# Money is the Root of Asset Bubbles*

Yu Awaya† Kohei Iwasaki‡ Makoto Watanabe§

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**Abstract**

This paper examines how monetary expansion causes asset bubbles. When there is no monetary expansion, a bubbly asset is not created due to a hold-up problem. Monetary expansion increases buyers' money holdings, and then, dealers are willing to buy a worthless asset from sellers, in hopes of selling it to buyers who may not know that it is worthless—a bubble now occurs.

**Keywords:** Bubbles, dealers, higher-order uncertainty, money

**JEL Classification Numbers:** D82, D83, D84, E44, E52, G12, G14

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†University of Rochester. Address: 208 Harkness Hall, Rochester, NY 14627, US. Email: YuAwaya@gmail.com

‡Kyoto University. Address: Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan. Email: iwasaki.kohei.7a@kyoto-u.ac.jp

§Kyoto University. Address: Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan. Email: makoto.wtnb@gmail.com

# 1 Introduction

As advocated by many policymakers and observers, e.g., Masaaki Shirakawa, the former governor of Bank of Japan (see Okina, Shirakawa, and Shiratsuka 2001) and Krugman (2015), bubbles occur when "too much money is chasing too few investment opportunities." While such a view is widely accepted not only among policymakers but also among the public, there has been few attempts in the economics academia to examine the exact mechanism that links too much money and bubbles.[1] Kindleberger (2000) states similar sentiments in a chapter entitled "Fueling the Flames: Monetary Expansion."

The objective of this paper is to fill this gap. We propose a new framework in which too much money causes bubbles. Specifically, we incorporate a finite-horizon asset-bubble framework into a workhorse model in monetary theory by Lagos and Wright (2005). In each period, a decentralized asset market opens where agents use money as a payment instrument. Sellers produce an asset that buyers may wish to obtain. Agents face higher-order uncertainty, that is, a lack of common knowledge. Even if all agents know that the asset is worthless, there is a situation where dealers intermediate the trade between buyers and sellers, knowing the asset is worthless, but without knowing whether buyers know that the asset is worthless. Then, dealers may have incentives to buy the asset from sellers in hopes of selling it to buyers.

In this framework, *too much money* is modeled as a consequence of an exogenous *monetary expansion*, with which the central bank issues money at the beginning of the decentralized market, and buyers receive the newly issued money as a lump-sum transfer. Hence, monetary expansion leads to a large amount of buyers' money holdings in the decentralized market, irrespective of their will.[2] The tractability of Lagos and Wright (2005) allows us to analyze such a monetary policy with a sound micro-founded

[1] Barlevy (2018) points out the importance of knowing how monetary (and other) policies affect bubbles.

[2] This is similar to the liquidity effects studied in, for example, Grossman and Weiss (1983) and Lucas (1990). Buyers have no incentive to refuse to get the money.

theory of money.

Our main result is, informally,

**Theorem.** *An asset bubble is fueled by monetary expansion.*

We first observe that without monetary expansion, asset bubbles do not occur. This is due to a *hold-up problem*. Since buyers know that dealers do not consume the asset, dealers have to accept any terms of trade from buyers once they obtain the asset and trade with buyers. In other words, not bringing much money to the decentralized market is a commitment device for buyers to not offer what they think the asset is really worth. Anticipating this, buyers then do not have incentives to bring money to the decentralized market, and as a result, dealers are unwilling to buy the asset from sellers—bubbles never occur.

Now, how does monetary expansion cause bubbles? Too much money, induced by monetary expansion, loosens buyers' budget constraints and hence their purse strings. This makes dealers—who know that the asset is worthless but do not know that buyers know it—more optimistic about buyers' payment. Thus, dealers are willing to buy the asset from sellers even if they know that it is worthless—a bubble now occurs.

Finally, too much money can arise endogenously rather than from the exogenous monetary expansion. We show this in an extended model where we introduce another goods market. Unlike in the baseline model, this extension also shows that money can have value even if the bubbly asset is not traded.

## Related Literature

This paper, of course, relates to the literature on both bubbles and money.

For the literature on bubbles, there are several approaches to studying bubbles (see, for example, Brunnermeier and Oehmke 2013 for a survey). Ours belongs to the one using higher-order uncertainty. Allen, Morris, and Postlewaite (1993) is the first to take the approach and show the existence of bubbles in a finite-horizon model. The model is further refined and developed in a sequence of papers by Conlon (2004, 2015), Doblas-Madrid (2012), Liu and Conlon (2018), Awaya, Iwasaki, and Watanabe (2022), Dong, Jia, and Wang (2022), and Liu, White, and Conlon (2023). None of these models has money explicitly, and we use a New Monetarist framework to add money to Awaya, Iwasaki, and Watanabe (2022), henceforth referred to as AIW. On top of this, another innovation relative to this literature—except for Araujo and Doblas-Madrid (2022) who show bubbles occur in a Walrasian market where prices convey information on the quality of assets—is to show that bubbles can occur in a robust equilibrium with a finite-state space even when prices are publicly observable.

This paper also belongs to the New Monetarist literature. See Lagos, Rocheteau, and Wright (2017) and Rocheteau and Nosal (2017) for recent surveys. Among New Monetarist models, environments in which money and assets with uncertain return coexist have been studied by, for example, Rocheteau (2011) and Geromichalos, Herrenbrueck, and Wang (2022). These models only consider first-order uncertainty, while higher-order uncertainty plays a key role in ours. Mattesini and Nosal (2016) and Lagos and Zhang (2019, 2020) also incorporate intermediaries into the Lagos and Wright (2005) environment. Other than the fact that they do not consider asymmetric information, another important difference is the intermediation mode. In their models, dealers are a platform that offers a marketplace to investors. In our model, dealers are *middlemen* who buy the asset from their own accounts and resell it to buyers. Monetary policy similar to ours is first introduced in Molico (2006) and then further explored by, for example, Wallace (2014). We consider an infinite-horizon discrete-time model, and each period has a finite-horizon decentralized market, where the asset bubbles may occur and collapse due to a lack of common knowledge. Rocheteau (2024) shows that the finite duration of fiat money emerges in an infinite-horizon continuous-time model of complete information.

Finally, in the existing New Keynesian frameworks, the results on the relationship between monetary expansion and bubbles are mixed. For instance, Galí (2014, 2021) shows that a monetary policy rule that raises interest rates in response to bubbles can paradoxically lead to larger bubbles. Later, Miao, Shen, and Wang (2019) argue Galí's model contains additional equilibria in which more aggressive rules dampen bubbles. Allen, Barlevy, and Gale (2023) show that a central bank that wants to dampen bubbles can always do so by raising interest rates aggressively enough. In contrast to these papers, we adopt a New Monetarist approach and are interested in the positive relationship between monetary expansion and the occurrence (rather than size) of bubbles. We show that such a positive relationship holds unambiguously (rather than as one possibility in multiple equilibria).

**Organization of the Paper**

The rest of the paper is organized as follows. Section B.1 introduces the model. Section 3 shows that, for all parameter values, there is no equilibrium in which bubbles occur without monetary expansion, while Section B.2 shows that, under certain conditions on parameters, there is an equilibrium in which bubbles occur with sufficiently large monetary expansion. Section 5 discusses some implications of our result, and Section 6 considers the extension. The Appendix contains some omitted proofs of Section 6 as well as the case where the asset is divisible.

# 2 Model

Time is discrete, continues forever, and is denoted by $t = 0, 1, \cdots$. Each period is divided into two subperiods as in the infinite-horizon monetary framework of Lagos and Wright (2005). In the first, agents interact in a decentralized asset market (DM). In the second, they interact in a frictionless centralized market (CM). These alternating markets induce an asynchronicity of expenditures and receipts crucial to any analysis of money, that is, agents want to make purchases in the DM while their incomes accrue in the CM, so they must bring money acquired in the past to trade. There are three

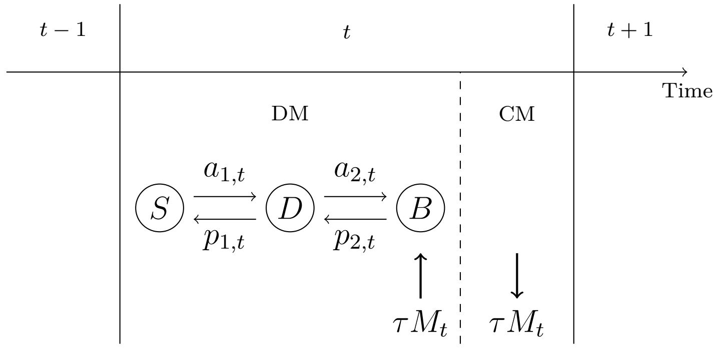


Figure 1: Timeline of the baseline model

types of infinitely lived agents: sellers, dealers, and buyers. They discount future at a common rate $\beta \in (0,1)$.[3] Their types are fixed over time, and the measure of each type is normalized to one.

We assume there is an intrinsically useless, durable and uncounterfeitable object—*money*. Money is assumed to be divisible, and let $M_t$ be its supply when the DM of period $t$ opens (before monetary expansion that will be described below). We assume that there is friction that hinders credit.

At the beginning of the DM of each period $t$, the central bank issues $\tau M_t$ units of money, where $\tau \geq 0$. Only buyers receive the newly issued money through a lump-sum transfer, and hence each buyer obtains $\tau M_t$ units in the DM (see Figure 1).[4]

In the subsequent CM, the central bank eliminates the newly issued fraction of money supply through a lump-sum tax $\tau M_t$. In other words, temporary monetary expansion is expected to be followed by temporary shrink. Hence, $M_t = M_{t+1}$ for each $t$. Assume that this monetary expansion is common knowledge, and anticipated by all agents.[5]

The assumption that the central bank takes back money in the CM allows us to

[3]For simplicity, there is no discounting between the DM and the CM.

[4]The assumption that only buyers get a transfer is for simplicity, and our result survives if sellers or dealers also obtain transfers. On the other hand, the timing of money injection plays a key role—monetary expansion would not cause bubbles if transfers were made in the CM.

[5]Our result would not hold if the monetary expansion was not expected.

isolate the effect of monetary expansion from *inflation*—if the central bank did not take back the money, such monetary expansion would result in inflation. In general, inflation reduces the liquidity of buyers, which works against the original effect of monetary expansion of increasing the liquidity. As found out in, e.g., Ikeda (2022), inflation tends to be low during asset market booms, notably in Japan's asset price bubble period of the late 1980s. The assumption is consistent with this fact.

We will compare the case where $\tau = 0$ (no monetary expansion) to the case where $\tau$ is sufficiently large.

## 2.1 Centralized Market

In the CM, money and a perishable good are traded. All agents enjoy $U(x)$ from consuming $x$ units of the good. Assume that $U'(x) > 0$ and $U''(x) < 0$ for each $x > 0$. The good is numeraire and produced one-for-one using labor, and hence its price and the real wage are equal to one. Agents suffer disutility $\ell$ from working for $\ell$ hours.

## 2.2 Decentralized Market

### 2.2.1 Economic Environment

In the DM, there exists an *indivisible* asset.[6] For simplicity, we assume that neither sellers nor dealers enjoy utility from consuming the asset, while our results hold if the utility is small enough. Buyers obtain utility $u > c$ with some probability and 0 with the remaining probability. Irrespective of whether it is consumed by buyers, the asset perishes after the DM.

In the DM, first, each seller meets a dealer for sure, trades the asset with the dealer, and leaves the DM. Then, the dealer meets a buyer for sure, trades the asset with

[6] Lotteries may be an efficient mechanism in the model with indivisible assets. We do not present the results with lotteries, but similar results hold with lotteries as well. The case where the asset is divisible is considered in the Appendix.

the buyer, and both the dealer and the buyer leave the DM. Thus, trade between sellers and buyers is sequential and must go through dealers. For sellers and dealers, their counterparties are drawn randomly from all dealers and all buyers, respectively. Money is used as the payment instrument, and we employ generalized Nash bargaining to determine the terms of trade.[7] We denote by $\theta_1 \in (0, 1)$ (resp. $\theta_2 \in (0, 1)$) the bargaining power of sellers (resp. dealers) in trade between sellers and dealers (resp. dealers and buyers).

We assume that buyers *can* observe the price between sellers and dealers. We will discuss its implication in Section 5.

### 2.2.2 Information Structure

We will construct an information structure that induces asset bubbles. Following the notion of *strong bubbles* by Allen, Morris, and Postlewaite (1993), asset bubbles are defined as follows.

**Definition 1.** *An asset bubble occurs* if the asset is traded for a positive amount of money, the price of money is positive, and all agents know that the consumption value of the asset for buyers is 0.

The information structure in the DM is (a simplified version of) the finite-horizon asset-bubble model of AIW. All parameters describing utilities, costs, etc., are common

[7] As in AIW, we consider a game form where a fictitious third party suggests the exchange ratio following the Nash bargaining solution, and then each agent either accepts or rejects the trade. The trade occurs only when both agents accept. While the agents have private information, the terms of trade that the third party proposes do not depend on it. Our results do not depend on Nash bargaining, but the bargaining solution has to be Pareto efficient. We also implicitly assume that buyers of the asset (that is, dealers in the seller-dealer meetings and buyers in the dealer-buyer meetings) are not allowed to only bring a fraction of their money holdings to their pairwise meetings. However, one can alternatively assume that buyers are allowed to do this, but a third party's proposal does not depend on the actual choice of money holdings. See Lebeau (2020) for further investigation on this issue.

knowledge except for the asset value (i.e., consumption value) for buyers. To describe the information structure, we introduce three states: $\omega^u_{SD}$, $\omega^0_E$, and $\omega^0_{SD}$.[8] The superscripts indicate whether the asset value for buyers is $u$ or 0. At $\omega^u_{SD}$, it is $u$; at the other states, it is zero. The subscripts signify who knows the asset value for buyers. At $\omega^u_{SD}$ and $\omega^0_{SD}$, only sellers and dealers know it; at $\omega^0_E$, every agent knows it. The set of states is

$$\Omega = \{\omega^u_{SD}, \omega^0_E, \omega^0_{SD}\}.$$

The state of each period realizes at the beginning of the DM in that period, and drawn independently across periods.

The information is the same among the agents of each type. Sellers' and dealers' partitions are the same:

$$\mathcal{P}_{SD} = \{\{\omega^u_{SD}\}, \{\omega^0_E, \omega^0_{SD}\}\}.$$

The first element, $\{\omega^u_{SD}\}$, corresponds to the case where the buyers' asset value is $u$, and sellers and dealers know it. The second element, $\{\omega^0_E, \omega^0_{SD}\}$, corresponds to the case where the buyers' asset value is zero, and sellers and dealers do not know whether buyers know it. Buyers' partition is

$$\mathcal{P}_B = \{\{\omega^u_{SD}, \omega^0_{SD}\}, \{\omega^0_E\}\}.$$

The first element, $\{\omega^u_{SD}, \omega^0_{SD}\}$, corresponds to the case where buyers do not know whether the asset value is $u$ or 0 for them. The second element, $\{\omega^0_E\}$, corresponds to the case where the asset value is 0, and buyers know it. An agent can distinguish any two states if those states belong to a different element of his or her partition, but cannot otherwise.

The common prior distribution over $\Omega$ does not depend on time and is denoted by $\mu$. Assume that $\mu(\omega) > 0$ for each $\omega \in \Omega$. At states $\omega^u_{SD}$ and $\omega^0_{SD}$, buyers believe that

[8] In the original AIW model, at least five states are required to generate a bubble. We thank Gadi Barlevy who pointed out later that only three states are sufficient. The specification adopted in this paper is the one pointed out by him. See also Liu, White, and Conlon (2023). We deliberately employ the simplest model to highlight the key idea.

the asset value is $u$ with probability

$$\psi_B = \frac{\mu(\omega^u_{SD})}{\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})}.$$

At state $\omega^u_{SD}$, dealers know that buyers do not know the asset value, and hence they believe that they can sell the asset to buyers for sure. At states $\omega^0_E$ and $\omega^0_{SD}$, dealers believe that buyers do not know the asset value with probability

$$\psi_D = \frac{\mu(\omega^0_{SD})}{\mu(\omega^0_E) + \mu(\omega^0_{SD})}.$$

We summarize how each state plays a role for the existence of bubbles. State $\omega^u_{SD}$ creates gains from trade of the asset. State $\omega^0_E$ establishes a situation where all agents know that the asset value for buyers is zero. State $\omega^0_{SD}$ constructs a case where the asset value is zero and buyers do not know it—only sellers and middlemen know the fact that the value is zero. These states are necessary ingredients of bubbles.

# 3 No Monetary Expansion

First, we will show that

**Proposition 1.** *There is no equilibrium in which an asset bubble occurs if there is no monetary expansion ($\tau = 0$).*

Note that in this case, money is not used and thus does not have value. Of course, this describes an extreme situation. In Section 6, we introduce a market of consumption goods and study asset bubbles in an environment where money has positive value in some equilibrium even without asset bubbles.

The reason for the nonexistence of bubbles is a *hold-up problem*. Note that dealers do not enjoy utility from consuming the asset. Since the bargaining between dealers and buyers is efficient, the dealers must give up the asset, *regardless of the amount of buyers' money holdings*. Since holding money across periods is costly, buyers do not have any incentives to bring money. Thus, when $\tau = 0$, buyers do not have money in the DM. Then, dealers do not have any incentives to buy the asset from sellers, and therefore there is no room for bubbles to exist.

The no-bubble result seems very different from the result of AIW who show bubbles occur for all parameter values. The key difference is the timing of the bargaining. In AIW, buyers can produce goods on the spot for dealers, and hence buyers' payments are unconstrained when buyers bargain with dealers. In the current model, buyers' payments are constrained to the amount of money that they bring to the DM when they bargain with dealers. In other words, buyers can commit smaller amounts of payment by bringing smaller amounts of money. Anticipating this, dealers do not buy the asset from sellers.

# 4 Monetary Expansion

In this section, we identify a necessary and sufficient condition for which an asset bubble occurs. Throughout the section, we assume that the gains from trade are sufficiently large so that

$$\frac{\theta_2 \psi_B u}{c} \geq \underline{\tau} \equiv \max\left\{\frac{1}{\psi_D}, \frac{1}{\beta[\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)]}\right\}. \tag{1}$$

As we will see later, it turns out that there is no equilibrium in which an asset bubble occurs if (1) is violated. Under (1), we have

**Theorem 1.** *There is an equilibrium in which asset bubbles occur at a state if and only if monetary expansion is sufficiently large ($\tau \geq \underline{\tau}$).*

Of course, Proposition 1 is a corollary of the theorem. Note that (1) is easier to be satisfied when the interest rate $(1 - \beta)/\beta$ is lower.

When these conditions are satisfied, we construct an equilibrium in which bubbles occur. This equilibrium has the following features (on-path).

- Money holdings: Sellers and buyers do not bring money from the CM, while dealers bring the minimum amount of money to make the sellers to produce the asset, $c/\phi_t$, where $\phi_t$ is the price of money. Buyers' money holdings in the DM are $\tau M_t$. Note that this amount is given by exogenous monetary expansion, rather than their choice.

- Trade between sellers and dealers: In all states, sellers produce the asset, and dealers pay $c/\phi_t$ in exchange for that.

- Trade between dealers and buyers: In states $\omega_{SD}^u$ and $\omega_{SD}^0$, dealers give the asset to buyers, and buyers pay $\min\{\tau M_t, \theta_2 \psi_B u/\phi_t\}$ units of money. In state $\omega_E^0$, no trade occurs.

- Occurrence of bubbles: At $\omega_E^0$, everyone knows the asset is worthless, but dealers still buy it.

The key idea is that if buyers hold enough money, dealers speculate that buyers will pay a large amount of money. This convinces sellers to produce the asset. While buyers do not have incentives to bring money themselves due to a hold-up problem, sufficiently large monetary expansion forces them to do so. Note that this occurs at $\omega_E^0$ as well. There, both dealers and buyers know that the asset is worthless, but dealers do not know the fact that buyers also know that. Thus, dealers speculate.

## 4.1 Bellman Equations

Let $V_t^S$, $V_t^D$, and $V_t^B$ be the DM value functions of sellers, dealers, and buyers, respectively. Similarly, we denote the CM value functions by $W_t^S$, $W_t^D$, and $W_t^B$. For each type $i \in \{S, D, B\}$, the Bellman equation for the CM is

$$W_t^i(m_t^i) = \max_{x_t^i, \ell_t^i, \widehat{m}_{t+1}^i} \left\{ U(x_t^i) - \ell_t^i + \beta V_{t+1}^i(\widehat{m}_{t+1}^i) \right\}$$
$$\text{subject to } x_t^i = \phi_t(m_t^i - \widehat{m}_{t+1}^i) + \ell_t^i - \frac{\phi_t \tau M_t}{3},$$

where $m_t^i$ and $\widehat{m}_{t+1}^i$ are money holdings when the CM opens and closes. Note that since buyers receive money in the next DM (i.e., money expansion), the one they choose in the CM, $\widehat{m}_{t+1}^B$, differs from the one they actually hold in the DM. Here, we have $-\phi_t \tau M_t/3$ in the budget constraint because measure 1 of buyers receive $\tau M_t$ units of money in the DM and the central bank takes back that amount of money from measure 3 of all agents through a lump-sum tax.

Assuming an interior solution for labor $\ell_t^i$,

$$\begin{aligned} W_t^i(m_t^i) &= \phi_t m_t^i - \frac{\phi_t \tau M_t}{3} + \max_{x_t^i} \left\{ U(x_t^i) - x_t^i \right\} \\ &\quad + \max_{\widehat{m}_{t+1}^i} \left\{ -\phi_t \widehat{m}_{t+1}^i + \beta V_{t+1}^i(\widehat{m}_{t+1}^i) \right\}. \end{aligned}$$

The optimal consumption in the CM is pinned down by

$$U'(x_t^i) = 1.$$

The optimal money holdings when the CM closes are determined independently of money holdings when the CM opens. Thus, we have the history independence of money holdings, and all agents of each type have the same unit of money at the end of the CM. Moreover, the CM value function is linear with slope $\phi_t$:

$$\frac{dW_t^i(m_t^i)}{dm_t^i} = \phi_t.$$

**Trade between Dealers and Buyers**

Now, we derive the terms of trade in seller-dealer meetings and dealer-buyer meetings, given money holdings of each type of agents. We start with dealer-buyer meetings and then consider seller-dealer meetings.

Consider DM trade between dealers and buyers. Let $a_{2,t}$ and $p_{2,t}$ be the amounts of the asset and money traded, respectively, that is, $p_{2,t}$ is the price of the asset. At $\omega_{SD}^u$ and $\omega_{SD}^0$, buyers do not know the asset value. Then, from the linearity of the CM value functions, dealers' surplus is

$$W_t^D(m_t^D + p_{2,t}) - W_t^D(m_t^D) = \phi_t p_{2,t},$$

and buyers' surplus is

$$\psi_B u a_{2,t} + W_t^B(m_t^B - p_{2,t}) - W_t^B(m_t^B) = \psi_B u a_{2,t} - \phi_t p_{2,t}.$$

Therefore, the terms of trade are determined by

$$\max_{a_{2,t}\in\{0,1\},p_{2,t}} (\phi_t p_{2,t})^{\theta_2}(\psi_B u a_{2,t} - \phi_t p_{2,t})^{1-\theta_2}$$
$$\text{subject to } a_{2,t} \le a_t^D \text{ and } p_{2,t} \le m_t^B,$$

where $a_t^D$ is the amount of dealers' asset holdings. Note that, implicitly, we also have incentive constraints that agents' surpluses must be nonnegative, $\phi_t p_{2,t} \ge 0$ and $\psi_B u a_{2,t} - \phi_t p_{2,t} \ge 0$.

If $\phi_t m_t^B > 0$, the solution to the bargaining problem between dealers and buyers takes the following form:

$$\begin{aligned} a_{2,t}(a_t^D, m_t^B) &= a_t^D \\ p_{2,t}(a_t^D, m_t^B) &= \begin{cases} m_t^B & \text{if } \phi_t m_t^B < \theta_2\psi_B u a_t^D, \\ \frac{\theta_2\psi_B u a_t^D}{\phi_t} & \text{if } \phi_t m_t^B \ge \theta_2\psi_B u a_t^D. \end{cases} \end{aligned}$$

Note that the amount of the asset traded is *independent* of the amount of buyers' money holdings. This is because dealers do not enjoy utility from consuming the asset, and therefore we can make buyers better off without hurting dealers by increasing the amount of the asset traded.[9] If $\phi_t m_t^B = 0$, the terms of trade $[a_{2,t}(a_t^D, m_t^B), p_{2,t}(a_t^D, m_t^B)]$ are any pairs $(a_{2,t}, p_{2,t})$ satisfying the constraints. At state $\omega_E^0$, buyers know that the asset value for buyers is 0, and thus $a_{2,t}(a_t^D, m_t^B)$ is any of 0 and $a_t^D$, and $p_{2,t}(a_t^D, m_t^B)$ is 0 if $\phi_t > 0$ and any number between 0 and $m_t^B$ if $\phi_t = 0$.

**Trade between Sellers and Dealers**

Next consider DM trade between sellers and dealers. For states $\omega_E^0$ and $\omega_{SD}^0$, let $a_{1,t}^0$ and $p_{1,t}^0$ be the amounts of the asset and money traded, that is, $p_{1,t}^0$ is the price of the

[9] The assumption that dealers do not obtain utility from assets is standard in the study of over-the-counter (OTC) markets initiated by Duffie, Gârleanu, and Pedersen (2005).

asset. At states $\omega_E^0$ and $\omega_{SD}^0$, dealers believe that buyers do not know the asset value with probability $\psi_D$. Then, sellers' surplus is

$$-ca_{1,t}^0 + W_t^S(m_t^S + p_{1,t}^0) - W_t^S(m_t^S) = \phi_t p_{1,t}^0 - ca_{1,t}^0,$$

and dealers' surplus is

$$\psi_D \phi_t p_{2,t}(a_{1,t}^0, M_t^B) + W_t^D(m_t^D - p_{1,t}^0) - W_t^D(m_t^D) = \phi_t[\psi_D p_{2,t}(a_{1,t}^0, M_t^B) - p_{1,t}^0].$$

The terms of trade are determined by

$$\max_{a_{1,t}^0 \in \{0,1\}, p_{1,t}^0} (\phi_t p_{1,t}^0 - ca_{1,t}^0)^{\theta_1} \{\phi_t[\psi_D p_{2,t}(a_{1,t}^0, M_t^B) - p_{1,t}^0]\}^{1-\theta_1}$$
$$\text{subject to } p_{1,t}^0 \leq m_t^D.$$

Again, implicitly, we also have incentive constraints that agents' surpluses must be nonnegative. This bargaining problem is more complicated than those in the models based on Lagos and Wright (2005) especially because dealers must take buyers' money holdings into account, when they trade with sellers.

Define $w^0(M_t^B) = \theta_1 \phi_t \psi_D p_{2,t}(1, M_t^B) + (1-\theta_1)c$, which is the value of money traded in the above problem if (i) the constraint, $p_{1,t}^0 \leq m_t^D$, does not bind and (ii) there are gains from trade between sellers and dealers, $\phi_t \psi_D p_{2,t}(1, M_t^B) - c > 0$. We divide the argument into three cases.

**Case 1:** Suppose that $\phi_t \psi_D p_{2,t}(1, M_t^B) > c$, that is, the gains from trade between sellers and dealers are positive. Then, the solution to the bargaining problem between sellers and dealers is

$$a_{1,t}^0(m_t^D, M_t^B) = \begin{cases} 1 & \text{if } \phi_t m_t^D > c, \\ 0 \text{ or } 1 & \text{if } \phi_t m_t^D = c, \\ 0 & \text{if } \phi_t m_t^D < c, \end{cases}$$

$$p_{1,t}^0(m_t^D, M_t^B) = \begin{cases} \frac{w^0(M_t^B)}{\phi_t} & \text{if } \phi_t m_t^D \geq w^0(M_t^B), \\ m_t^D & \text{if } c < \phi_t m_t^D < w^0(M_t^B), \text{ or } \phi_t m_t^D = c \text{ and } a_{1,t}^0(m_t^D, M_t^B) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

**Case 2:** Suppose that $\phi_t\psi_D p_{2,t}(1, M_t^B) = c$, that is, there are no gains from trade between sellers and dealers. Then,

$$
\begin{aligned}
a_{1,t}^0(m_t^D, M_t^B) &= \begin{cases} 0 \text{ or } 1 & \text{if } \phi_t m_t^D \geq c, \\ 0 & \text{if } \phi_t m_t^D < c, \end{cases} \\
p_{1,t}^0(m_t^D, M_t^B) &= \begin{cases} \frac{c}{\phi_t} & \text{if } a_{1,t}^0(m_t^D, M_t^B) = 1, \\ 0 & \text{if } a_{1,t}^0(m_t^D, M_t^B) = 0. \end{cases}
\end{aligned}
$$

**Case 3:** Suppose that $\phi_t\psi_D p_{2,t}(1, M_t^B) < c$, that is, there are losses from trade between sellers and dealers. Then, $a_{1,t}^0(m_t^D, M_t^B) = 0$, and $p_{1,t}^0(m_t^D, M_t^B) = 0$ if $\phi_t > 0$ and $p_{1,t}^0(m_t^D, M_t^B)$ is any number between 0 and $m_t^D$ if $\phi_t = 0$.

For state $\omega_{SD}^u$, let $a_{1,t}^u$ and $p_{1,t}^u$ be the amounts of the asset and money traded, respectively, and define $w^u(M_t^B) = \theta_1\phi_t p_{2,t}(1, M_t^B) + (1-\theta_1)c$. Then, we can obtain the terms of trade $[a_{1,t}^u(m_t^D, M_t^B), p_{1,t}^u(m_t^D, M_t^B)]$ by replacing $\psi_D$ with 1 in $[a_{1,t}^0(m_t^D, M_t^B), p_{1,t}^0(m_t^D, M_t^B)]$ derived above.

### Bellman Equations for the DM

Using the linearity of the CM value functions, we define the Bellman equations for the DM. For sellers,

$$
\begin{aligned}
V_t^S(m_t^S) &= W_t^S(m_t^S) + \mu(\omega_{SD}^u)[\phi_t p_{1,t}^u(M_t^D, M_t^B) - c a_{1,t}^u(M_t^D, M_t^B)] \\
&\quad + [\mu(\omega_E^0) + \mu(\omega_{SD}^0)][\phi_t p_{1,t}^0(M_t^D, M_t^B) - c a_{1,t}^0(M_t^D, M_t^B)].
\end{aligned}
$$

Sellers always trade with dealers.

For dealers,

$$
\begin{aligned}
V_t^D(m_t^D) &= W_t^D(m_t^D) + \phi_t\mu(\omega_{SD}^u)\{p_{2,t}[a_{1,t}^u(m_t^D, M_t^B), M_t^B] - p_{1,t}^u(m_t^D, M_t^B)\} \\
&\quad + \phi_t\{\mu(\omega_{SD}^0)p_{2,t}[a_{1,t}^0(m_t^D, M_t^B), M_t^B] - [\mu(\omega_E^0) + \mu(\omega_{SD}^0)]p_{1,t}^0(m_t^D, M_t^B)\}.
\end{aligned}
$$

Dealers buy the asset from sellers, and at states $\omega_{SD}^{u}$ and $\omega_{SD}^{0}$, can sell it to buyers.

Finally, for buyers,

$$\begin{aligned} V_t^B(m_t^B) &= W_t^B(m_t^B) + \mu(\omega_{SD}^u)\{ua_{1,t}^u(M_t^D, M_t^B) - \phi_t p_{2,t}[a_{1,t}^u(M_t^D, M_t^B), m_t^B]\} \\ &\quad + \mu(\omega_{SD}^0)\{-\phi_t p_{2,t}[a_{1,t}^0(M_t^D, M_t^B), m_t^B]\}. \end{aligned}$$

Buyers purchase the asset from dealers with probability $\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)$, and then, cannot enjoy utility from the purchased asset with probability $\mu(\omega_{SD}^0)$.

## 4.2 Equilibrium

We will derive an equilibrium where an asset bubble occurs. Let $z_t = \phi_t M_t$, which is called the *value of money in period* $t$.

Note that as is usual in Lagos-Wright models, it is costly to hold money, and so agents who do not use money in the DM do not buy money in the CM. In our model, this means that neither sellers nor buyers have incentives to buy a positive amount of money in the CM, and hence each dealer has $M_t$ units of money at the beginning of period $t$. Then, we have $\phi_t M_t^D = z_t$. Moreover, since buyers have $\tau M_t$ units of money in the DM, $\phi_t M_t^B = \tau z_t$.

**Lemma 1.** *Dealers' money holdings satisfy* $m_t^D \leq c/\phi_t$ *in any equilibrium with* $\phi_t > 0$.

This is because in all the cases in the previous section, choosing $m_t^D > c/\phi_t$ does not increase $a_{1,t}^0$ or $a_{1,t}^u$ and hence bringing more money only incurs additional cost.

For an asset bubble to occur, we must have $a_{1,t}^0(M_t, \tau M_t) = 1$, and this can occur only in Cases 1 or 2 in the previous section. The condition for which $a_{1,t}^0(M_t, \tau M_t) = 1$ can be rewritten as

$$\frac{\phi_t p_{2,t}(1, \tau M_t)}{c} \geq \frac{1}{\psi_D}. \tag{2}$$

With this condition, we also have $a_{1,t}^u(M_t, \tau M_t) = 1$ because $\psi_D < 1$.

Observe that in order for $a_{1,t}^0(M_t, \tau M_t) = 1$ to occur, we must have $m_t^D \geq c/\phi_t$ and thus $m_t^D = c/\phi_t$. Thus, $z_t = c$, which means $\phi_t = c/M_t$, and $p_{1,t}^0(M_t, \tau M_t) =$

$p^u_{1,t}(M_t, \tau M_t) = c/\phi_t$ for each $t$. Note that the prices are the same in all states, and thus buyers do not learn anything about the state from prices. This implies that our bubble equilibrium is robust even when prices are observable. See Section 5 for more on the observability of prices.

Now,

$$p_{2,t}(1, \tau M_t) = \begin{cases} \tau M_t & \text{if } \tau < \frac{\theta_2 \psi_B u}{c}, \\ \frac{\theta_2 \psi_B u}{\phi_t} & \text{if } \tau \geq \frac{\theta_2 \psi_B u}{c}. \end{cases}$$

If $\tau < \theta_2 \psi_B u/c$, then (2) is rewritten as

$$\tau \geq \frac{1}{\psi_D}.$$

If $\tau \geq \theta_2 \psi_B u/c$, it is

$$\frac{\theta_2 \psi_B u}{c} \geq \frac{1}{\psi_D}.$$

Therefore, (2) holds if and only if

$$\min\left\{\tau, \frac{\theta_2 \psi_B u}{c}\right\} \geq \frac{1}{\psi_D}.$$

Finally, dealers must have incentives to bring money to the DM, that is, we must have the following condition:

$$\phi_t M_{t+1} \leq \beta\{\phi_{t+1} M_{t+1} + [\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]\phi_{t+1} p_{2,t+1}(1, \tau M_{t+1}) - \phi_{t+1} M_{t+1}\}. \quad (3)$$

The left-hand side, $\phi_t M_{t+1}$, is the cost of bringing money to the DM. In the right-hand side, $\phi_{t+1} M_{t+1}$ is the resale value of money, $[\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]\phi_{t+1} p_{2,t+1}(1, \tau M_{t+1})$ is the benefit from trade using money, and $-\phi_{t+1} M_{t+1}$ is the payment. We have $M_t = M_{t+1}$ and $\phi_t M_t = c$, and hence, (3) is rewritten as

$$\frac{\phi_{t+1} p_{2,t+1}(1, \tau M_{t+1})}{c} \geq \frac{1}{\beta[\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]}.$$

Thus, (3) holds if and only if

$$\min\left\{\tau, \frac{\theta_2 \psi_B u}{c}\right\} \geq \frac{1}{\beta[\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]}.$$

This inequality is of course violated when $\tau = 0$, and Proposition 1 follows.

### Occurrence of Asset Bubbles

We will explain how an asset bubble occurs in this equilibrium. Consider state $\omega_E^0$. At this state, the asset value for buyers is 0, and every agent knows it. Hence, the fundamental value of the asset is 0. However, since dealers' knowledge is

$$\mathcal{P}_{SD} = \{\{\omega_{SD}^u\}, \{\omega_E^0, \omega_{SD}^0\}\},$$

dealers do not know that buyers know that the asset value is 0. Moreover, with monetary expansion, buyers have too much money in the sense that they can buy the same unit of the asset with any smaller amount of money than the one injected by the monetary expansion. To obtain such money from buyers, dealers buy the asset from sellers in hopes of selling it to buyers, but buyers do not purchase it from dealers because buyers know that the asset is worthless. That is, an asset bubble occurs in trade between sellers and dealers, and bursts in trade between dealers and buyers.

This completes the proof of the theorem.

# 5 Discussion

### Observability of Prices

Prices in trade between sellers and dealers are the same across all the states, and therefore buyers do not learn the asset value from its price. Thus, our result survives irrespective of whether past prices are observable or not. This is in sharp contrast to AIW who need to assume that past prices are unobservable to establish robust bubbles. More precisely, in their Appendix A, AIW show that bubbles occur when past prices are observable but this is true only for some knife-edge parameter values. In the current paper, bubbles occur in an open, nonempty set of parameter values.

The fact that one cannot infer states from prices echoes policymakers' difficulty. For example, Bernanke and Gertler (2012) write: "Trying to stabilize asset prices per se is problematic for a variety of reasons, not the least of which is that it is nearly impossible to know for sure whether a given change in asset values results from fundamental factors, non-fundamental factors, or both."

Of course, the robustness result of the current model depends on many assumptions. One of them is that the cost incurred to produce the asset is the same across states. The price is just enough to cover sellers' cost, and that is independent of the state by this assumption.

### Intermediation Mode

In our model, intermediaries (i.e., dealers) do trade using their own accounts and make profits by flipping. There is another mode of intermediaries.[10] In particular, platforms (or brokers) just connect buyers and sellers, or investors and interdealer markets, and make profits by brokerage fees. This difference is crucial.

In our model, dealers must hold the asset when they trade with buyers and thus their holding of the asset is sunk. This opens the room for the hold-up problem—buyers do not bring money and hence there is no bubble without monetary expansion. Such a hold-up would never occur if intermediaries were brokers. When there is monetary expansion, bubbles occur because dealers *buy* the asset even when they know that it is worthless. If intermediaries are brokers, trade never occurs if buyers know that the asset is worthless.

### Policy Implications

Is it better to burst bubbles? In our model, creation of worthless assets—and therefore bubbles—is just a waste, but *ex ante* (without knowing which state will be realized) creation of asset is welfare-improving. Hence, a policy to curb bubbles by setting a small $\tau$ regardless of the state is welfare deteriorating. On the other hand, if the central

[10] See Gautier, Hu, and Watanabe (2023) for more details.

bank can choose $\tau$ *after* observing the state, then the optimal monetary policy would be to choose sufficiently large $\tau$ when the asset has value ($\omega = \omega_{SD}^{u}$), and sufficiently small $\tau$ (to prevent bubbles) when the asset has no value ($\omega \in \{\omega_{SD}^{0}, \omega_{E}^{0}\}$).[11] In this sense, our model echoes the view of the wait-and-see approach advocated by, for example, Bernanke and Gertler (2012).[12]

### Role of Credit

Throughout the paper, we assumed that credit cannot be used. It is of course well-known that fiat money is valued for liquidity when there are frictions that hinder credit (Kocherlakota 1998). If credit is viable and agents have unlimited access to it, then money is not needed as a median of exchange and so monetary expansion cannot create asset bubbles. On the other hand, even when we allow for credit trade, if there exists a tight credit limit, then money plays a role (Gu, Mattesini, and Wright 2016) and, following the same logic as we have shown, monetary expansion causes asset bubbles.

## 6 Extension

The baseline model explains how monetary expansion causes asset bubbles in a simple manner. In this section, we introduce another market for goods and demonstrate that exogenous monetary expansion is just one way to induce bubbles, and too much money and bubbles can result from buyers' endogenous decision. This extension also allows us

[11]Other bubble-bursting policies can be announcing the state, or prohibiting creation or trading of assets. Of course, similar arguments hold and their welfare implications heavily depend on whether the policy can depend on states.

[12]Some papers point out that bubbles are detrimental. For example, Dong, Jia, and Wang (2022) show that bubbles create misallocation of talent. Grossman and Yanagawa (1993) and Guerron-Quintana, Hirano, and Jinnai (2023) demonstrate that asset bubbles crowd out investment. In Allen, Barlevy, and Gale (2022), bubbles cause costly default. See Barlevy (2018) for further discussion. None of these channels are present in the current paper.

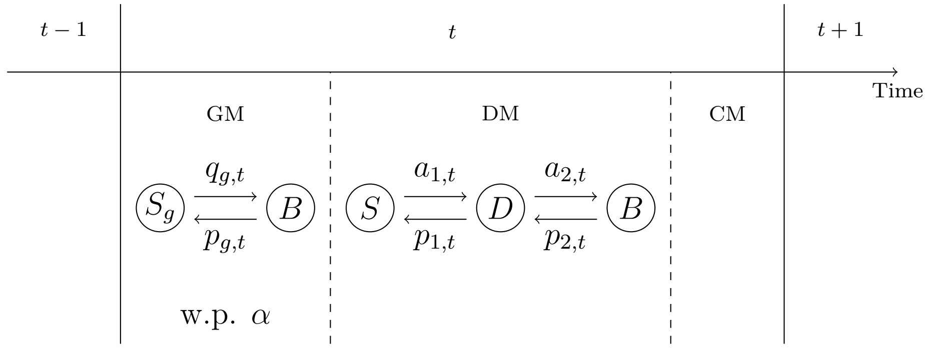


Figure 2: Timeline of the extended model

to show that the price of money is positive even without asset bubbles.

Now, each period is divided into three subperiods. In the first, agents interact in a decentralized goods market (GM). The second and third subperiods are the same as the DM and the CM in the baseline model, respectively. In particular, the state is determined at the beginning of the DM. There are additional measure 1 of agents, and we call them goods sellers. They enter only the GM and the CM, and do not trade in the DM.

The GM opens with probability $\alpha \in [0, 1]$, and the probability is serially independent. The GM has perishable indivisible goods. Each goods seller can produce only one unit of them at a cost $c_g > 0$, and buyers enjoy utility $u_g > c_g$ from consuming it. In the GM, there are only meetings between goods sellers and buyers. If $\alpha = 0$, the extended model would be exactly the same as the baseline one. As in the DM, money is used as the payment instrument, and the terms of trade are determined by generalized Nash bargaining, where $\theta_g \in (0, 1)$ is the bargaining power of goods sellers in trade between goods sellers and buyers. We assume that, at the beginning of the DM, all agents know whether or not the GM opened. See Figure 2.

Our result of this section, whose proof is relegated to the Appendix, is the following.

**Theorem 2.** *In the extended model with $\tau = 0$, there is an equilibrium in which asset* *bubbles occur with positive probability at a state if and only if*

$$\min\left\{\frac{c_g}{c}, \frac{\theta_2\psi_B u}{c}\right\} \geq \max\left\{\frac{1}{\psi_D}, \frac{1-\alpha\beta}{(1-\alpha)\beta[\mu(\omega_{SD}^u)+\mu(\omega_{SD}^0)]}\right\}$$

*and*

$$\frac{\alpha u_g}{c_g} + (1-\alpha)\left\{1 - [\mu(\omega_{SD}^u)+\mu(\omega_{SD}^0)]\min\left\{1, \frac{\theta_2\psi_B u}{c_g}\right\}\right\} \geq \frac{1}{\beta}.$$

The first condition is similar to that in the baseline model and requires that the gains from trade between (asset) sellers and dealers should be sufficiently large and dealers should have incentives to bring money to the DM. Note that this condition does not hold if $\alpha = 1$. This is because, when $\alpha = 1$, the amount of buyers' money holdings after the GM is equal to zero and dealers do not have incentives to bring money to the DM. Thus, for the existence of bubbles, we need $\alpha < 1$.

The second condition means that buyers must have incentives to bring money to the GM. Note that this condition does not hold if $\alpha = 0$ since, in that case, the GM never opens and buyers do not bring money to the GM. Hence, a bubble occurs only if $\alpha > 0$. As long as $\alpha > 0$, the condition is satisfied for sufficiently large gains from trade of GM goods, $u_g/c_g$.

Observe that even though we assume $\tau = 0$, asset bubbles occur with positive probability. Observe also that even when asset bubbles do not occur, buyers obtain money in the CM to purchase goods in the GM as long as the GM opens with positive probability and the gains from trade of GM goods are sufficiently large. Therefore, money can have a positive value without asset bubbles.

Does monetary expansion (money injection to buyers at the beginning of DM) still fuel bubbles in the extended model? Note that when $\tau = 0$, a bubbly asset is created and traded only when the GM is not opened—if the GM is opened, buyers spend all their money in the GM and do not bring money to the DM. When $\tau$ is sufficiently large, bubbles can occur even when GM is opened. Thus, we have

**Corollary 1.** *The probability that asset bubbles occur is increasing in* $\tau$*.*

# A Appendix: Details of the Extended Model

Let $G_t^{S_g}$ and $G_t^B$ be the GM value functions of goods sellers and buyers, respectively, and we denote the CM value function of goods sellers by $W_t^{S_g}$ (hereafter, superscript $S_g$ indicates goods sellers). For goods sellers, the Bellman equation for the CM is

$$\begin{aligned} W_t^{S_g}(m_t^{S_g}) &= \max_{x_t^{S_g}, \ell_t^{S_g}, \widehat{m}_{t+1}^{S_g}} \left\{ U(x_t^{S_g}) - \ell_t^{S_g} + \beta G_{t+1}^{S_g}(\widehat{m}_{t+1}^{S_g}) \right\} \\ & \quad \text{subject to } x_t^{S_g} = \phi_t(m_t^{S_g} - \widehat{m}_{t+1}^{S_g}) + \ell_t^{S_g}. \end{aligned}$$

For buyers, the Bellman equation for the CM is

$$\begin{aligned} W_t^B(m_t^B) &= \max_{x_t^B, \ell_t^B, \widehat{m}_{t+1}^B} \left\{ U(x_t^B) - \ell_t^B + \beta G_{t+1}^B(\widehat{m}_{t+1}^B) \right\} \\ & \quad \text{subject to } x_t^B = \phi_t(m_t^B - \widehat{m}_{t+1}^B) + \ell_t^B - \frac{\phi_t \tau M_t}{3}. \end{aligned}$$

Since buyers enter the GM, $V_{t+1}^B$ is replaced with $G_{t+1}^B$. For (asset) sellers and dealers, the Bellman equations for the CM are the same as those in the baseline model.

At the beginning of the DM, all agents know whether or not the GM opened. If it opened, the amount of buyers' money holdings in the DM is equal to the amount that they bring from the CM minus the payment for goods sellers in the GM; otherwise, the amount of buyers' money holdings in the DM is the same as the amount that they bring from the CM. All agents know this fact about buyers' money holdings. Other than this feature, there is no change in the DM.

For trade in the GM, let $q_{g,t}$ and $p_{g,t}$ be the amounts of goods and money traded, respectively. Since goods sellers do not enter the DM, their surplus is

$$-c_g q_{g,t} + W_t^{S_g}(m_t^{S_g} + p_{g,t}) - W_t^{S_g}(m_t^{S_g}) = \phi_t p_{g,t} - c_g q_{g,t}.$$

Buyers' surplus is

$$u_g + V_t^B(m_t^B - p_{g,t}) - V_t^B(m_t^B)$$

because they trade in the DM after the GM. Unlike the difference in $W_t^{S_g}$, the difference in $V_t^{S_g}$ takes a complicated form, and we leave it as it is. The terms of trade are determined by

$$\max_{q_{g,t}\in\{0,1\},p_{g,t}} (\phi_t p_{g,t} - c_g q_{g,t})^{\theta_g}[u_g + V_t^B(m_t^B - p_{g,t}) - V_t^B(m_t^B)]^{1-\theta_g}$$
$$\text{subject to } p_{g,t} \leq m_t^B.$$

As in the baseline model, we also have incentive constraints that agents' surpluses must be nonnegative. Each solution to the bargaining problem depends on a buyer's money holdings in this trade, $m_t^B$, in addition to the other buyers' money holdings, $M_t^B$, and dealers' money holding, $M_t^D$. Define the Bellman equations for the GM as follows. For goods sellers, it is

$$G_t^{S_g}(m_t^{S_g}) = W_t^{S_g}(m_t^{S_g}) + \alpha[p_{g,t}(M_t^B, M_t^B, M_t^D) - c_g q_{g,t}(M_t^B, M_t^B, M_t^D)].$$

For buyers, it is

$$G_t^B(m_t^B) = \alpha\{u_g q_{g,t}(m_t^B, M_t^B, M_t^D) + V_t^B[m_t^B - p_{g,t}(m_t^B, M_t^B, M_t^D)]\} + (1-\alpha)V_t^B(m_t^B).$$

We will derive an equilibrium where, with probability $\alpha$, asset bubbles occur ($a_{1,t}^0 = a_{1,t}^u = 1$, $p_{1,t}^0 = p_{1,t}^u > 0$, and $\phi_t > 0$) and buyers purchase goods if the GM opens ($q_{g,t} = 1$ and $p_{g,t} > 0$). As in the baseline model, goods sellers and (asset) sellers do not have incentives to obtain money in the CM, and dealers' money holdings must satisfy $\phi_t M_t^D = c$, which implies that $p_{1,t}^0 = p_{1,t}^u$. Buyers still do not have incentives to bring money to the DM due to the hold-up problem. However, they may have incentives to bring money to the GM to purchase goods. Note that if buyers still have money at the end of the GM, they may have to pay some fraction of the money to dealers in the DM. Hence, we have

$$V_t^B(m_t^B - p_{g,t}) - V_t^B(m_t^B) \geq -\phi_t p_{g,t},$$

and, as a result, there are always gains from GM trade:

$$u_g - c_g + \phi_t p_{g,t} + V_t^B(m_t^B - p_{g,t}) - V_t^B(m_t^B) > 0.$$

By a similar argument about dealers' money holdings, buyers' money holdings must satisfy $\phi_t M_t^B = c_g$. Therefore, we obtain

$$z_t = \phi_t M_t = \phi_t (M_t^D + M_t^B) = c + c_g.$$

If the GM did not open, buyers have $c_g/\phi_t$ units of money in the DM. In this case, as in the baseline model, for $a_{1,t}^0 = a_{1,t}^u = 1$ to hold, we must have

$$\frac{\phi_t p_{2,t}\left(1, \frac{c_g}{\phi_t}\right)}{c} \geq \frac{1}{\psi_D}.$$

Now,

$$p_{2,t}\left(1, \frac{c_g}{\phi_t}\right) = \begin{cases} \frac{c_g}{\phi_t} & \text{if } c_g < \theta_2 \psi_B u, \\ \frac{\theta_2 \psi_B u}{\phi_t} & \text{if } c_g \geq \theta_2 \psi_B u. \end{cases}$$

The above condition is rewritten as

$$\min\left\{\frac{c_g}{c}, \frac{\theta_2 \psi_B u}{c}\right\} \geq \frac{1}{\psi_D}.$$

Dealers buy the asset from (asset) sellers exactly when the GM does not open. Hence, dealers' incentive constraint is

$$\phi_t M_{t+1}^D \leq \beta\left\{\alpha \phi_{t+1} M_{t+1}^D + (1-\alpha)[\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)]\phi_{t+1} p_{2,t+1}\left(1, \frac{c_g}{\phi_{t+1}}\right)\right\}.$$

Since $\phi_t M_t^D = c$, the above incentive constraint holds if and only if

$$\min\left\{\frac{c_g}{c}, \frac{\theta_2 \psi_B u}{c}\right\} \geq \frac{1-\alpha\beta}{(1-\alpha)\beta[\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)]}.$$

Now, in addition to dealers, we need to consider buyers' incentive constraint:

$$\begin{aligned} \phi_t M_{t+1}^B \quad \leq \quad & \beta\Big\{\phi_{t+1} M_{t+1}^B + \alpha(u_g - \phi_{t+1} M_{t+1}^B) \\ & -(1-\alpha)[\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)]\phi_{t+1} p_{2,t+1}\left(1, \frac{c_g}{\phi_{t+1}}\right)\Big\}. \end{aligned}$$

The left-hand side, $\phi_t M_{t+1}^B$, is the cost of bringing money to the GM. In the right-hand side, $\phi_{t+1} M_{t+1}^B$, is the resale value of money, and $\alpha(u_g - \phi_{t+1} M_{t+1}^B)$ is the expected value of the benefit minus the payment in GM trade. If the GM does not open, buyers may have to pay some fraction of their money holdings to dealers in the DM. The remaining term captures this cost. Since $\phi_t M_t^B = c_g$, the incentive constraint is rewritten as

$$\frac{\alpha u_g}{c_g} + (1-\alpha)\left\{1 - [\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)] \min\left\{1, \frac{\theta_2 \psi_B u}{c_g}\right\}\right\} \geq \frac{1}{\beta}.$$

# B Appendix: Divisible Assets

This section considers a variant of the model where the asset is divisible. We will show that, as in the model of the main text, monetary expansion causes asset bubbles.

The major differences between the indivisible and divisible asset cases can be summarized as follows.

- Robustness: In the indivisible asset case, bubbles occur on generic parameters; in the divisible asset case, they occur only on knife-edge parameters satisfying the condition described below, equation (4). This is exactly as happens in the existing finite state-space models building on Allen, Morris, and Postlewaite (1993).
- Value of money: In the indivisible asset case, it is pinned down by the production cost of the asset, $z_t = c$; in the divisible asset case, it is determined by the Euler equation. See equation (5) below.
- Bargaining problem: In the indivisible asset case, sellers' decision is binary—either to produce or not; In the divisible asset case, sellers can choose any amount of production.

## B.1 Model

We will amend the model as follows. Each seller can produce $a$ units of the asset at cost of $c(a)$. Neither sellers nor dealers enjoy utility from consuming the asset, but buyers obtain utility $u(a)$ with some probability and 0 with the remaining probability from consuming $a$ units of the asset. Assume that $u(0) = c(0) = 0$ and, for each $a > 0$, $u'(a) > 0$, $c'(a) > 0$, and $u''(a) \leq 0 \leq c''(a)$ with at least one of the inequalities strict. For gains from trade of the asset to be positive but finite, assume also that $u'(0)/c'(0) = \infty$ and there exists $\overline{a} > 0$ such that $u(\overline{a}) = c(\overline{a})$.

The state space is modified as follows:

$$\Omega = \{\omega_N^u, \omega_E^0, \omega_{SD}^0, \omega_B^0\}.$$

Among these, $\omega_E^0$ and $\omega_{SD}^0$ are the same as those in the main text—the asset has no value, and everyone and only sellers and dealers know it at $\omega_E^0$ and $\omega_{SD}^0$, respectively. At $\omega_N^u$, the asset value for buyers is $u(a)$ but no agent knows it. Finally, at $\omega_B^0$, the asset value for buyers is 0 and only buyers know it. At $\omega_N^u$ and $\omega_{SD}^0$, buyers believe that the asset value for buyers is $u(a)$ with probability

$$\psi_B = \frac{\mu(\omega_N^u)}{\mu(\omega_N^u) + \mu(\omega_{SD}^0)}.$$

Note that at $\omega_N^u$ and $\omega_B^0$, dealers are *good* in the sense that they do not know the asset value for buyers. At states $\omega_E^0$ and $\omega_{SD}^0$, dealers are *bad* in the sense that they know that the asset value for buyers is 0. For the existence of bubbles, good and bad dealers must behave in the same manner because, otherwise, buyers learn the asset value by looking at dealers' behavior. Therefore, we assume that the probabilities of good and bad dealers are equal,

$$\frac{\mu(\omega_N^u)}{\mu(\omega_N^u) + \mu(\omega_B^0)} = \frac{\mu(\omega_{SD}^0)}{\mu(\omega_E^0) + \mu(\omega_{SD}^0)}, \tag{4}$$

and let $\psi_D$ be the probability. This assumption admittedly causes bubbles to be less robust to parameter changes.

Everything else is unchanged, and for the same reason as in the indivisible asset case, bubbles do not occur when $\tau = 0$. In the next section, we will argue that bubbles occur when $\tau$ is sufficiently large.

## B.2 Monetary Expansion

In this section, we will show that

**Theorem 3.** *If monetary expansion is sufficiently large, there is an equilibrium in which asset bubbles occur at a state.*

### B.2.1 Bargaining Solution

We will derive the bargaining solutions by focusing on the case where the value of buyers' money holdings in the DM is positive, $\phi_t M_t^B > 0$. In the dealer-buyer meetings, the bargaining solutions are largely similar to those in the indivisible asset case—let $a_{1,t}$ be the amount that sellers produce, and we can think as if the asset is indivisible and its value is $u(a_{1,t})$. On the other hand, in the seller-dealer meetings, the bargaining solutions are very different from those in the indivisible asset case.

The CM Bellman equations are analogous to those in the indivisible asset case and hence omitted. Thus, the envelope condition is unchanged, and we can use the linearity of the CM value function in what follows.

First, consider DM trade between dealers and buyers. At $\omega_N^u$ and $\omega_{SD}^0$, the solution to the bargaining problem between dealers and buyers takes the following form:

$$\begin{aligned} a_{2,t}(a_t^D, m_t^B) &= a_t^D, \\ p_{2,t}(a_t^D, m_t^B) &= \begin{cases} m_t^B & \text{if } \phi_t m_t^B < \theta_2 \psi_B u(a_t^D), \\ \frac{\theta_2 \psi_B u(a_t^D)}{\phi_t} & \text{if } \phi_t m_t^B \geq \theta_2 \psi_B u(a_t^D). \end{cases} \end{aligned}$$

At $\omega_E^0$ and $\omega_B^0$, the asset is not traded with a positive amount of money.

Second, consider DM trade between sellers and dealers. At $\omega_N^u$ and $\omega_B^0$, dealers believe that buyers do not know the asset value with probability $\mu(\omega_N^u)/[\mu(\omega_N^u)+\mu(\omega_B^0)]$. Similarly, at $\omega_E^0$ and $\omega_{SD}^0$, dealers believe that buyers do not know the asset value with probability $\mu(\omega_{SD}^0)/[\mu(\omega_E^0) + \mu(\omega_{SD}^0)]$. From (4), these probabilities are the same, and therefore the bargaining problem between sellers and dealers does not depend on states.

For each state, let $a_{1,t}$ and $p_{1,t}$ be the amounts of the asset and money traded. Note that in the indivisible-asset case, we do not have to impose this kind of assumptions because, in that case, dealers pay the same amount of money to sellers regardless of states.

From the bargaining solution between dealers and buyers, if $\phi_t M_t^B \geq \theta_2 \psi_B u(a_{1,t})$, sellers' surplus is

$$\phi_t p_{1,t} - c(a_{1,t})$$

and dealers' surplus is

$$\psi_D \phi_t p_{2,t}(a_{1,t}, M_t^B) - \phi_t p_{1,t} = \theta_2 \psi_D \psi_B u(a_{1,t}) - \phi_t p_{1,t}.$$

Hence, the efficient quantity $a_1^*$ is the solution to $\theta_2 \psi_D \psi_B u'(a_{1,t}) = c'(a_{1,t})$. As in the indivisible asset case, buyers do not have incentives to bring money to the DM, and so $M_t^B = \tau M_t$ in equilibrium. To simplify the exposition of our results, throughout the rest of this appendix, we focus on the case where monetary expansion is sufficiently large so that

$$\tau z_t = \phi_t M_t^B \geq \theta_2 \psi_B u(a_1^*).$$

Then, since sellers and dealers do not trade any larger amount of the asset than $a_1^*$, we have $\phi_t M_t^B \geq \theta_2 \psi_B u(a_{1,t})$.

To describe the bargaining solution between sellers and dealers, define

$$v(a) = \frac{\theta_1 c'(a) \theta_2 \psi_D \psi_B u(a) + (1-\theta_1) c(a) \theta_2 \psi_D \psi_B u'(a)}{\theta_1 c'(a) + (1-\theta_1) \theta_2 \psi_D \psi_B u'(a)},$$

and denote by $\widetilde{a}_1(\phi_t m_t^D)$ the solution $a_{1,t}$ to $\phi_t m_t^D = v(a_{1,t})$. The function $v$ takes the standard form for the generalized Nash bargaining in the New Monetarist models, and $v(a_{1,t})$ is the amount of dealers' real balances to obtain $a_{1,t}$ units of the asset from sellers. Since $v$ is increasing, the existence and uniqueness of $\widetilde{a}_1(\phi_t m_t^D)$ are guaranteed. Then, the solution to the bargaining problem between sellers and dealers takes the following form:

$$a_{1,t}(m_t^D, M_t^B) = \begin{cases} \widetilde{a}_{1,t}(\phi_t m_t^D) & \text{if } \phi_t m_t^D < v(a_1^*), \\ a_1^* & \text{if } \phi_t m_t^D \geq v(a_1^*), \end{cases}$$

$$p_{1,t}(m_t^D, M_t^B) = \begin{cases} m_t^D & \text{if } \phi_t m_t^D < v(a_1^*), \\ \frac{v(a_1^*)}{\phi_t} & \text{if } \phi_t m_t^D \geq v(a_1^*). \end{cases}$$

If dealers have a sufficient amount of money to buy the efficient quantity of the asset from sellers ($\phi_t m_t^D \geq v(a_1^*)$), the quantity of the asset traded is the efficient quantity $a_1^*$. Otherwise, dealers pay all their money and receive $\widetilde{a}_{1,t}(\phi_t m_t^D)$ determined by $v$.

### B.2.2 Bellman Equations

Now, we define the Bellman equations for the DM. For sellers, it is

$$V_t^S(m_t^S) = W_t^S(m_t^S) + \phi_t p_{1,t}(M_t^D, M_t^B) - c[a_{1,t}(M_t^D, M_t^B)].$$

Sellers do not use money in DM trade and always have DM trade with dealers. For dealers,

$$V_t^D(m_t^D) = W_t^D(m_t^D) + \phi_t\{[\mu(\omega_N^u) + \mu(\omega_{SD}^0)]p_{2,t}[a_{1,t}(m_t^D, M_t^B), M_t^B] - p_{1,t}(m_t^D, M_t^B)\}.$$

Dealers use money when they buy the asset from sellers, and with probability $\mu(\omega_N^u) + \mu(\omega_{SD}^0)$, can sell the asset to buyers. Finally, for buyers,

$$\begin{aligned} V_t^B(m_t^B) &= W_t^B(m_t^B) + \mu(\omega_N^u)\{u[a_{1,t}(M_t^D, M_t^B)] - \phi_t p_{2,t}[a_{1,t}(M_t^D, M_t^B), m_t^B]\} \\ &\quad + \mu(\omega_{SD}^0)\{-\phi_t p_{2,t}[a_{1,t}(M_t^D, M_t^B), m_t^B]\}. \end{aligned}$$

Buyers purchase the asset from dealers by money with probability $\mu(\omega_N^u) + \mu(\omega_{SD}^0)$, and then, cannot enjoy utility from the purchased asset with probability $\mu(\omega_{SD}^0)$.

### B.2.3 Equilibrium

To define equilibrium, we will derive a difference equation of $z_t$. As in the indivisible asset case, only dealers bring money from the CM, and so we have $\phi_t M_t^D = z_t$. Moreover, since buyers have $\tau M_t$ units of money in the DM, $\phi_t M_t^B = \tau z_t$.

We start from the following observation. The terms of trade between sellers and dealers are affected by a change in dealers' money holdings $\widehat{m}_{t+1}^D$, and hence there is a positive marginal benefit by bringing an additional unit of money to the DM if and only if $\phi_t \widehat{m}_{t+1}^D < v(a_1^*)$, which becomes $z_t < v(a_1^*)$ in equilibrium. From the first-order condition with respect to $\widehat{m}_{t+1}^D$ in the CM,

$$\begin{aligned} \phi_t &= \beta \frac{\partial V_{t+1}^D}{\partial \widehat{m}_{t+1}^D} \\ &= \beta \phi_{t+1} \left\{ 1 + [\mu(\omega_N^u) + \mu(\omega_{SD}^0)] \frac{\partial p_{2,t+1}}{\partial a_{1,t+1}} \frac{\partial a_{1,t+1}}{\partial \widehat{m}_{t+1}^D} - \frac{\partial p_{1,t+1}}{\partial \widehat{m}_{t+1}^D} \right\}. \end{aligned}$$

Here,

$$\frac{\partial p_{2,t+1}}{\partial a_{1,t+1}} = \frac{\theta_2 \psi_B u'(a_{1,t+1})}{\phi_{t+1}}.$$

Moreover,

$$\frac{\partial p_{1,t+1}}{\partial \widehat{m}_{t+1}^D} = \begin{cases} 1 & \text{if } z_t < v(a_1^*), \\ 0 & \text{if } z_t > v(a_1^*), \end{cases}$$

and

$$\frac{\partial a_{1,t+1}}{\partial \widehat{m}_{t+1}^D} = \begin{cases} \frac{\phi_{t+1}}{v'(a_{1,t+1})} & \text{if } z_t < v(a_1^*), \\ 0 & \text{if } z_t > v(a_1^*). \end{cases}$$

Therefore,

$$\phi_t = \begin{cases} \beta \phi_{t+1} [\mu(\omega_N^u) + \mu(\omega_{SD}^0)] \frac{\theta_2 \psi_B u'[v^{-1}(z_{t+1})]}{v'[v^{-1}(z_{t+1})]} & \text{if } z_t < v(a_1^*), \\ \beta \phi_{t+1} & \text{if } z_t \geq v(a_1^*). \end{cases}$$

Multiplying $M_t = M_{t+1}$ to both sides of this equation, we derive $z_t = g(z_{t+1})$, where

$$g(z_{t+1}) = \begin{cases} \beta z_{t+1} [\mu(\omega_N^u) + \mu(\omega_{SD}^0)] \frac{\theta_2 \psi_B u'[v^{-1}(z_{t+1})]}{v'[v^{-1}(z_{t+1})]} & \text{if } z_t < v(a_1^*), \\ \beta z_{t+1} & \text{if } z_t \geq v(a_1^*). \end{cases} \tag{5}$$

Equilibrium is defined as follows.

**Definition 2.** An *equilibrium* $(z_t)_{t=0}^{\infty}$ is a solution to the difference equation $z_t = g(z_{t+1})$ that is bounded ($\lim_{t\to\infty} \beta^t z_t = 0$) and nonnegative ($z_t \geq 0$ for each $t$).

To demonstrate that asset bubbles occur in some equilibrium, consider, for example, a *stationary monetary equilibrium*, where $z_t = z_{t+1}$ and $z_t > 0$ for each $t$. In stationary monetary equilibrium, there must exist some $z^S > 0$ such that

$$\frac{1}{\beta[\mu(\omega_N^u) + \mu(\omega_{SD}^0)]\theta_2\psi_B} = \frac{u'[v^{-1}(z^S)]}{v'[v^{-1}(z^S)]}.$$

The analysis for the existence of $z^S$ is qualitatively the same as the one by Gu and Wright (2016). Their result guarantees the existence (and generic uniqueness) of $z^S$. In this stationary monetary equilibrium, asset bubbles occur at $\omega_E^0$ for the same reason as that in the indivisible asset case. This completes the proof of the theorem.

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