Buy Big or Buy Small? Procurement Policies, Firms' Financing, and the Macroeconomy*

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Abstract

This paper provides a framework to study how different public procurement contract allocation systems affect the macroeconomy. We build a dataset for Spain that merges public procurement, credit register, and firm-level data, and provide evidence consistent with procurement contracts acting as collateral for firms. We build a firm-dynamics model with asset- and earnings-based borrowing constraints and a government purchasing goods and services from firms. Procurement policies that promote the participation of small firms have sizeable macroeconomic effects, but the impact on GDP is ambiguous and depends on how policies are implemented and the type and strength of financial frictions.

JEL Classifications: E22; E23; E62; G32

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1 Introduction

Governments play a key role in economic activity. They are large employers, they set taxes and transfers, and they purchase goods and services from the private sector. These purchases are made by awarding *public procurement contracts* to private firms. The size of public procurement varies over time and across countries, but it consistently represents a large fraction of GDP —12.8% in the OECD countries, 14% in EU countries, and 9.3% in the United States.[1]

Because of its large size and high level of discretion, many governments use the public procurement process to allocate resources to specific firms. In the U.S., for example, the Small Business Act aims to "ensure that a fair proportion of federal contracts is awarded to small business".[2] Similarly, in the EU, promoting the participation of small firms is at the core of the European Commission's agenda for public procurement regulation.[3] A common justification for these policies is that small firms may face demand or financial constraints, preventing them from reaching their full potential and hence negatively affecting aggregate economic outcomes. Given the wide presence of these types of policies worldwide, it is surprising how little we know about their macroeconomic impact. This paper aims to fill this gap by asking if it is possible to achieve better aggregate outcomes when changing the composition of firms from which governments buy their goods and services.

Our research question is the following: how would the level of aggregate output, TFP, and provision of public goods be affected if governments change the composition of firms from which they buy while keeping total expenditure fixed? In order to address this question, we carry out an analysis that integrates a novel firm-level dataset with a macroeconomic model of firm dynamics. Our dataset merges administrative data on public procurement, credit allocation at the bank-firm level, and firm outcomes for the Spanish economy over the 2000-2013 period. Our model builds on the canonical framework of firm dynamics with financial frictions (e.g., Midrigan and Xu, 2014) and incorporates two novel elements. First, there is a government that purchases goods and services from different private sector firms. Firms that are willing to sell to the government must make a risky investment in advance, which reflects the costs of preparing a good proposal and increasing the chances to win the

[1] See EU Commission's web and OECD for details.

[2] See Report from Congressional Research Service for details. Policies promoting small firms' participation in public procurement are widely spread worldwide. Some other examples are Australia and Brazil.

[3] See the Public Sector Directive 2014/24/EU for details. There has been strong support from the European Parliament for explicit regulation that discriminates in favor of small firms: "From this House, we must insist that...administrative bodies incorporate terms into their tender specifications that facilitate positive discrimination in favor of SMEs and remove contractual provisions that hinder their participation." See EU Parliament Debate.

contract. Second, we allow for earnings-based borrowing constraints where the pledgeability of firms' earnings from procurement may differ from that from selling to the private sector.[4]

We study the interplay between procurement and the macroeconomy by calibrating the model to reproduce several macro and, crucially, micro moments related to firm selection into procurement and to firm dynamics after winning procurement contracts (treatment).

We document a selection pattern where firms that participate in procurement are 72% larger ex-ante in terms of value-added. We refer to this difference as the "procurement size premium." Our model generates this procurement size premium through two firm-level state variables: productivity (TFPQ) and net worth. As is standard in heterogeneous firm models, the value of participating in a given market —the procurement market in this case— depends on firms' ability to deliver large projects (e.g., Melitz, 2003, in the context of international trade). In our model, this ability uniquely depends on firms' TFPQ in the case of financially unconstrained firms. However, for constrained firms, it also depends on their financing capacity, which itself depends on firms' net worth (e.g., Chaney, 2016, also in the context of international trade). In our baseline calibration, where we match the 72% value-added procurement size premium, our model implies a procurement premium of 35% in terms of TFPQ and 44% in terms of net worth.

Regarding treatment, the model reproduces several facts related to the evolution of firms' credit and sales which we document by estimating local projections with our micro data.

First, we show a positive and persistent effect of participating in procurement on firms' credit. Importantly, the effect on impact is completely explained by an increase in loans for which no tangible collateral is posted. This finding points towards the importance of earnings-based constraints. The subsequent effects over time, however, are partly also explained by loans that require capital as collateral. This finding points towards the presence of asset-based constraints playing an important role, and the fact that firms' ability to accumulate capital increases as a result of participating in procurement. In the calibration, we reproduce a structural regression in which the change in financially constrained firms' leverage, i.e., total credit divided by fixed assets, depends on two variables: the change in total earnings divided by fixed assets and the change in earnings from procurement divided by fixed assets. The coefficient associated with the former ratio pins down the parameter that governs the pledgeability of firms' earnings from selling to the private sector, whereas the coefficient associated with the latter pins down the difference in the pledgeability of earnings from procurement relative to the pledgeability of private earnings.[5] We run this

[4]Other recent papers that embed earning-based borrowing constraints into macro models are Drechsel (2023), Brooks and Dovis (2020), and Li (2022). The importance of earning-based borrowing constraint is also supported by direct evidence, see Lian and Ma (2020).

[5]This structural identification of the earnings-based constraints is similar to the one used by Li (2022)

regression for firms that are likely to be financially constrained, both in the model and in the data, and find that firms can pledge 10% of their annual earnings from selling to the private sector and 15% of their annual earnings from procurement.

Second, we also show that sales to the private sector decline upon obtaining a procurement contract (a "crowding-out" effect) and increase afterward (a "crowding-in" effect) leading to firm growth in the medium run. In our model, this crowding-out effect on impact of the procurement shock occurs because financially constrained firms have to split their scarce collateral to serve both procurement and private sector operations. The fact that government sales can be collateralized partly alleviates but does not eliminate this problem. Importantly, we show that the crowding-out effect is particularly stronger for firms that are more likely to be financially constrained, which suggests that the financial frictions at play in the model are also relevant in the data.

To assess the interplay between procurement and the macroeconomy, we use our calibrated model to perform some *expenditure-neutral* counterfactual experiments that consist of reallocating procurement contracts across firms while keeping government expenditure unchanged. In particular, we compare our benchmark economy with counterfactual economies in which a higher share of procurement contracts is allocated to small firms. Our preferred counterfactual, which consists of promoting small firms' participation by directly targeting them in the procurement allocation system, aims to reproduce the "set-aside" policies for small businesses implemented by the U.S Small Business Administration.

Our main result is that promoting the participation of small firms in government procurement generates sizeable macroeconomic effects through three main channels. First, the policy generates the positive effect of helping small firms grow and overcome financial constraints. The high pledgeability of procurement contracts together with the extra profits they generate reinforces the self-financing channel previously emphasized in the literature (Moll, 2014). In this respect, public procurement is a policy tool that helps small firms overcome financial frictions and achieve closer to optimal size in the long run. We find that through this channel, the policy reform would increase GDP by around 3% in partial equilibrium.

Second, there is an unintended consequence related to the change in capital accumulation incentives of relatively big firms (those for which the expected value of procurement decreases) and their responses to general equilibrium price changes. In particular, one of the reasons firms accumulate financial wealth in our model is the possibility of obtaining a public procurement contract in the future. A procurement contract represents a large demand shock in response to which firms want to expand their capital stock. Intuitively, productive firms want to have enough net worth to build capacity in case an opportunity for a big

for the case of private sector earnings only.

procurement contract is realized. A procurement policy that aggressively targets smaller firms will hence remove savings incentives for middle-sized and large firms, whose chances of obtaining a procurement contract are diminshed. This channel shrinks the positive aggregate effect on GDP by 2 percentage points, from 3% to 1%.

Third, there is a second unintended consequence of the reform related to the government's underlying production of public goods. By buying relatively more from the pool of small firms, the government also buys from firms that are fundamentally less productive and thus charge higher prices. Under a constant government expenditure, this phenomenon inevitably translates into a higher price index paid by the government and a lower provision of public goods. When accounting for this reduction in the amount of real final goods provided by the government, the overall effect on GDP shrinks further from 1% to 0.05%.

We also conduct an alternative policy experiment that consists in promoting small firms' participation by reducing the average size of contracts. This reform aims to mimic the European Commission's strategy to increase the presence of small firms in public procurement in Europe.[6] Our main finding is that this alternative policy counterfactual would reach a higher number of small firms than in our preferred counterfactual but would actually generate a fall in aggregate GDP of 0.85%. The reason for this decline in GDP is that the reduction in incentives of relatively big firms to accumulate capital is particularly strong when the size of procurement contracts falls. Our results thus imply that the particular way in which the promotion of small firms is implemented is crucial to understand its aggregate effects.

1.1 Related literature

Governments may harm the long-run economic performance of countries through the implementation of policies that distort the allocation of resources across firms. Some examples are credit subsidies to state-owned-enterprises (Song, Storesletten and Zilibotti, 2011), size-dependent policies (Guner, Ventura and Xu, 2008; García-Santana and Pijoan-Mas, 2014), labor market regulations (Garicano, Lelarge and Reenen, 2016), tariffs (Berthou, Chung, Manova and Sandoz, 2019), or capital markets regulation (Bau and Matray, 2023). However, one of the central roles that governments play in modern economies, i.e., being buyers of goods and services from private sector firms, has been overlooked. We focus on this by analyzing specific size-dependent procurement policies aimed at helping small firms in the presence of financial frictions.[7]

Our finding that the type of financial frictions matters in understanding the effects of

[6] See Trybus (2014) for details.

[7] Note that we do not claim that these policies are the most effective way of alleviating financial frictions, but the effects of these prevalent policies may depend on the extent and type of financial frictions.

procurement on the macroeconomy is also related to recent papers that show that the type of financial frictions, i.e., earnings- vs. asset-based, and not only their severity, plays a crucial role in explaining important economic outcomes: the gains from trade liberalization (Brooks and Dovis, 2020), aggregate productivity (Li, 2022), macroeconomic fluctuations (Drechsel, 2023; Drechsel and Kim, 2022), and the transmission of monetary policy (Caglio, Darst and Kalemli-Özcan, 2021).

Our focus on firm-level financial frictions as a channel through which public procurement can affect the macroeconomy builds on work that quantifies the effects of financial constraints on aggregate output and productivity. Some examples are Buera, Kaboski and Shin (2011), Midrigan and Xu (2014), Gopinath, Kalemli-Özcan, Karabarbounis and Villegas-Sanchez (2017), Leibovici (2021), David and Venkateswaran (2019), David, Schmid and Zeke (2022), and Catherine, Chaney, Huang, Sraer and Thesmar (2022).[8] A few papers in this literature have studied the interplay of financial frictions with different forms of taxation (Erosa and González, 2019; Itskhoki and Moll, 2019; Guvenen, Kuruşçu, Kambourov, Ocampo and Chen, 2019) but none has focused on the expenditure side of government policies.

There is practically no literature that analyzes how the microeconomic aspects of public procurement can affect the macroeconomy. One recent exception is Cox, Muller, Pasten, Schoenle and Weber (2021), who document several new facts using micro-level data on public procurement contracts awarded by the U.S. federal government, and study how accounting for these facts— in particular that government spending is concentrated in sectors with stickier prices— can affect the short run fiscal transmission mechanism in a New Keynesian model. Our interest differs in that we focus on quantifying the long-run macroeconomic effects of different procurement allocation systems through their impact on firm dynamics.

Our results on the treatment effects of winning procurement contracts on firms are related to the recent literature analyzing the relationship between public procurement and firm dynamics. Ferraz, Finan and Szerman (2016) and Lee (2021) use quasi-experimental designs for Brazil and South Korea, respectively, to show that firms winning procurement contracts have a positive and permanent effect on firms' performance. Hebous and Zimmermann (2021) document for the U.S. a positive relationship between winning a procurement contract and firm investment.[9] Our results are consistent with this body of research. We further provide novel evidence on loan acceptances and on the fact that only non-collateralized credit increases, which along with the other empirical facts that we document, can be taken as evidence of earnings-based financial constraints that are alleviated with procurement

[8] See Buera, Kaboski and Shin (2015) for a survey of this literature.

[9] Cappelletti and Giuffrida (2021) use data for Italy to show that firms that receive public procurement contracts survive longer, a dimension of the data that we do not explore.

projects. Additionally, our results on the short-run crowding-out of sales to the private sector by procurement sales are related to recent papers that investigate within-firm spillover effects across markets, like Almunia, Antràs, López-Rodríguez and Morales (2021) and Kohn, Leibovici and Szkup (2016) with domestic versus foreign markets and Alfaro-Ureña, Manelici and Vasquez (2022) with multinational corporations versus other buyers.[10]

The rest of the paper is structured as follows. Section 2 describes the construction of the dataset and provides summary statistics. Section 3 provides our empirical evidence organized in three main stylized facts. Section 4 presents the model of firm dynamics with procurement. Section 5 discusses how we parameterize the model. Section 6 describes our benchmark economy. Section 7 provides the main quantitative results and Section 8 presents different robustness exercises. Section 9 concludes. After concluding, we provide an extensive Appendix with additional details on the data and the model.

2 Data and Summary Statistics

Our empirical work is based on merging three large datasets at the firm level. First, we construct a novel dataset on Spanish public procurement contracts published by the official bulletin of the Spanish Central Government (*Boletín Oficial del Estado*, BOE) over the 2000-2013 period. We have information on the type of good or service provided, the institution awarding the contract, the initial bidding and final price of the contract, the type of procedure used to allocate the contract, and the firm(s) that won the contract. Second, we use more standard firm-level data on balance sheets and income statements of the quasi-universe of Spanish companies between 2000 and 2016, a dataset that is maintained by the Banco de España and taken from the Spanish Commercial Registry. And third, we use credit register data for Spain, which contain detailed information on all outstanding loans over 6,000 euros to non-financial firms granted by all banks operating in Spain (including whether or not non-personal collateral was posted on a particular loan). Additionally, the credit register data set contains rich information on loan applications. Appendix A provides details about the different data sources and samples that we use.

Main sample. We merge the three data sets to build a panel of firms at the annual frequency (which is the frequency of observation of our firm-level data) with detailed information on credit and procurement activity. For our sample we only keep those firms that get at least one procurement project over the sample period.[11] This leaves 12,000 unique firms

[10] Our paper also relates to the vast micro/IO literature that analyzes the factors determining the efficiency of procurement outcomes. Some examples are Bandiera, Prat and Valletti (2009), Decarolis (2018), Decarolis, Giuffrida, Iossa, Mollisi and Spagnolo (2020), and Lotti, Muço, Spagnolo and Valletti (Forthcoming).

[11] Note that because all our empirical work uses firm-level fixed effects, the relevant variation is within-firm

observed for an average of 12 years. Alternatively, in the Appendix, we also build a sample at a quarterly frequency and a smaller sample for projects with data on the full ranking of bidders. These two samples deliver a tighter identification of the effects of procurement on credit growth, but cannot be used to examine the effects of procurement on firm sales.

Procurement vs. non-procurement firms. We find that firms participating in procurement are significantly larger and older on average, but there is considerable overlap in the support of the size and age distribution for procurement and non-procurement firms (see Table A.V in the Appendix). For example, the average number of employees of a procurement firm is around 6 times larger than for the rest of the firms (73.56 vs. 12.75), total sales are 7 times larger (8.9 million euros vs. around 1.2 million), and procurement firms are 9 years older (20 vs 11 years). Yet, around 25% of procurement firms have less than 16 employees, have revenues that are lower than 1.1 million euros, and are 12 or fewer years old. We also find that conditional on having at least one procurement project, there is a lot of variation on the importance of these projects as a fraction of firms' total revenue. The ratio of procurement value to total revenues is 0.20 on average, with the 25th, 50th, and 75th percentiles being 0.01, 0.03, and 0.10 respectively. Finally, we observe large differences between procurement and non-procurement firms in terms of their composition of credit. In particular, procurement firms seem to rely more on non-collateralized credit (86% vs 71% on average) despite holding higher levels of assets.

3 Motivating Empirical Evidence

We begin by documenting three key facts related to the effects of procurement on firms' outcomes. First, we show a positive relationship between obtaining a procurement contract and firms' credit growth, which in the short run comes entirely from credit for which no tangible collateral is posted. Second, obtaining a procurement contract is related to a persistent increase in firm revenues, and this increase in revenues happens alongside a short-run *decline* in sales to the private sector (crowding-out), followed by an increase in private sector sales above the level before the procurement shock (crowding-in). Third, the initial crowding-out is particularly stronger for firms more likely to be financially constrained.

3.1 Main facts

To study the relationship between participation in procurement and firms' dynamics, we estimate local projection panel regressions (Jordà, 2005). In particular, we regress the cumulative difference of variable x, $\Delta_h \log(x_{it+h}) \equiv \log(x_{it+h}) - \log(x_{it-1})$ on the regressor PROC_{it},

variation for those firms that get at least one procurement project in the sample period.

(a) total credit **(b)** non-collateral credit **(c)** collateral credit

Notes: This figure shows the cumulative impact of the estimate of β_2^h from regression (1) for different time horizons $h = 0, 1, 2, 3, 4$, as well as its 5% confidence intervals. Panel (a) shows the results for the case of x being firms' total credit. Panel (b) shows the results for the case of x being firms' non-collateralized credit. Panel (c) shows the results for the case of x being firms' collateralized credit. As mentioned above, we have run these regressions with around 12,000 firms that have at least one contract during the sample period. Standard errors clustered at the firm level.

Figure 1. Procurement effect on credit

firm fixed effects α_i, sector×year fixed effects α_{st}, and lagged x:

$$\Delta_h \log(x_{it+h}) = \alpha_i + \alpha_{st} + \beta_1^h \text{PROC}_{it} + \beta_2^h \log\ x_{it-1} + \varepsilon_{it+h} \tag{1}$$

where i denotes firm and $h = 0, 1, ..., H$ denotes the horizon at which the impact of procurement is estimated. The regressor PROC_{it} is a dummy variable that takes value 1 if the firm obtained a procurement contract in year t. The inclusion of firm fixed effects implies that we are comparing the differential within-firm growth of credit across periods with and without procurement. Next, we focus on how procurement affects the evolution of several variables at the firm level.

Fact 1. Credit growth. We show the results from running the regressions given by equation (1), with x referring to credit, in our sample at an annual frequency in Figure 1, where we plot the estimated β_1^h coefficients and their associated confidence intervals. Our main result is that winning a procurement contract is associated with a persistent effect on firms' credit. On impact ($h = 0$), winning a procurement contract is associated with an increase in firms' total credit of around 5.5 percentage points (see panel (a) of Figure 1). We find that this effect is persistent, which implies that firms increase their credit beyond the duration of the procurement contract

We also find that the effect on impact is entirely driven by non-collateralized credit: the effect on non-collateralized and collateralized credit is around 8 and 0 percentage points, respectively, when $h = 0$ (see panels (b) and (c) of Figure 1). Finally, we find that, although absent on impact, the effect of procurement on collateralized credit gets bigger over time. For example, the 5 years ($h = 4$) cumulative effect is around 6 percentage points. This final result is consistent with the idea that firms use their extra profits earned in procurement to increase their net worth and therefore increase their borrowing capacity in the future.[12]

A limitation of our data is that it does not contain information about the duration of each contract. One potential concern is that the estimated "persistent" effect of procurement on credit results from long contracts. To have a sense of the duration of contracts in Spain, we use a different dataset, Tenders Electronic Daily (TED), which provides information on the estimated length of contracts.[13] For the years 2018-2019, for example, around 75% of the contracts have a duration of one year or less, 89% have a duration of two years or less, and 93% have a duration of three years or less. These numbers likely represent an upper bound, given that very large contracts, which tend to last longer, are over-represented in TED. As a reference for a different country, Cox et al. (2021) find that the median contract in the U.S. has a duration of 31 days and 90% of contracts last less than one year.

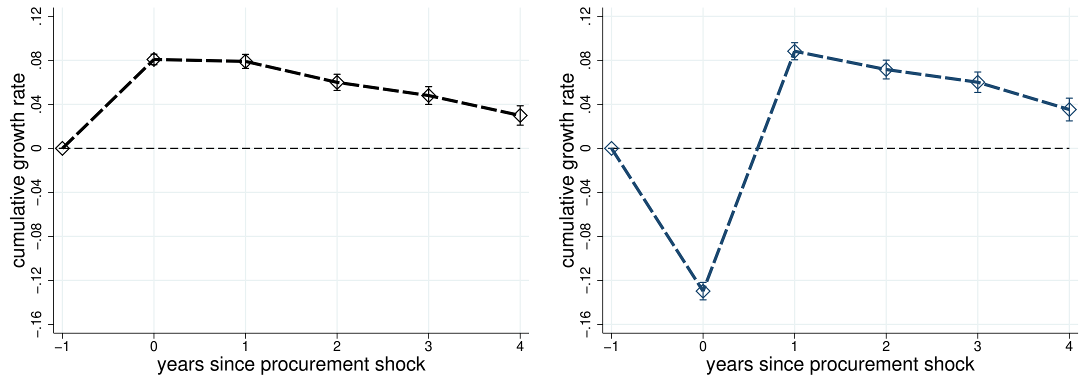
Fact 2. Sales composition and sales growth. We next look at the evolution of firms' total sales and their composition. In particular, we run regression (1) for the case of x = total sales (see panel (a) of Figure 3) and x = sales to the private sector (see panel (b) of Figure 3). Several important results emerge from running these regressions. First, as expected, we find that firms' total sales increase after winning a procurement contract, by around 8 percentage points.

Second, we find that this effect is persistent, similar to the previous result on credit. Third, we find that sales to the private sector fall right after a firm wins a procurement contract, which means that procurement generates a short-run crowding-out of sales to the private sector. In the model we will present below, this crowding-out will happen because financial constraints limit firms' production capacity, and hence firms decrease the amount of output sold to the private sector in order to deliver their procurement contracts. Finally, we find that this crowding-out effect disappears and actually gets reversed over time, meaning that procurement crowds in sales to the private sector in the longer run. One interpretation of this fact is that the extra revenues of the procurement contract can be used for the financially constrained firms to accumulate wealth and finance larger operations in the future.

Fact 3. Heterogeneous effects of crowding-out of private sales. The initial crowding-out of sales to the private sector may arise due to firms' lack of borrowing capacity to serve the private and public sectors at the same time, which the extra credit obtained

[12] An alternative explanation for the estimated "persistent" effect of procurement on credit is that obtaining a contract in a given year may increase the probability of obtaining more contracts in subsequent periods. While it is true that there is persistence in procurement, the persistent effect of procurement on credit is very similar for a sub-sample of firms that only obtain one contract during our sample period.

[13] TED is the online version of the supplement to the Official Journal of the EU dedicated to European public procurement. See García-Santana and Santamaría (2022) for details.



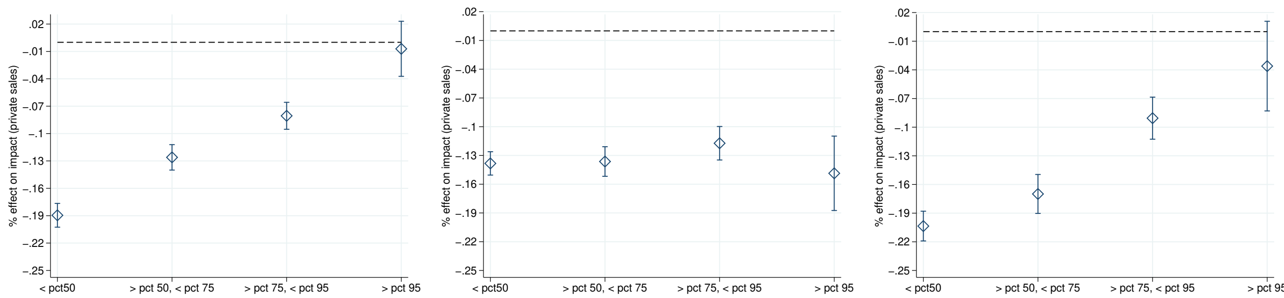
(a) total sales **(b)** sales to the private sector

Notes: This figure shows the cumulative impact of the estimate of β_2^h from regression (1) for different time horizons $h = 0, 1, 2, 3, 4$, as well as its 5% confidence intervals. Panel (a) shows the results for the case of x being firms' total sales. Panel (b) shows the results for the case of x being firms' sales to the private sector. Standard errors clustered at the firm level.

Figure 2. Procurement effect on sales

with the procurement contract only partially offsets. However, it could also arise due to capacity constraints or decreasing returns to scale at the firm level. If the effect is at least partly explained by financial frictions, we should find that firms more likely to be financially constrained exhibit a larger crowding-out. Indeed, our simple static production problem in Section 4.6 shows this to be the case (see Appendix E.4). In the data, it is hard to identify which firms are more financially constrained. However, three conventional proxies for how financially constrained a firm is, are size (measured by assets), leverage (measured by credit/assets), and age. We test the hypothesis that the crowding-out effect of private-sector sales is greater for more financially constrained firms by interacting the effect of procurement on dummy variables for several percentiles of the fixed assets, leverage, and age distribution. We present these results in Figure 3. In terms of assets and age, we find a negative monotonic relationship between them and the extent of the crowding-out effect on private sales. In terms of leverage, we find that the crowding-out effect is particularly strong for firms whose leverage is above the 95th percentile.

Heterogeneous effects of procurement on credit growth. In Appendix B, we provide evidence of the heterogeneous effects of procurement on firms' credit growth. We find that the empirical relationship between the change in firms' credit after procurement and proxies for financial constraints at the firm level is non-monotonic. However, as we discuss in Appendix B, this evidence is not very informative. Models of financial constraints would have conflicting predictions on which firms should increase credit more, depending on whether



(a) distribution of fixed assets **(b)** distribution of leverage **(c)** distribution of age

Notes: This figure shows the effect on impact, i.e., $h = 0$ (as well as its 10% confidence intervals), of public procurement on sales to the private sector for different percentiles of the distribution of assets (panel a), leverage (panel b), and age (panel c). Standard errors clustered at the firm level.

Figure 3. Heterogeneous effects on sales to the private sector

the proxies for financial frictions in the data represent the likelihood of being financially constrained or the severity of the financial constraints conditional on these being binding.

3.2 Robustness

Appendix C provides further evidence on the relationship between firms' participation in procurement and credit growth. We summarize the evidence here.

Procurement and credit using quarterly frequency. To provide tighter identification of the effect of procurement on credit, in Appendix C.1 we exploit higher frequency credit data and estimate the procurement effect on credit using quarterly data. Importantly, these specifications allow us to control for *firm×year* fixed effects, which absorb all firm characteristics, observable or unobservable, varying at the yearly frequency. First, we find that winning a procurement contract is associated with an annualized increase in credit of around 5.5 pct, which is similar in terms of magnitude to the effects on impact, i.e., $h = 0$, estimated above in Section 3.1. And second, we also find that the effect is entirely driven by non-collateral credit.

Winners vs. the rest. In Appendix C.2, we use the sub-sample of procurement projects where we have information on all bidders as well as the final ranking (see Appendix A.1 for details on this information). Doing so allows us to run regressions where we can identify the association between a firm's ranking in a given auction and its ensuing credit growth. First, we find that winning firms experience significantly larger growth in credit relative to losing firms within the same auction. Second, we find that, again, all the effect is explained by non-collateral credit. Third, we find that the ranking itself does not seem to matter in explaining the evolution of credit. In particular, when excluding winners, we find no differential effect on credit growth between the firms that were ranked second vs. firms that were ranked third and fourth within the same auction. And fourth, we show all the previous results are not driven by the presence of pre-trends in the evolution of credit for winners vs. the rest.

Loan applications. Finally, in Appendix C.3, we exploit data on loan applications to see whether the acceptance probability of a credit application increases when firms get a procurement contract. In particular, we can see whether a firm has applied to a given bank and whether the loan application has been accepted or rejected throughout our sample period. We use this information to help identify an increase in firms' borrowing capacity. To do so, we run regressions at the firm-bank level and relate the probability of firms obtaining a loan to whether they have received a procurement contract. We find that the probability of receiving a bank loan conditional on having applied for it increases by approximately 2 percentage points in the quarter that a firm wins a procurement project, which is consistent with the idea that procurement represents a credit supply shock for the firm.

3.3 Discussion

There are two possible interpretations of the observed credit growth upon winning a procurement contract. The first one is that the arrival of a procurement project represents a *credit demand shock* at the firm level, without financial frictions playing any relevant role. This will be the case if firms tend to borrow in advance to increase production. This interpretation may be consistent with the initial crowding-out of sales to the private sector (if firms face short-run capacity constraints or decreasing returns to scale in production) and with the medium-run crowding-in of sales to the private sector (if firms invest in production capacity to serve the public sector and later use it for their private sector operations). The second interpretation is that procurement represents a *credit supply shock* at the firm level. This will be the case if procurement contracts can be pledged for extra borrowing and a sizeable fraction of firms doing procurement are credit-constrained. This would explain why the increase of credit at impact does not require collateral to be posted. The existence of credit constraints for firms participating in procurement has been documented elsewhere, see Abad et al. (2023) for quasi-experimental evidence in Spain. It is also consistent with our evidence of the effect of procurement on sales: (a) the short-run crowding-out of sales to the private sector (as financially constrained firms do not have the financial capacity to completely serve both demands), (b) the crowding-out being stronger for firms more likely to be financially constrained, and (c) the medium-run crowding-in of sales to the private sector (procurement gives firms extra profits, which can be saved and used to expand production capacity in the future).

While both interpretations are plausible, our results point to the *credit supply shock* interpretation playing a relevant role because (i) the short-run increase in credit is limited to non-collateral credit, (ii) the short-run crowding-out of private sales being larger for firms more likely to be constrained, and (iii) the additional evidence on the acceptance of loan applications.[14] However, to gauge the relative importance of the credit *demand* and *supply* interpretations, we need a formal model structure as that presented in Section 4.

4 The Model

We set up a model of privately held heterogeneous firms. We build on standard models of firm dynamics with collateral constraints —as Midrigan and Xu (2014), Moll (2014), or Buera and Moll (2015)— and extend this setting to allow for (a) a public sector demanding goods from private firms, (b) earnings-based borrowing constraints with different degrees of pleadgeability for procurement and private sector sales, (c) downward-sloping demands in both the private and public sectors, and (d) a choice to compete for procurement projects. In the model, financial frictions distort the economy through different channels. First, for a given selection of firms into procurement, financial frictions lead to lower aggregate capital, misallocation of capital across firms, and misallocation of capital within firms across sectors. And second, financial frictions also distort the selection of firms into procurement, giving more value to firm net worth at the expense of firm productivity.

4.1 Technology

Time is discrete and we omit the subscript t unless it is strictly needed. The economy is populated by a continuum of size 1 of heterogeneous infinitely-lived households indexed by i. Each household is also an entrepreneur running a firm that produces a differentiated intermediate good y_i. There are two final goods in the economy: the "private sector" good, Y_p, used by households to consume, invest in productive capital, or prepare applications for procurement projects, and the "public sector" good Y_g, purchased by the government to produce (useless) public consumption.

Final goods. The two final goods are assembled by two final good producers combining the differentiated intermediate goods y_i through the following CES aggregators:

$$Y_p = \left(\int_{[0,1]} y_{ip}^{\frac{\sigma_p - 1}{\sigma_p}} \, di \right)^{\frac{\sigma_p}{\sigma_p - 1}} \quad \text{and} \quad Y_g = m_g^{\frac{1}{1-\sigma_g}} \left(\int_{I_g} y_{ig}^{\frac{\sigma_g - 1}{\sigma_g}} \, di \right)^{\frac{\sigma_g}{\sigma_g - 1}} \quad \text{with } \sigma_p, \sigma_g > 1 \tag{2}$$

where I_g is the subset of goods purchased by the public sector and m_g is the measure of

[14] Also, indirect inference motivated by our structural model, as seen in Section 5, provides evidence for procurement earnings relaxing firms' credit constraints.

this set. Note that Y_g is adjusted by m_g to prevent a love-for-variety effect.[15] We also note that I_g (and the implied m_g) is a policy variable and the identity of firms in this set is discussed below. The final goods producers are perfectly competitive and choose the optimal demand of intermediate goods y_{ip} and y_{ig}, respectively, to maximize profits taking intermediate good prices p_{ip} and p_{ig}, final good prices P_p and P_g, and the set I_g as given. We assume that firms compete independently in each sector and face the downward-sloping demands, $p_{ip} = B_p y_{ip}^{-1/\sigma_p}$ and $p_{ig} = B_g y_{ig}^{-1/\sigma_g}$, where for convenience we define $B_p \equiv P_p Y_p^{1/\sigma_p}$ and $B_g \equiv m_g^{-1/\sigma_g} P_g Y_g^{1/\sigma_g}$. The prices p_{ip} and p_{ig} faced by the private and public sector producers in the purchase of the same intermediate good i may differ because intermediate good i producer has monopoly power over its variety and may be selling different quantities to each market. Y_g is the demand of the public good from the government and is a policy variable in the model, while Y_p is the demand of the private good from the households and it is determined in equilibrium. The aggregate prices P_p and P_g of the private and public goods are given by the usual aggregators:

$$P_p = \left(\int_{[0,1]} p_{ip}^{1-\sigma_p} di \right)^{\frac{1}{1-\sigma_p}} \quad \text{and} \quad P_g = \left(\int_{I_g} \frac{1}{m_g} p_{ig}^{1-\sigma_g} di \right)^{\frac{1}{1-\sigma_g}} \tag{3}$$

We will use the final private good as the numeraire, so we set $P_p = 1$ in what follows.

Intermediate inputs. The intermediate inputs are produced by heterogeneous firms. At any period in time, these firms are characterized by their idiosyncratic stochastic productivity s_i, their capital stock k_i (which depreciates at rate δ), their debt level l_i (when $l_i > 0$ the firm is a net borrower), and whether they currently hold a procurement project $d_i = 1$ or not $d_i = 0$. Output y_i is given by a simple CRS production function, $y_i = f(s_i, k_i) = s_i k_i$, that depends on capital k_i and managerial productivity s_i.[16] The firm-specific s_i follows a first-order Markov process, specified in more detail below. If a procurement project is active ($d_i = 1$) a fraction of output u_i, chosen by the firm, is sold to the private sector and a fraction $1-u_i$ is sold to the public sector, otherwise all output is sold to the private sector. Our simple production function implies that u_i is also the fraction of capital used for the production of the private sector variety, that is, $k_{ip} = u_i k_i$.

4.2 Participation in public procurement

The government has control over the subset I_g of goods purchased by the public sector, and a choice of the subset I_g naturally implies its measure m_g. In order to introduce structure

[15] Governments purchase only a fraction of goods and services provided by the private economy mainly because their needs are different than the needs of private households and firms. By removing 'love-for-variety' we eliminate this trivial effect from the analysis of the effects of the number of contracts offered.

[16] In Section 8.2 we explore a version of our model with decreasing returns to scale in production.

in this choice, we consider that the government follows a simple stochastic rule for the allocation of procurement contracts based on the quality of the proposals. In particular, we assume that firms who wish to sell to the government next period ($d_{it+1} = 1$) must invest an amount of private sector good $b_{it} > 0$ today. This quantity may reflect the costs of learning how the process works, the actual costs of preparing a proposal, or the costs of establishing connections with government officials. There is always uncertainty in the outcome of the application, which reflects the fact that "equally capable" firms usually compete in the same auction with only one winner.[17]

The probability of being able to sell to the government next period depends on the amount invested,

$$Pr\left(d_{it+1} = 1 \mid b_{it}\right) = g\left(b_{it}\right) = 1 - e^{-\eta_0 b_{it}^{\eta_1}} \tag{4}$$

with $\eta_0 > 0$ and $1 > \eta_1 > 0$ to ensure positive and diminishing returns. Also notice that $\lim_{b \to 0} \frac{\partial g(b)}{\partial b} \to \infty$, so there will always be an interior solution in the optimal choice of b_{it}. This probability function captures in reduced form the competition for procurement projects. The slope parameter η_1 controls the extent of decreasing returns to investment b_{it}, and it hence regulates firm selection into procurement. With higher η_1, larger investments generate larger probabilities of winning a procurement project compared to smaller investments, thereby making firms that value a procurement project more (and hence invest a larger amount b_{it}) more likely to win the contract. The level parameter η_0 is an equilibrium object that ensures that the fraction of firms obtaining a procurement project equals the measure m_g of goods purchased by the public sector. Hence, the probability of procurement depends on firms' own actions through b_{it} as well as on the actions of all other firms through the equilibrium object η_0.[18] The winners of the competition for procurement form the set I_g in that period.

4.3 Entry and exit

A fraction $1 - \theta$ of households die every period and are replaced by the same number of new households running new firms. To avoid changing the composition of the goods produced in the economy, the entrant households produce the varieties left vacant by the exiting households. Dying households leave accidental bequests that for simplicity are taken by

[17] In practice, each government agency has its own criterion to decide the final ranking of firms, which may be based on a number of attributes as price, quality, and technical requirements. Therefore, firms always face uncertainty about how the public entity awarding the contract will perceive them and their competitors fulfilling these attributes.

[18] Alternatively, we could have followed a more structural approach in modelling the competition for public contracts. For instance, in a different setting, Michelacci and Pijoan-Mas (2012) model competition for jobs with a job finding probability depending on individual human capital relative to the average human capital of the economy. Yet, our formulation is flexible and does not require taking a stand on the complex procurement competition process.

the government. Entrant households start with a joint distribution of financial wealth and productivity Γ_0 and with no procurement project. The wealth of the entrants is provided by the government. Alternatively, we could have assumed that all accidental bequests go to the newborns, but we want to break this link in order to have the flexibility to choose the amount of financial wealth for entrants.

4.4 Preferences and constraints

Firms are owned by entrepreneurs. Entrepreneurs have CRRA preferences over consumption with curvature μ, and their objective is to maximize the discounted sum of utilities.[19] They obtain income only from their firm so their budget constraint is given by:

$$c_{it} + b_{it} + k_{it+1} - l_{it+1} \leq p_{ipt} y_{ipt} + p_{igt} y_{igt} + (1-\delta) k_{it} - (1+r_t) l_{it} - \text{tax}_{it} \tag{5}$$

where $\text{tax}_{it} = \tau \left[p_{ipt} y_{ipt} + p_{igt} y_{igt} - (r_t + \delta) k_{it} \right]$ denotes the proportional taxes on profits paid by entrepreneur i at time t. The tax function is purposely simple because we focus on expenditure-neutral counterfactuals.[20] As it is standard in the literature, we only allow for one-period debt contracts l_t that pay a risk-free interest rate r_t. The amount of debt is limited by the repayment capacity of the firm through a combination of earnings-based and asset-based collateral constraints. In particular, the amount of debt of a firm coming into $t+1$ is limited by:

$$l_{it+1} \leq \varphi_a \, k_{it+1} + \varphi_p \, p_{ipt+1} y_{ipt+1} + \varphi_g \, p_{igt+1} y_{igt+1} \tag{6}$$

If $\varphi_a = 0$, $\varphi_p = 0$, and $\varphi_g = 0$ no external finance is available and all production needs to be self-financed. With $\varphi_a > 0$ the firm can leverage up against fixed capital. With $\varphi_p > 0$ and $\varphi_g > 0$ firms can borrow against the earnings generated in the private and the public sector respectively.[21] There are no reasons to assume that φ_p and φ_g should be equal to each other. Through the lens of our model, we will later find that $\varphi_g > \varphi_p$. This may happen for several reasons that we do not explicitly model. For instance, it may indicate that governments pay with a higher probability than the average contract in the private sector. Having a government contract in hand could reduce uncertainty about a firm's near-future earnings, thus potentially lowering its risk of defaulting.

[19] Modeling firms as run by entrepreneurs with curvature in preferences over consumption could also be justified by firms' dividend-smoothing motives, as empirically documented, e.g. Leary and Michaely (2011).

[20] Note that this simple tax function allows for tax deductibility of depreciation δk_{it} and of the (interest) opportunity cost of capital $r_t k_{it}$. Since $l_{it} \leq k_{it}$ whenever the firm's net worth is non-negative, we are implicitly allowing the firm to deduct more than just the interest payments on debt $r_t l_{it}$.

[21] An alternative and more structural borrowing constraint would limit repayment $(1+r_{t+1}) l_{it+1}$ explicitly by a fraction of undepreciated capital $(1-\delta) k_{it+1}$ plus earnings: $(1+r_{t+1}) l_{it+1} \leq \tilde{\varphi}_a (1-\delta) k_{it+1} + \tilde{\varphi}_p \, p_{ipt+1} y_{ipt+1} + \tilde{\varphi}_g \, p_{igt+1} y_{igt+1}$. In steady state with constant r this specification would be equal to (6) with the redefinitions: $\varphi_a \equiv \frac{(1-\delta)\tilde{\varphi}_a}{1+r}$, $\varphi_p \equiv \frac{\tilde{\varphi}_p}{1+r}$, and $\varphi_g \equiv \frac{\tilde{\varphi}_g}{1+r}$. In counterfactual exercises, increases (decreases) in the equilibrium r would tighten (loosen) the borrowing constraints. Our formulation ignores this second-order effect.

4.5 Timing and state space

Regarding the non-procurement part of the model, we follow the timing convention commonly used in the firm dynamics literature. First, we assume that resources devoted to consumption are spent at the beginning of each period t. Second, we assume that production in $t+1$ is carried out using capital installed at the end of period t. Third, we assume that the household survival shock and the firms' $t+1$ productivity are revealed (in this order) before firms decide how much capital to install for the next period, k_{it+1}, and how much debt to issue for next period, l_{it+1}. Regarding the variables related to procurement, we follow a similar logic. The amount of resources devoted to increasing the probability of being active in procurement in $t+1$, i.e., b_{it}, is spent at the beginning of each period t. Whether or not the firm is successful and becomes active in procurement in $t+1$, i.e, $d_{it+1} = 1$, is revealed at the same time as $t+1$ productivity and right after the survival shock. This means that procurement applications of dying households are ignored by the government and hence dying households are not awarded a procurement project that cannot be delivered.

Alternative specification. These assumptions on timing simplify the state-space dimensionality of the problem. In particular, let $a_{it+1} \equiv k_{it+1} - l_{it+1}$ be the firm's net worth to be carried to the next period in units of private good today. Then we can redefine the budget constraint as

$$c_{it} + b_{it} + a_{it+1} \leq (1-\tau)\left[p_{ipt}y_{ipt} + p_{igt}y_{igt} - (r_t+\delta)k_{it}\right] + (1+r_t)a_{it} \tag{7}$$

The collateral constraint becomes

$$k_{it} \leq \phi_a a_{it} + \phi_p p_{ipt}y_{ipt} + \phi_g p_{igt}y_{igt} \tag{8}$$

where the parameters in the borrowing constraint are re-defined as:

$$\phi_a \equiv \frac{1}{1-\varphi_a} \in [1,\infty), \quad \phi_p \equiv \frac{\varphi_p}{1-\varphi_a} \in [0,\infty), \quad \phi_g \equiv \frac{\varphi_g}{1-\varphi_a} \in [0,\infty) \tag{9}$$

Hence, the production decisions (capital and sales composition) are intratemporal, while the accumulation of net worth and the investment in procurement are intertemporal. This allows to split the firm's problem in two: a *static production* problem and a *dynamic consumption-saving* problem. In the following, we describe them in turn. We present the full definition of the steady state equilibrium, including a discussion of the governments' budget constraint and its calibrated net financial position D in Appendix D.

4.6 The static production problem

The intratemporal production problem is characterized by firm productivity s, firm net worth a, and the availability of a procurement project d. For simplicity we drop the firm subindex i.

Firms with $d = 0$ only have to choose their optimal size k subject to the borrowing constraint, while firms with $d = 1$ also decide on the fraction of output $u \in [0,1]$ sold to the private sector. We can write the formal maximization problem for the firm of type $(s, a, d = 1)$ as:

$$\pi(s,a,1) = \max_{k,u} \{p_p y_p + p_g y_g - (r+\delta) k\}$$
$$\text{subject to:}$$
$$p_p y_p = B_p [\quad u \quad sk]^{\frac{\sigma_p - 1}{\sigma_p}}; \quad p_g y_g = B_g [(1-u) sk]^{\frac{\sigma_g - 1}{\sigma_g}}$$
$$k \in [0, \phi_a a + \phi_p p_p y_p + \phi_g p_g y_g]; \; u \in [0,1]$$

while for the firm of type $(s, a, d = 0)$ all the terms $p_g y_g$ trivially disappear and $u = 1$. Let λ be the multiplier of the intratemporal borrowing constraint and let's consider the general case with $d = 1$. The optimal choices are described by the following FOC:

$$(1+\lambda\phi_p)\frac{\partial p_p y_p}{\partial u} + (1+\lambda\phi_g)\frac{\partial p_g y_g}{\partial u} = 0 \tag{10}$$
$$(1+\lambda\phi_p)\frac{\partial p_p y_p}{\partial k} + (1+\lambda\phi_g)\frac{\partial p_g y_g}{\partial k} = r + \delta + \lambda \tag{11}$$
$$\lambda \geq 0, \; \phi_a a + \phi_p p_p y_p + \phi_g p_g y_g - k \geq 0, \; \lambda [\phi_a a + \phi_p p_p y_p + \phi_g p_g y_g - k] = 0 \tag{12}$$

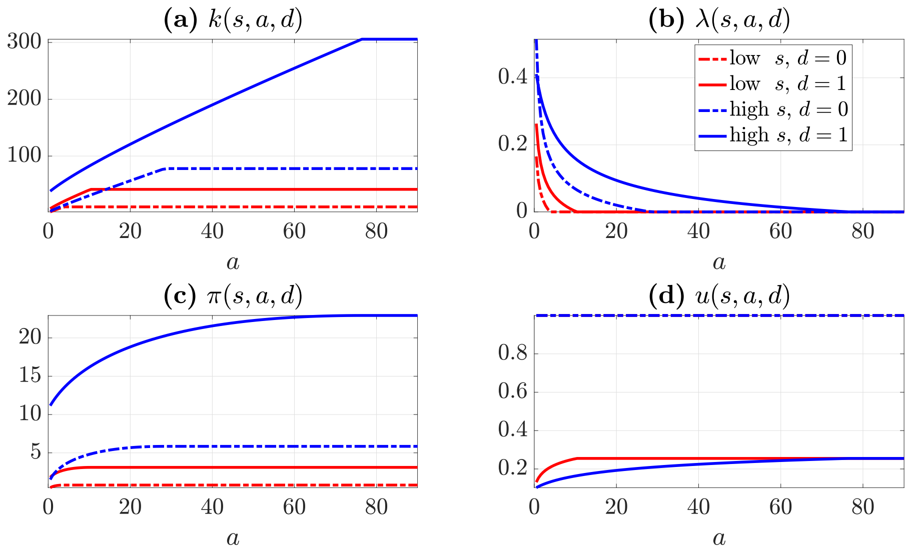
These optimality conditions show how financial frictions distort the two decisions faced by firms: production composition and firm size. Equation (10) characterizes the composition of sales. Let $k_p = uk$ and $k_g = (1-u)k$. A more intuitive way of writing equation (10) is:

$$\frac{\text{MRPK}_p}{\text{MRPK}_g} = \frac{1+\lambda\phi_g}{1+\lambda\phi_p}$$

where $\text{MRPK}_p \equiv \partial p_p y_p / \partial k_p$ and $\text{MRPK}_g \equiv \partial p_g y_g / \partial k_g$ are the marginal revenue products of each firm in each sector, see Appendix E for details. With $\lambda = 0$, the optimal choice requires the equalization of the marginal revenues obtained from each sector. Because of the concave revenue functions in both sectors, there is always an interior solution to this problem. With binding financial constraints ($\lambda > 0$), production is shifted towards the sector whose output can be better collateralized. For instance, if procurement contracts offer better collateral value than sales to the private sector ($\phi_g > \phi_p$) the optimal choice requires lower marginal revenues from public procurement relative to the private sector, which happens when production is shifted towards the public sector and away from the private sector. This generates misallocation of capital *within firms* and sectoral misallocation in the aggregate.

Equation (11) determines optimal firm size, which can be rewritten as

$$\text{MRPK} = u\left(\frac{r+\delta+\lambda}{1+\lambda\phi_p}\right) + (1-u)\left(\frac{r+\delta+\lambda}{1+\lambda\phi_g}\right)$$



Notes: This figure shows the solution to the firm's problem. Panel (a) shows the size of the firm represented by the amount of capital $k(s, a, d)$; Panel (b) shows the multiplier of the financial constraint $\lambda(s, a, d)$; Panel (c) shows the profits $\pi(s, a, d)$. All of them are plotted against firm's productivity s, for two different levels of net worth, and for the cases $d = 0$ and $d = 1$.

Figure 4. Solution of the static profit maximization problem

where MRPK $\equiv \partial \left(p_p y_p + p_g y_g\right) / \partial k$ is the marginal revenue product of each firm. With $\lambda = 0$ the optimal choice requires to equalize the marginal revenue product of capital to its cost, which is just $r + \delta$. With binding financial constraints ($\lambda > 0$), the effective cost of capital is $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ for sales to the private sector and $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$ for sales to the public sector. The multiplier of the financial constraint λ has two opposite effects on the cost of capital: on the one hand it increases the cost of capital as in standard asset-based financial constraints, but on the other hand, it decreases the cost of capital because a fraction of the generated output can also be collateralized. We will restrict ϕ_p and ϕ_g as indicated in Assumption 1 below to ensure that the earnings-based constraints cannot self-finance the optimal capital of the unconstrained problem, that is, to ensure that the financial constraints are binding for at least the entrepreneurs with zero net worth. Otherwise, all firms would be unconstrained, see Lemma 2 and Proposition 2 in Appendix E.

Assumption 1 *The model parameters satisfy the following boundary constraints:* $\phi_p < \frac{\sigma_p - 1}{\sigma_p} \left(r + \delta\right)^{-1}$ *and* $\phi_g < \frac{\sigma_g - 1}{\sigma_g} \left(r + \delta\right)^{-1}$.

Because $\sigma_p, \sigma_g > 1$, Assumption 1 imposes that the values of ϕ_p and ϕ_g are also below $\left(r + \delta\right)^{-1}$. This implies that the effective costs of capital for the private and public sector, $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$, are monotonically increasing in λ, reflecting the fact that financially constrained firms operate with less capital, see Lemma 1 and Proposition 1 in Appendix E. This has two consequences. First, to the extent that a significant fraction of firms are financially constrained, aggregate capital will be inefficiently low. Second, because λ depends on the firm state variables (s, a, d), there will be heterogeneity in the cost of capital across firms, which will generate misallocation of capital *across firms* and lower aggregate TFP in both the private sector and the procurement sector.

Static policy functions. In Appendix E we characterize analytically the solution to for both non-procurement ($d = 0$) and procurement firms ($d = 1$) whenever $\sigma_g = \sigma_p$. In Figure 4, we illustrate the numerical solution for both cases with the parameterization discussed in Section 5. Panels (a) and (d) show the optimal choices $k(s, a, d)$ and $u(s, a, d)$, respectively. Panels (b) and (c) show the associated shadow value of the financial constraint $\lambda(s, a, d)$ and a profit function $\pi(s, a, d)$, respectively.

First, as is common in standard models of firm dynamics with collateral constraints, constrained firms with no procurement see their capital and profits increase with net worth (while the shadow value of the borrowing constraints declines) until the point in which the financial constraints stop binding and net worth plays no role. Second, different from models with only asset-based collateral constraints, financially constrained firms without procurement increase capital and profits when productivity increases. This happens through the earnings-based constraint, which allows more productive firms to generate more revenues at the same level of net worth and hence expand production. Note also that more productive firms are more financially constrained at any level of net worth (their shadow value of the borrowing constraint is larger) because the expansion of borrowing possibilities with s is lower than the increase in the optimal size. Third, looking at firms with procurement, the fraction of output sold by constrained firms to the private sector is decreasing in productivity s and increasing in net worth a, which simply says that more financially constrained firms, conditional on participating, have a higher fraction of their capital allocated to the production of goods sold to the government. This last result is true under $\phi_g > \phi_p$ and it would be the opposite if $\phi_g < \phi_p$. Finally, note also that capital, profits, and the shadow value of the borrowing constraint for firms with procurement evolve with s and a as in the case without procurement.

4.7 A procurement shock

We can also analyze the static effect of a procurement shock by comparing the solutions of the $d = 1$ and $d = 0$ cases at any value of the state variables s and a, details are in Appendix E.4. For unconstrained firms, a procurement shock leaves operations in the private sector unchanged, $y_p(s, a, 1) = y_p(s, a, 0)$, increases firm size to serve the public demand, $k(s, a, 1) - k(s, a, 0) > 0$, and increases profits, $\pi(s, a, 1) - \pi(s, a, 0) > 0$. This is due to the constant returns to scale production assumption and the absence of adjustment costs. The profit gain from procurement $\pi(s,a,1) - \pi(s,a,0) > 0$ is independent from a and increases with s as more productive firms can deliver larger projects.

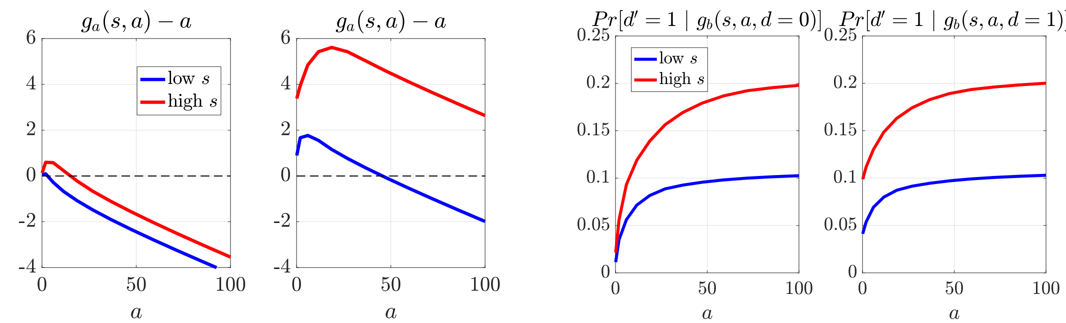
For constrained firms things are more complex. A procurement shock tightens the financial constraint whenever $\phi_g \leq \phi_p$. With $\phi_g = \phi_p$ this is because the firm with $d = 1$ has two demands to serve, which are equally pledgeable, and has the same net worth to finance capital in the two different markets. As a result the firm scales down the operations in the private sector to free up collateral for the production in the public sector, which generates a negative within-firm private sector spillover of the procurement contract, that is, $y_p(s,a,1) < y_p(s,a,0)$. When $\phi_g < \phi_p$ the financial situation is aggravated because the public sector demand can be self-financed to a lesser extent than the private sector one and the negative private sector spillover is larger. Instead, when $\phi_g > \phi_p$ public procurement may alleviate the firm financial situation because the public sector demand can be self-financed to a larger extent. However, this will only be relevant for firms with little or no wealth, which will be less constrained when obtaining a procurement project than they were originally and will use the extra financing capacity coming from the public sector to scale up operations in the private sector, generating a positive spillover. This is precisely stated in Proposition 13 in Appendix E.4. In our numerical exercises with $\phi_g > \phi_p$, with a realistic calibration, and endogenously accumulated net worth distributions, however, a procurement shock always increases firm size, makes the firm more constrained, and always generates a *negative* spillover on the private sector sales for constrained firms that obtain procurement. Finally, a procurement shock also increases profits among constrained firms. For the empirically relevant case $\phi_g > \phi_p$, this is more so for the more productive and the wealthier firms, because these two variables determine the capacity to deliver large projects. The only exception is for the firms with little or no wealth discussed above, in which case the increase of profits with procurement actually falls with net worth (Proposition 15 in Appendix E.4).

4.8 The dynamic problem

The *dynamic consumption-saving* problem can be written in recursive form:

$$
\begin{aligned}
&V(s,a,d) = \max_{c,b,a'} \left\{ u(c) + \beta\theta\, \mathbb{E}_{s',d'|s,b}\left[V(s',a',d')\right] \right\} \\
&\quad \text{subject to:} \\
&\quad \mathbb{E}_{s',d'|s,b}\left[V(s',a',d')\right] = g(b)\, \mathbb{E}_{s'|s} V(s',a',1) + (1 - g(b))\, \mathbb{E}_{s'|s} V(s',a',0) \\
&\quad c + b + a' = (1+r)\, a + (1-\tau)\, \pi(s,a,d) \quad \text{and} \quad a' \geq 0
\end{aligned}
$$

The first constraint says that the expected firm's value for tomorrow is an average of the firm's value under procurement, i.e., $d' = 1$, and no procurement, i.e., $d' = 0$, weighted by



Notes: The first and second panels show the net saving rules $g_a(s,a,d))$ for firms of different levels of productivity and net worth, both for non-procurement and procurement firms respectively. The third and fourth panels show the endogenous probability of obtaining procurement contracts evaluated at the optimal rules $g_b(s,a,d)$ for firms with different levels of productivity and net worth, both for non-procurement and procurement firms respectively.

Figure 5. Decision rules

the endogenous probability of procurement $g(b)$. This probability is why the expectations operator $\mathbb{E}$ depends on b in addition to s. The FOC for the choices of a' and b are:

$$u_c(c) \geq \beta\theta\,\mathbb{E}_{s',d'|s,b}\left[\left(1+r+(1-\tau)\frac{\partial\pi(s',a',d')}{\partial a'}\right)u_c(c')\right] \tag{13}$$

$$u_c(c) = \beta\theta\frac{\partial g(b)}{\partial b}\mathbb{E}_{s'|s}\left[V(s',a',1)-V(s',a',0)\right] \tag{14}$$

The first equation is the standard Euler equation that emerges in models of heterogeneous firms with financial constraints. In future states where the firm is not constrained in the static production problem $\partial\pi(s',a',d')/\partial a' = 0$ and the return to accumulate wealth is just r. Instead, in states where the firm is financially constrained in the static production problem there is an extra return $\partial\pi(s',a',d')/\partial a' = \phi_a\lambda(s',a',d') > 0$ above r to accumulating net worth, which is given by the increase in (after tax) profits due to relaxing the firm's collateral constraint, see Appendix E. The second equation determines the optimal spending in b: the entrepreneur will equalize its marginal utility of consumption to the marginal return of b, which is given by the expected increase of the firm's value coming from the possibility of selling to the government. Because of the properties of $g(b)$ and because $E_{s'|s}[V(s',a',1)-V(s',a',0)]>0$) the right-hand side declines with b.[22]

Decision rules. Figure 5 illustrates the net saving decision $a'-a$ of firms without and with procurement (first and second panel respectively). At low levels of net worth there is a hump-shaped relationship between net savings and net worth that is driven by the tradeoff between smoothing consumption vs. relaxing future borrowing constraints, a feature present in similar models like Midrigan and Xu (2014). At larger levels of wealth, the saving behavior follows the logic in Aiyagari (1994): net savings decrease monotonically with net worth and

[22] Proposition 15 in Appendix E shows that $\pi(s,a,1)-\pi(s,a,0)>0$, and $V(s,a,1)-V(s,a,0)$ inherits this property as d plays no other role than increasing profits π in the value function.

there is a target level of wealth that is larger for larger productivity s. This figure also shows large differences between procurement and non-procurement firms in terms of saving decisions. In particular, procurement firms save more conditional on their current net worth a and productivity shock s. This difference is driven by the fact that profits are higher for firms that are active in procurement, which relaxes their budget constraint and hence allows them to save more without sacrificing too much consumption.

Figure 5 also shows the function $g(b)$ evaluated at the actual choice of b for firms with different levels of net worth and productivity, both for non-procurement (third panel) and procurement firms (fourth panel). The first thing to notice is that high-net-worth firms invest more resources in increasing their probability of being able to sell to the government. This emerges as a result of an interesting trade-off: in the dynamic problem there are two competing options for saving and increasing future wealth. On the one hand, households can accumulate wealth a', which also relaxes the asset-based constraint and increases profits next period (right-hand side of Equation (13)). On the other hand, they can alternatively invest in applications for procurement projects that will increase expected future earnings, and potentially also relax the earnings-based constraint if $\phi_g > \phi_b$ (right-hand side of (14)). Appendix E shows that $\frac{\partial^2 \pi(s,a,d)}{\partial a^2} < 0$, which means that the return of accumulating net worth is lower for firms with more net worth (see Corollaries 3, 6, and 7). The profit premium of a procurement project $\pi(s,a,1) - \pi(s,a,0)$ increases with net worth for constrained firms, see Proposition 15 in Appendix E.4, and so does $V(s,a,1) - V(s,a,0)$, which means that the return of investment in procurement is larger for firms with more net worth. This higher profit premium for higher a happens because selling to the government does not relax the borrowing constraints completely, which means that firms still rely on their own assets for determining the size of their procurement contracts. This reflects a "size effect": the bigger the procurement projects the firm expects to be able to deliver, the higher the expected profits that participating in procurement generates. Therefore, we obtain the result that the investment in procurement projects increases with firm net worth.

The second thing to notice is that there are almost no differences in g between procurement and non-procurement firms for given s ad a. The reason is that, conditional on b, the probability of obtaining contracts tomorrow is independent from whether the firm is active in procurement today. However, we notice that procurement firms spend a bit more on b at low levels of a, which simply reflects that these firms have more available resources.

5 Calibration

The model period is one year. We classify the model parameters into four different blocks. The first block contains parameters related to preferences, technology, and productivity that we set to predetermined values. We calibrate the parameters in the other three blocks such that in equilibrium the model matches several moments measured in the Spanish economy. Our model matches well all the targeted moments. Panel A in Table 1 shows the definition of the parameters as well as their inferred values. Panel B shows the description of moments and their value in the data and in the model. In the remainder of this section, we discuss our calibration choices and identification.

Block #1: preferences and technology We set the relative risk aversion coefficient μ equal to 2, the CES elasticities σ_p and σ_g both equal to 3, the discount factor β equal to 0.94, and the annual depreciation rate δ equal to 0.10, which are all within the range of standard values in the literature. Finally, in this block we also include the parameters governing firms' idiosyncratic productivity. We assume that the log of a firm's productivity process s evolves over time according to an AR(1) process with Gaussian shocks and unconditional mean $\bar{s} \equiv \mathbb{E}[\log(s)]$. We set the autocorrelation coefficient ρ_s equal to 0.80 and the standard deviation of the innovations σ_s equal to 0.30, as estimated by Ruiz-García (2020) using the same dataset of firms. We discretize the process following the Rouwenhorst method, with $N_s = 5$ states. Our model matches quite well the moments that are usually used in the literature to pin down these two parameters. In our sample the one-year autocorrelation of firms' log sales is 0.89 (which compares to 0.82 implied by our model) and the standard deviation of firms' sales growth is 0.57 (which compares to 0.49 implied by our model).

Block #2: financial constraints. Our model contains three parameters governing firms' financial constraints: ϕ_a, ϕ_p, and ϕ_g. We choose a value of ϕ_a so that the model matches the credit-to-capital ratio observed in our micro-level data, 0.55. Regarding parameterizing ϕ_p and ϕ_g, we proceed as follows. Given the credit constraint in equation (6), and after dividing by k and taking first differences, changes in firms' leverage for constrained firms are given by:

$$\Delta\left(\frac{l_{it}}{k_{it}}\right) = \underbrace{\varphi_p}_{\beta_1} \Delta\left(\frac{p_{it}y_{it}}{k_{it}}\right) + \underbrace{(\varphi_g - \varphi_p)}_{\beta_2} \Delta\left(\frac{p_{igt}y_{igt}}{k_{it}}\right) \tag{15}$$

where l_{it}/k_{it} is the firms' leverage, i.e., total credit divided by fixed assets; $p_{it}y_{it}/k_{it}$ is the firms' total average product of capital, measured as total value added (minus wages) divided by total fixed assets; $p_{igt}y_{igt}/k_{it}$ is the firms' value added (minus wages) coming from selling to the government divided by the firm's total stock of capital. We can therefore recover φ_g

Panel A: parameters		(1) Value	Panel B: Moments	Data	Model
Block 1					
μ	CRRA coefficient	2.00			
σ_p	CES private sector	3.00			
σ_p	CES government	3.00	PREDETERMINED		
β	Discount factor	0.94			
δ	Depreciation rate	0.10			
ρ_s	AR(1) correlation	0.80			
σ_s	AR(1) variance	0.30			
Block 2				Data	Model
ϕ_a	borrowing const. (a)	2.81	Credit/K	0.55	0.55
ϕ_p	borrowing const. ($p_p y_p$)	0.27	reg. coefficient (φ_p)	0.30	0.30
ϕ_g	borrowing const. ($p_g y_g$)	0.41	reg. coefficient (φ_g)	0.22	0.22
Block 3					
η_0	probability function (level)	0.78	Consistency of $g(b)$ with m_g	–	
η_1	probability function (slope)	0.62	Procurement premium	0.72	0.71
Y_g	demand shifter	0.83	Share of procurement in GDP	0.12	0.12
m_g	measure of procurement goods	0.12	Percentage of procurement firms	12%	12%
Block 4					
D	Government lending	0.86	Interest rate	5%	5%
$\bar{s}$	Productivity shifter	-6.51	K/Y (aggregate)	3.88	3.84
θ	Survival probability	0.95	Exit rate	5%	5%

Notes: This table summarizes our baseline calibration. Government wealth D is expressed as a fraction of total credit in the model economy. In column (1), we show the parameter values in our baseline calibration.

Table 1. Calibration

and φ_p using indirect inference by setting the coefficients β_1 and β_2 from an OLS regression of equation (15) to be equal in the data and the model.[23] Table 2 presents the results from running the empirical counterpart of equation (15) for firms with at least one procurement contract in our sample, using the same sample that we use in Section 3. Columns (1) and (2) show the estimated coefficients when including and not including firm FEs, respectively. Because equation (15) should hold with equality only for firms whose financial constraint is binding, in columns (3) and (4), we run the same regression using a sample of firms that are likely to be financially constrained. In particular, we run the regressions for firms with leverage ratios above the median, which, through the lens of our model, is a strong proxy

[23]The construction of "output", i.e., $p_{it}y_{it}$ in the data requires some explanation. Following the recent literature on earnings-based constraints (e.g., Drechsel (2023)), we assume that the flow variable that firms can collateralize is EBITDA: sales net of overhead and labor costs, without subtracting investment, interest payments or taxes. Because we do not have labor in our model, that variable is equal to the firm's value added $p_{ipt}y_{ipt} + p_{igt}y_{igt}$. However, in the data, we compute the counterpart of that variable as: $p_{ipt}y_{ipt} + p_{igt}y_{igt} = \text{VA}_i - \text{wage bill}_i$. Because we do not observe the use of factors separately for what is used to deliver sales to the private vs. the government sector. To compute the value added generated by selling to the government, we assume that the intermediate goods and the labor share in total expenditure is constant within the firm, i.e., it does not change depending on whether the firm sells to the private sector or the government.

	All firms		> Median leverage	
	(1)	(2)	(3)	(4)
$\Delta p_{it}y_{it}/k_{it}$	0.208^a	0.211^a	0.303^a	0.292^a
	(0.001)	(0.001)	(0.001)	(0.001)
$\Delta p_{igt}y_{igt}/k_{it}$	0.182^a	0.161^a	0.229^b	0.191^b
	(0.055)	(0.052)	(0.100)	(0.089)
Observations	61,445	62,442	29,528	30,037
R-squared	0.238	0.084	0.296	0.118
Sector×year FE	Yes	Yes	Yes	Yes
Firm FE	Yes	No	Yes	No

Notes: This table presents results from estimating the relationship between the change in firm's leverage and the change in its average product of capital and the change in its earnings coming from selling to the government divided by the firm's total stock of capital. Regression (15) is estimated with firms obtaining at least one procurement project over 2000-13 using annual data. Standard errors are clustered at the firm level; ^{a} indicates significance at the 1% level, ^{b} at the 5% level, and ^{c} at the 10% level.

Table 2. Change in Leverage and Procurement

for whether the firm is financially constrained. We find the same two results across all the specifications. First, the estimated coefficient β_1 is positive, which is evidence of earnings-based borrowing constraints ($\varphi_p > 0$). Second, the estimated coefficient β_2 is also positive, which is evidence that revenues from the public sector can be pledged to a higher extent than earnings from the private sector ($\varphi_g > \varphi_p$).

We take column (4) as our preferred specification. This delivers estimates $\beta_1 = 0.292$ and $\beta_2 = 0.191$. For the model to generate these regression coefficients and a credit-to-capital ratio of 0.55 we need $\varphi_p = 0.10$, $\varphi_g = 0.15$, and $\varphi_a = 0.64$. Hence, firms can pledge 10% of their annual earnings from selling to the private sector, 15% of their annual earnings from selling to the government, and 64% of their capital stock. Using equation (9), these numbers translate into $\phi_p = 0.27$, $\phi_g = 0.41$, and $\phi_a = 2.81$, which means that firms can increase their capital by 27% of their annual earnings in the private sector and by 41% of their annual earnings from selling to the government.[24] This is the result of a multiplier effect: firms can borrow against their revenues, allowing them to buy more capital, which can be partly collateralized to obtain further credit. This is an important interaction and captures how earnings-based constraints affect a firm's ability to grow also depends on the value of φ_a.

Block #3: participation and size of procurement. There are four parameters driving the size and composition of procurement: Y_g, m_g, η_0, and η_1. The parameters Y_g and m_g are policy parameters governing the relative size of procurement in the economy and the fraction of goods bought by the government. To capture the size of procurement

[24]We note that both ϕ_p and ϕ_g satisfy Assumption 1, which means that capital cannot be self-financed through the earnings-based constraints, see Lemma 2 in Appendix E.

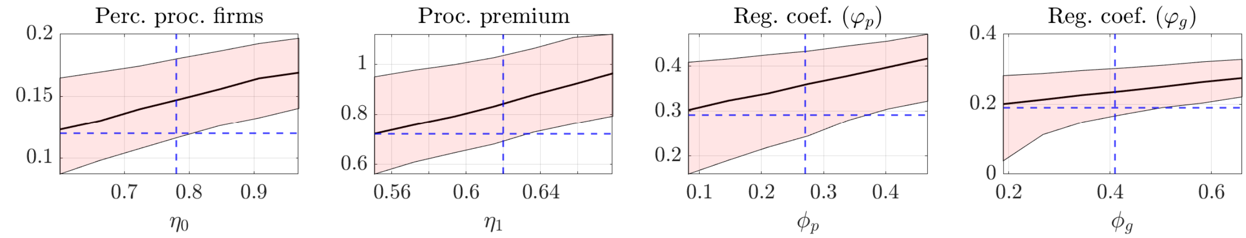
in the aggregate economy, we set Y_g to match the share of procurement in GDP equal to 12.1%, which is the value we measure in the Spanish national accounts in the year 2006. To capture the number of firms that sell to the government, we proceed as follows. To have a comprehensive estimate of the actual number of firms active in procurement, we make use of a recently available dataset provided by the Spanish Government, *Plataforma de Contratación del Sector Público (PLACSP)*, where all types of public agencies in Spain are required to upload their procurement activity. Although the platform includes information since 2013, compliance was only strong starting in March 2018, when introducing information on all contracts became compulsory by law.[25] Using information for the year 2019 (when the coverage is already high and before COVID), we measure around 75,000 firms with at least one procurement contract in the platform, which represents around 12% of the total number of active firms in the data. In our baseline calibration, we thus set m_g equal to 12%.

We calibrate the level parameter η_0 to ensure that the fraction of firms doing procurement equals the fraction of goods bought by the government, m_g, which is the equilibrium condition (e) (see Appendix D). Regarding the curvature parameter η_1, we identify it by matching the selection pattern of firms into procurement observed in the data. We proceed as follows. In the data, we select firms with no procurement contracts between 1999 and 2005. Then, we classify as procurement firms those firms that obtain at least one contract in 2006. We define the "procurement premium" as the relative difference in size (measured by value added) between procurement and non-procurement firms in 2005 (only exploiting variation across firms within the same 4-digit industry). That is, we want the model to match the ex-ante difference in size between procurement and non-procurement firms. We measure this procurement premium to be 72%.

The intuition why the parameter η_1 affects the selection of firms into procurement is as follows. When η_1 approaches zero, the probability function $g(b)$ exhibits strong diminishing marginal returns in b: the marginal increase in probability falls quickly as firms invest more. This makes differences in b across firms inconsequential for their probability of selling to the government, and hence generates very little selection, with complete randomness in allocation when $\eta_1 = 0$. Conversely, when η_1 approaches 1, the diminishing marginal returns are small: the marginal increase in probability falls slowly with b as firms invest more. This implies that differences in b translate into big differences in the probability of participating in procurement, which generates a strong selection pattern.

Block #4: rest of the parameters. We calibrate the survival probability $\theta = 0.95$ to the firms' exit rate in Spain of 0.05. We use firms' average productivity level $\bar{s}$ to match the

[25] In particular, that law ("Ley 9/2017, de 8 de noviembre, de Contratos del Sector Público") was aimed to adopt the European Parliament directives 2014/23/UE and 2014/24/UE.



Notes: This figure plots the relationship between percentiles of a given moment (vertical axis) and a specific parameter (horizontal axis). The thick black line refers to the median, and the thinner black lines above and below refer to the 85th and 15th percentile respectively. The horizontal dashed blue line represents the targeted value of the moment while the vertical one represents the actual calibrated parameter value. These lines have been generated by solving the steady-state of the model using combinations of parameters drawn from a 8-dimensional hypercube.

Figure 6. Identification of the main parameters

capital-to-output ratio observed in our firm-level data, which is 3.88. This delivers $\bar{s} = -6.51$. The reason why this moment is informative of the average productivity in the economy has to do with our AK assumption on firms' technology. Finally, we target an equilibrium interest rate equal to 5%. To obtain this, we use government's wealth D and obtain that the amount of government lending represents around 86% of the total credit in the economy.

Identification of the main parameters. Our calibration strategy consists of choosing values for 10 parameters so that the model matches 10 moments from the data. Two of these parameters (θ and m_g) are set without the need to solve the model, which leaves 8 parameters to be chosen by a simulated method of moments. In the previous sub-sections we have discussed the intuition for identification of each parameter. In this section we provide a more formal argument about the *global* identification of our parameters. To do so, we proceed as follows. First, we draw a large random sample of parameter combinations based on a 8-dimensional hypercube. Second, for each vector or parameters, we solve for the model's steady-state equilibrium and calculate the relevant moments. Third, we plot how the 25th percentile, the 50th, and the 75th percentile of a given moment changes as we move along the values of any given parameter. This figure shows how a particular moment is affected by a specific parameter letting other parameters vary randomly. The steeper the slope of the relationship between the parameter values and percentiles of the moment, the stronger the identification.[26]

In Figure 6, we show these relationships for the main parameters of our model, η_0, η_1, φ_p, and φ_g, together with the value of the targeted moment in the data (horizontal dashed blue line). The figure shows that the fraction of firms selling to the government, the procurement premium, and the two estimated coefficients in the leverage regression,

[26]Note that this is different from the matrix with the derivatives of the model-generated moments with respect to the calibrated parameters at some given point in the parameter space, which gives information about *local* identification.

Equation (15), are very informative about their associated parameters. In Appendix F, we show these relationships for all possible combinations of moments and parameters.

6 The Benchmark Economy

This section describes three relevant dimensions of our benchmark economy: the selection pattern of firms into procurement, the treatment effect of a procurement shock on firm dynamics, and the macroeconomic consequences of procurement.

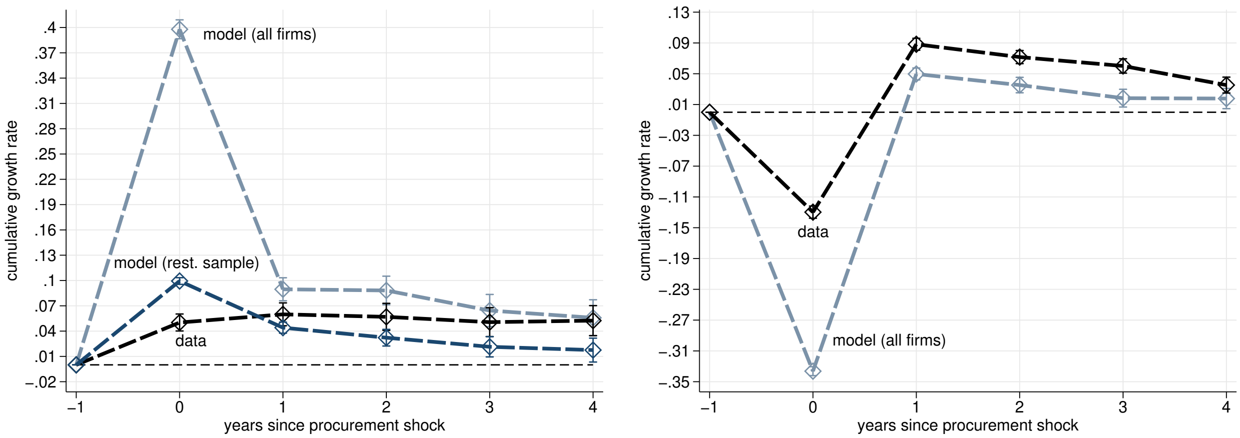
6.1 Selection

In our calibration, we use η_1 to target one moment related to selection: the "procurement premium" in value added of 72% (the relative value added of procurement vs. non-procurement firms before they obtain a procurement project). As discussed in Section 4.6, the value of procurement, $V(s, a, d = 1) - V(s, a, d = 0)$, is increasing in firms' ability to deliver large projects, which is determined by productivity s for unconstrained firms and also by net worth a for constrained firms. Hence, in the model firms self-select into procurement based on their productivity s and their net worth a. When we compute the "procurement premium" for s and a we find that procurement firms are ex-ante 35% more productive and hold ex-ante 44% more net wealth. These premia will be affected in our counterfactuals, which will be important to understand the aggregate consequences of changing the allocation of procurement to firms.

6.2 Treatment and other un-targeted moments

In our calibration, we use ϕ_g to target one moment related to the treatment effect of procurement: the change in leverage resulting from increasing sales to the government for firms likely to be financially constrained. What we do next is to examine the model-predicted dynamic responses of credit and sales after a procurement shock, and compare them to the evidence presented in Section 3. To do so, we use model-simulated data to estimate the same local projection regressions that we run in the data. Importantly, none of this evidence was targeted in our calibration.

Credit. In Panel (a) of Figure 7, we show the estimated cumulative effects on credit of a procurement shock for different time horizons, both in the model and in the data. The first thing to note is that, when using "all firms" to run the regression in the model, the implied effect on impact is distinctly higher compared to its counterpart in the data (~ 39% vs. ~ 5%). Starting in the first year after the procurement shock, however, the cumulative effects



(a) credit

(b) sales to the private sector

Notes: This figure shows the cumulative impact of the estimate of β_2^h (and its associated 10% confidence bands) from regression (1) for different time horizons $h = 0, 1, 2, 3, 4$. Panel (a) shows the results for the case of x being firms' total sales. Panel (b) shows the results for the case of x being firms' sales to the private sector. The line "data" shows the estimated coefficients using the real data, i.e., the same results as in panel (a) of Figure 1. The line "model (all firms)" shows the results when using all firms in our simulated sample. The line "model (rest. sample)" shows the results when restricting the simulated sample to observations where credit growth is below the top 10% and above the bottom 10%.

Figure 7. Procurement effect on credit and private sales

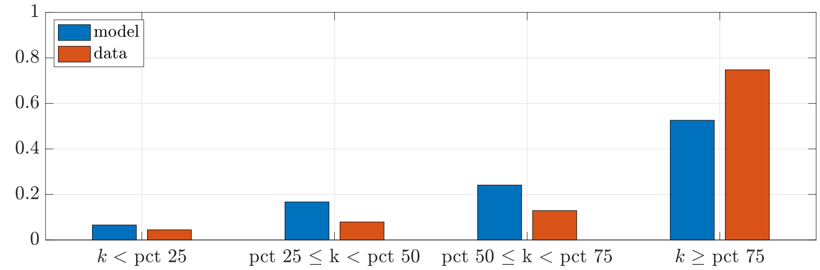
in the model and the data become similar, converging at somewhere around 5% in the year 3 after the shock. In that respect, both in the model and in the data, we observe a similar long-lasting effect of procurement on firm credit.

We note that the sharp peak observed at $h = 0$ is mostly driven by a set of very few unconstrained firms that dramatically increase credit to expand their operations at the time of the shock, and decrease it afterward. That is, procurement represents a large demand shock for these firms, and by the timing assumptions in our model, the only source to finance the firm's capital expansion is debt. For example, within the set of firms that are unconstrained throughout the simulated sample, 25% of them have their credit increased by 4 times after receiving a procurement shock. These unconstrained firms operate with a capital stock that is very similar to their net worth (with credit being close to zero). When they get a procurement contract, their optimal capital stock increase at impact leads to very large percentage changes in credit. On the other hand, for constrained firms, the highest increase in credit observed in our model-simulated sample is around 85%. Our model performs significantly better once we remove the extremely responsive firms from our simulated sample. For example, the increase in credit at $h = 0$ becomes around 9.5% once we remove the bottom and top 10% observations in terms of the change in credit. In reality, the unconstrained firms have access to other sources of funds, such as drawing down their cash positions or retaining more earnings (paying fewer dividends), so one should not expect to see such a large unconditional spike in the data. Given that the focus of our study is on the interplay between financial frictions (and thus, financially constrained firms) and public procurement, most importantly the dynamic "crowding-in" benefits of winning public procurement contracts, we take the results in Panel (a) at horizons $h \geq 1$ as evidence that the model performs remarkably well in this regard.

Sales to the private sector. We next look at the evolution of firms' sales to the private sector in Panel (b) of Figure 7. Similar to the data, our model predicts a "crowding-out" effect on impact and a "crowding-in" effect from $h = 1$ onwards. As discussed in Section 4.6, this is because at impact constrained firms have to split their scarce collateral between the two sectors (a financial problem that is only partially alleviated by the extra credit generated by selling to the government). However, the new profits generated from procurement allow the firm to accumulate more net worth over time. This higher level of net worth will ease the firm's financial constraint (lowering λ) and hence allow it to increase output in the private sector in the subsequent periods. Similarly to the effect of procurement on credit, our model overpredicts the "crowding-out" effect at impact (~35% in the model vs. ~13% in the data) but does a better job in replicating the "crowding-in" effect from $h = 1$ onwards (firms increase sales to the private sector by around 8% at $h = 1$ and around 2-3% at $h = 4$ both in the model and the data).

Endogenous persistence in sales to the government. We also find that the procurement contract has a persistent positive effect on b, which is the result of an increase in the firm's net worth. In particular, we find an increase of around 8, 6, and 3% estimated for one, two, and three years, respectively, after the firm obtains a contract (see Figure A.XV in Appendix G). This result identifies a channel through which the model endogenously generates some persistence in firms' participation in procurement. To have a sense of how strong this channel is as compared to the data, we compute the probability of obtaining a contract over the next three years, i.e., $t + 1$, $t + 2$, or $t + 3$, conditional on a firm having a contract in t. In the data, this number is quite high, 75%, which compares to its model counterpart of 42%. That is, our model generates more than one-half of the persistence observed in the data.

Distribution of government purchases across firms. Finally, we also show how the model distribution of procurement sales across firms compares to its counterpart in the data. As explained above, firms with higher s and a will self-select into procurement. Although we control this selection pattern by calibrating η_1 to the observed procurement premium, we do not explicitly target any moment of the distribution of firms' sales to the government.



Notes: This figure shows the share of government purchases accounted for by procurement firms, i.e., firms with at least one procurement contract, falling into different parts of the distribution of k. In the data, we have used data for the year 2006. Very similar patterns are observed in different years.

Figure 8. Distribution of government purchases across procurement firms

To see how well the model does along this dimension, we look at the share of government purchases accounted for by firms of different sizes as measured by their levels of assets k. In Figure 8, we compute the share of procurement sales accounted for by firms in different size groups, based on k. First, the model captures well the fact that bigger firms account for higher shares of government purchases, which is a combination of these firms selecting more into procurement and getting bigger contracts. Second, the model does an overall good job of generating the shares that we observe in the data, especially so given our admittedly simple procurement allocation system. That being said, the model falls somewhat short of the large concentration of procurement by the biggest firms. In the data, procurement firms above the 75th percentile in the k distribution account for 74% of total government purchases, which compares to 53% in the model.

6.3 The macroeconomy

Aggregating output over firms we get that $Y_p = \text{TFP}_p K_p$ and $Y_g = \text{TFP}_g K_g$, where TFP_p and TFP_g are weighted averages of firm-level productivities s and distortions λ, see Appendix H for details. We can define GDP in units of the private sector good as $Y \equiv Y_p + P_g Y_g$, which leads to $Y = \text{TFP}K$, where aggregate capital is $K \equiv K_p + K_g$ and aggregate productivity is $\text{TFP} \equiv \frac{K_p}{K}\text{TFP}_p + \frac{K_g}{K} P_g \text{TFP}_g$. In column (1) of Table 3 we report all these objects for our benchmark economy.

Productivity. We find significant differences in aggregate productivity across the two sectors: TFP in the procurement sector is 18.6% higher than in the private sector (0.306 vs. 0.258). The difference in sectoral TFP can arise for two reasons: (a) selection of firms into procurement based on s and (b) different misallocation of capital across firms in the production of goods in each sector (see Appendix H for details). In our economy, the main reason for the higher productivity in the public sector is the selection of more productive firms into procurement, which more than offsets the slightly higher misallocation of capital in the production of procurement goods. Indeed, absent misallocation, TFP in the procurement sector would be 19.6% higher than in the private sector.

The selection of more productive firms into procurement is due to their ability to deliver larger projects. In particular, the average s is almost 30% higher in procurement. The higher misallocation in procurement arises due to the fact that firms producing for both the government and the public sector are *ex-post* more financially constrained (the average λ is twice as high for procurement firms). This is the result of procurement firms facing a much larger demand, which is harder to finance despite the selection of wealthier firms into procurement (the average a is around 80% larger for procurement firms) and despite $\phi_g > \phi_p$. A way to quantify the extent of misallocation in each sector is by looking at the variance in the log MRPK across firms, which is around 40% higher in the procurement sector (0.045 vs. 0.033). As a result, the aggregate TFP gains of equalizing MRPK across firms are 6.6% in procurement and 5.7% in the private sector.

Relative prices. We next look at the price of public goods relative to private goods, which will be important to understand some of the results from the policy counterfactuals. The relative price can be written as:

$$\frac{P_g}{P_p} = \frac{\overline{\text{MRPK}}_g}{\overline{\text{MRPK}}_p} \frac{\text{TFP}_p}{\text{TFP}_g} \tag{16}$$

where $\overline{\text{MRPK}}_g$, $\overline{\text{MRPK}}_p$, TFP_g, and TFP_p are the weighted average marginal revenue products and sectorial TFP's (see Appendix H for derivations). As in standard multi-sector models, the ratio of relative prices is inversely related to sectorial TFPs. However, equation (16) also implies that the relative price is positively related to the ratio of average marginal revenue products in each sector. That is, a relatively high sectorial "wedge" —arising from a suboptimal allocation of capital into the sector— is associated with a higher relative price. Because firms active in procurement are on average more financially constrained, i.e., have a higher λ, the average wedge in the procurement sector is higher. In particular, in our benchmark economy, $\overline{\text{MRPK}}_g$ is around 7% higher than $\overline{\text{MRPK}}_p$. However, as mentioned above, TFP_g is 21% higher than TFP_p. All together the relative price of public goods is $P_g/P_p = 0.901$.

7 Policy Experiments

Our empirical evidence in Section 3 and the quantitative results in Section 6 show that procurement contracts help firms grow out of their financial constraints. At the same time, in Section 5 we have seen that smaller firms, typically the most constrained, participate less in procurement. This suggests that making procurement contracts available to smaller firms may lead to aggregate output gains. For this reason, in this section we quantify the aggregate effects of reforming the public procurement allocation system through *expenditure-neutral* changes that favor small firms. Recall that aggregate GDP (in units of the private good) is $Y \equiv Y_p + P_g Y_g$. As mentioned above, the counterfactuals we run are *expenditure-neutral*, meaning that $P_g Y_g$ will remain constant by construction. That is, changes in Y_p will drive all the changes in GDP. However, although $P_g Y_g$ remains constant, Y_g and P_g are allowed to change. Because we also want to study the impact of the reform on the "real" provision of public goods, we use an alternative definition of GDP where we keep P_g constant at its level in the benchmark calibration. We refer to this alternative measure as "real" GDP, indicating that we measure it at relative prices from the baseline economy.

7.1 Counterfactual 1: Targeting the selection pattern

We first run an experiment that encourages the participation of smaller firms in procurement. This counterfactual aims to reproduce the type of "set-aside" policies for small businesses implemented, for example, by the U.S. Small Business Administration. Policies of this type could be those that facilitate access to the competition for procurement contracts —like better publicity or direct assistance to prepare the application— or measures to provide more transparency of the whole process.

To evaluate this type of policies, we reduce the parameter η_1 so that the model generates a procurement size premium of 50%, as opposed to the 72% in the baseline calibration. That is, we solve for a new economy in which the procurement system gives relatively lower weight to firms' investment in b, making it easier for small firms to participate. We also change η_0 and Y_g so that the fraction of procurement firms m_g and total government expenditure $P_g Y_g$ remain unchanged.

We present the main results from this exercise in column (2) of Table 3, which shows the relative change of some relevant variables compared to their counterparts in the benchmark economy.[27] As mentioned before, we report the two measures of GDP: "nominal" GDP, which uses the relative price of procurement P_g in the reformed economy, and "real" GDP, which keeps the price P_g of the benchmark economy. We also report changes in the levels of capital in the two sectors and the aggregate, together with changes in variables related to misallocation and aggregate TFP.

Aggregate output. We find that the reform increases nominal GDP by around 1.07%.[28]

[27] We report the difference, not the relative change, for variables that are already in shares, i.e., the percentage of procurement firms and the share of procurement in GDP, as well as for the interest rate r, the tax τ, and the parameters η_0 and η_1.

[28] This compares to a 12% GDP increase from eliminating financial frictions in our economy ($\phi_a \to \infty$).

Since we keep $P_g Y_g$ constant in our experiment, this increase comes entirely from a 1.21% increase in Y_p. Importantly, we find only a tiny increase in "real" GDP of 0.05%. The reason is that, due to the policy experiment, the relative price of procurement P_g increases (making the provision of public goods more costly), and hence the total amount of public sector goods Y_g falls. The fall in Y_g is apparent in the real measure of GDP growth but not in the nominal (as nominal expenditure remains constant). We will come back to the increase in P_g below.

TFP vs. K. We can decompose the increase in "nominal" GDP into that coming from capital accumulation vs. TFP. We find that most of the increase in GDP (around 75%) is accounted for by an increase in the aggregate stock of capital K, which increases by 0.78%. In Section 7.3 below, we will provide more details that will help understand the evolution of the stock of capital. The rest is explained by an increase of 0.28% in aggregate "nominal" TFP, which is the result of a slight increase in TFP_p (0.09%), a large reduction in TFP_g (6.85%), and the above-mentioned increase in P_g. When keeping constant P_g to its value in the benchmark economy, our model predicts a decrease in "real" TFP (3.49%) due to the large fall in TFP_g.[29]

Why do TFP_p and TFP_g change? The increase in TFP_p is the result of the beneficial effects of procurement on wealth accumulation of small firms. In the new steady state, firms that had a relatively high MRPK_{ip} in the benchmark economy are more likely to be active in procurement, which allows them to accumulate more assets and hence operate with a higher level of capital in the private sector (see Section 6). This reallocation of procurement contracts towards relatively high MRPK_{ip} firms implies a reduction in the dispersion of MRPK_{ip} and hence in misallocation.

The decrease in TFP_g is explained by a change in the selection of firms into procurement and hence by the change in the composition of procurement firms. In particular, procurement firms in this counterfactual economy have lower relative productivity s (7.98% less) and net worth a (14.78% less). As a result, procurement firms are relatively less productive and relatively more constrained (as reflected by a larger λ). This leads to the decline in TFP_g due to both lower s (lower first-best productivity) and lower a (more misallocation).

A reduction in the efficiency of public goods provision: Why does P_g increase? As explained above, the relative price of public goods depends on the ratio of sectorial wedges times the inverse of relative sectorial TFPs, see equation (16). We have just seen that the ratio $\text{TFP}_p/\text{TFP}_g$ increases substantially, first and foremost because selection along productivity s into procurement worsens. Moreover, because the counterfactual procurement system also selects firms that have relatively lower net worth a, and are thus more financially constrained, all else equal, it leads to a relative increase in the procurement sectorial wedge,

[29] We derive aggregate TFP in Appendix H.

as captured by $\overline{\text{MRPK}}_g$. Both of these forces raise P_g. This is an important result that highlights an interesting tradeoff for the government: by targeting small firms, governments also end up buying from firms that are small not because they are constrained but because they are fundamentally less productive. This fact compounds the force that buying from a relatively more constrained firm, even at identical productivities, implies a restricted supplied quantity and higher price. Under constant government expenditures, all of this necessarily implies an underlying lower provision of public goods.

7.2 Counterfactual 2: Decreasing contract size

Our policy experiment in the previous section is only one possible way for governments to promote the participation of small firms into procurement. An alternative policy would be to decrease the size of procurement contracts, which is at the core of the European Commission's agenda for public procurement regulation. We therefore perform a second experiment that consists of reducing the size of contracts while keeping the same level of expenditure P_gY_g. In practice, we solve for a counterfactual economy in which the fraction of firms from which the government buys, m_g, increases from 12% to 20%.[30] We do so by increasing η_0 and adjusting Y_g so that P_gY_g remains unchanged. We also keep η_1 unchanged.

We show the results from running this policy experiment in column (3) of Table 3. In contrast to the previous counterfactual, we find that the reform reduces nominal GDP by around 0.85%. Out of this decline, around 70% is explained by a fall in aggregate capital and the rest by a decline in aggregate TFP. As in the previous counterfactual, we will explain the behavior of capital accumulation below (see Section 7.3).

Also different from the first counterfactual, P_g remains virtually unchanged. As in the previous counterfactual, $\text{TFP}_p/\text{TFP}_g$ goes up, which should raise P_g. However, the average wedge in the procurement sector $\overline{\text{MRPK}}_g$ decreases by a similar amount, 3.10%. As in the previous counterfactual, the policy allocates procurement contracts to smaller firms, which leads to a pool of procurement firms that are both less productive and have less net worth. However, different from the previous counterfactual this policy also reduces the average contract size. This makes procurement firms relatively less constrained (instead of relatively more constrained as in the previous counterfactual) because they have a smaller demand to serve (the average λ of procurement firms declines by 20%) and hence the wedge in the procurement sector is smaller.

[30]This represents a large change in the average size of contracts. In the counterfactual economy, the average size of the contract is 60% of that in the benchmark economy. The European Commission is not explicit about by how much governments should decrease the size of the contracts: "[...] Such division could be done on a quantitative basis, making the size of the individual contracts better correspond to the capacity of SMEs [...]." (see the Public Sector Directive 2014/24/EU for details.)

	(1) Benchmark (levels)	(2) Count 1 % change	(3) Count 2 % change
Output			
Y_p	6.035	1.21%	-0.97%
Y_g	0.904	-8.55%	0.05%
GDP	6.850	1.07%	-0.85%
real GDP	6.850	0.05%	-0.85%
Capital			
K_p	23.395	1.11%	-1.07%
K_g	2.956	-1.82%	3.24%
$K_p + K_g$	26.351	0.78%	-0.59%
Productivity			
TFP_p	0.258	0.09%	0.10%
TFP_g	0.306	-6.85%	-3.09%
TFP	0.260	0.28%	-0.27%
real TFP	0.534	-3.49%	-1.55%
$\overline{\text{MRPK}_p}$	0.258	0.07%	0.05%
$\overline{\text{MRPK}_g}$	0.276	1.89%	-3.10%
TFP_p gain	0.057	-1.76%	-1.87%
TFP_g gain	0.066	9.14%	-12.09%
Prices/tax			
P_g/P_p	0.901	9.38%	-0.01%
r	0.050	0.0010	0.0001
τ	-0.079	0.103	-0.032
Size of procurement			
% firms	0.120	0.000	0.080
Share GDP	0.119	-0.001	0.001
η_0	0.783	-0.212	0.563
η_1	0.624	-0.128	0.000
Selection			
ratio mean s	1.289	-7.98%	-2.70%
ratio mean a	1.776	-14.78%	-8.52%
ratio mean λ	2.082	12.32%	-20.13%

Notes: Column (1) shows the main results from the benchmark economy. Column (2) shows the results from running counterfactual 1, which consists of changing η_0, η_1, and Y_g so that the procurement size premium decreases from 72% to 50%, while keeping unchanged the % of procurement firms and the procurement expenditure P_gY_g. Column (3) shows the results from running counterfactual 2, which consists of changing η_0 and Y_g so that the model generates an increase of procurement firms from 12% to 20% while keeping unchanged the procurement expenditure P_gY_g, which decreases the average size of contracts accordingly.

Table 3. Counterfactuals

	(0)	(1)	(2)
	Benchmark (levels)	**Long-run PE** (% change)	**Full GE** (% change)
Panel A: Count. 1			
Y_p	6.035	3.51%	1.21%
K	26.351	3.02%	0.78%
GDP	6.850	3.08%	1.07%
r	0.050	0.0000	0.0010
Panel B: Count. 2			
Y_p	6.035	0.47%	-0.97%
K	26.351	0.77%	-0.59%
GDP	6.850	0.42%	-0.85%
r	0.050	0.0000	0.0001

Notes: This table shows the results from running different versions of our model. Columns (0) and (2) show the value of the variables both in our benchmark economy and in the new steady state. Column (1) refers to the "Long-run partial equilibrium effect." Importantly, in column (1), we solve the model by using the η_0 and η_1 that we use to compute the new steady states, and adjust Y_g so that P_gY_g remains unchanged.

Table 4. Channels

7.3 Understanding the effects on capital accumulation

The main factor driving changes in aggregate GDP in the two experiments is capital accumulation. In the first counterfactual, a higher K explains around 73% of the rise in GDP. In the second counterfactual, a lower K explains around 70% of the fall in GDP. To provide more details on the specific mechanisms driving changes in K, we solve an intermediate version of the two counterfactual economies that aims to isolate the different channels at play. Table 4 again reports statistics of our benchmark economy in column (0), of this intermediate exercise in column (1), and the complete counterfactual economy in column (2).

Long-run partial equilibrium effect. Financially constrained firms that get procurement projects increase their revenues and can accumulate net worth at a faster pace and hence increase their private sector activity in the long run. In the intermediate step ("Long-run PE"), we isolate the aggregate effects of this strengthening of the self-financing mechanism. To do so, we solve for a new steady state partial equilibrium economy under the new η_0 and η_1, adjusting Y_g so that P_gY_g remains unchanged, keeping the same r and τ as in the benchmark, and imposing that the policy functions of the dynamic problem $c(s,a,d)$, $a'(s,a,d)$ and $b(s,a,d)$ remain unchanged (as a ratio of the entrepreneurs' cash on hand $(1+r)a+(1-\tau)\pi(s,a,d)$). This economy delivers a joint distribution Γ of firms over the state space $\mathbf{X}$ that is different from the benchmark. Our goal is to quantify the mechanical accumulation effect that income from procurement generates on directly affected firms, without taking into account adjustments in dynamic decisions or GE effects. Column

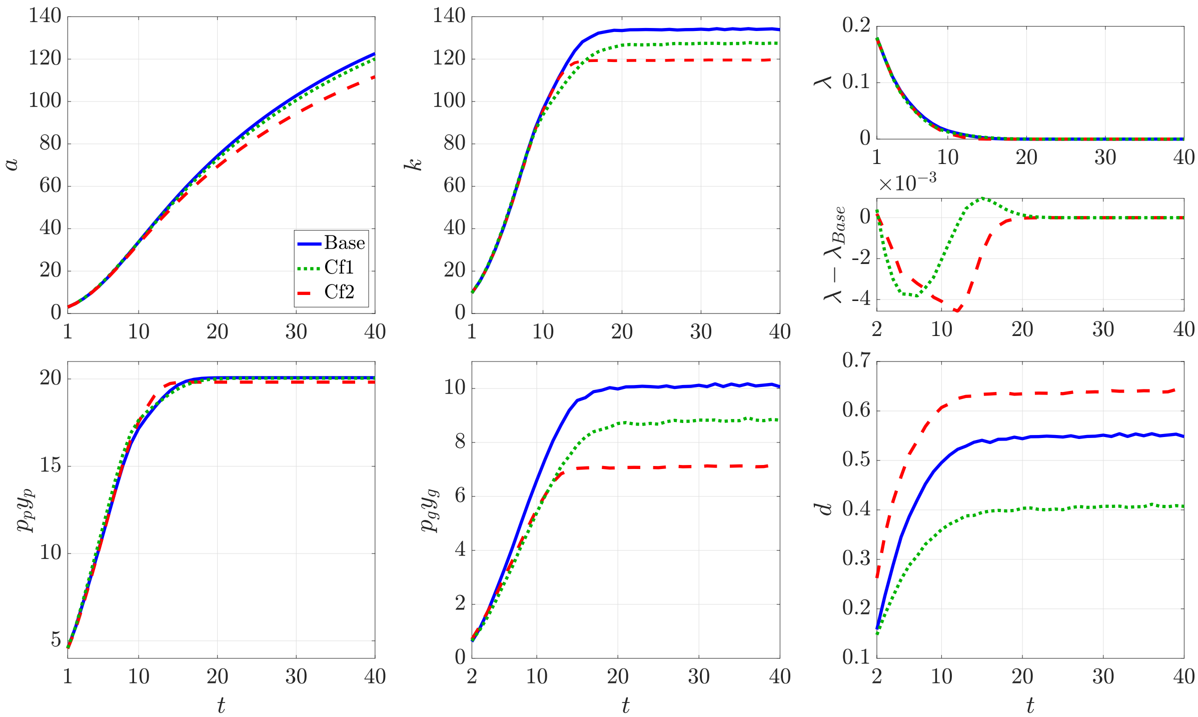
(1) of Table 4 shows a significant positive effect of this channel on the macroeconomy. In counterfactual 1, the implied capital stock and GDP are 3.02% and 3.08% higher than in the benchmark. In counterfactual 2, K and GDP are 0.77% and 0.42% higher than in the benchmark. Hence, if we keep the policy functions of the dynamic problem and the interest rate unchanged, reforming the procurement allocation system in a way that favors small firms generates a positive aggregate effect because it allows more constrained firms to accumulate more net worth, grow out of financial constraints, and hence produce more in the long run.

Full effect. Once we incorporate the aggregate effects of the changes in the dynamic policy functions and prices in general equilibrium, we find that the reforms reduce the incentives for big firms to accumulate assets over time, which shrinks (in counterfactual 1) or even turns negative (in counterfactual 2) the output gains associated to the reforms. One of the reasons why firms accumulate assets in our model is the possibility of obtaining a public procurement contract in the future. Obtaining a contract represents a demand shock in response to which firms want to expand their capital stock. This expectation increases the return to wealth accumulation, leading to wealth accumulation even for relatively large firms. When obtaining a procurement contract is less likely or the average contract size is lower, the expected return to wealth accumulation diminishes and so does wealth accumulation.

In Figure 9, we show how this reduction in firms' incentives manifests in our model. This figure simulates the average life cycle profile of a cohort of firms in the benchmark economy (solid blue line), in counterfactual 1 (dotted green line), and in counterfactual 2 (dashed red line). In particular, we take a large number of newborn firms that draw the highest productivity shock every period and obtain procurement projects according to the probability $g(b)$ and their choices of b. We focus on firms that draw the highest productivity state in every period to capture the fact that changes in saving incentives across the different economies will be especially apparent in firms that expect to operate at large scales.

The three economies exhibit similar patterns. As they age, firms become larger — accumulate more net worth, operate with more capital, and sell more both in the private and procurement sectors — and less financially constrained. The panels for $p_g y_g$ and d show the differences in the procurement allocation systems across the three economies. The probability of participating in procurement, given by d, is the highest in counterfactual 2 (there is a higher number of contracts) and the lowest in counterfactual 1 (the number of contracts is the same but high s firms are less likely to get them). The revenues from procurement ($p_g y_g$), are highest under the benchmark economy, despite the fact that the probability of getting a contract is significantly higher in counterfactual 2. This is because contracts are considerably bigger in the benchmark than in counterfactual 2.

The most important insights from this figure come from the evolution of a and k over



Notes: This figure shows the "average" life cycle profile of a large number of firms, all drawing the highest productivity level s in every period, simulated in our model under three different scenarios: the benchmark economy (blue solid line), counterfactual 1 (green dotted line), and counterfactual 2 (red dashed line).

Figure 9. Firms' life cycle profiles

the firms' life cycle. Net worth accumulation by high s firms is the highest in the benchmark economy. In counterfactual 1, these firms prefer to accumulate slightly less because, although the size of contracts is still big, it is less likely for high s firms to obtain them.[31] The difference becomes especially visible in counterfactual 2, where firms' net worth and capital accumulation is significantly lower. In this counterfactual, firms reach their optimal size at an age that is noticeably earlier than in the other two cases. To see the intuition for this result, refer back to the Euler equation (13). The expected return of accumulating net worth (self-financing motive) is given by the term $\frac{\partial \pi(s',a',d')}{\partial a'}$, which is equal to $\phi_a \lambda'$ (see Appendix E.2 and E.3). That is, the expected value of the financial constraint multiplier represents an extra return to asset accumulation. Thus, firms that expect to be more financially constrained next period have stronger incentives to accumulate assets today, all else equal. In counterfactual 2 contracts are smaller, and hence expected λ' in case of obtaining a contract is smaller too.

Panel $\lambda - \lambda_{Base}$ in Figure 9 compares the λ's in the two counterfactual economies to the benchmark. The λ's tend to be smaller in the two counterfactuals for high s firms, and

[31]Instead, in counterfactual 1, all firms with lower s, accumulate *more* net worth compared to the baseline.

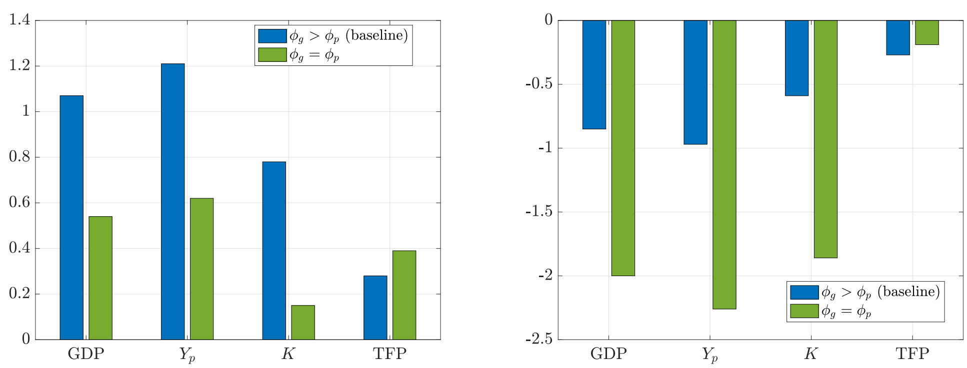
particularly so in counterfactual 2. This is driven by the fact that, in counterfactual 2 the procurement contracts are smaller, so firms do not need much financial capacity to expand in order to service them. In counterfactual 1 the old high-productivity firms are less likely to win procurement contracts as these are being reallocated to firms with lower a and s.

8 Robustness exercises

We close the paper by looking at two important variations of our benchmark economy.

8.1 The importance of $\phi_g > \phi_p$

Our firm-level panel regressions show that leverage increases with firm earnings, and more so when earnings come from the public sector (see Table 2). Through the lens of our model, this evidence implies that earnings from the public sector can be pledged to obtain credit to a larger extent than earnings from the public sector ($\phi_g > \phi_p$). In this section we want to assess the role played by this asymmetry. To do so, we compare the macroeconomic effects of the policy reforms in a world where $\phi_g = \phi_p$. In particular, we decrease ϕ_g such that $\phi_g = \phi_p = 0.27$ (the value of ϕ_p in our baseline economy) and run the same counterfactuals as we did in Section 7.1 and Section 7.2.



(a) Counterfactual 1 **(b)** Counterfactual 2

Notes: This figure shows the % changes (y-axis) in the main aggregates as the result of conducting Counterfactual 1 and Counterfactual 2, both in our baseline economy and in the same economy but under $\phi_g = \phi_p = 0.27$. Notice that the numbers associated with the blue bars ($\phi_g > \phi_p$) are the same as reported in columns (2) and (3) in Table 3.

Figure 10. Policy reforms under $\phi_p = \phi_g$

Panels (a) and (b) in Figure 10 report the % changes of the main aggregates in the $\phi_g > \phi_p$ vs. $\phi_g = \phi_p$ cases for Counterfactual 1 and Counterfactual 2 exercises respectively.

Our main finding is that if government contracts were less pledgeable, changes in the procurement system that facilitate the presence of small firms would be associated with worse macroeconomic outcomes. In the case of keeping the average size of contracts but increasing the strength of diminishing returns to b, i.e., Counterfactual 1, the increase in nominal GDP would be 0.54% in the $\phi_g = \phi_p$ case (vs. 1.07% in the $\phi_g > \phi_p$ case). In the case of reducing the average size of contracts, i.e., Counterfactual 2, the fall in nominal GDP would be -2.00% in the $\phi_g = \phi_p$ case (vs. -0.85% in the $\phi_g > \phi_p$ case).

The reason for these results is as follows. When $\phi_g = \phi_p$, the private sector negative spillover of procurement in the short run is larger because there is no extra financing through public earnings to alleviate the problem of scarce collateral (see Proposition 13 in Appendix E). In addition, by reducing the extent to which borrowing capacity increases when participating in procurement, the long-run positive effects also weaken. Overall, procurement is less effective in helping constrained firms increase their production.

8.2 Decreasing returns to scale

Our second exercise assesses the sensitivity of our main results to firms' technology. In particular, instead of assuming a CRS production function, we consider: $y = f(s,k) = sk^{\alpha}, \alpha \in (0,1]$, where $\alpha = 1$ would correspond to our baseline CRS economy.

As discussed in Section 4.7, with CRS the static problem of firms that are not financially constrained can be solved independently for the private and public sectors. How much an unconstrained firm sells to the government does not affect firms' decisions in selling to the private sector and hence the crowding-out effect of procurement to private sales is absent. This result breaks under DRS. As we show in Appendix I, the elasticity of private sales to an increase in B_g (a demand shock in the public sector) for unconstrained firms is given by:

$$\frac{\partial \log p_p y_p}{\partial \log B_g} = \left(\frac{(\sigma-1)(\alpha-1)}{\sigma-\alpha(\sigma-1)}\right)\left(\frac{1}{B_p+B_g}\right) \tag{17}$$

This elasticity is < 0 when $\alpha < 1$ and $= 0$ when $\alpha = 1$. Therefore, the overall short-run crowding-out effect of procurement to private sales would be bigger under DRS: in addition to the crowding-out coming from financial frictions there would be the crowding-out coming from DRS, which would affect all firms (constrained and unconstrained).

There are three reasons why we use CRS as our preferred technology. First, our model already generates an average crowding-out effect which is large enough compared to the data (see Figure 7). Second, we find evidence consistent with zero (or close to zero) crowding-out effect for unconstrained firms (see Figure 3). And third, most of our analytical results for the intra-temporal production problem can only be obtained with CRS. In principle, it is not obvious under which of the two technology assumptions we should expect larger gains from reallocating procurement contracts toward small firms. On the one hand, in an economy without financial frictions, a bigger share of small firms in government procurement would be justified by the fact that the largest firms will be relatively less productive with DRS than in a world with CRS. On the other hand, under financial frictions, a lower share of firms will be financially constrained under DRS because the firm optimal size will be lower.

To assess the importance of CRS for our policy counterfactuals, we carry out a similar calibration as the one explained in Section 5 but under the assumption that $\alpha = 0.85$ (Atkeson and Kehoe, 2005) and implement Counterfactual 1. The aggregate gains of promoting the participation of small firms are slightly higher under DRS (see Figure A.XVII in Appendix I). For example, in terms of GDP, the gains would be 1.07% under CRS vs. 1.53% under DRS. This result is mostly explained by a larger effect on capital accumulation under DRS: 1.43 vs. 0.78%.

9 Conclusion

In this paper, we quantify the macroeconomic impact of changes in the public procurement allocation system. To do so, we use a comprehensive framework that builds on three steps: selection, treatment, and the interplay between procurement and the macroeconomy. We use our framework to evaluate some of the policy reforms that are currently implemented in the US or are in the industrial policy agenda of the European Commission. In particular, we quantify the long-run macroeconomic effects of a size-dependent expenditure-neutral policy reform that consists of facilitating small firms' participation by either targeting them in the allocation process or by breaking down big projects into smaller ones.

Our results point towards the presence of long-run positive effects for directly affected firms, but also suggest the existence of important changes in large firms' dynamic behaviors that could shrink the expansionary effects of changes in the procurement allocation system, or even make the effects negative. Our findings show that both the sign and size of these effects and hence the overall macroeconomic impact of this type of policies crucially depends on the severity and type of financial frictions in the economy. But the effects also depend on the type of reform, which determines how the change in procurement harms larger firms. These findings suggest that the optimal procurement allocation system in a country would depend on the specific institutional characteristics of the economy.

We view our contribution as part of a broader research agenda on the macroeconomic effects of government procurement, a policy that is surprisingly understudied. This paper only investigates the long-run consequences of expenditure-neutral changes in the procurement allocation system. Issues like the short-term consequences of reforms, or the potential implications for the effectiveness of fiscal policy are still unexplored. We would like to emphasize that pursuing this research agenda will deliver important policy implications.

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Online Appendix

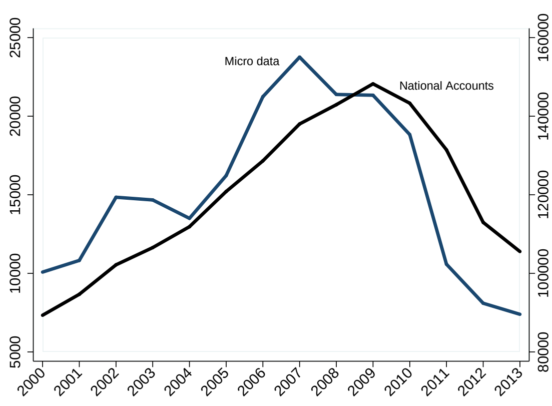
A Details on the data

A.1 Public procurement data

According to the System of National Accounts (SNA), "Government consumption expenditures and gross investment", i.e., G, measures the fraction of GDP, or final expenditures, that is accounted for by the government sector. Public procurement is defined in the *System of National Accounts (SNA)* as the sum of intermediate consumption (e.g., purchases of goods like medical consumables and services like accounting services), gross capital formation (e.g., building new roads), and social transfers in kind via market producers (e.g., medicines). Roughly speaking, one can think of public procurement as "government consumption expenditures and gross investment" (the G part of GDP) minus "compensation of employees" and "consumption of fixed capital." The size of public procurement varies across countries and over time. For OECD countries during 2007-2017, public procurement represented roughly 12% of GDP and 30% of G, on average.

Figure A.XI shows the evolution of procurement value as measured with our micro data and compares it to the counterpart from national accounts. On average, our micro data accounts for around 13% of total government procurement as measured in Spanish national accounts. Our micro data reproduces well the cyclical aspect of public procurement expenditure, increasing during the boom and decreasing during the recession.

Figure A.XI. Evolution of Public Procurement in Spain, 2000-13



Notes: This figure shows the evolution of public procurement in Spain over 2000-13. The blue line ("Micro data", left y-axis) is computed by aggregating the individual projects scraped from the BOE, https://www.boe.es/. The black line ("National accounts", right y-axis) is measured from Spanish national accounts.

Main sample of projects published in BOE. According to Spanish law, all procurement contracts above a certain threshold awarded by public institutions must be published in official bulletins.[32] If the contract is awarded by the central government, the information on this contract must be published in the *Agencia Estatal Boletín Oficial del Estado* (BOE), which is the official bulletin of the central government of Spain. In contrast, if the entity that awards the contract is a regional government or a municipality, the information about this contract can alternatively be published at their respective regional or local bulletin. We construct a novel dataset on Spanish public procurement contracts by scraping the BOE website over the 2000-2013 period. Each contract provides information on the type of contract (kind of good or service provided), the awarding institution, the type of procedure used to allocate the contract, and the firm(s) that won the contract. In total, we scraped more than 150,000 projects over 2000-2013, which we assign to the month that the project was awarded. Of these, 130,633 projects have a value assigned to them that we were able to recover. The sum of all these projects totals around 220 billion euros. On average, our micro data account for around 13% of total public procurement as measured in National Accounts. Despite the level differences, our micro data are able to capture the overall evolution of public procurement over time, which increased from 9.9 to 13.8 percent between 2000 and 2009 and decreased from 13.8 to 10.0 percent between 2010 and 2013; see Figure A.XI.

Small sample of projects with information on bidders. The BOE website does not provide the identity of the firms that competed for the project but did not win. This is a limitation of our dataset because it does not allow us to construct a well-defined control group. To overcome this limitation, we construct a sample of procurement projects for which we have detailed information about the awarding process. Although we did not find any government agency that provided information about the awarding process during our main sample period (2000-2013), we could identify around 50 agencies that started providing detailed information about their projects starting in 2013. Putting all these agencies together, we were able to uncover the identity of the firms competing for the same projects as well as their final rankings for around 1,000 contracts over the 2013-2016 period.

A.2 Balance sheet and credit data

We use the balance sheets and income statements of the quasi-universe of Spanish companies between 2000 and 2016, a dataset that is maintained by the Banco de España and taken from the Spanish Commercial Registry. For each firm and year, this dataset includes information on the firm's name, fiscal identifier, sector of activity (4-digit NACE Rev. 2 code), age, net operating revenue, material expenditures, number of employees, labor expenditures, total

[32] The thresholds above which the contract must be advertised in official bulletins depend on the type of contract. In the case of supplies and services, for example, the threshold is 60,000 euros.

fixed assets, and total assets. The final sample covers around 85-90% of non-financial firms for all size categories in terms of both turnover and number of employees.

Table A.V. Descriptive evidence from the final merged dataset, year 2006

	mean		**25th pctile**		**50th pctile**		**75th pctile**	
	Proc	NoProc	Proc	NoProc	Proc	NoProc	Proc	NoProc
Age	20.42	10.95	12.00	5.00	17.00	10.00	24.00	15.00
Employment	73.56	12.75	16.00	3.00	45.00	6.00	155.0	12.00
Sales	8.96	1.19	1.14	0.10	4.22	0.28	16.89	0.86
Procurement/Sales	0.20	0.00	0.01	0.00	0.03	0.00	0.10	0.00
Fixed Assets	3.80	0.85	0.21	0.03	0.82	0.14	3.58	0.50
Net worth	3.92	0.43	0.36	0.01	1.27	0.07	6.12	0.30
Credit	2.51	0.57	0.11	0.03	0.48	0.08	2.32	0.30
Coll. Credit (share)	0.14	0.29	0.00	0.00	0.00	0.00	0.14	0.74

Notes: This table presents summary statistics from our merged dataset for the year 2006, separately for firms with at least one procurement contract ($n = 2,411$) vs. the rest of the firms ($n = 406,261$). The variable *Employment* measures the number of full-time workers employed by the firm; the variable *Sales* is just firm's revenue measured in millions of euro; *Procurement/Sales* measures the value of all the procurement projects awarded to a firm in a given year divided by total revenue in that year; *Assets* measures the value of fixed assets; *Credit* measures the value of all firm's outstanding loans in millions of euro; *Coll. Credit (share)* is the share of *Credit* collateralized against firm's assets; *Def. Credit (share)* is the share of defaulted credit over total *Credit*; *age* measures the age of the firm. We winsorize the 1% tails of all variables.

The *Central de Información de Riesgos* (CIR) is maintained by the Banco de España in its role as primary banking supervisory agency, and contains detailed monthly information on all outstanding loans over 6,000 euros to non-financial firms granted by all banks operating in Spain since 1984. Given the low reporting threshold, virtually all firms with outstanding bank debt appear in the CIR. In addition to the total amount of credit, CIR also contains information on whether or not a non-personal collateral ("Garantía real") was posted for a particular loan. These collaterals include assets like real estate, land, machinery, securities, deposits, and merchandise (i.e., hard collateral). With this information, we can hence assess whether a particular loan for a bank-firm pair was granted on the basis of tangible collateral.

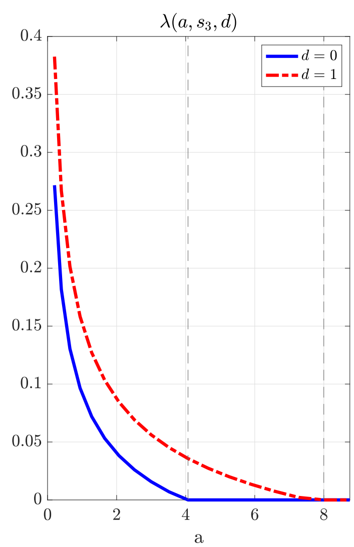
Loan applications. Besides the information on outstanding loans, we also have information about loan applications at the firm-bank level. The construction of this dataset is as follows. Spanish banks can request information about a firm whenever this firm "seriously" approaches them to obtain credit.[33] Because banks already have information about the firms with which they have a credit relationship, banks only request information on firms that have never received a loan from them or that ended the credit relationship before the current request. By matching the loan applications with the information on outstanding

[33] The Law stipulates that a bank can not request information about the firm without its consent, which indicates the seriousness of the approach

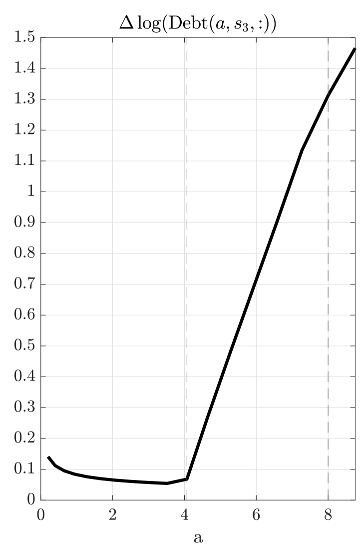
loans from CIR, we can infer whether the loan was granted or not.

B Heterogeneous effects of credit growth

We investigate the heterogeneous effects of procurement on firms' credit growth. Before looking at the data, we first shed light on this relationship using our model's calibrated version. In panel (a) of Figure A.XII, we show how the Lagrange multiplier associated with the financial constraint, $\lambda(s, a, d)$, changes across firms with different levels of net-worth, a.[34] We show this relationship for firms with procurement, i.e., $d = 1$, and without procurement, i.e., $d = 0$. As discussed in Section 4.6 of the main text, higher net-worth (a) firms are less constrained, and, conditional on a, procurement firms ($d = 1$) are more likely to be constrained because of their higher demand. The dashed lines divide the graph into three regions. The region to the left is one in which firms, with and without procurement, are financially constrained. In the middle region, only firms with procurement are constrained. In the region to the right, all firms are unconstrained.



(a) Lagrange multiplier λ



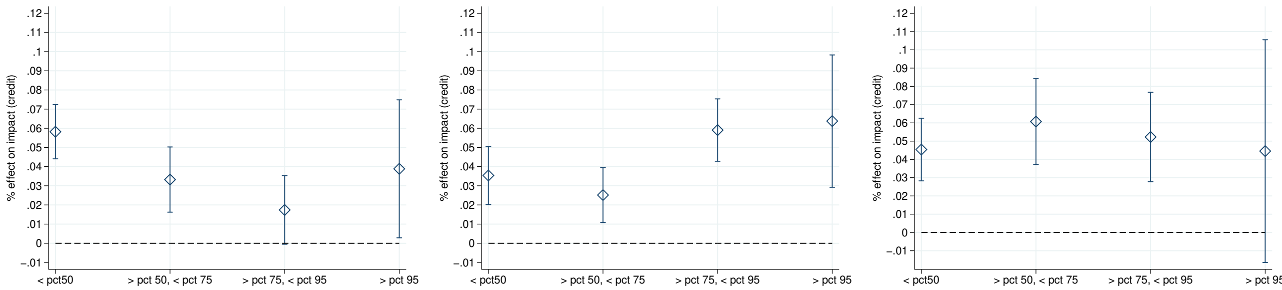
(b) change in log credit

Notes: Panel (a) shows the Lagrangian multiplier associated to the financial constraint, λ, for firms with different levels of net-worth in our model, both for firms with and without procurement ($d = 0$ and $d = 1$). The dotted lines organize the graph in three regions. In Panel (b) the effect on impact, i.e., $h = 0$, for different levels of firms' financial constraints as proxied by a (and implicitly given by λ).

Figure A.XII. Het. effects of credit (calibrated model)

In panel (b), we show the change in credit on impact ($h = 0$), of a firm becoming active

[34] We produce this graph fixing the level of firms' productivity at the middle point in our productivity grid, i.e., s_3. For other levels of s, the graph would simply be an identical, scaled, version of Figure A.XII.



(a) distribution of fixed assets **(b)** distribution of leverage **(c)** distribution of age

Notes: This figure shows the effect on impact, i.e., $h = 0$ (as well as its 10% confidence intervals), of public procurement on firms' credit for different percentiles of the distribution of assets (panel a), leverage (panel b), and age (panel c). Standard errors clustered at the firm level.

Figure A.XIII. Heterogeneous effects of procurement on credit

in procurement ($d = 0 \to d = 1$), as a function of net worth a, i.e., the inverse of financial constraints all else equal. The main takeaway from this graph is that the model predicts a non-monotonic relationship between firms' financial constraints and the effect of procurement on credit. When the financial constraint is binding both before and after the procurement shock (left region), the model predicts that less financially constrained firms (higher a and hence lower λ) exhibit a smaller increase in credit when becoming active in procurement. However, the impact for firms in the other two groups (when transitioning from unconstrained to constrained or remaining unconstrained) increases with net worth. As a result, the model exhibits the u-shaped relationship between net credit growth and net worth.

In Section 6.2 we discuss the intuition for why unconstrained firms exhibit a larger increase in credit due to the procurement shock. The idea is that these firms, precisely because they are unconstrained, can expand freely by increasing credit. On the contrary, even if procurement allows them to borrow more, constrained firms remain constrained, limiting the amount of credit they can raise after the demand shock implied by procurement.

We next look at the heterogeneous effects of procurement on firms' credit in the data. Similarly to Section 3.1 in the main text, we show the impact effect, i.e., $h = 0$ (as well as its 10% confidence intervals), of public procurement on firms' credit for different percentiles of the distribution of assets (panel a), leverage (panel b), and age (panel c). See Figure A.XIII. As in the model, the relationship between proxies for firms' financial constraints and the change in credit is non-monotonic. With fixed assets and leverage as proxies, the empirical evidence is reasonably consistent with the u-shaped relationship predicted by the model.

C Additional evidence

C.1 Effects on impact using quarterly data

To document a stronger identification of the effect of procurement on credit growth, we use quarterly data. We first regress firms' credit growth on a dummy variable for procurement:

$$\Delta \log l_{it} = \alpha_{iy} + \alpha_{st} + \beta_1 \mathrm{PROC}_{it} + \beta_2 \log l_{it-1} + \varepsilon_{it} \tag{C.1}$$

where the dependent variable $\Delta \log l_{it}$ is the annualized quarterly growth of credit (loans) of firm i between quarter $t-1$ and quarter t defined as $\Delta \log l_{it} \equiv \log l_{it} - \log l_{it-1}$. The regressor PROC_{it} is a dummy variable that takes value one if the firm obtained a procurement contract in quarter t. We include the firm's lagged credit at $t-1$ to control for the fact that firms with large outstanding loan volumes may mechanically have less room for credit growth than firms with smaller outstanding loan levels.[35] We further include a stringent set of fixed effects. In particular, we use firm×year fixed effects, α_{iy}, in order to capture firm-level characteristics that vary over time at the yearly (y) level. Importantly, these fixed effects help control for several factors that may otherwise bias the estimation. First, as they vary at the year level, they pick up the overall firm-level trend of credit growth and thus helps assuage the concern of any potential bias arising from differences in trends pre/post "treatment" by procurement events. Second, these fixed effects control for any firm-level variables that may change at annual level such as productivity or demand. We further include 4-digit sector×quarter effects, α_{st}, which control for both sector and macroeconomic conditions that vary over time. Thus, identification of the key parameter of interest, β_1, comes from the variation of a firm's credit growth across quarters within a year conditional on obtaining a procurement contract.

Table A.VI, column (1), presents the results of this regression for the main sample. The estimate of β_1 is positive and significant at the one-percent level.[36] The estimated coefficient implies that winning a procurement contract in a quarter translates into an increase of credit growth of 5.5 percentage points annually.

C.2 Effects on impact using data on all bidders

We next use the sample of procurement projects where we have information on all bidders as well as the final ranking. Doing so allows us to run regressions analogous to (C.1), except that we can identify the association between a firm's ranking in a given auction and its ensuing credit growth. To be more precise, we run two regressions similar to specification

[35] The estimation results without lagged credit are similar and are available upon request.

[36] We cluster standard errors at the firm-level in all regressions unless otherwise noted.

Table A.VI. Credit Growth and Procurement

	All firms	Bidders only	
		First	Second
	(1)	(2)	(3)
PROC_{it}	0.055^a	0.073^a	-0.061
	(0.004)	(0.028)	(0.049)
$\log(\text{Credit}_{it-1})$	-0.410^a	-0.175^a	-0.229^a
	(0.001)	(0.043)	(0.044)
Observations	700,780	8,310	3,683
R-squared	0.786	0.360	0.458
Sector×quarter FE	Yes	No	No
Firm×year FE	Yes	Yes	Yes
Quarter FE	No	Yes	Yes
Auction FE	No	Yes	Yes

Notes: Results from estimating the relationship between total credit growth and procurement participation (PROC) by regression (C.1): with firms obtaining at least one procurement project over 2000-13 in column (1), and with firms who participated in procurement contests over 2013–15 in columns (2) and (3), where the PROC dummy indicates the winning firm ('First') in column (2) and the runner-up firm ('Second') in column (3). All regressions use quarterly data. Standard errors clustered at the firm level; ^{a} indicates significance at the 1% level, ^{b} at the 5% level, and ^{c} at the 10% level.

(C.1) at the auction level. In the first regression, we include all bidders and the PROC variable indicates which firm wins the auction ('First' place). Table A.VI, column (2), shows the results. We find that the winner of a procurement contract has higher credit growth relative to the firms it competes against in a given auction. Note that identification of the coefficient is exploiting the full time series of bidders, so the comparison is based on the within-auction group of firms but also with respect to each firm's annual credit growth given the inclusion of firm×year effects. The coefficient on the winner is 0.073, which indicates that winning the auction is associated to a 7.3 percentage points higher credit growth annually.

While the bidder firms' sample is more restrictive than the full sample, we are reassured that we are picking up an unbiased "procurement effect" for a few reasons. First, the point estimates of PROC in columns (1) and (2) are remarkably similar, even though the sample and variation exploited are slightly different. Second, we are able to control for firm×year effects in both regressions, thus helping dilute firm-specific productivity or demand effects at the annual level. Third, an identification threat would be that productivity or demand shocks at the quarterly level are correlated with the concession of procurement projects such that winning the contract may be a proxy for these shocks. Therefore, our estimate may capture the effect of being ranked above other firms as opposed to the effect of obtaining the procurement contract. In Column (3) of Table A.VI we drop the winner of the procurement contest and the PROC dummy now indicates which firm was runner-up ('Second' place).

We run this second regression to make sure that winning the contract, as opposed to the relative ranking, is what is really associated with differences in credit growth across auction participants. The estimated coefficient on PROC implies that there is no statistical difference in quarterly credit growth for the firm that placed second relative to other losers of the auction. Fourth, as in any diff-in-diffs type of environment, it could be the case that winner firms are on different credit trajectories than non-winner firms. In principle, this should be captured by the firm×year effects. We provide evidence below (see Figure A.XIV) which shows that this is not the case. In particular, we show that the evolution of credit growth for winners and non-winners was similar before the auction and that it diverged afterwards.

We next decompose the increase in credit associated with winning a procurement contract into that coming from collateralized vs. non-collateralized credit, which will help us motivate the type of financial constraint we use in our model. To this end, we use the information on the composition of firms' loans, which indicates whether these loans require collateral or not to be posted by a firm to receive financing from a bank. We therefore run a similar regression as (C.1), constructing the dependent variable at the firm×credit-type×quarter level, and split the estimation between collateralized and non-collateralized credit growth.

Table A.VII presents the main results, where c denotes the additional collateral/non-collateral dimension that we exploit in the data. Looking at the main sample, we see that a procurement contract is not significantly correlated with the growth rate of collateralized credit in column (1). However, when turning to column (2) we see a positive and significant association with a firm obtaining a procurement contract and non-collateralized credit growth. The results with the bidders sample, in columns (3) and (4), mimic the findings for the main sample. That is, a firm winning a contract experiences significantly larger growth in non-collateralized loans relative to losing firms, but there is no differential for collateralized loan growth. Regressions for the second vs. the rest samples in columns (5) and (6) do not yield any significant estimates. Overall, these findings point to the growth rate in overall credit associated with obtaining a procurement contract observed in Table A.VI being driven by the growth in loans that do not require tangible-assets backing.

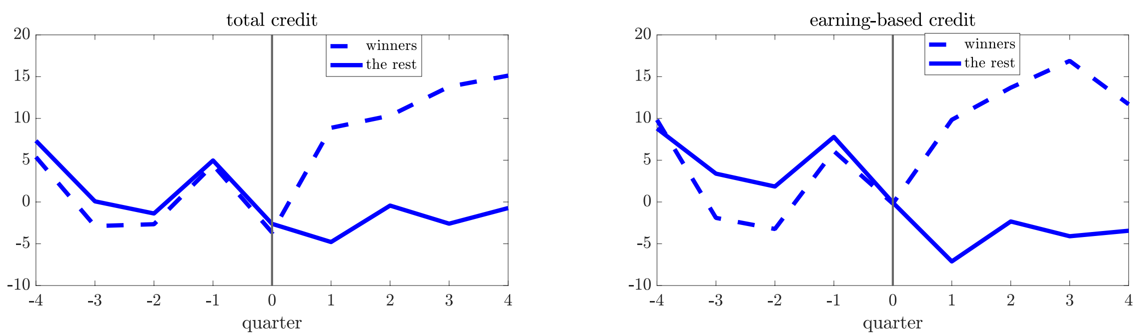
Pre-trends for winners vs. the rest. Graphically, the right panel in Figure A.XIV shows the average growth of credit without collateral of firms that win a procurement project in quarter 0 before and after winning the project, and compares it to the rest of firms. Again, there is a similar evolution of credit growth before procurement (parallel trends) and a clear (and persistent) divergence after that.

Table A.VII. Composition of Credit Growth and Procurement

	All firms		Bidders only			
			First		Second	
	Collat.	NoCollat.	Collat.	NoCollat.	Collat.	NoCollat.
	(1)	(2)	(3)	(4)	(5)	(6)
PROC_{it}	0.001	0.070^a	-0.011	0.080^b	-0.019	-0.058
	(0.006)	(0.005)	(0.029)	(0.031)	(0.044)	(0.057)
$\log(\text{Credit}_{ict-1})$	-0.474^a	-0.421^a	-0.449^a	-0.192^a	-0.461^a	-0.254^a
	(0.003)	(0.001)	(0.073)	(0.040)	(0.064)	(0.044)
Observations	224,011	557,873	2,690	8,110	1,423	3,606
R-squared	0.791	0.764	0.357	0.368	0.435	0.435
Sector×quarter FE	Yes	Yes	No	No	No	No
Firm×year FE	Yes	Yes	Yes	Yes	Yes	Yes
Quarter FE	No	No	Yes	Yes	Yes	Yes
Auction FE	No	No	Yes	Yes	Yes	Yes

Notes: Results from estimating the relationship between collateralized (Collat.) and non-collateralized (NonCollat.) credit growth and procurement participation (PROC) by regression (C.1) with firms obtaining at least one procurement project over 2000-13 in columns (1) and (2), and with firms who participated in procurement contests over 2013-15 in columns (3)-(6) respectively, where the PROC dummy indicates the winning firm ('First') in columns (3)-(4) and the runner-up firm ('Second') in columns (5)-(6). All regressions use quarterly data. Standard errors clustered at the firm level; ^{a} indicates significance at the 1% level, ^{b} at the 5% level, and ^{c} at the 10% level.

Figure A.XIV. Credit Growth: bidders sample



Notes: These graphs plot the evolution of the average change in credit for winning vs. non-winning firms, before and after the quarter in which the auction takes place (Quarter=0). The left panel is for all credit. The right panel is for non-collateral credit only.

C.3 Loan applications

In this Section we ask whether firms are able to use their procurement contracts to access credit more easily at the extensive margin. A unique piece of information contained in the Banco de España's credit register allows us answer this question: the information on the loan application process for firms and banks. In particular, we can see whether a firm has applied to a given bank and whether the loan application has been accepted or rejected throughout our sample period. We use this information to help identify an increase in firms' borrowing capacity. To do so, we run regressions at the firm-bank level and relate the probability of firms obtaining a loan to whether they have received a procurement contract using the following linear probability specification:

$$\text{Loan granted}_{ibt} = \alpha_{ib} + \alpha_{bt} + \alpha_{st} + \beta \text{PROC}_{it} + \varepsilon_{ibt} \tag{C.2}$$

where the variable 'Loan granted' is a dummy variable equal to 1 when firm i receives a loan from bank b in quarter t conditional on the firm applying for it during that same quarter. We include firm×bank fixed effects, α_{ib}, which implies that we are identifying the coefficient β on the procurement variable via the variation within a firm-bank relationship over time. We further control for overall bank credit supply in a given period with bank×quarter fixed effect α_{bt}, and for macroeconomic events with sector×quarter fixed effects α_{st}.

Table A.VIII. Probability of a New Loan and Procurement

	All firms	
	(1)	(2)
PROC_{it}	0.024^a	0.023^b
	(0.008)	(0.011)
Observations	36,857	26,924
R-squares	0.395	0.628
Firm×bank FE	Yes	Yes
Bank×quarter FE	No	Yes
Sector×quarter FE	No	Yes

Notes: Results from estimating the relationship between loan participation and procurement participation (PROC) by regression (C.2) with firms obtaining at least one procurement project over 2000-13 using quarterly data. Standard errors clustered at the firm level; ^{a} indicates significance at the 1% level, ^{b} at the 5% level, and ^{c} at the 10% level.

Table A.VIII shows the results from running this regression. We include only firm×bank fixed effects in column (1), and add the time-varying bank and sector fixed effects in column (2). Overall, regardless of the specification, the probability of receiving a bank loan conditional on having applied for it increases by approximately 2 percent in the quarter that a firm wins a procurement project.

D Steady state equilibrium

Let $\mathbf{X} \equiv S \times A \times \{0,1\}$ be the state space of the household problem, $\mathbf{X}_1 \equiv S \times A \times \{1\}$ the subset of the state space for firms with a procurement project, $\mathcal{X}$ a σ-algebra generated by $\mathbf{X}$, and Γ a probability measure over $\mathcal{X}$. Given government policy parameters Y_g and m_g and a distribution of entrants Γ_0, we define the steady state equilibrium of the model as

(i) firms' policy functions, (ii) the probability measure Γ, (iii) total amount of final private good Y_p, (iv) interest rate r, (v) tax rate τ, (vi) relative price of the final public good P_g, and (vii) the procurement probability shifter η_0, so that:

a) Entrepreneurs solve their optimization problem

b) The probability measure Γ is stationary

c) The market for the private good clears:

$$\int_{\mathbf{X}} p_p(s,a,d)\,u(s,a,d)\,y(s,a,d)\,d\Gamma = Y_p = \int_{\mathbf{X}} \left[b(s,a,d) + c(s,a,d) + \delta k(s,a,d)\right] d\Gamma$$

d) The market for the public good clears: $\int_{\mathbf{X}_1} p_g(s,a,1)\left[1 - u(s,a,1)\right] y(s,a,1)\,d\Gamma = P_g Y_g$

e) The probability of obtaining procurement projects is consistent with the measure of goods bought by the public sector: $\int_{\mathbf{X}} Pr\left(d' = 1 \mid b(s,a,d)\right) d\Gamma = \int_{\mathbf{X}_1} d\Gamma = m_g$

f) The budget constraint of the government holds:

$$P_g Y_g = rD + \tau \int_{\mathbf{X}} \pi(s,a,d)\,d\Gamma + (1-\theta)\left[\int_{\mathbf{X}} a'(s,a,d) d\Gamma - \int_{\mathbf{X}} a d\Gamma_0\right]$$

g) By Walras law, the credit market clears: $D = \int_{\mathbf{X}} \left[k(s,a,d) - a(s,a,d)\right] d\Gamma$

Several comments are in order. First, the parameter η_0 driving the average probability of a procurement project is an equilibrium object that ensures meeting equilibrium condition (e). It captures in reduced form the competition for projects. Second, the government can accumulate financial wealth D, which serves as an aggregate counterpart for the loans of entrepreneurs such that loans do not need to be in zero net supply in condition (g). Indeed, we calibrate D to match an interest rate of $r = 5\%$ given the total amount of debt relative to capital held by firms in the data. Third, condition (f) establishes that the government budget constraint in steady state is such that procurement is financed by taxes, plus interest revenues from the stationary amount of government wealth D, plus accidental bequests left by dying entrepreneurs, minus the initial net worth provided by the government to newly born entrepreneurs (which is dictated by the exogenously fixed distribution of entrants Γ_0). Finally, we note that the aggregate objects determined in general equilibrium that are relevant for the optimization problem of households are Y_p, r, τ, P_g, and η_0.

E Details on the static production problem

In this Appendix we characterize the solution of the static production problem. First, in Section E.1 we derive the results that serve to restrict the parameters ϕ_p and ϕ_g such that the problem is well-behaved. Then, in Section E.2 we characterize analytically the solution to the production problem for firms without procurement ($d = 0$), which is useful to understand the interaction of asset based and earnings based financial constraints. Next, in Section E.3 we characterize analytically some of the solutions to the production problem for firms with procurement ($d = 1$) for the case $\sigma_p = \sigma_g$. Finally, in Section E.4 we show analytically the effect of a procurement shock for the case $\sigma_p = \sigma_g$, that is, the differences in allocations and profits between a firm with $(s, a, d = 1)$ and a firm with $(s, a, d = 0)$.

Before going to all these results, we start the Appendix by rewriting the FOC of the static production problem as follows. First, note that because the FOC for u states that the marginal revenue per unit of output sold —including its value as collateral— has to be equalized across the two sectors, and using the fact that $\frac{\partial p_p y_p}{\partial k} / \frac{\partial p_p y_p}{\partial u} = u/k$ and $\frac{\partial p_g y_g}{\partial k} / \frac{\partial p_g y_g}{\partial (1-u)} = (1-u)/k$ we can write the FOC for k as:

$$\text{MRPK}_p \equiv \frac{\partial p_p y_p}{\partial k_p} = \frac{r + \delta + \lambda}{1 + \lambda \phi_p} \tag{E.1}$$

or as

$$\text{MRPK}_g \equiv \frac{\partial p_g y_g}{\partial k_g} = \frac{r + \delta + \lambda}{1 + \lambda \phi_g} \tag{E.2}$$

or combining them both, $\text{MRPK} \equiv \frac{\partial [p_p y_p + p_g y_g]}{\partial k} = u \left(\frac{r+\delta+\lambda}{1+\lambda\phi_p} \right) + (1-u) \left(\frac{r+\delta+\lambda}{1+\lambda\phi_g} \right)$, where note that $\frac{\partial p_p y_p}{\partial k_p} = \frac{\partial p_p y_p}{\partial k} \frac{1}{u}$ and $\frac{\partial p_g y_g}{\partial k_g} = \frac{\partial p_g y_g}{\partial k} \frac{1}{1-u}$. That is, the revenue marginal product of capital in each sector (MRPK_p and MRPK_g) is equal to the capital cost of each sector and the revenue marginal product of capital for the whole firm (MRPK) is a weighted average of the capital costs in the two sectors, with the weights given but the cost shares of each sector.

It will be useful later on to use the actual revenue functions and substitute in equations (E.1) and (E.2) to obtain,

$$\left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{p_p y_p}{k} \frac{1}{u} = \frac{r + \delta + \lambda}{1 + \phi_p \lambda} \tag{E.3}$$

$$\left(\frac{\sigma_g - 1}{\sigma_g} \right) \frac{p_g y_g}{k} \frac{1}{1-u} = \frac{r + \delta + \lambda}{1 + \phi_g \lambda} \tag{E.4}$$

and using the production function one can write them as

$$\left(\frac{\sigma_p - 1}{\sigma_p} \right) p_p \, s = \frac{r + \delta + \lambda}{1 + \lambda \phi_p} \tag{E.5}$$

$$\left(\frac{\sigma_g - 1}{\sigma_g} \right) p_g \, s = \frac{r + \delta + \lambda}{1 + \lambda \phi_g} \tag{E.6}$$

Finally, dividing these two equations we get an expression for the optimal relative prices,

$$\frac{p_p}{p_g} = \frac{1+\lambda\phi_g}{1+\lambda\phi_p}\frac{(\sigma_g - 1)/\sigma_g}{(\sigma_p - 1)/\sigma_p} \tag{E.7}$$

Note that whenever $\sigma_p = \sigma_g$, $p_g/p_p = 1$ for firms without binding financial frictions ($\lambda = 0$). For firms with binding financial frictions ($\lambda > 0$) $p_g/p_p < 1$ ($p_g/p_p > 1$) whenever $\phi_g > \phi_p$ ($\phi_g < \phi_p$) because production is shifted towards the sector that provides better collateral, and $p_g/p_p = 1$ whenever $\phi_g = \phi_p$.

E.1 Some preliminary results

Lemma 1 *The terms $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$ describing the cost of capital for the production of the private sector and the public sector goods respectively, are (a) strictly below $1/\phi_p$ and $1/\phi_g$ respectively, (b) increasing in λ, and (c) strictly above $r+\delta$ when $\lambda > 0$, if and only if $\phi_p < (\delta + r)^{-1}$ and $\phi_g < (\delta + r)^{-1}$ respectively.*

Proof: Part (a) is straightforward:

$$\frac{r+\delta+\lambda}{1+\lambda\phi_p} < \frac{1}{\phi_p} \Leftrightarrow \phi_p(r+\delta+\lambda) < (1+\lambda\phi_p) \Leftrightarrow \phi_p(r+\delta) < 1 \Leftrightarrow \phi_p < (r+\delta)^{-1}$$

For part (b) note that

$$\frac{d}{d\lambda}\left(\frac{r+\delta+\lambda}{1+\lambda\phi_p}\right) \propto (1+\lambda\phi_p) - \phi_p(r+\delta+\lambda) > 0 \Leftrightarrow \phi_p(r+\delta) < 1 \Leftrightarrow \phi_p < (r+\delta)^{-1}$$

Finally, part (c) is proved by noting that $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ equals $r+\delta$ whenever $\lambda = 0$ and its derivative w.r.t. λ is positive, see part (b). The same arguments apply for $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$. ■

Proposition 1 *Holding s constant, more constrained firms sell less to the private sector, sell less to the public sector, and demand less capital if both $\phi_p < (\delta + r)^{-1}$ and $\phi_g < (\delta + r)^{-1}$.*

Proof: Let's combine the FOC (E.3) with the demand equation to produce the expression,

$$y_p = \left(\frac{\sigma_p - 1}{\sigma_p} B_p s \frac{1+\lambda\phi_p}{r+\delta+\lambda}\right)^{\sigma_p}$$

Then, by virtue of Lemma 1 y_p falls with λ whenever $\phi_p < (\delta + r)^{-1}$. The case for y_g is analogous. Finally, note that total output is split between private sector and public sector sales, that is, $y_p + y_g = f(s,k) = sk$, so the derivative of capital with respect to λ is just,

$$\frac{dk}{d\lambda} = \frac{1}{s}\left(\frac{dy_p}{d\lambda} + \frac{dy_g}{d\lambda}\right)$$

which is negative given the previous results in this Proposition. ■

Lemma 2 *The optimal unconstrained capital for the private and the public sector respectively cannot be self-financed through its own revenues if and only if* $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < 1$ *and* $\phi_g \frac{\sigma_g}{\sigma_g - 1}(r+\delta) < 1$ *respectively.*

Proof: The optimal unconstrained solution for the private sector capital is given by equation (E.3) when $\lambda = 0$, which implies $\frac{p_p y_p}{k}\frac{1}{u} = \frac{\sigma_p}{\sigma_p - 1}(r+\delta)$. When $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < 1 \Leftrightarrow \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < \phi_p^{-1}$ this leads to $\frac{p_p y_p}{k}\frac{1}{u} < \phi_p^{-1} \Leftrightarrow \phi_p p_p y_p < uk$, that is, the optimal unconstrained capital for the private sector, uk, cannot be self-financed through its own revenues. The proof for the public sector capital is analogous by use of the FOC (E.4) ■

Proposition 2 *Entrepreneurs with zero net worth are financially constrained if both* $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < 1$ *and* $\phi_g \frac{\sigma_g}{\sigma_g - 1}(r+\delta) < 1$.

Proof: Note that if both $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < 1$ and $\phi_g \frac{\sigma_g}{\sigma_g - 1}(r+\delta) < 1$, then following Lemma 2 both $\phi_p p_p y_p < uk$ and $\phi_g p_g y_g < (1-u)k$. Adding them up leads to $\phi_p p_p y_p + \phi_g p_g y_g < k$, which implies that the capital of the unconstrained solution cannot be financed through revenue based constraints and hence entrepreneurs with zero net worth are constrained. ■

Lemma 3 *The term* $\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k}$ *describing the share of capital that can be self-financed through revenues is positive and strictly smaller than one for constrained firms.*

Proof: That this term is positive is straightforward. To show that it is lower than one, note that for constrained firms the borrowing constraint holds with equality. Hence, for $a \geq 0$ it must be that $k \geq \phi_p p_p y_p + \phi_g p_g y_g$ or $\phi_p \frac{p_p y_p}{k} + \phi_g \frac{p_g y_g}{k} \leq 1$ (with strict equality for $a = 0$). Given our revenue function, the marginal products are proportional to the average products $\frac{\partial p_p y_p}{\partial k} = \left(\frac{\sigma_p - 1}{\sigma_p}\right)\frac{y_p p_p}{k}$ and $\frac{\partial p_g y_g}{\partial k} = \left(\frac{\sigma_g - 1}{\sigma_g}\right)\frac{y_g p_g}{k}$, so we can rewrite

$$\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k} = \phi_p \left(\frac{\sigma_p - 1}{\sigma_p}\right)\frac{p_p y_p}{k} + \phi_g \left(\frac{\sigma_g - 1}{\sigma_g}\right)\frac{p_g y_g}{k}$$

Note that $\sigma_p > 1$ and $\sigma_g > 1$ implies $\frac{\sigma_p - 1}{\sigma_p} < 1$ and $\frac{\sigma_g - 1}{\sigma_g} < 1$ (the marginal products are below the average products), and hence it is the case that $\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k} < \phi_p \frac{p_p y_p}{k} + \phi_g \frac{p_g y_g}{k} \leq 1$ ■

Lemma 4 *The term* $\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u}$ *describing the increase in credit that can be achieved by reallocation output to the private sector has the sign of* $(\phi_p - \phi_g)$ *for constrained firms.*

Proof: Using the FOC for u, we can write:

$$\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u} = \frac{\partial p_p y_p}{\partial u}\left[\phi_p - \phi_g \frac{1+\lambda\phi_p}{1+\lambda\phi_g}\right] = \frac{\partial p_p y_p}{\partial u}\phi_g\left[\frac{\phi_p}{\phi_g} - \frac{\lambda^{-1}+\phi_p}{\lambda^{-1}+\phi_g}\right]$$

Note that with $\phi_p > \phi_g$ ($\phi_p < \phi_g$), this expression is positive (negative) when λ tends to zero, it decreases (increases) monotonically with λ, and tends to zero when λ tends to infinity. ■

E.2 Firms without procurement

We start analyzing the production problem for firms without procurement ($d = 0$).

E.2.1 Unconstrained firms

With $\lambda = 0$ the FOC for k becomes, $\frac{\partial p_p y_p}{\partial k} = r + \delta$, i.e., firms must equalize the marginal revenue product of capital to the cost of capital. This equation defines the optimal demand of capital $k^* (s, a, 0)$ for every entrepreneur of type $(s, a, d = 0)$. In particular, one gets $\frac{\sigma-1}{\sigma} \frac{p_p y_p}{k} = r + \delta$ and substituting for the revenue function yields the optimal demand for capital

$$k^* (s, a, 0) = \left[\left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{B_p}{r + \delta} \right]^{\sigma} s^{\sigma - 1} \tag{E.8}$$

Next, note that profits are given by $\pi = p_p y_p - (r + \delta) k$, which given the optimal choice of capital can be written as $\pi = \frac{1}{\sigma_p} p_p y_p$ or $\pi = \frac{1}{\sigma_p - 1} (r + \delta) k$. Substituting optimal capital demand to the revenue function gives $p_p y_p = B_p \left[\left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{B_p}{r+\delta} \right]^{\sigma_p - 1} s^{\sigma_p - 1}$, which can be substituted back to the profit function to obtain:

$$\pi^* (s, a, 0) = \frac{1}{\sigma_p} \left[\left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{1}{r + \delta} \right]^{\sigma_p - 1} B_p^{\sigma} s^{\sigma_p - 1} \tag{E.9}$$

Hence, capital demand and profits increase monotonically with the shock s and are independent from net worth a.

E.2.2 Constrained firms

If the firm is constrained, then $\lambda > 0$ and the FOC of the problem are:

$$(1 + \lambda \phi_p) \frac{\partial p_p y_p}{\partial k} = r + \delta + \lambda \tag{E.10}$$

$$k = \phi_a a + \phi_p p_p y_p \tag{E.11}$$

which determine k and λ. In particular, the borrowing constraint (E.11) defines capital demand $k (s, a, 0)$, the FOC (E.10), delivers the shadow value of the constraint $\lambda (s, a, 0)$, and the objective function delivers the profit function $\pi (s, a, 0)$. The next propositions characterize the derivatives of these functions with respect to the state variables a and s.

Totally differentiating equation (E.11) in turn with respect to a and s yields,

$$\frac{\partial k(s,a,0)}{\partial a} = \phi_a \left(1-\phi_p \frac{\partial p_p y_p}{\partial k}\right)^{-1} \tag{E.12}$$

$$\frac{\partial k(s,a,0)}{\partial s} = \phi_p \frac{\partial p_p y_p}{\partial s}\left(1-\phi_p \frac{\partial p_p y_p}{\partial k}\right)^{-1} \tag{E.13}$$

With $\phi_p = 0$ we are in the case without earnings-based collateral constraints and these derivatives are just equal to ϕ_a and 0 respectively: higher net worth allows to operate with more capital but higher productivity does not. With $\phi_p > 0$ both derivatives are positive, that is, constrained firms with more net worth or higher productivity operate with more capital. Indeed, in this case $\frac{\partial k(s,a,0)}{\partial a} > \phi_a$ because an increase in net worth has a multiplier effect through the increase in revenues and the easing of the earnings-based financial constraint (see Lemma 3). This is stated in the next proposition:

Proposition 3 *The derivative of $k(s,a,0)$ w.r.t. a is positive, while the derivative of $k(s,a,0)$ w.r.t. s is positive as long as $\phi_p > 0$ (and zero othwewise).*

Proof: The derivatives of $k(s,a,0)$ with respect to a and s are given by equation (E.12) and (E.13). $\phi_a \geq 1$ and Lemma 3 states that $\phi_p \frac{\partial p_p y_p}{\partial k} < 1$, so the derivative with respect to a is strictly positive. For the derivative with respect to s, note additionally that $\frac{\partial p_p y_p}{\partial s} > 0$. Hence, this derivative is strictly positive (zero) if $\phi_p > 0$ ($\phi_p = 0$). ■

Note also that the derivatives of capital with respect to a and s are higher for more constrained firms (higher λ) because the multiplier effect of the earnings-based constraints is larger for firms with higher marginal product of capital, that is, the increase in capital demand with net worth a or productivity s is larger for more financially constrained firms.

Corollary 1 *The derivatives of $k(s,a,0)$ w.r.t. a and s increase with λ.*

Proof: The derivatives are characterized by equations (E.12) and (E.13). Using the FOC (E.10) and the fact that $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$ we can further rewrite them as

$$\frac{\partial k(s,a,0)}{\partial a} = \phi_a \left(1-\phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \tag{E.14}$$

$$\frac{\partial k(s,a,0)}{\partial s} = \phi_p \frac{k}{s}\frac{r+\delta+\lambda}{1+\lambda\phi_p}\left(1-\phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \tag{E.15}$$

To prove this corollary it is enough to show that the term $(r+\delta+\lambda)/(1+\lambda\phi_p)$ in equations (E.31) and (E.32) increases with λ, which is proved in Lemma 1. ■

Next, (E.10) allows to recover $\lambda(s,a,0)$. It can be shown that $\lambda(s,a,0)$ declines with a — wealthier entrepreneurs can finance larger amounts of capital and are hence less constrained—

and increases with s —s increases optimal capital by more than it increases the amount of capital that can be self-financed through revenues. This is stated formally in Proposition 4.

Proposition 4 *The derivative of $\lambda(s,a,0)$ w.r.t. a is negative, while w.r.t. s it is positive as long as $a > 0$ (and zero otherwise).*

Proof: Equation (E.10) can be rewritten as $\frac{\partial p_p y_p}{\partial k} = \frac{r+\delta+\lambda}{1+\lambda\phi_p}$. The r.h.s, the cost of capital, increases with λ, see Lemma 1. Hence, the sign of the derivative of $\lambda(s,a,0)$ with respect to a or s equals the sign of the derivative of $\frac{\partial p_p y_p}{\partial k}$ with respect to a or s. We start by obtaining an expression of the marginal revenue product of capital by use of the revenue function:

$$\frac{\partial p_p y_p}{\partial k} = \frac{\sigma_p - 1}{\sigma_p}\frac{p_p y_p}{k} = \frac{\sigma_p - 1}{\sigma_p} B_p s^{\frac{\sigma_p - 1}{\sigma_p}} k^{-\frac{1}{\sigma_p}}$$

where $\frac{\partial p_p y_p}{\partial k}$ declines with $k(s,a,0)$. For net worth a it is straightforward to see that $\lambda(s,a,0)$ declines with a because $k(s,a,0)$ increases with a, see Proposition 3. For the shock s we take the derivative of the marginal revenue product of capital w.r.t. s, and asking it to be non-negative delivers:

$$\frac{\partial^2 p_p y_p}{\partial k \partial s} \propto \left[(\sigma_p - 1) - \frac{\partial k}{\partial s}\frac{s}{k}\right] \geq 0$$

where the first term reflects the positive direct effect of s on the marginal revenue product of capital for fixed capital, while the second term reflects the negative indirect effect of s on the marginal revenue product of capital through its induced increase in the choice of capital. Using $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$, equation (E.13) shows that

$$\frac{\partial k}{\partial s}\frac{s}{k} = \phi_p \frac{\partial p_p y_p}{\partial k}\left(1 - \phi_p \frac{\partial p_p y_p}{\partial k}\right)^{-1}$$

Then, we can rewrite

$$(\sigma_p - 1) - \frac{\partial k}{\partial s}\frac{s}{k} \geq 0 \quad \Leftrightarrow \quad \phi_p \frac{\partial p_p y_p}{\partial k} \leq \frac{\sigma_p - 1}{\sigma_p} \quad \Leftrightarrow \quad k \geq \phi_p p_p y_p$$

where the last step uses the fact that $\frac{\partial p_p y_p}{\partial k} = \frac{\sigma_p - 1}{\sigma_p}\frac{p_p y_p}{k}$. Note that whenever a firm has zero net worth it will be able to self-finance capital up to the point $k = \phi_p p_p y_p$. In this case the derivative of $\lambda(s,a,0)$ with respect to s will be zero. Whenever a firm owns $a > 0$ then capital k is going to be above $\phi_p p_p y_p$ and the derivative of $\lambda(s,a,0)$ w.r.t. s is positive. ■

Next, with Corollary 1 and Proposition 4, one can also show that $\frac{\partial^2 k(s,a,0)}{\partial a^2} < 0$ (the increase in capital due to an increase in net worth is larger for firms with less net worth) and that $\frac{\partial^2 k(s,a,0)}{\partial a \partial s} > 0$ (the increase in capital due to an increase in net worth is larger for firms with higher productivity), see Corollary 2.

Corollary 2 *The derivative of $\partial k(s,a,0)/\partial a$ w.r.t. a is negative, while the derivative of $\partial k(s,a,0)/\partial a$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: By the chain rule we can write

$$\begin{aligned}
\frac{\partial^2 k(s,a,0)}{\partial a^2} &= \frac{\partial^2 k(s,a,0)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,0)}{\partial a} \\
\frac{\partial^2 k(s,a,0)}{\partial a \partial s} &= \frac{\partial^2 k(s,a,0)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,0)}{\partial s}
\end{aligned}$$

The first derivative in the r.h.s. of these expressions is positive by Corollary 1. Hence, the sign of the derivatives $\frac{\partial^2 k(s,a,0)}{\partial a^2}$ and $\frac{\partial^2 k(s,a,0)}{\partial a \partial s}$ is the same as the sign of the derivatives $\frac{\partial \lambda(s,a,0)}{\partial a}$ and $\frac{\partial \lambda(s,a,0)}{\partial s}$ described in Proposition 4. ■

Finally, we can also characterize the derivatives of the profit function $\pi(s,a,0)$, as

$$\frac{\partial \pi(s,a,0)}{\partial a} = \left[\frac{\partial p_p y_p}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,0)}{\partial a} \tag{E.16}$$

$$\frac{\partial \pi(s,a,0)}{\partial s} = \left[\frac{\partial p_p y_p}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,0)}{\partial s} + \frac{\partial p_p y_p}{\partial s} \tag{E.17}$$

We can substitute the partial derivatives of capital w.r.t. a and s described by (E.12) and (E.13) into equations (E.16) and (E.17) respectively. Then, using the FOC in (E.11) yields

$$\frac{\partial \pi(s,a,0)}{\partial a} = \phi_a \lambda(s,a,0) \tag{E.18}$$

$$\frac{\partial \pi(s,a,0)}{\partial s} = (1+\phi_p \lambda(s,a,0))\frac{\partial p_p y_p}{\partial s} \tag{E.19}$$

Profits increase with a since more net worth allows to increase constrained capital. They increase with s for two reasons. First, there is the direct increase of revenues with s for given capital. Second, if $\phi_p > 0$, a larger s implies higher revenues and hence more borrowing and higher capital. For this, see the next Proposition, with a Corollary on second derivatives.

Proposition 5 *The derivatives of $\pi(s,a,0)$ w.r.t. a and s are positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (E.18) and (E.19). These derivatives are positive because $\lambda(s,a,0) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ (see the revenue function). ■

Corollary 3 *The derivative of $\partial \pi(s,a,0)/\partial a$ w.r.t. a is negative, while the derivative of $\partial \pi(s,a,0)/\partial s$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: Using equation (E.18) we can write the second derivatives as $\frac{\partial^2\pi(s,a,0)}{\partial a^2} = \phi_a \frac{\partial\lambda(s,a,0)}{\partial a}$ and $\frac{\partial^2\pi(s,a,0)}{\partial a \partial s} = \phi_a \frac{\partial\lambda(s,a,0)}{\partial s}$. Then, one only needs to check the signs of the derivatives of λ in Proposition 4. ■

E.2.3 Binding constraints

Finally, we need to characterize the set of entrepreneurs that are financially constrained. Under Assumption 1, Proposition 2 says that $k(s,0,0) < k^*(s,0,0)$, and we have shown that $\frac{\partial k(s,a,0)}{\partial a} > 0$ and that $k^*(s,a,0)$ is invariant in a. Hence, for every s there will be a unique threshold $\underline{a}(s,0)$ satisfying $k(s,a,0) = k^*(s,a,0)$ such that for every s entrepreneurs with $a \geq \underline{a}(s,0)$ are unconstrained while entrepreneurs with $a < \underline{a}(s,0)$ are constrained.

E.3 Firms with procurement

We now analyze the production problem for firms with procurement ($d = 1$), given $\sigma_p = \sigma_g = \sigma$.

E.3.1 Unconstrained firms

With $\lambda = 0$ the FOC for k and u become $\frac{\partial p_p y_p}{\partial u} + \frac{\partial p_g y_g}{\partial u} = 0$ and $\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} = r + \delta$, which states that unconstrained firms allocate output between the two sectors to equalize the marginal revenues and choose capital such that the marginal revenue product of capital equals the capital costs. These two equations determine the optimal capital demand $k^*(s,a,1)$ and allocation of output in the private sector $u^*(s,a,1)$ for entrepreneurs of type $(s,a,d=1)$. In particular, the FOC for k can be written as $\frac{\sigma-1}{\sigma}\frac{p_p y_p + p_g y_g}{k} = r + \delta$. Substituting for the revenue functions yields the optimal demand for capital $k^*(s,a,1) = \left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma}\left(B_p^{\sigma} + B_g^{\sigma}\right)s^{\sigma-1}$. Using the FOC for u implies $\frac{p_p y_p}{ku} = \frac{p_g y_g}{k(1-u)}$ where substituting the revenue functions yields:

$$u^*(s,a,1) = \left(1 + \left(\frac{B_g}{B_p}\right)^{\sigma}\right)^{-1} \tag{E.20}$$

Clearly $k^*(s,a,1)$ increases monotonically with the shock s and is invariant with the net worth a, while $u^*(s,a,1)$ is independent from both s and a and is only determined by the relative demands B_p/B_g. Next, note that profits are given by $\pi = p_p y_p + p_g y_g - (r+\delta)k$, which given the condition for the optimal choice of capital can be written as $\pi = \frac{1}{\sigma}(p_p y_p + p_g y_g)$ or $\pi = \frac{1}{\sigma-1}(r+\delta)k$. Substituting the optimal capital demand into the revenue function gives total revenues as $p_p y_p + p_g y_g = \left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma-1}\left(B_p^{\sigma} + B_g^{\sigma}\right)s^{\sigma-1}$, which can be substituted back into the profit function to obtain

$$\pi^*(s,a,1) = \frac{1}{\sigma}\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma-1}\left(B_p^{\sigma} + B_g^{\sigma}\right)s^{\sigma-1} \tag{E.21}$$

The profit function increases with productivity s and is invariant with assets a.

E.3.2 Constrained firms.

We now explain how to determine $k(s,a,1)$, $u(s,a,1)$, and $\lambda(s,a,1)$ for constrained firms. The characterization of these functions is simple whenever $\phi_g = \phi_p$ and more involved when not. To characterize $u(s,a,1)$ let's start by noting that the FOC for u can be rewritten as in (E.7) and that after substituting prices we obtain,

$$\frac{u}{1-u} = \left(\frac{B_p}{B_g}\right)^{\sigma}\left(\frac{1+\lambda\phi_p}{1+\lambda\phi_g}\right)^{\sigma} \tag{E.22}$$

To characterize $k(s,a,1)$ we totally differentiate the binding borrowing constraint with respect to a and s in turn, which gives,

$$\frac{\partial k}{\partial a} = \left[\phi_a + \left(\phi_p\frac{\partial p_p y_p}{\partial u} + \phi_g\frac{\partial p_g y_g}{\partial u}\right)\frac{du}{da}\right]\left[1-\left(\phi_p\frac{\partial p_p y_p}{\partial k} + \phi_g\frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{E.23}$$

$$\frac{\partial k}{\partial s} = \left[\left(\phi_p\frac{\partial p_p y_p}{\partial s} + \phi_g\frac{\partial p_g y_g}{\partial s}\right) + \left(\phi_p\frac{\partial p_p y_p}{\partial u} + \phi_g\frac{\partial p_g y_g}{\partial u}\right)\frac{du}{ds}\right]\left[1-\left(\phi_p\frac{\partial p_p y_p}{\partial k} + \phi_g\frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{E.24}$$

Finally, the derivatives of the profit function $\pi(s,a,1)$ are given by

$$\frac{\partial \pi(s,a,1)}{\partial a} = \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,1)}{\partial a} + \left[\frac{\partial p_p y_p}{\partial u} + \frac{\partial p_g y_g}{\partial u}\right]\frac{\partial u(s,a,1)}{\partial a} \tag{E.25}$$

$$\begin{aligned}\frac{\partial \pi(s,a,1)}{\partial s} &= \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,1)}{\partial s} \\ &+ \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right]\frac{\partial u(s,a,1)}{\partial s} + \frac{\partial p_p y_p}{\partial s}\end{aligned} \tag{E.26}$$

Now, substituting (E.1), (E.2), and (E.23) into (E.25) and using the FOC for u we obtain

$$\frac{\partial \pi(s,a,1)}{\partial a} = \phi_a\lambda(s,a,1) \tag{E.27}$$

while substituting (E.1), (E.2), and (E.24) into (E.26) and using the FOC for u we obtain

$$\frac{\partial \pi(s,a,1)}{\partial s} = (1+\phi_p\lambda(s,a,1))\frac{\partial p_p y_p}{\partial s} + (1+\phi_g\lambda(s,a,1))\frac{\partial p_g y_g}{\partial s} \tag{E.28}$$

Profits increase with a because more net worth allows to increase capital and hence profits. Profits increase with s for two reasons. First, there is the direct increase of revenues with s for given capital. Second, if $\phi_p > 0$ and/or $\phi_g > 0$ the increase in revenues with s allows to increase capital, which in turn increases profits.

For the case $\phi_g = \phi_p$ it can be shown that $u(s,a,1) = u^*(s,a,1)$ —as revenues from both sectors are equally pledgeable— and hence $u(s,a,1)$ is invariant in a and s. This makes the problem analogous to the case without procurement ($d = 0$), and hence the derivatives of $k(s,a,1)$, $\lambda(s,a,1)$, and $\pi(s,a,1)$ with respect to a and s are as in the $d = 0$ case. This can be seen in the next propositions.

Proposition 6 *When $\phi_g = \phi_p$, the optimal choice of $u(s,a,1)$ is as in the unconstrained case and it is hence independent from a and s.*

Proof: Equation (E.22) clearly shows that whenever $\phi_g = \phi_p$ the optimal solution for u for constrained firms is equal to the one for unconstrained firms, see equation (E.20). This means that $u(s,a,1)$ is independent from s and a and only determined by the relative demands B_p/B_g of each sector. ■

Proposition 7 *When $\phi_g = \phi_p$, the derivative of $k(s,a,1)$ w.r.t. a is positive, while the derivative of $k(s,a,1)$ w.r.t. s is positive as long as $\phi_p > 0$ (and zero othwewise).*

Proof: Note that with $\phi_g = \phi_p$ the optimality condition for u implies that $\frac{\partial p_g y_g}{\partial u} = -\frac{\partial p_p y_p}{\partial u}$ and hence we can rewrite equations (E.23) and (E.24) as follows,

$$\frac{\partial k}{\partial a} = \phi_a \left[1 - \phi_p\left(\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{E.29}$$

$$\frac{\partial k}{\partial s} = \phi_p\left(\frac{\partial p_p y_p}{\partial s} + \frac{\partial p_g y_g}{\partial s}\right)\left[1 - \phi_p\left(\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{E.30}$$

Given $\phi_a \geq 1$ and $\phi_p > 0$ both $\partial k/\partial a$ and $\partial k/\partial s$ are positive by Lemma 3. If $\phi_p = 0$ then $\partial k/\partial s = 0$. ■

Corollary 4 *When $\phi_g = \phi_p$, the derivatives of $k(s,a,1)$ w.r.t. a and s increase with λ.*

Proof: The FOC for k can be written as $\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} = \frac{r+\delta+\lambda}{1+\lambda\phi_p}$. Then, using the fact that $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$ we can rewrite equations (E.29) and (E.30) as

$$\frac{\partial k(s,a,1)}{\partial a} = \phi_a \left(1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \tag{E.31}$$

$$\frac{\partial k(s,a,1)}{\partial s} = \phi_p \frac{k}{s}\frac{r+\delta+\lambda}{1+\lambda\phi_p}\left(1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \tag{E.32}$$

To prove this corollary it is enough to show that the term $(r+\delta+\lambda)/(1+\lambda\phi_p)$ in equations (E.31) and (E.32) increases with λ, which is proved in Lemma 1. ■

Proposition 8 *When $\phi_g = \phi_p$, the derivative of $\lambda(s,a,1)$ w.r.t. a is negative, while the derivative of $\lambda(s,a,1)$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: Note that the FOC for k_p is given by equation (E.1). Because u is invariant in a and s, see Proposition 6, the proof of Proposition 8 for the case $d = 0$ carries over. ■

Corollary 5 *When $\phi_g = \phi_p$, the derivative of $\partial k(s,a,1)/\partial a$ w.r.t. a is negative, while the derivative of $\partial k(s,a,1)/\partial a$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: By the chain rule we can write

$$\frac{\partial^2 k(s,a,1)}{\partial a^2} = \frac{\partial^2 k(s,a,1)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,1)}{\partial a}$$
$$\frac{\partial^2 k(s,a,1)}{\partial a \partial s} = \frac{\partial^2 k(s,a,1)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,1)}{\partial s}$$

The first derivative in the r.h.s. of these expressions is positive by Corollary 4. Hence, the sign of the derivatives $\frac{\partial^2 k(s,a,1)}{\partial a^2}$ and $\frac{\partial^2 k(s,a,1)}{\partial a \partial s}$ is the same as the sign of the derivatives $\frac{\partial \lambda(s,a,1)}{\partial a}$ and $\frac{\partial \lambda(s,a,1)}{\partial s}$ described in Proposition 8. ■

Proposition 9 *When $\phi_g = \phi_p$, the derivatives of $\pi(s,a,1)$ w.r.t. a and s are positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (E.27) and (E.28). These derivatives are positive because $\lambda(s,a,1) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ and $\frac{\partial p_g y_g}{\partial s} > 0$ (see the revenue functions). ■

Corollary 6 *When $\phi_g = \phi_p$, the derivative of $\partial \pi(s,a,1)/\partial a$ w.r.t. a is negative, while the derivative of $\partial \pi(s,a,1)/\partial s$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: Using (E.27) we can write the second derivatives as, $\frac{\partial^2 \pi(s,a,1)}{\partial a^2} = \phi_a \frac{\partial \lambda(s,a,1)}{\partial a}$ and $\frac{\partial^2 \pi(s,a,1)}{\partial a \partial s} = \phi_a \frac{\partial \lambda(s,a,1)}{\partial s}$. The signs of the derivatives of λ follow from Proposition 8. ■

The case $\phi_g > \phi_p$ is more involved because $u(s,a,1)$ changes with a and s. It can be shown that firms with more net worth are less constrained and hence run larger firms and sell a higher fraction of output to the private sector, which offers lower collateral value. More productive firms are able to run larger firms thanks to the earnings-based constraints but are more constrained —because their optimal capital is even larger— and hence sell a lower fraction of output to the private sector. This means that firms with larger s sell a larger quantity to the public sector but they may either sell a larger or smaller quantity to the private sector. This is proved in the following propositions.

Lemma 5 *The sign of the derivative of u w.r.t. λ is the same as the sign of $(\phi_p - \phi_g)$, that is, more constrained firms shift their output relatively towards the sector whose revenues provide better collateral.*

Proof: Equation (E.22) implies $du/d\lambda < 0$ when $\phi_g > \phi_p$ and the opposite when $\phi_g < \phi_p$. ■

Proposition 10 *When $\phi_g > \phi_p$, the derivatives of $u(s,a,1)$, $k(s,a,1)$, and $\lambda(s,a,1)$ w.r.t. a are positive, positive, and negative respectively,*

Proof: First note that, following Lemma 5, $du/d\lambda < 0$ when $\phi_g > \phi_p$ and that Proposition 1 says that $dk/d\lambda < 0$. That is, more constrained entrepreneurs tilt production towards the sector with higher collateral value and run smaller firms. Next, using the FOC (E.3) and (E.4), the demand equations, and the production function we can write,

$$k_p = \left(\frac{\sigma_p - 1}{\sigma_p} B_p \frac{1 + \lambda\phi_p}{r + \delta + \lambda}\right)^{\sigma_p} s^{\sigma_p - 1} \quad \text{and} \quad k_g = \left(\frac{\sigma_p - 1}{\sigma_p} B_g \frac{1 + \lambda\phi_g}{r + \delta + \lambda}\right)^{\sigma_p} s^{\sigma_p - 1}$$

Adding them up, by the chain rule, let us express: $\frac{\partial k}{\partial a} = \frac{\partial k}{\partial \lambda}\frac{\partial \lambda}{\partial a}$. Also, using equation (E.22) and the chain rule we can write $\frac{\partial u}{\partial a} = \frac{\partial u}{\partial \lambda}\frac{\partial \lambda}{\partial a}$. These two expressions state that $\frac{\partial k}{\partial a}$ and $\frac{\partial u}{\partial a}$ should have the same sign because both k and u fall with λ. Given this, equation (E.23) implies that $\frac{\partial k}{\partial a} > 0$ and $\frac{\partial u}{\partial a} > 0$. To see why, recall that by Lemma 3 the denominator is positive. In addition, the term $\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u}$ is negative whenever $\phi_g > \phi_p$ see Lemma 4. Hence, for $\frac{\partial k}{\partial a} < 0$ we would need $\frac{\partial k}{\partial u} > 0$. That is, given that higher a allows to increase capital through ϕ_a, for higher a to lead to lower capital it must be that entrepreneurs with higher a tilt production towards the sector with lower collateral value. But this would require the signs of $\frac{\partial k}{\partial a}$ and $\frac{\partial u}{\partial a}$ to be different. Instead, $\frac{\partial k}{\partial a} > 0$ can be obtained with $\frac{\partial u}{\partial a} > 0$. It follows that, because $\frac{\partial k}{\partial \lambda} < 0$ and $\frac{\partial k}{\partial a} > 0$, it must be the case that $\frac{\partial \lambda}{\partial a} < 0$. ■

Proposition 11 *When $\phi_g > \phi_p$, the derivatives of $u(s,a,1)$, $k(s,a,1)$, and $\lambda(s,a,1)$ w.r.t. s are negative, positive, and positive respectively,*

Proof: First note that, following Lemma 5, $du/d\lambda < 0$ when $\phi_g > \phi_p$ and that Proposition 1 says that $dk/d\lambda < 0$. That is, more constrained entrepreneurs tilt production towards the sector with higher collateral value and run smaller firms. Next, by the chain rule (see proof of Proposition 10) we can write $\frac{dk}{ds} = \frac{\partial k}{\partial \lambda}\frac{\partial \lambda}{\partial s} + \frac{\partial k}{\partial s}$ and $\frac{du}{ds} = \frac{\partial u}{\partial \lambda}\frac{\partial \lambda}{\partial s}$. We learn two things from here. First, $\frac{dk}{ds} \leq 0$ requires $\frac{\partial \lambda}{\partial s} > 0$ (because $\frac{\partial k}{\partial s} > 0$ and $\frac{\partial k}{\partial \lambda} < 0$). Second, $\frac{\partial \lambda}{\partial s} > 0$ requires $\frac{du}{ds} < 0$ (because $du/d\lambda < 0$). But equation (E.24) shows that if $\frac{du}{ds} < 0$ then it must be $\frac{dk}{ds} > 0$ so this enters a contradiction. Therefore, $\frac{dk}{ds} > 0$. Note that from equation (E.24) $\frac{dk}{ds} > 0$ can be achieved with any sign of $\frac{du}{ds}$. Now, regarding the derivatives of $u(s,a,1)$ and $\lambda(s,a,1)$ with respect to s, two different things can happen. If $\frac{\partial \lambda}{\partial s} \geq 0$ then $\frac{du}{ds} \leq 0$ (this is an if and only if statement), and then $\frac{dk}{ds} > 0$ according to equation (E.24). Instead, if $\frac{\partial \lambda}{\partial s} < 0$ then $\frac{du}{ds} > 0$ (again an if and only if statement) and we can have both $\frac{dk}{ds} > 0$ or $\frac{dk}{ds} < 0$ according to equation (E.24). ■

Proposition 12 *When $\phi_g > \phi_p$, the derivatives of $\pi(s,a,1)$ w.r.t. a and s are positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (E.27) and (E.28). These derivatives are positive because $\lambda(s,a,1) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ and $\frac{\partial p_g y_g}{\partial s} > 0$ (see the revenue functions). ■

Corollary 7 *When $\phi_g > \phi_p$, the derivative of $\partial\pi(s,a,1)/\partial a$ w.r.t. a is negative, while the derivative of $\partial\pi(s,a,1)/\partial s$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: Using (E.27) we can write the second derivatives as, $\frac{\partial^2\pi(s,a,1)}{\partial a^2} = \phi_a \frac{\partial\lambda(s,a,1)}{\partial a}$ and $\frac{\partial^2\pi(s,a,1)}{\partial a \partial s} = \phi_a \frac{\partial\lambda(s,a,1)}{\partial s}$ The signs of the derivatives of λ follow from Propositions 10 and 11. ■

E.4 A procurement shock

Finally, in this Section we analyze how firm choices change upon arrival of a procurement project for the case $\sigma_p = \sigma_g = \sigma$. To do so, we compare the choices of firms in the $(s,a,1)$ state with firms in the $(s,a,0)$ state.

E.4.1 Unconstrained firms

For unconstrained firms, the increase in total capital is given by $\frac{k^*(s,a,1)}{k^*(s,a,0)} = 1 + \left(\frac{B_g}{B_p}\right)^\sigma = \frac{1}{u^*(s,a,1)}$ which implies that $u^*(s,a,1)\,k^*(s,a,1) = k^*(s,a,0)$. Hence, the amount of capital used in the private sector for the unconstrained firm with a procurement project equals the capital stock it was using without procurement. This means that unconstrained firms do not change their private sector operations and increase their capital stock to meet the extra demand. The increase in capital $k^*(s,a,1) - k^*(s,a,0)$ is given by $\left(\frac{B_g}{B_p}\right)^\sigma k^*(s,a,0)$. Because $k^*(s,a,0)$ increases with s and is independent from a, so does the capital increase with procurement. We can also see that the value of a procurement contract increases with firm productivity s and is independent from firm net worth a. This can be seen by use of the expression $\pi = \frac{1}{\sigma-1}(r+\delta)\,k$, which implies that $\pi^*(s,a,1) - \pi^*(s,a,0)$ is proportional to the capital increase $k^*(s,a,1) - k^*(s,a,0)$. This could have also be seen by combining equations (E.9) and (E.21), which allows to express $\pi^*(s,a,1) - \pi^*(s,a,0) = \frac{1}{\sigma}\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma-1} B_g^\sigma s^{\sigma-1}$.

E.4.2 Constrained firms

For financially constrained firms, the effects of a procurement shock are more intricate, depending on the size of ϕ_g relative to ϕ_p and the net worth of the firm.

Lemma 6 *A procurement shock generates a private sector negative spillover if and only if the procurement shock makes the firm more constrained, that is, $k_p(s,a,1) < k(s,a,0) \Leftrightarrow \lambda(s,a,1) > \lambda(s,a,0)$.*

Proof: The FOC for optimal k_p given $d = 1$ is given by equation (E.1), where recall $\frac{\partial p_p y_p}{\partial k}\frac{1}{u} = \frac{\partial p_p y_p}{\partial k_p}$. The FOC for the optimal choice of k for a firm with $d = 0$ is given by the same equation (E.1) when $u = 1$. The right-hand side of equation (E.1) increases with λ (see Lemma 1), so more constrained firms have a higher marginal product of capital and a lower level of capital in the private sector. Hence, $k_p(s,a,1) < k(s,a,0) \Leftrightarrow \lambda(s,a,1) > \lambda(s,a,0)$. ∎

Lemma 7 *A procurement shock generates a private sector negative spillover for constrained firms if and only if the chosen production for the public sector cannot be self-financed, that is, if and only if* $\phi_g p_g(s,a,1)\, y_g(s,a,1) < k_g(s,a,1)$

Proof: The demand for capital of constrained firms, with or without procurement, implies

$$\begin{aligned} k_p(s,a,0) - \phi_p p_p(s,a,0)\, y_p(s,a,0) &= \phi_a a \\ k_p(s,a,1) - \phi_p p_p(s,a,1)\, y_p(s,a,1) &= \phi_a a - \left[k_g(s,a,1) - \phi_g p_g(s,a,1)\, y_g(s,a,1)\right] \end{aligned}$$

Importantly, the left-hand side of these equations increases with k_p. To see how, note that the derivative of the left-hand side w.r.t. k_p is equal to $1 - \phi_p \frac{\partial p_p y_p}{\partial k_p} = 1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}$ according to equation (E.1). Now, $\phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p} < 1$ according to Lemma 1, so the derivative is positive. Hence, if $k_p(s,a,1) < k_p(s,a,0)$ then $[k_p(s,a,1) - \phi_p p_p(s,a,1)\, y_p(s,a,1)] < [k_p(s,a,0) - \phi_p p_p(s,a,0)\, y_p(s,a,0)]$ which requires $\phi_g p_g(s,a,1)\, y_g(s,a,1) < k_g(s,a,1)$. ∎

Proposition 13 *When* $\phi_g \le \phi_p$, *a procurement shock for constrained firms generates a private sector negative spillover, that is,* $k_p(s,a,1) < k(s,a,0)$, *makes the firm more constrained, that is,* $\lambda(s,a,1) > \lambda(s,a,0)$, *and production in the government sector cannot be self-financed, that is,* $\phi_g p_g(s,a,1)\, y_g(s,a,1) < k_g(s,a,1)$. *When* $\phi_g > \phi_p$ *the same will happen, with the exception of firms with very small net worth for which the opposite will happen.*

Proof: To prove the first part, let's rewrite the borrowing constraint for $d = 0$ firms as

$$1 = \phi_a \frac{a}{k(s,a,0)} + \phi_p \frac{p_p(s,a,0)\, y_p(s,a,0)}{k(s,a,0)} \tag{E.33}$$

and for $d = 1$ firms as

$$\begin{aligned} 1 &= \phi_a \frac{a}{k_p(s,a,1) + k_g(s,a,1)} + \phi_p \frac{p_p(s,a,1)\, y_p(s,a,1)}{k_p(s,a,1)} \\ &+ (1 - u(s,a,1)) \left[\phi_g \frac{p_g(s,a,1)\, y_g(s,a,1)}{k_g(s,a,1)} - \phi_p \frac{p_p(s,a,1)\, y_p(s,a,1)}{k_p(s,a,1)}\right] \end{aligned} \tag{E.34}$$

If $\phi_g = \phi_p$, firms with $d = 1$ equalize the average product in the public and private sectors, see equations (E.3) and (E.4), so that the third term in equation (E.34) disappears. In this case, if $k_g(s,a,1) = 0$ then equations (E.33) and (E.34) are identical and $k_p(s,a,1) = k(s,a,0)$.

However, because the marginal revenue product in the public sector goes to infinity when $k_g(s,a,1) = 0$, it must be that $k_g(s,a,1) > 0$ and hence comparison of equations (E.33) and (E.34) requires $k_p(s,a,1) < k(s,a,0)$. If $\phi_g < \phi_p$, then the third term in equation (E.34) is negative. This can be easily seen by multiplying both sides of equation (E.3) by ϕ_p and both sides of equation (E.4) by ϕ_g. Then whenever $k_g > 0$ and hence $(1-u) > 0$, equation (E.34) requires $k_p(s,a,1) < k(s,a,0)$ to hold. The second and third parts of the Proposition come from Lemma 6 and Lemma 7 respectively. Finally, for the case $\phi_g > \phi_p$ the third term in equation (E.34) is positive. If $a = 0$ this requires $k_p(s,a,1) > k(s,a,0)$ for equation (E.34) to hold as the first term in the right-hand side of equation (E.34) disappears. For $a > 0$, the first term in the right-hand side of equation (E.34) reappears and offsets this force. More specifically, as a increases, λ falls by Proposition 10, and thus $\frac{p_g(s,a,1)y_g(s,a,1)}{k_g(s,a,1)}$ decreases. In the limit, if a becomes sufficiently large and exceeds the net worth level $a_g^*(s)$ above which the procurement firm is unconstrained, $\frac{p_g(s,a,1)y_g(s,a,1)}{k_g(s,a,1)}$ falls to the unconstrained level of $\frac{\sigma}{\sigma-1}(r+\delta)$, which is strictly smaller than $\frac{1}{\phi_g}$ by Assumption 1. This means that there exists a cutoff level $\bar{a}_g(s)$ such that if $a \in (\bar{a}_g(s), a_g^*(s))$, then $\phi_g p_g(s,a,1)y_g(s,a,1) < k_g(s,a,1)$. And by Lemma 7, this means that the spillover is negative for a in this interval. ■

Proposition 14 *Whenever $\phi_g \geq \phi_p > 0$, a procurement shock generates an increase in firm size, i.e., $k(s,a,1) > k(s,a,0)$, $\forall a, s$. Whenever $\phi_g < \phi_p$ the opposite may happen. In the particular case of $\phi_g = \phi_p = 0$, a procurement shock does not change firm size.*

Proof: We prove the $\phi_g \geq \phi_p > 0$ case by contradiction by showing that if $k(s,a,1) \leq k(s,a,0)$, then the borrowing constraint for the firm with $d = 1$ would not bind, which could not be optimal; so it must be that $k(s,a,1) > k(s,a,0)$. To se why, we start with the case $k(s,a,1) = k(s,a,0)$. In this situation, the firm with $d = 1$ optimally chooses $u(s,a,1) < 1$ because the marginal revenue product of revenues in the public sector tends to infinity as u tends to 1. This generates more revenues and because $\phi_g \geq \phi_p > 0$, Lemma 4 guarantees that this also generates more (unused) borrowing capacity, so it cannot be optimal. If $k(s,a,1) < k(s,a,0)$ and $u(s,a,1) = 1$ this again generates slack in the borrowing constraint because of Lemma 3, and cannot be optimal. But lowering u generates the same or further slack when $\phi_g \geq \phi_p > 0$, see Lemma 4. So $k(s,a,1) < k(s,a,0)$ cannot be optimal either. Note that the argument by contradiction requires that $\phi_g \geq \phi_p > 0$ such that when the firm with $d = 1$ substitutes private revenues with public revenues the borrowing capacity increases. When $\phi_g < \phi_p$, instead, the contrary happens because selling to the government limits the borrowing capacity of the firm, and the proof does not hold. For example, it can be shown that with $0 = \phi_g < \phi_p$ we will have $k(s,a,1) < k(s,a,0)$. Using the financial constraint, the difference in the capital that can be financed with $d = 1$ and $d = 0$ when $\phi_g = 0$ is given by,

$$k(s,a,1) - k(s,a,0) = \phi_p \left[p_p(s,a,1) \, y_p(s,a,1) - p_p(s,a,0) \, y_p(s,a,0) \right]$$

Proposition 13 says that there is a negative private sector spillover, $p_p(s,a,1)\, y_p(s,a,1) < p_p(s,a,0)\, y_p(s,a,0)$ whenever $\phi_g < \phi_p$, and thus $k(s,a,1) < k(s,a,0)$. Finally, note that with $\phi_g = \phi_p = 0$, $k(s,a,1) = k(s,a,0)$ as constrained firms' k is determined only by a. ■

Proposition 15 *Having access to procurement generates extra profits, that is,* $\pi(s,a,1) > \pi(s,a,0)\ \forall s,a$. *Whenever* $\phi_g \leq \phi_p$, *the value of procurement is increasing in net worth; whenever* $\phi_g > \phi_p$, *the value of procurement is generally increasing in net worth except for firms with very low net worth when the opposite will happen. The value of procurement is increasing in firm productivity whenever* $\phi_g \geq \phi_p$.

Proof: The first part is trivial. A firm with $d = 1$ has profits equal to

$$\pi(s,a,1) = p_p(s,a,1) \, y_p(s,a,1) + p_g(s,a,1) \, y_g(s,a,1) - (r+\delta) \, k(s,a,1)$$

and can always replicate the profits of a firm with $d = 0$ by choosing $u(s,a,1) = 1$. Because of our functional form assumptions, the marginal revenue product of capital in the public sector, $\partial p_g y_g / \partial k_g$, tends to infinity whenever $u(s,a,1) = 1$, so it means that it is optimal for any firm with $d = 1$ to choose $u(s,a,1) < 1$ and increase profits compared to the case $u(s,a,1) = 1$ and therefore compared to the case of no procurement. For the second part we want to show that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial a} > 0$. Equations (E.18) and (E.27) imply

$$\frac{\partial \left[\pi(s,a,1) - \pi(s,a,0) \right]}{\partial a} = \phi_a \Big[\lambda(s,a,1) - \lambda(s,a,0) \Big] > 0$$

and the sign of $\lambda(s,a,1) - \lambda(s,a,0)$ is given by Proposition 13. Finally, for the third part we want to show that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} > 0$ whenever $\phi_g \geq \phi_p$. (E.19) and (E.28) imply

$$\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} = (1 + \phi_p \lambda(s,a,1)) \frac{\partial p_p y_p}{\partial s} + (1 + \phi_g \lambda(s,a,1)) \frac{\partial p_g y_g}{\partial s} - (1 + \phi_p \lambda(s,a,0)) \frac{\partial p_p y_p}{\partial s}$$

Note that $\frac{\partial p_p y_p}{\partial s} = \frac{k_p}{s} \frac{\partial p_p y_p}{\partial k_p} = \frac{k_p}{s} \frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and an analogous expression holds for the public good. Substituting these expressions in the above equation gives

$$\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} = \frac{r+\delta+\lambda(s,a,1)}{s} \left[k_p(s,a,1) + k_g(s,a,1) \right] - \frac{r+\delta+\lambda(s,a,0)}{s} k_p(s,a,0)$$

With $\phi_g \geq \phi_p$, Proposition 14 states that $k_p(s,a,1) + k_g(s,a,1) > k_p(s,a,0)$. Therefore, whenever $\lambda(s,a,1) > \lambda(s,a,0)$ we can guarantee that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} > 0$. According to Proposition 13 this will generally happen, except for very low a when $\lambda(s,a,1) < \lambda(s,a,0)$. However, in this case we can still show the statement to be true by showing that $\frac{r+\delta+\lambda(s,a,1)}{s}k_p(s,a,1) > \frac{r+\delta+\lambda(s,a,0)}{s}k_p(s,a,0)$. To show this, we take the FOC for k_p in equation (E.1) to obtain an expression $\lambda = \frac{\frac{\partial p_p y_p}{\partial k_p}-(r+\delta)}{1-\phi_p\frac{\partial p_p y_p}{\partial k_p}}$. Then adding $(r+\delta)$ in both sides, rearranging, and multiplying by k_p in both sides we obtain

$$(r+\delta+\lambda)k_p = [1-\phi_p(r+\delta)]\frac{\frac{\partial p_p y_p}{\partial k_p}}{1-\phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p$$

Using our functional form for the revenue function, we can rewrite the last terms as:

$$\frac{\frac{\partial p_p y_p}{\partial k_p}}{1-\phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p = \frac{\sigma-1}{\sigma}\frac{B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{\sigma-1}{\sigma}}}{1-\phi_p\frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}}$$

Taking the derivative of this object w.r.t. k_p, we have:

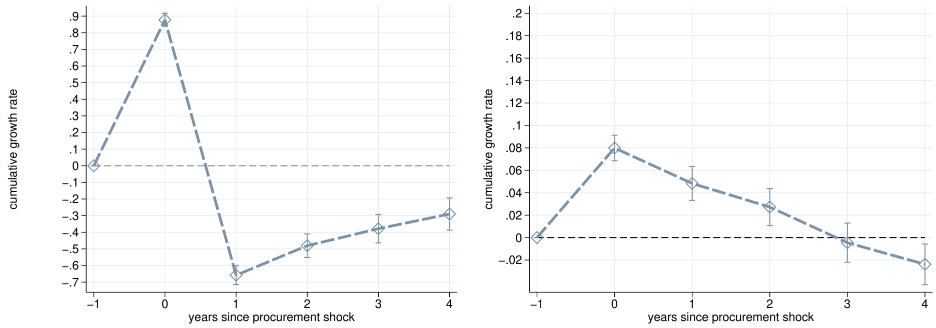
$$\begin{aligned}
\frac{\partial}{\partial k_p}\left(\frac{\frac{\partial p_p y_p}{\partial k_p}}{1-\phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p\right) &\propto \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left[1-\phi_p\frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right]-B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{\sigma-1}{\sigma}}\frac{\sigma-1}{\sigma}\frac{1}{\sigma}\phi_p B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}-1} \\
&= \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left\{1-\phi_p B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right\} \\
&= \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left\{1-\phi_p\frac{p_p y_p}{k_p}\right\} > 0
\end{aligned}$$

where the last inequality follows from the fact that by Lemma 3, $\phi_p\frac{p_p y_p}{k_p} < 1$ for constrained firms. This establishes that for constrained firms, the term $(r+\delta+\lambda)k_p$ must be higher whenever k_p is higher. Therefore, if $\lambda(s,a,1) < \lambda(s,a,0)$, the k_p FOC, implies that $k_p(s,a,1) > k_p(s,a,0)$, which in turns implies $[r+\delta+\lambda(s,a,1)]k_p(s,a,1) > [r+\delta+\lambda(s,a,0)]k_p(s,a,0)$. And trivially, this implies $\frac{r+\delta+\lambda(s,a,1)}{s}[k_p(s,a,1)+k_g(s,a,1)] - \frac{r+\delta+\lambda(s,a,0)}{s}k_p(s,a,0)$, proving the statement whenever $\phi_g \geq \phi_p$ and $\lambda(s,a,1) < \lambda(s,a,0)$. ■

F Identification of model parameters

Our strategy consists of internally calibrating 8 parameters so that the model matches 8 moments. We want to show that our choice of targets is justified by the fact that each of these targets is "particularly informative" of a particular parameter.

First, we draw many parameter combinations based on a 8-dimensional hypercube. Second, we solve for the model's steady state and calculate the relevant moments for all parameter combinations. And third, we plot how the 25th percentile, the 50th, and the 75th percentile of a given moment changes as we move along the vector of its associated parame-



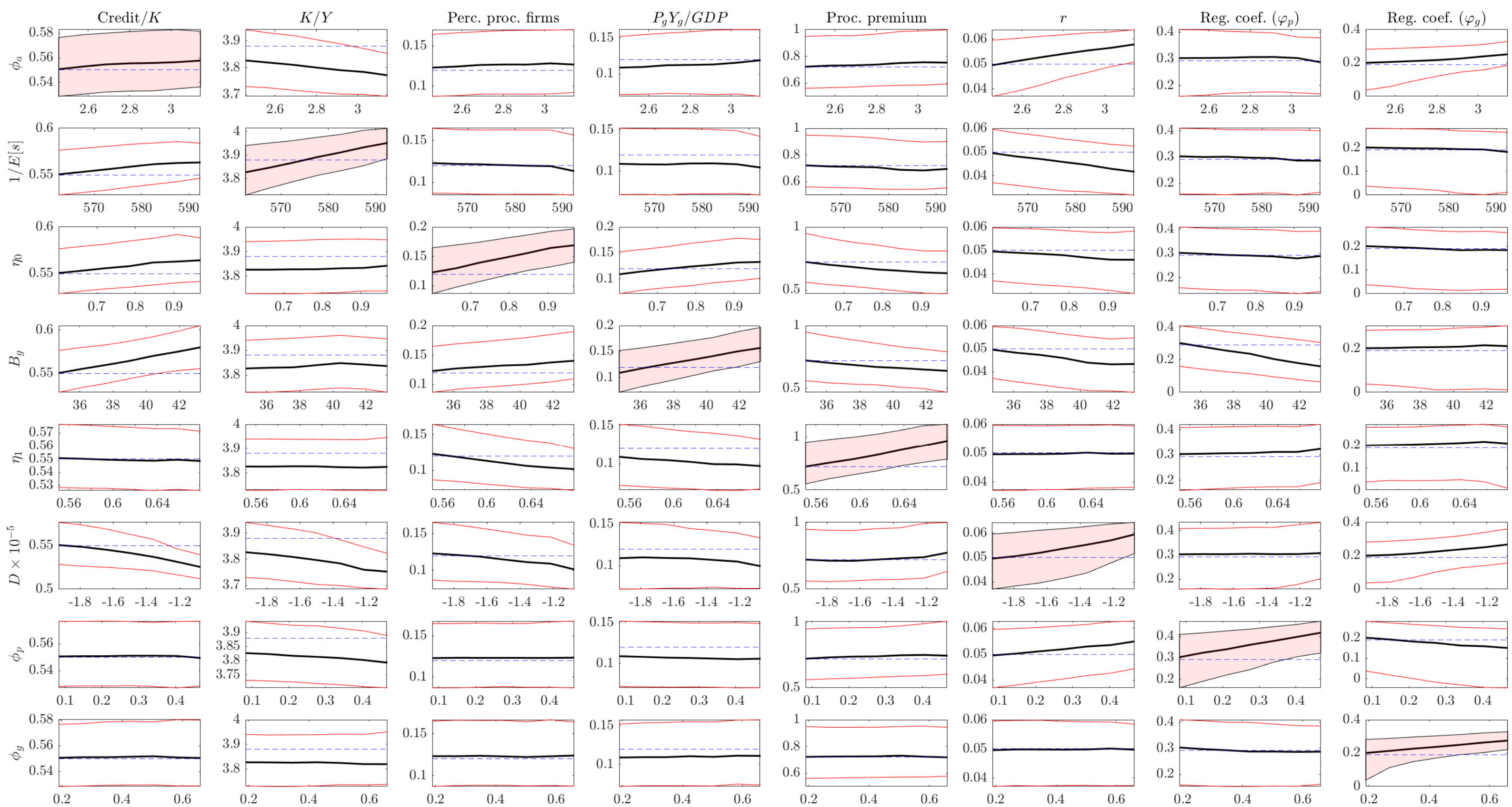
(a) λ **(b)** b

Notes: This figure shows the cumulative estimated impact (β_1^h) of obtaining a procurement contract for horizons $h = 0, 1, 2, 3, 4$. The left panel shows the effects on λ. The right panel shows the effects on b.

Figure A.XV. Treatment effects of procurement in the baseline calibration

ter. Intuitively, this figure shows how a particular moment is affected by a specific parameter letting other parameters move. The steeper the slope of the relationship between the parameter values and percentiles of the moment, the stronger the identification. In Figure A.XVI, we show that each moment is especially informative of one parameter.

Figure A.XVI. Identification of the model parameters



Notes: This figure plots the relationship between percentiles of a given moment and a specific parameter. The thick black line refers to the median, and the thiner black lines above and below refer to the 85th and 15th percentile respectively. The dashed blue line represents the targeted value of the moment. These lines have been generated by solving the steady-state of the model using combinations of parameters drawn from a 8-dimensional hypercube.

G Further details on treatment effects in benchmark

To analyze dynamic treatment effects of procurement in our model and in the data, we estimate the same local projection panel regressions as in Section 3.1. See Figure A.XV.

H Aggregate output, productivity, and prices

GDP and aggregate TFP. We can define GDP in the model as

$$Y \equiv Y_p + P_g Y_g = \mathrm{TFP}_p K_p + P_g \mathrm{TFP}_g K_g = \mathrm{TFP} K \tag{H.1}$$

where $K \equiv K_p + K_g$ and aggregate TFP in units of the private goods is defined by,

$$\mathrm{TFP} \equiv \frac{K_p}{K}\mathrm{TFP}_p + \frac{K_g}{K} P_g \mathrm{TFP}_g \tag{H.2}$$

Sectorial TFP. The TFP for the private and public sectors are given by,

$$\mathrm{TFP}_p \equiv \frac{Y_p}{K_p} = \left[\int_{[0,1]} \left(s_i \frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}}\right)^{\sigma_p - 1} di\right]^{\frac{1}{\sigma_p - 1}}, \quad \mathrm{TFP}_g \equiv \frac{Y_g}{K_g} = \left[\int_{I_g} \frac{1}{m_g} \left(s_i \frac{\overline{\mathrm{MRPK}}_g}{\mathrm{MRPK}_{ig}}\right)^{\sigma_g - 1} di\right]^{\frac{1}{\sigma_g - 1}} \tag{H.3}$$

$$\text{where} \quad \frac{1}{\overline{\mathrm{MRPK}}_p} \equiv \int_{[0,1]} \frac{p_{ip} y_{ip}}{P_p Y_p} \frac{1}{\mathrm{MRPK}_{ip}} di, \quad \frac{1}{\overline{\mathrm{MRPK}}_g} \equiv \int_{I_g} \frac{1}{m_g} \frac{p_{ig} y_{ig}}{P_g Y_g} \frac{1}{\mathrm{MRPK}_{ig}} di \tag{H.4}$$

Absent financial frictions there would be no heterogeneity in $\overline{\mathrm{MRPK}}_p$ and $\overline{\mathrm{MRPK}}_g$ and optimal TFP in the private and public sectors (conditional on selection) would be,

$$\mathrm{TFP}_p^* = \left[\int_{[0,1]} s_i^{\sigma_p - 1} di\right]^{\frac{1}{\sigma_p - 1}} \quad \text{and} \quad \mathrm{TFP}_g^* = \left[\int_{I_g} \frac{1}{m_g} s_i^{\sigma_g - 1} di\right]^{\frac{1}{\sigma_g - 1}} \tag{H.5}$$

Relative price of public sector good. Using the definitions of P_g and P_p , the relative price can be written as,

$$\frac{P_g}{P_p} = \frac{\left[\int_{I_g} \frac{1}{m_g} p_{ig}^{1-\sigma_g} di\right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} \quad p_{ip}^{1-\sigma_p} di\right]^{\frac{1}{1-\sigma_p}}} = \frac{\left[\int_{I_g} \frac{1}{m_g} \left(\frac{1}{s_i}\mathrm{MRPK}_{ig}\right)^{1-\sigma_g} di\right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} \quad \left(\frac{1}{s_i}\mathrm{MRPK}_{ip}\right)^{1-\sigma_p} di\right]^{\frac{1}{1-\sigma_p}}}$$

which follows from the definition of MRPK_{ip}, and the production function as,

$$\mathrm{MRPK}_{ip} \equiv \frac{\partial p_{ip} y_{ip}}{\partial k_{ip}} = \frac{\sigma_p - 1}{\sigma_p} \frac{p_{ip} y_{ip}}{k_{ip}} = \frac{\sigma_p - 1}{\sigma_p} p_{ip} s_i \quad \Rightarrow \quad p_{ip} = \frac{\sigma_p}{\sigma_p - 1} \frac{1}{s_i} \mathrm{MRPK}_{ip}$$

and the same applies for MRPK_{ig}. Next multiplying and dividing by $\overline{\mathrm{MRPK}}_g$ in the numerator and by $\overline{\text{MRPK}}_p$ in the denominator we obtain,

$$\frac{P_g}{P_p} = \frac{\overline{\text{MRPK}}_g}{\overline{\text{MRPK}}_p} \frac{\left[\int_{I_g} \frac{1}{m_g} \left(\frac{1}{s_i} \frac{\overline{\text{MRPK}}_g}{\text{MRPK}_{ig}}\right)^{1-\sigma_g} di\right]^{\frac{1}{\sigma_g - 1}}}{\left[\int_{[0,1]} \left(\frac{1}{s_i} \frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}}\right)^{\sigma_p - 1} di\right]^{\frac{1}{1-\sigma_p}}} = \frac{\overline{\text{MRPK}}_g}{\overline{\text{MRPK}}_p} \frac{\text{TFP}_p}{\text{TFP}_g}$$

Relative sectoral TFP. Given the definition of TFP_p in equation (H.3), we can write

$$\begin{aligned} \text{TFP}_p &= \left[m_g \int_{I_g} \frac{1}{m_g} \left(s_i \frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}}\right)^{\sigma_p - 1} di + (1 - m_g) \int_{I_g^c} \frac{1}{1 - m_g} \left(s_i \frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}}\right)^{\sigma_p - 1} di\right]^{\frac{1}{\sigma_p - 1}} \\ &= \left[m_g \text{TFP}_{p,I_g}^{\sigma_p - 1} + (1 - m_g)\, \text{TFP}_{p,I_g^c}^{\sigma_p - 1}\right]^{\frac{1}{\sigma_p - 1}} \end{aligned}$$

where we have defined TFP_{p,I_g} and TFP_{p,I_g^c} as the average TFP in the private sector within the set of procurement (I_g) and non-procurement (I_g^c) firms respectively. Then, dividing by TFP_g in both sides we get the expression for $\text{TFP}_p/\text{TFP}_g$:

$$\frac{\text{TFP}_p}{\text{TFP}_g} = \left[m_g \left(\frac{\text{TFP}_{p,I_g}}{\text{TFP}_g}\right)^{\sigma_p - 1} + (1 - m_g) \left(\frac{\text{TFP}_{p,I_g^c}}{\text{TFP}_g}\right)^{\sigma_p - 1}\right]^{\frac{1}{\sigma_p - 1}} \tag{H.6}$$

The first term in equation (H.6) reflects the within-firm misallocation. With $\sigma_g = \sigma_p$ this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0\ \forall i$). Instead, if $\phi_g > \phi_p$ firms switch their output relatively towards the public sector and the dispersion of MRPK_{ig} declines, which makes $\text{TFP}_{p,I_g}/\text{TFP}_g$ fall. The second term in equation (H.6) reflects both between-firm misallocation and selection into procurement. If firms with higher s self-select into procurement, then $\text{TFP}_{p,I_g^c}/\text{TFP}_g$ declines. If there is more dispersion in MRPK_{ip} between non-procurement firms than in MRPK_{ig} between procurement firms, then $\text{TFP}_{p,I_g^c}/\text{TFP}_g$ is lower. In short, absent financial frictions the only reason for $\text{TFP}_p/\text{TFP}_g \neq 1$ would be the selection of firms into procurement. In the first best (no financial frictions and the government selects the firms with highest s) we would have $\text{TFP}_p/\text{TFP}_g < 1$.

Relative sectoral $\overline{\textbf{MRPK}}$. Given the definition of $\overline{\text{MRPK}}_p$ in (H.4), we can write

$$\begin{aligned} \overline{\text{MRPK}}_p &= \left[\frac{R_{p,I_g}}{P_p Y_p} \int_{I_g} \frac{p_{ip} y_{ip}}{R_{p,I_g}} \text{MRPK}_{ip}^{-1} di + \frac{R_{p,I_g^c}}{P_p Y_p} \int_{I_g^c} \frac{p_{ip} y_{ip}}{R_{p,I_g^c}} \text{MRPK}_{ip}^{-1} di\right]^{-1} \\ &= \left[\frac{R_{p,I_g}}{P_p Y_p} \overline{\text{MRPK}}_{p,I_g}^{-1} + \frac{R_{p,I_g^c}}{P_p Y_p} \overline{\text{MRPK}}_{p,I_g^c}^{-1}\right]^{-1} \end{aligned}$$

where R_{p,I_g} and R_{p,I_g^c} denote total revenues in the private sector by procurement firms and non-procurement firms respectively. Then, dividing by $\overline{\text{MRPK}}_g$ in both sides we obtain the expression for $\overline{\text{MRPK}}_p / \overline{\text{MRPK}}_g$

$$\frac{\overline{\text{MRPK}}_p}{\overline{\text{MRPK}}_g} = \left[\frac{R_{p,I_g}}{P_p Y_p} \left(\frac{\overline{\text{MRPK}}_{p,I_g}}{\overline{\text{MRPK}}_g} \right)^{-1} + \frac{R_{p,I_g^c}}{P_p Y_p} \left(\frac{\overline{\text{MRPK}}_{p,I_g^c}}{\overline{\text{MRPK}}_g} \right)^{-1} \right]^{-1} \tag{H.7}$$

Whenever $\overline{\text{MRPK}}_p \neq \overline{\text{MRPK}}_g$ there is misallocation of capital across sectors. The first term in equation (H.7) reflects the effects of within-firm misallocation on this between-sector misallocation. With $\sigma_g = \sigma_p$ this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0 \forall i$). Instead, if $\phi_g > \phi_p$ firms switch their output relatively towards the public sector and hence $\overline{\text{MRPK}}_{p,I_g} > \overline{\text{MRPK}}_g$. The second term in equation (H.7) reflects both between-firm misallocation and selection into procurement.

I Decreasing Returns to Scale

We characterize the crowding-out effect of procurement to private sales for unconstrained firms under decreasing returns to scale. We also present the result of running our Counterfactual 1 in an economy in which there are DRS. The firm solves the problem:

$$\pi = \max_{k, y_p, y_g} \left\{ B_p^{\frac{1}{\sigma}} y_p^{\frac{\sigma-1}{\sigma}} + B_g^{\frac{1}{\sigma}} y_g^{\frac{\sigma-1}{\sigma}} - (r+\delta)k \right\} \tag{I.1}$$

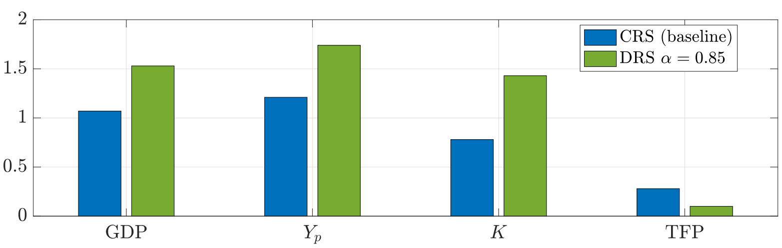
$$\text{s.t.} \quad y_p + y_g \leq s \; k^{\alpha} \tag{I.2}$$

The optimal solution of the firm implies a private sector revenue given by:

$$p_p y_p = B_p^{\frac{1}{\sigma}} y_p^{\frac{\sigma-1}{\sigma}} = B_p (B_g + B_p)^{\frac{(\sigma-1)(\alpha-1)}{\sigma-\alpha(\sigma-1)}} \left[\left(\frac{\sigma-1}{\sigma} \right) \frac{\alpha}{r+\delta} \right]^{\frac{\alpha(\sigma-1)}{\sigma-\alpha(\sigma-1)}} s^{\frac{\sigma-1}{\sigma-\alpha(\sigma-1)}} \tag{I.3}$$

The elasticity of $p_p y_p^*$ with respect to B_g is given by $\frac{\partial \; log \; p_p y_p^*}{\partial \; log \; B_g} = \left(\frac{(\sigma-1)(\alpha-1)}{\sigma-\alpha(\sigma-1)} \right) \left(\frac{1}{B_p + B_g} \right) < 0$.

Figure A.XVII. Counterfactual 1: CRS vs. DRS



Notes: This figure shows the effects of running Counterfactual 1 both in our CRS (baseline) economy and a recalibrated economy with DRS ($\alpha = 0.85$). The y-axis shows the percentage changes relative to the benchmark economy.