Evidence Disclosure in Competitive Markets

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Intro and motivation

The goal is to build a model of **competitive equilibrium** with **concealable hard information/evidence**

Examples of concealable evidence in the real world:

- real estate market: surveyors' reports
- labor market: employee's qualifications or outside offers; quantifiable, provable working conditions
- any market (e.g., pharma): objective cost or demand data, evidence concerning the quality/ effectiveness of the product

Model: when evidence is obtained (by chance & for free) and disclosed by either side of the market it becomes commonly known, but not being informed is not provable ... this info structure is embedded in a model of a "competitive market"

Related literature (skippable)

Dye (1985), Jung & Kwon (1988): concealable hard evidence (a type of information structure) introduced in the accounting literature

Shin (1994a,b), Shavell (1994), Shin (2003), Acharya, DeMarzo, and Kremer (2011), **DeMarzo, Kremer, and Skrzypacz (2019)**: almost exhaustive list of applications in bilateral trade and asset markets

Our own papers in 2014 and 2019: dynamic bargaining model with concealable hard evidence on both sides, all offers made by one side

Almost no studies (except Shin 1994a and our two papers) consider two-sided asymmetric information (in the form of hard evidence) concerning a state of nature potentially affecting both sides

Outline

General model on a single slide; two lemmas on disclosure

Specification 1: Evidence concerns a demand shock

- non-monotonic disclosure rules under general conditions

Specification 2: Evidence affects both sides of the market
(but everything is linear, simple, stylized)

- explicit solutions for equilibrium prices, quantities, welfare
- adverse-selection style comparative statics effects
    - e.g., as agents become more informed, suspicion that one's trading partner is hiding inconvenient evidence increases; this may lead to less trade, less social surplus, without change in the market price

Conclusions, extensions

1. Model

Economy with one good (quantity traded = $x \in \mathbb{R}_+$) and money

Initially unknown state of nature: $\theta \in [0,1]$ with pdf $f > 0$

Single buyer B with marginal benefit $v(x,\theta)$ such that $\partial v / \partial x < 0$

Single seller S with marginal cost $c(x,\theta)$ such that $\partial c / \partial x > 0$

Game (a model of a competitive market with concealable hard info):

(1) B, S privately learn θ with probability r_B, r_S respectively

(2) B, S choose whether to disclose or conceal θ when informed (ignorance cannot be proven, θ becomes public if disclosed)

(3) price p is set to maximize social surplus given all public info

(4) B, S submit demand x_B, supply x_S, given p and all private info; the quantity traded is $x = \min\{x_B, x_S\}$

Housekeeping

More on strategic actors:

- B_θ is a buyer (he), S_θ a seller (she) who has privately observed θ
- $B_\emptyset$ is a buyer, $S_\emptyset$ a seller who is (at least initially) uninformed
- the price is set by auctioneer A (it, also a player) with no private information; A's objective is to maximize social surplus (CS+PS)

First-best quantity and price, $x^*(\theta)$ and $p^*(\theta)$, are defined by

$$v\left(x^*(\theta), \theta\right) = p^*(\theta) = c\left(x^*(\theta), \theta\right)$$

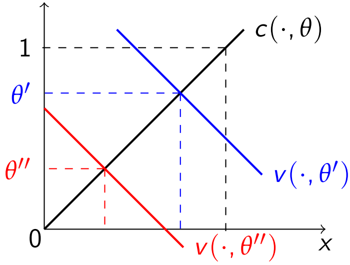
Without loss of generality order θ so that p^* is strictly increasing

No restriction on x^*, hence θ can be demand or supply shock or both

Illustration: two simple examples

$\theta \in [0,1]$ in both examples (say, uniformly distributed)

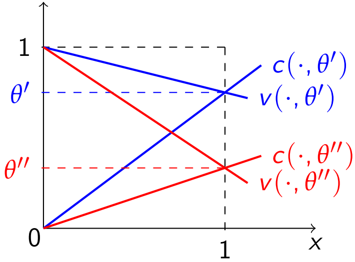
$v(x,\theta) = 2\theta - x$
$c(x,\theta) = x$



first-best: $p^*(\theta) = x^*(\theta) = \theta$

θ is pure demand shock

$v(x,\theta) = 1 - (1-\theta)x$
$c(x,\theta) = \theta x$



first-best: $p^*(\theta) = \theta$, $x^*(\theta) \equiv 1$

θ affects both v and c

Equilibrium objects and conditions (slide 1 of 2)

Standard notion of **extended Perfect Bayesian Equilibrium** (ePBE)

Equilibrium objects:

- **disclosure rules** for B_θ and S_θ; e.g., $\mathcal{D}_B = \{\theta : B_\theta \text{ discloses } \theta\}$, $\mathcal{D}_S = \{\theta : S_\theta \text{ discloses } \theta\}$
- **beliefs** over θ and types of other players contingent on history seen (B and S observe private histories, A sees public history)
- **price** contingent disclosure or no disclosure, latter denoted by $\widehat{p}$
- final-stage **demand** and **supply** contingent on price and beliefs

Refinements are not applicable nor needed: θ not being disclosed is always on equilibrium path; disclosure makes θ commonly known

- but: ePBE requires B and S <u>not</u> to update if A sets non-equilibrium price

Equilibrium objects and conditions (slide 2 of 2)

Equilibrium conditions:

- disclosure rules of B_θ and S_θ are **optimal** given anticipated contingent prices (after disclosure or no disclosure) and trade
- price is set **optimally** by A (to maximize expected social welfare) conditional on all public information (the value of θ if disclosed, A's updated beliefs if not), and anticipated final-stage quantities
- final-stage demand and supply of B_θ, $B_\emptyset$, S_θ, $S_\emptyset$ are **optimal** given their privately-observed histories and the prevailing price
- beliefs are **consistent**, price and actions are anticipated correctly

Circularity: contingent continuation play (the price A is expected to set after disclosure and no disclosure, anticipated trade) determine disclosure rules, which in turn induce beliefs and continuation play

Continuation after disclosure

If θ is disclosed then, unsurprisingly,

– at any price p set by A, the buyer's and seller's final-stage demand and supply, $x_B(p,\theta)$ and $x_S(p,\theta)$, must satisfy

$$v(x_B(p,\theta),\theta) = p = c(x_S(p,\theta),\theta)$$

– anticipating this, the social welfare maximizing auctioneer sets

$$p = p^*(\theta)$$

so that first-best $x^*(\theta)$ will be demanded, supplied, hence traded

Easy case: when θ is disclosed, first-best price $p^*(\theta)$ and trade $x^*(\theta)$

Continuation after no disclosure

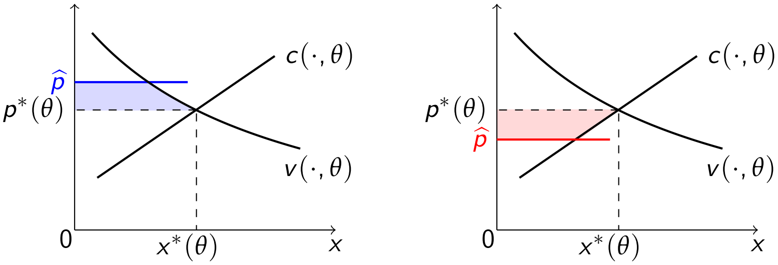
If θ is not disclosed then

- A, $B_\emptyset$ and $S_\emptyset$ update beliefs on θ and the types of agents they face (e.g., A and $B_\emptyset$ know that S is either $S_\emptyset$, or S_θ with $\theta \notin \mathcal{D}_S$)
- A is anticipated to set no-disclosure price $\widehat{p}$, which it must find optimal at this stage (A could set any other p), knowing that
  - ◊ B_θ and S_θ will demand and supply $x_B(p,\theta)$ and $x_S(p,\theta)$, respectively, satisfying $v(x_B(p,\theta),\theta) = p = c(x_S(p,\theta),\theta)$
  - ◊ $B_\emptyset$ will demand $x_B(p,\emptyset)$ and $S_\emptyset$ will supply $x_S(p,\emptyset)$ that is part of a continuation equilibrium after p, given their beliefs

Non-trivial equilibrium objects yet TBD: disclosure rules $\mathcal{D}_B$ and $\mathcal{D}_S$, no-disclosure price $\widehat{p}$, final-stage quantities $x_B(p,\emptyset)$ and $x_S(p,\emptyset)$

Lemma 1: when to disclose

Lemma 1. B_θ discloses θ if $p^*(\theta) < \widehat{p}$; S_θ discloses θ if $p^*(\theta) > \widehat{p}$



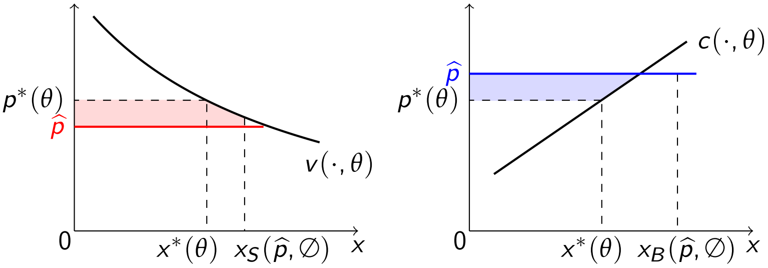
B_θ gains from disclosing

S_θ gains from disclosing

Implication: if B and S both learn θ then at least one discloses it

Lemma 2: when to conceal

Lemma 2. B_θ conceals if $p^*(\theta) > \widehat{p}$ **and** $x_S(\widehat{p}, \emptyset) \geq x^*(\theta)$, whereas
S_θ conceals if $p^*(\theta) < \widehat{p}$ **and** $x_B(\widehat{p}, \emptyset) \geq x^*(\theta)$



B_θ gains from concealing

S_θ gains from concealing

Informed agent conceals θ when expecting more trade at better price

No Full-Disclosure Equilibrium

Theorem 1. If $r_B, r_S \in (0,1)$ and $p^*(\theta)$ is strictly increasing (assumed) then there is no full-disclosure equilibrium: $\mathcal{D}_B \cap \mathcal{D}_S \neq [0,1]$

Proof idea (for simplicity assume v and c affine in θ):

- in FDE, after no disclosure, all players keep prior beliefs
- hence A sets the ex-ante optimal (market-clearing) price
- if there is θ with $x^*(\theta)$ less than the ex-ante first-best trade then by Lemma 2 one of the agents conceals, contradiction
- otherwise it can be shown that B_θ with θ just above prior mean has first-order gain from concealing provided p^* is increasing

We have an example of FDE in an environment with constant $p^*(\theta)$

Equilibrium existence

Theorem 2. Under general conditions on f, v, c, there is an equilibrium with objects $\mathcal{D}_B$, $\mathcal{D}_S$, $\widehat{p}$, $\{ x_B(p, \emptyset), x_S(p, \emptyset)$ for all $p \}$

Notes on the proof (see paper for details):

- disclosure sets $\mathcal{D}_B$, $\mathcal{D}_S$ are not generally intervals; a technical assumption ensures they are finite collections of intervals
- the analysis of the final-stage quantity-setting game is simplified by assuming θ affects either v or c but not both
- we construct $\mathcal{D}_B$, $\mathcal{D}_S$ and $\{x_B(p, \emptyset), x_S(p, \emptyset) : p \in P\}$ for any $\widehat{p} \in P = [p^*(0), p^*(1)]$ using a fixed-point argument
- let $\phi(\widehat{p})$ be the optimal no-disclosure price given $\hat{p}$ anticipated at disclosure stage; it is continuous on P, hence it has a fixed point

2. Demand shock

- assume $\theta \geq 0$ with a finite expected value
- $c(x)$ does not depend on θ, whereas $v(x, \theta)$ is increasing & affine in θ (the latter is just a normalization)
- c strictly increasing, v strictly decreasing in x
- $p^*(\theta) \equiv c(x^*(\theta)) \equiv v(x^*(\theta), \theta)$ therefore strictly increasing
- assume that $v''_{\theta x}$ (second cross partial) is non-positive, moreover, $\frac{\mathrm{d}}{\mathrm{d}\theta} p^*(\theta)$ is weakly decreasing in θ

There is a partial-disclosure equilibrium where S discloses θ iff it is greater than a threshold (which is below $\widehat{\theta}$ such that $p^*(\widehat{\theta}) = \widehat{p}$), whereas B uses a non-monotonic disclosure rule with exactly two thresholds (disclose $\theta < \widehat{\theta}$ and also θ above a threshold $> \widehat{\theta}$)

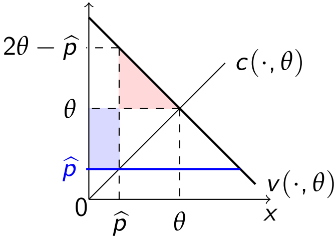
Example with non-interval disclosure set

Let $v(x, \theta) = 2\theta - x$, $c(x, \theta) = x$, so $p^*(\theta) = x^*(\theta) = \theta$, let $\theta \sim \mathrm{U}[0, 1]$

Assume $r_B = 0$ and $\frac{3}{4} < r_S < 1$

Equilibrium objects: $\mathcal{D}_S = [\widehat{p}, 1]$, $\widehat{p} = \beta = \frac{(1-r_S)/2 + r_S \widehat{p}^2/2}{1 - r_S + r_S \widehat{p}}$

Final stage: $x_S(p, \cdot) = p$, $x_B(p, \theta) = 2\theta - p$, $x_B(p, \emptyset) = 2\beta - p$
Interim stage $p = \widehat{p} = \beta$ checks out optimal



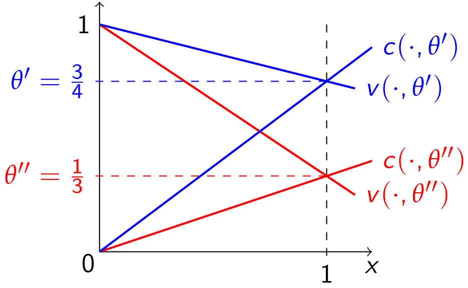
Red triangle > blue rectangle
$\Leftrightarrow \frac{1}{2}(2\theta - \widehat{p} - \theta)(\theta - \widehat{p}) > (\theta - \widehat{p})\widehat{p}$
$\Leftrightarrow \theta > 3\widehat{p}$
Hence $\mathcal{D}_B = [0, \widehat{p}] \cup [3\widehat{p}, 1]$
though literally B never informed

If $r_S \gg 3/4$ and $r_B \approx 0$ then $\widehat{p} \approx \beta < 1/3$ and $\mathcal{D}_B$ is not connected

3. Worked example with θ affecting both v and c

$$v(x,\theta) = 1-(1-\theta)x \qquad c(x,\theta)=\theta x \qquad \theta \sim \mathrm{U}[0,1]$$

$r_B, r_S \in (0,1)$ either not too large or not far from each other



Hence $p^*(\theta)=\theta$ and $x^*(\theta)=1$

If $x<1$ is traded then deadweight loss is $\frac{1}{2}(1-x)^2$ irrespective of θ

Equilibrium derivation

If A sets price p then informed agents demand and supply

$$x_B(p,\theta) = \frac{1-p}{1-\theta} \quad \text{and} \quad x_S(p,\theta) = \frac{p}{\theta}$$

If either agent discloses θ then A sets $p^*(\theta)$ and $x^*(\theta)$ is traded

Method of deriving the rest

- suppose that agents anticipate no-disclosure price $\widehat{p}$
- conjecture $\mathcal{D}_B = [0, b]$ and $\mathcal{D}_S = [s, 1]$; by Lemma 1 $s \le \widehat{p} \le b$
- find uninformed demand and supply, $x_B(p, \emptyset)$ and $x_S(p, \emptyset)$
- given $\mathcal{D}_B$, $\mathcal{D}_S$, x_B, x_S, let A set optimal no-disclosure price
- confirm $\mathcal{D}_B$, $\mathcal{D}_S$ are optimal and anticipated $\widehat{p}$ same as set by A

Checking equilibrium in the final stage (1)

Fix $\mathcal{D}_B = [0, b]$, $\mathcal{D}_S = [s, 1]$ and consider any $p \in [0, 1]$

If $B_\emptyset$ demands x, his expected payoff is proportional to

$$(1 - r_S) \int_0^1 (1-p)\left[x \wedge x_S(p, \emptyset)\right] - \tfrac{1}{2}(1-\theta)\left[x \wedge x_S(p, \emptyset)\right]^2 \, \mathrm{d}\theta$$

$$+ r_S \int_0^s (1-p)\left(x \wedge \frac{p}{\theta}\right) - \tfrac{1}{2}(1-\theta)\left(x \wedge \frac{p}{\theta}\right)^2 \, \mathrm{d}\theta$$

$S_\emptyset$ supplies x to maximize

$$(1 - r_B) \int_0^1 p\left[x \wedge x_B(p, \emptyset)\right] - \tfrac{1}{2}\theta\left[x \wedge x_B(p, \emptyset)\right]^2 \, \mathrm{d}\theta$$

$$r_B \int_b^1 p\left(x \wedge \frac{1-p}{1-\theta}\right) - \tfrac{1}{2}\theta\left(x \wedge \frac{1-p}{1-\theta}\right)^2 \, \mathrm{d}\theta$$

Not pretty, but solvable

Checking equilibrium in the final stage (2)

Consistent solutions $x_B(p, \emptyset)$ and $x_S(p, \emptyset)$ can be found for all p

$$x_B(p, \emptyset) = \frac{1-p}{1-\beta} \text{ for all } p \geq \widehat{p}$$

$$x_S(p, \emptyset) = \frac{p}{\sigma} \text{ for all } p \leq \widehat{p}$$

β is $B_\emptyset$'s expectation of θ conditional on S not disclosing given $\mathcal{D}_S$ and σ is $S_\emptyset$'s analogous expectation of θ given B not disclosing:

$$\beta = \frac{1 - r_S + r_S s^2}{2(1 - r_S + r_S s)} < \frac{1}{2} \text{ and } \sigma = \frac{1 - r_B b^2}{2(1 - r_B b)} > \frac{1}{2}$$

For p near $\widehat{p}$ we have $x_B(p, \emptyset) \equiv x_S(p, \emptyset) \equiv x_B(p, \beta) \wedge x_S(p, \sigma)$ the market clears at $\widehat{p}$ with uninformed traders, maximal trade

Solving the auctioneer's problem

By Lemma 1, if both traders observe θ then θ is disclosed; then by Lemma 0 first-best price is set, first-best trade $\Rightarrow$ no deadweight loss

Otherwise, if $x < 1$ is traded then DWL = $\frac{1}{2}(1-x)^2$

Hence for fixed $\widehat{p}$, b, s, β, σ the auctioneer minimizes

$$\begin{aligned} D(p) = {} & (1-r_B)(1-r_S)\left[1 - x_B(p,\emptyset) \wedge x_S(p,\emptyset)\right]^2 \\ & + (1-r_B)r_S \int_0^s \left(1 - x_B(p,\emptyset) \wedge \frac{p}{\theta}\right)^2 \mathrm{d}\theta \\ & + r_B(1-r_S) \int_b^1 \left(1 - \frac{1-p}{1-\theta} \wedge x_S(p,\emptyset)\right)^2 \mathrm{d}\theta \end{aligned}$$

Not pretty, again...

Solving the auctioneer's problem

If r_B and r_S are not too large, or if they are nearly equal to each other at any value, then only the first term matters ...

... optimized by setting p so that $x_B(p, \emptyset) = x_S(p, \emptyset)$ and as large as possible, which occurs at $p = \widehat{p}$ where

$$x_B(\widehat{p}, \emptyset) = \frac{1 - \widehat{p}}{1 - \beta} = \frac{\widehat{p}}{\sigma} = x_S(\widehat{p}, \emptyset)$$

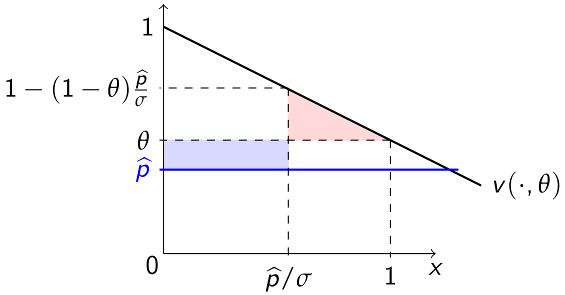
equivalently written as $\widehat{p}/(1 - \widehat{p}) = \sigma/(1 - \beta)$

[The first term of $D(p)$ has a "kink" at $p = \widehat{p}$, hence the second and third terms can be ignored if r_B, r_S are not too large and/or $r_B \approx r_S$; this requires a formal proof, which we have but omit here]

Verifying disclosure rules

Consider B_θ. By Lemma 1 he discloses $\theta < \widehat{p}$

Suppose $\theta > \widehat{p}$. By Lemma 1 agent B_θ knows that after no disclosure he will face $S_\emptyset$ because S_θ discloses $\theta > \widehat{p}$. Hence $x_S = \widehat{p}/\sigma$



B_θ discloses if the red triangle is larger than the blue rectangle

Verifying disclosure rules

From the comparison on the previous slide, B_θ with $\theta > \widehat{p}$ discloses

$$\theta < b = 1 - \frac{2\widehat{p}(1-\widehat{p})\sigma}{\widehat{p}^2 + \sigma^2}$$

which pins down b in $\mathcal{D}_B = [0, b]$

Analogous reasoning for S_θ with $\theta < \widehat{p}$ yields that S_θ discloses

$$\theta > s = \frac{2\widehat{p}(1-\widehat{p})(1-\beta)}{(1-\widehat{p})^2 + (1-\beta)^2}$$

which pins down s in $\mathcal{D}_S = [s, 1]$

Derivation complete, ready to summarize equilibrium

Proposition 1: Equilibrium for r_B, r_S not too large

B_θ discloses $\theta \in \mathcal{D}_B = [0, b]$ and S_θ discloses $\theta \in \mathcal{D}_S = [s, 1]$ with formulas for b and s given on the previous slide

After no disclosure, the beliefs of $B_\emptyset$ and $S_\emptyset$ concerning θ are

$$\beta = \frac{1 - r_S + r_S s^2}{2(1 - r_S + r_S s)} < \frac{1}{2} \quad \text{and} \quad \sigma = \frac{1 - r_B b^2}{2(1 - r_B b)} > \frac{1}{2}$$

Auctioneer (optimally) sets no-disclosure price $\widehat{p}$ such that

$$\frac{\widehat{p}}{1 - \widehat{p}} = \frac{\sigma}{1 - \beta} = \frac{s}{1 - b}$$

Final stage quantities at any p are $x_B(p, \theta) = \frac{1-p}{1-\theta}$, $x_S(p, \theta) = \frac{p}{\theta}$

$$x_B(p, \emptyset), x_S(p, \emptyset) \leq \min\left\{\frac{1 - p}{1 - \beta}, \frac{p}{\sigma}\right\}$$

Comparative statics

Suppose B is better able to identify θ ($r_B \uparrow$). What happens to $\widehat{p}$, b, s?

Proposition 2. If $r_B \uparrow$ then $\sigma \uparrow$ (S more suspicious), B must disclose more hence $b \uparrow$; following no disclosure A thinks θ is higher, so $\widehat{p} \uparrow$ (unfavorable for B), therefore S discloses less, $s \uparrow$

From now on focus on fully symmetric case, $r_B = r_S = r \in (0, 1)$

Proposition 3. In equilibrium $\widehat{p} = 1/2$, $b = 1 - s$, and $\beta = 1 - \sigma$. As $r \uparrow$ equilibrium $b \uparrow$ and $\sigma \uparrow$, $s \downarrow$ and $\beta \downarrow$; $x_B(\widehat{p}, \emptyset) = x_S(\widehat{p}, \emptyset) \downarrow$; more disclosure but more suspicion, less trade after no disclosure

More comparative statics

Is more "hard but concealable" information, $r\uparrow$, socially beneficial?

- more disclosure (learn θ more often, disclose more θs)
- but more suspicion, less trade conditional on no disclosure

Overall welfare effect is negative for low r, positive for high r

No welfare loss as $r \to 0$ nor as $r \to 1$ (but no full disclosure either!)
welfare losses arise from suspicion, not uncertainty per se

Proposition 4. Transform distribution of θ making it more extreme: go from $U[0,1]$ to $U\{0,1\}$. Equilibrium welfare $\downarrow$ for all r

4. Conclusions, slides reserve

Model of competitive market with concealable hard information

- **concealable hard info** well motivated (documentary evidence), but we do not study information design (optimal persuasion)
- **competitive** in that B and S are price-taker in the final stage, market sets contingent prices to maximize social welfare
- yet fully strategic, rational, **no commitment** (players make all inferences, no mechanism design)

In contrast to partial/ general equilibrium with incomplete "soft" information, here no issue with existence, no off-path beliefs either

Effects (eg., Props 2-4) work through affecting or **putting pressure on no-disclosure price**, i.e., via market interaction

Plans

Extensions, variants contemplated so far:

- pure exchange instead of partial equilibrium model (one version done, can say more if interested)
- auctioneer maximizing weighted sum of surpluses (representing bargaining power and departing from competitive market)
- auctioneer could also act as intermediary in disclosure (and conceal if beneficial), not just as price setter
- multiple buyers and sellers, possibly heterogeneous
- application to political competition, other environments (e.g., multi-dim communication with concealable hard evidence)