T}_{b,z}\right) dz\right] + D_{G,t} + Q_{E,t}, \tag{C.20}$$

where $C_t \equiv \mu_b C_{b,t} + (1-\mu_b) C_{s,t}$.

Let $Q_{C,0} \equiv \mathbb{E}_0 \left[\int_0^\infty \frac{\eta_t}{\eta_0} C_t dt\right]$ denote the value of the aggregate consumption claim and $Q_{H,0} \equiv \mathbb{E}_0 \left[\int_0^\infty \frac{\eta_t}{\eta_0} \left(\frac{W_t}{P_t} N_t + \mu_b \tilde{T}_{b,t}\right) dt\right]$ denote the value of borrowers' human wealth (after transfers). These claims satisfy the following pricing conditions:

$$r_{C,t} = \lambda_t \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_{C,t} - Q^*_{C,t}}{Q_{C,t}}, \qquad r_{H,t} = \lambda_t \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_{H,t} - Q^*_{H,t}}{Q_{H,t}}, \tag{C.21}$$

where $r_{C,t} \equiv \frac{C_t}{Q_{C,t}} + \frac{\dot{Q}_{C,t}}{Q_{C,t}} - (i_t - \pi_t)$ and $r_{C,t} \equiv \frac{\frac{W_t}{P_t} N_t + \mu_b \tilde{T}_{b,t}}{Q_{H,t}} + \frac{\dot{Q}_{H,t}}{Q_{H,t}} - (i_t - \pi_t)$.

Linearizing the pricing condition, we obtain

$$\dot{q}_{C,t} - \rho q_{C,t} = -\frac{C}{Q_C} c_t + i_t - \pi_t - r_n + r_C p_{d,t}, \tag{C.22}$$

where we used the fact that $\frac{C}{Q_C} = r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_C - Q_C^*}{Q_C} = \rho - \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_C^*}{Q_C}$.

Integrating the expression above forward, we obtain

$$q_{C,0} = \frac{C}{Q_C} \int_0^\infty e^{-\rho t} c_t dt - \int_0^\infty e^{-\rho t} \left(i_t - \pi_t + r_C p_{d,t}\right) dt. \tag{C.23}$$

Similarly, the initial price of the claim on human wealth is given by

$$q_{H,0} = \frac{Y}{Q_H} \int_0^\infty e^{-\rho t} \left[(1-\alpha)(w_t - p_t + n_t) + T_t\right] dt - \int_0^\infty e^{-\rho t} \left(i_t - \pi_t + r_H p_{d,t}\right) dt. \tag{C.24}$$

The linearized intertemporal budget constraint is given by

$$Q_C q_{c,0} = Q_H q_{H,0} + D_G q_{L,0} + Q_E q_{E,0}. \tag{C.25}$$

We can write the expression above as follows

$$\int_0^\infty e^{-\rho t} c_t dt - \frac{Q_C}{Y}\int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_C p_{d,t}\right) dt = \int_0^\infty e^{-\rho t}\left[(1-\alpha)(w_t - p_t + n_t) + T_t\right] dt$$
$$- \frac{Q_H}{Y}\int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_H p_{d,t}\right) dt + \frac{D_G}{Y} q_{L,0} + \int_0^\infty e^{-\rho t}\hat{\Pi}_t dt$$
$$- \frac{Q_E}{Y}\int_0^\infty e^{-\rho t}[i_t - \pi_t - r_n + r_E p_{d,t}] dt \tag{C.26}$$

Rearranging the expression above, we obtain

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t}\left[\hat{\Pi}_t + (1-\alpha)(w_t - p_t + n_t) + T_t\right] dt + \frac{D_G}{Y} q_{L,0}$$
$$\frac{Q_C - Q_H - Q_E}{Y}\int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n\right) dt + \int_0^\infty e^{-\rho t}\left[\frac{Q_C}{Y} r_C - \frac{Q_H}{Y} r_H - \frac{Q_E}{Y} r_E\right] p_{d,t} dt. \tag{C.27}$$

From the aggregate IBC in the no-disaster and disaster state, we obtain $Q_C = Q_H + D_G + Q_E$ and $Q_C^* = Q_H^* + D_G \frac{Q_L^*}{Q_L} + Q_E^*$. We then obtain the following condition

$$\frac{Q_C}{Y} r_C - \frac{Q_H}{Y} r_H - \frac{Q_E}{Y} r_E = \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \left[Q_C - Q_C^* - (Q_H - Q_H^*) - (Q_E - Q_E^*)\right] \frac{1}{Y} = \frac{D_G}{Y} r_L. \tag{C.28}$$

We can then write the discount value of consumption as follows:

$$\int_0^\infty e^{-\rho t} c_t dt = \Omega_0, \tag{C.29}$$

where

$$\Omega_0 \equiv \int_0^\infty e^{-\rho t}\left[\hat{\Pi}_t + (1-\alpha)(w_t - p_t + n_t) + T_t\right] dt + \overline{d}_G q_{L,0} + \overline{d}_G \int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_L p_{d,t}\right) dt. \tag{C.30}$$

The price of the consumption claim in the stationary equilibrium satisfies the condition

$$\frac{C}{Q_C} - r_n = \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \left[1 - \frac{Q_C^*}{Q_C}\right] \Rightarrow Q_C = \frac{C + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{C^*}{r_n^*}}{\rho} \tag{C.31}$$

C.4 Wealth effects and Slutsky compensation

In this subsection, we show that Ω_0 corresponds to (minus) the sum of the *Slutsky wealth compensation* for each household. Therefore, Ω_0 captures the aggregate wealth effect associated with the monetary shock. We start by defining the Slutsky wealth compensation in the context of our dynamic economy and then proceed to compute its value for individual households and for the household sector.

Mas-Colell et al. (1995) define Slutsky wealth compensation as the difference between the amount of wealth required to purchase the initial consumption bundle at the new prices and the initial wealth. For a type-j saver, this corresponds to the amount

$$\mathcal{W}_j \equiv \frac{1}{Y}\mathbb{E}_0\left[\int_0^{t^*}\frac{\eta_t}{\eta_0}C_j dt + \int_{t^*}^{\infty}\frac{\eta_t^*}{\eta_0}C_j^* dt\right] - \frac{B_{j,0}}{Y}, \tag{C.32}$$

where we normalized the difference by Y. Notice that $B_{j,0} = \mathbb{E}_0\left[\int_0^{t^*}\frac{\eta_t}{\eta_0}C_{j,t}dt + \int_{t^*}^{\infty}\frac{\eta_t^*}{\eta_0}C_{j,t}^* dt\right]$.

Similarly, we can define the Slutsky wealth compensation for borrowers:

$$\mathcal{W}_b \equiv \frac{1}{Y}\mathbb{E}_0\left[\int_0^{t^*}\frac{\eta_t}{\eta_0}C_b dt + \int_{t^*}^{\infty}\frac{\eta_t^*}{\eta_0}C_b^* dt\right] - \frac{B_{b,0}}{Y} - \frac{Q_{H,0}}{\mu_b Y}, \tag{C.33}$$

where $Q_{H,0}$ denotes human wealth at time 0 for all borrowers, so $Q_{H,0}/\mu_b$ denotes the value for an individual borrower. Notice that $\mathcal{W}_b$ is compensating not only for the change in real interest rates, but also the change in labor income.

Let $\mathcal{W}$ denote the aggregate value of $\mathcal{W}_j$ across all households. Using the market clearing condition for bonds and equities, we obtain

$$\mathcal{W} = \frac{1}{Y}\mathbb{E}_0\left[\int_0^{t^*}\frac{\eta_t}{\eta_0}C dt + \int_{t^*}^{\infty}\frac{\eta_t^*}{\eta_0}C^* dt\right] - \frac{D_{G,0}}{Y} - \frac{Q_{E,0}}{Y} - \frac{Q_{H,0}}{Y}. \tag{C.34}$$

Using the aggregate IBC, we obtain

$$\mathcal{W} = \frac{1}{Y}\mathbb{E}_0\left[\int_0^{t^*}\frac{\eta_t}{\eta_0}C dt + \int_{t^*}^{\infty}\frac{\eta_t^*}{\eta_0}C^* dt\right] - \frac{1}{Y}\mathbb{E}_0\left[\int_0^{t^*}\frac{\eta_t}{\eta_0}C_t dt + \int_{t^*}^{\infty}\frac{\eta_t^*}{\eta_0}C_t^* dt\right]. \tag{C.35}$$

Up to a first-order approximation, we can write the expression above as

$$\mathcal{W} = -\int_0^{\infty} e^{-\rho t} c_t dt = -\Omega_0. \tag{C.36}$$

Therefore, Ω_0 corresponds to (minus) the sum of the Slutsky wealth compensation for all households.

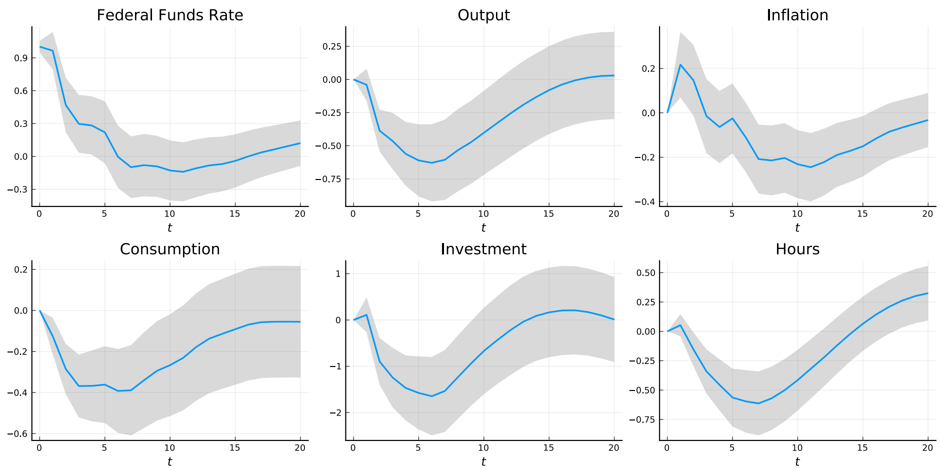

Figure D.1: Estimated IRFs.

D Estimation of Fiscal Response to a Monetary Shock

We estimate the empirical IRFs using a VAR identified by a recursiveness assumption, as in Christiano et al. (1999), extended to include fiscal variables. The variables included are: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, the federal funds rate, the 5-year constant maturity rate, and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The identification assumption of the monetary shock is as follows: the only variables that react contemporaneously to the monetary shock are the federal funds rate, the 5-year rate and the value of government debt. All other variables, including tax revenues and expenditures, react with a lag of one quarter.

Data sources. The data sources are: **Nominal GDP:** BEA Table 1.1.5 Line 1; **Real GDP:** BEA Table 1.1.3 Line 1, **Consumption Durable:** BEA Table 1.1.3 Line 4; **Consumption Non Durable:** BEA Table 1.1.3 Line 5; **Consumption Services:** BEA Table 1.1.3 Line 6; **Private Investment:** BEA Table 1.1.3 Line 7; **GDP Deflator:** BEA Table 1.1.9 Line 1; **Capacity Utilization:** FRED CUMFNS; **Hours Worked:** FRED HOANBS; **Nominal Hourly Compensation:** FRED COMPNFB; **Civilian Labor Force:** FRED CNP16OV; **Nominal Revenues:** BEA Table 3.1 Line 1; **Nominal Expenditures:** BEA Table 3.1 Line 21; **Nominal Transfers:** BEA Table 3.1 Line 22; **Nominal Gov't Investment:** BEA Table 3.1 Line 39; **Nominal Consumption of Net Capital:** BEA Table 3.1 Line 42; **Effective Federal Funds Rate (FF):** FRED FEDFUNDS; **5-Year Treasury Constant Maturity Rate:** FRED DGS5; **Market Value of Government Debt:** Hall et al. (2018).

	(1) Revenues	(2) Interest Payments	(3) Transfers & Expenditures	(4) Debt in T	(5) Initial Debt	(1) - (2) - (3) + (4) - (5) Residual
Data	-26	68.88	-12.09	2.91	-49.74	30.13
	[-72.89,20.89]	[30.01,107.75]	[-48.74,24.56]	[-12.79,18.62]	[-68.03,-31.46]	[-4.74,65]

Table D.1: The impact on fiscal variables of a monetary policy shock

Note: Calculations correspond to a a 100 bps unanticipated interest rate increase. Confidence interval at 95% level.

All the variables are obtained from standard sources, except for the real value of debt, which we construct from the series provided by Hall et al. (2018). We transform the series into quarterly frequency by keeping the market value of debt in the first month of the quarter. This choice is meant to avoid capturing changes in the market value of debt arising from changes in the *quantity* of debt after a monetary shock instead of changes in *prices*.

VAR estimation. Figure D.1 shows the results. As is standard in the literature, we find that a contractionary monetary shock increases the federal funds rate and reduces output and inflation on impact. Moreover, the contractionary monetary shock reduces consumption, investment, and hours worked.

The Government's Intertemporal Budget Constraint. The fiscal response in the model corresponds to the present discounted value of transfers over an infinite horizon, that is, $\sum_{t=0}^{\infty} \tilde{\beta}^t T_t$, where $\tilde{\beta} = \frac{1-\lambda}{1+\rho_s}$. We next consider its empirical counterpart. First, we calculate a truncated intertemporal budget constraint from period zero to $\mathcal{T}$:

$$\underbrace{b_y b_0}_{\substack{\text{debt}\\ \text{revaluation}}} = \sum_{t=0}^{\mathcal{T}} \tilde{\beta}^t \left[\underbrace{\tau y_t + \tau_t}_{\text{tax revenue}} - \underbrace{\tilde{\beta}^{-1} b_y (i_{t-1}^m - \pi_t - r^n)}_{\text{interest payments}} \right] - \underbrace{T_{0,\mathcal{T}} + \tilde{\beta}^{\mathcal{T}} b_y b_{\mathcal{T}}}_{\substack{\text{other transfers/expenditures}\\ \text{\& final debt}}} \tag{D.1}$$

The right-hand side of (D.1) is the present value of the impact of a monetary shock on fiscal accounts. The first term represents the change in revenues that results from the real effects of monetary shocks. The second term represents the change in interest payments on government debt that results from change in nominal rates. The last two terms are adjustments in transfers and other government expenditures, and the final debt position at period $\mathcal{T}$, respectively. In particular, $T_{0,\mathcal{T}}$ represents the present discounted value of

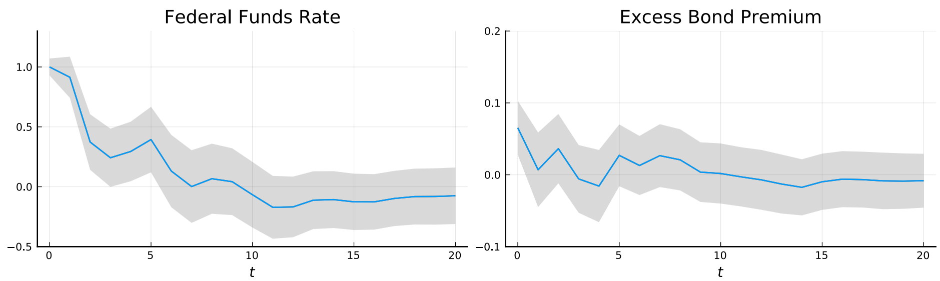

Figure D.2: IRFs for the federal funds rate and excess bond premium.

transfers from period 0 through $\mathcal{T}$. Provided that $\mathcal{T}$ is large enough, such that (y_t, τ_t, i_t) have essentially converged to the steady state, then the value of debt at the terminal date, $b_{\mathcal{T}}$, equals (minus) the present discounted value of transfers and other expenditures from period $\mathcal{T}$ onward. Hence, the last two terms combined can be interpreted as the present discounted value of fiscal transfers from zero to infinity. Finally, the left-hand side represents the revaluation effect of the *initial* stock of government debt.

Table D.1 shows the impact on the fiscal accounts of a monetary policy shock, both in the data and in the estimated model. We first apply equation (D.1) to the data and check whether the difference between the left-hand side and the right-hand side is different from zero. The residual is calculated as

$$\text{Residual} = \text{Revenues - Interest Payments - Transfers + Debt in } \mathcal{T} \text{ - Initial Debt}$$

We truncate the calculations to quarter 60, that is, $\mathcal{T} = 60$ (15 years) in equation (D.1). The results reported in Table D.1 imply that we cannot reject the possibility that the residual is zero and, therefore, we cannot reject the possibility that the intertemporal budget constraint of the government is satisfied in our estimation.

The adjustment of the fiscal accounts in the data corresponds to the patterns we observed in Figure 2. The response of initial debt is quantitatively important, and it accounts for the bulk of the adjustment in the fiscal accounts.

EBP. To estimate the response of the corporate spread in the data, we add the EBP measure of Gilchrist and Zakrajšek (2012) into our VAR (ordered after the fed funds rate). Since the EBP is only available starting in 1973, we reduce our sample period to 1973:1-2007:7. The estimated IRFs are in line with those obtained for the longer sample. We find a significant increase of the EBP on impact, of 6.5 bps, in line with the estimates in the literature.

Appendix References

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